

**Multiphase Microfluidics**  
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**Lecture – 13**  
**Taylor Flow: Heat Transfer**

Hello. So, in this lecture we will be talking about heat transfer in the Taylor flow regime.

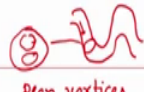
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Heat Transfer in Taylor Flow: Applications

- Electronics cooling
- Exothermic gas-liquid reactions
- Compact heat exchangers
  - Aerospace Engineering
  - Offshore Petroleum Engineering

Heatric - Meggitt PLC Inc  
(PCHEx)  
Printed Circuit Heat Exchangers

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So, as we know that in microchannels, the surface area to volume ratio that is surface area density is large when you compare with large diameter channels or conventional channels. So, what happens that one can get large surface area for heat transfer or large interfacial area for heat transfer, and that results in enhanced heat transfer performance in microchannel, additionally the diffusion length, because in a channel when the flow is happening during the convective heat transfer the radial conduction happens from the general factor to the wall.

So, that is the diffusion length or the, that is the conduction length and as the channel size becomes smaller, this conduction length becomes smaller. So, the heat transfer rate for the same conductivity will be increased. Now looking at some of the applications in which the heat transfer in microchannel is being used increasingly are electronics cooling. So, in electronics cooling, the cooling of different parts of computer or other

hardwares as we know of from Moore's law that in about a year a number of chips on a motherboard on a number of chips increases almost every year

So, that increases the requirement for the cooling of the electronic equipment that increases that the larger heat flux, when the number of chips are increasing, then the largest heat flux need to be removed. So, at one stage the number of chips that can be put are limited by the heat flux that can be removed from the surface. So, it is important to develop technologies which can enhance the heat transfer performance for electronics cooling, and one of those technologies is based on microchannels. So, in that case when we have single phase flow. Now the performance can be further increased if in place of gas cooling one uses liquid cooling, because thermal mass of the liquid is more than that of gas.

Further if we want to increase the performance further, then additional means of heat transfer mixing is required, because in microchannels we always have flow to be laminar or in most of the cases, the flow is generally laminar. So, there is no mixing offered by the turbulent nature of the flow. So, people have devised different active and passive means of mixing one of them being using the, say serpentine channel which causes the flow to have a secondary flow in the secondary direction, say if this is the cross section of the channel, then in addition to the flow in the streamline direction. There is another flow in the cross section of the channel at that and that enhances the mixing in the channel. So, these are called dean vortices

So, that can be one of the solutions. Another solution is where people use bevel in different arrangements to enhance the radial mixing by convective means and so on so forth. So, further on if we want to increase the further heat flux or for a single phase flow or liquid only flow, if the heat flux is increased at one point, it will start evaporating and in that case we will have a boiling in microchannels.

So, that becomes important, or one can have another means of mixing that the flow can be segmented which is generate a Taylor flow regime, using bubbles or droplets. So, if one uses bubbles than one is just segmenting the liquid flow and enhancing the leak internal recirculation, and the heat transfer increase significantly 2 to 3 times depending on the flow conditions

In comparison to gas liquid segmented flow or gas liquid Taylor flow the heat transfer is further increased, if one have liquid liquid flow, because now both the phases will have internal recirculations and both the phases will also increase or will contribute to the heat transfer. So, the heat transfer can be further increased, when one compare with the gas liquid flow ok..

Another application is exothermic gas liquid reactions. So, a number of applications which require or for which the microchannels are being used for, are for the reactions which are highly exothermic, because it is relatively safer to carry out these reactions in microchannels.

One of the reasons being that higher heat flux can be handled in small diameter channels, and additionally when we have gas liquid reactions, the mass transfer is also enhanced by the, by carrying out these reactions, because the interfacial area has increased significantly, when we compare with normal channels or conventional channels, then they have applications as compact heat exchangers in aerospace engineering applications. As we can guess that the design of aerospace equipments is limited by that one, should have the highest performance with the minimum mass. So, that more and more payload for a space applications on a more and more load in the aeronautical applications can be taken there

So, one would like to have the lowest mass possible, and that will be achieved that, can be achieved by having high interfacial or high surface area density. So, it is being explored for application in aerospace engineering, then oxide offshore petroleum engineering, where again one would like to have compact heat exchangers. So, the offshore industry is using the compact heat exchangers. One of the companies that supply the heat exchangers is name as Heatric, and it is a part of a company called Meggitt PLC. So, you can look at some of the applications of these compact heat exchangers, the heat exchangers that Heatric offer are known as PCHE or printed circuit heat exchangers ok

So, printed circuit heat exchangers, what they are that in that is one of the way to a scale up the microchannels for industrial applications. So, in a metallic plate, a number of channels are hatched, which are of the millimeter size or smaller dimensions, and then another plate which has the same kind of channel it diffused, bonded on this plate and

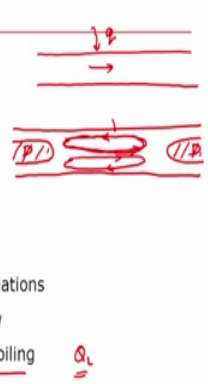
then these flat plates are stacked together. They may be thousands of channels in a plate and thousands of plates or hundreds of these plates can be stacked together and then the flow happens in this

Now, one of the challenges, in such cases will be the distribution of the fluids in the manifold, but that is not our topic of discussion today. So, these printed circuit heat exchangers are already being used for a number of applications; say for example, of floor applications right different companies ok.

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### Heat Transfer in Taylor Flow

- Heat flux removed from the channel wall
- Better heat transfer rates than liquid-only flow
  - The continuous phase is segmented
  - Internal recirculations in both the phases
- Enhanced mixing caused by the internal recirculations
- Negligible effect in case of gas-liquid Taylor flow
- Further heat flux removal during evaporation and boiling



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So, coming to heat transfer in Taylor flow regime. Generally it is when we are looking at it, it is convective heat transfer that is, because the flow is happening and, the heat is either entering in the channel and being removed by the fluid or it can be the other way around that the heat is being taken away in case of say exothermic reactions

So, they can offer better heat transfers than liquid and single phase flow as I just said, because the typical structure in a Taylor flow is, we have the dispersed phase this maybe bubbles or droplets and in between these bubbles, there is internal recirculations in the liquid slugs..

So, this internal recirculation causes that the hot fluid from the wall, that comes towards the centre and then efficient mixing happens. So, it enhances the heat transfer with respect to the liquid on the flow of core subject to the condition that if these dispersed

phase is bubble and then the volume of bubble should be smaller. So, that the efficient heat transfer is happening

Now, because in gas liquid flow, they do not contribute to the heat transfer and as I said earlier that, this heat flux that can be removed, can be further increased by phase change, where one get Q l or the latent heat that can be removed. So, one can remove further heat flux or higher heat flux by evaporation and boiling ok.


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**Single phase Heat Transfer in a Channel**

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- Forced Convection
- Nusselt number :  $Nu = \frac{h d}{k}$ 

$\left\{ \begin{array}{l} R d \rightarrow \text{channel diameter / Hydraulic diameter} \\ k \rightarrow \text{conductivity of the phase in contact with the wall} \end{array} \right.$
- Heat transfer coefficient  $h = \frac{Q_w}{A_w (T_w - T_b)}$
- Mean temperature  $T_m \text{ or } T_b = \frac{\int \rho u c_p T dA}{\int \rho u c_p dA}$



$T_b \text{ or } T_m$   $k \frac{dT}{dr}|_w$

BC:-  $\text{Const } q_w - HBC$   
 $\text{Const } T_w - TBC$   
Mixed BC  
 $q_w = h A (T_w - T)$

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So, let us look at some of the basics, because this course is not about heat transfer. So, let us remind ourselves some of the basics of convective heat transfer. So, when we talk about forced convection, it is often represented by a non dimensional number which is known as Nusselt number and it is abbreviated as Nu, given as hd over k, where h is heat transfer coefficient, d can be taken as channel diameter or in case of non circular cross sections, one can take as take this as hydraulic diameter and k conductivity of the fluid..

Now in case of two phase fluids, it is important to understand which phase of conductivities should be used in this expression, because this is a comparison of. If you can rearrange this h over k over d. So, the convective heat transfer divided by the conductive heat transfer in the radial direction. So, the conductivity of the phase in contact with the wall; that is what should be used now..

So, we know what  $d$  is the diameter of the channel, and what  $k$  is the conductivity of the fluid which is the property of the fluid. and now we can find it out or we can calculate by using different formula available in transport phenomena. Now  $h$  is the heat transfer coefficient which is defined as  $Q_{\text{wall}} / (A_w (T_{\text{wall}} - T_{\text{bulk}}))$ , it is important to remember that this is not the cross sectional area, but the area of the channel. So, for a cylindrical channel, it will be  $2 \pi r l Q_w / A_w (T_{\text{wall}} - T_{\text{bulk}})$ . So,  $T_{\text{bulk}}$  or  $T_{\text{mean}}$

So, generally there can be different kind of boundary conditions one can have on the wall. So, in general one encounters the constant wall heat flux constant or constant  $h$  boundary condition, which is called this constant wall heat flux. So, it is called  $h$  boundary condition that the heat flux is constant. The another boundary condition is constant wall temperature. So, in that, in this case it is called  $T$  boundary condition or the third boundary condition is mixed boundary condition, where one will have a relationship between  $Q_{\text{wall}}$  is equal to  $h A (T_{\text{wall}} - T)$  and

So, in this case. So, if we known that this can be used as a mixed boundary condition, one can see that  $Q_{\text{wall}}$  from Fourier's law of heat transfer is equal to  $k d T / d r$  at the wall. So, this is basically the boundary condition in which the gradient of the temperature is being defined. Now the question comes when we are defining the heat transfer coefficient  $T_{\text{wall}}$  is wall temperature, and this  $T$  which is bulk temperature or mean temperature..

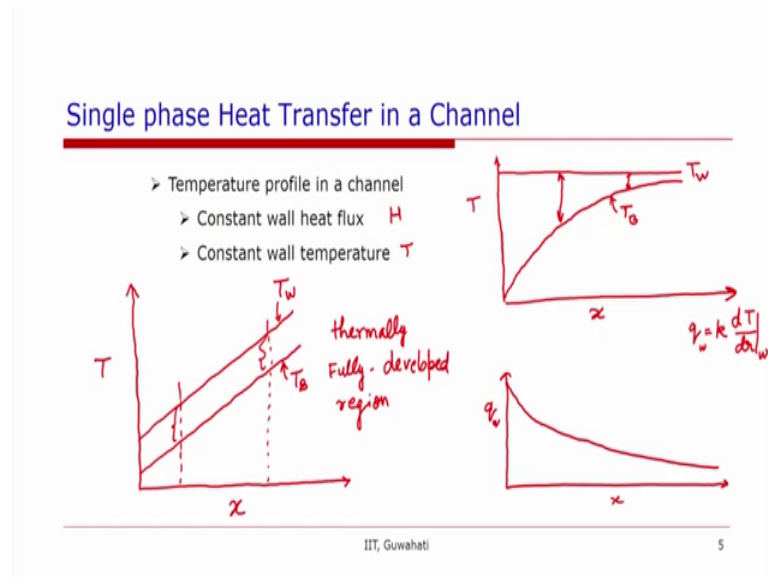
What this temperature is, because if we look at the flow in a channel you can see here is heat and drink at the channel wall the, the temperature profile may look something like this which suggest that the temperature will be highest near the wall, and then it will be wearing and become minimum at the centre and then again increase towards the wall.

So, the, clearly the temperature varies according to the with the radius of the channel. So, the which temperature, should we take say in this case the mean temperature is defined as equal to  $T_{\text{mean}}$  or  $T_{\text{bulk mean}}$  is equal to  $\int \rho u C_p T dA / \int \rho u C_p dA$ .

So, where  $\rho$  is the density of the fluid  $u$  is the local velocity of the fluid,  $C_p$  is the heat capacity and  $T$  is the local temperature of the fluid and  $dA$  is the area of the cross section, it has to be integrated over the area, it has to be integrated over the area. So, what you

can observe from this that the only difference between the integrals at the top and bottom is  $T$ . So, this is basically mass weighted average of the fluid ok. So, that is bulk mean temperature.

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Now, because this is flow in a channel. So, we will remind our self the temperature profiles for, to often use boundary conditions for  $h$  boundary condition and for  $T$  boundary condition. So, in a constant wall heat flux case as one can see or one can speculate that the temperature off the wall will be increasing and the temperature of the fluid will be increasing.

We have this in the fully developed regime from the, when say a fully developed region, this is thermally fully developed region. So, in the family fully developed region, we will a little later, we will talk about how do we define the family fully developed region in a in a channel. So, this is the upper line is for  $T_w$  and the line below is  $T_{bulk}$  and we can say that they are parallel to each other, and the slope of these lines can be found by writing the heat balance equation. So, the temperature of the wall and temperature of the fluid both increases linearly, and the difference between them is constant everywhere for the fully developed flow case

Now, for a constant wall heat flux case, this will not be the case, but what we will have is the temperature and  $x$ . So, the temperature of the wall will be a constant, because that is what the boundary condition is, and then as the fluid start flowing into it. The

temperature of the fluid will increase and then approach towards the temperature of the wall, and that will be achieved only at infinity..

So, in this case we can see the difference between the temperatures keep decreasing, as we move in the axial direction and  $Q$  is equal to  $k d T$  over  $d r$  at the wall  $Q_{wall}$ . So, we can see that  $Q_{wall}$  in this case will keep decreasing almost exponentially ok. So, these are the typical temperature and heat wall, wall, heat flux profiles for single phase flow in a channel.

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**Single phase Heat Transfer in a Channel**


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➤ Thermally fully-developed flow:  
 ➤ Non-dimensional temperature

*Hydrodynamically fully developed flow*  $\frac{dv_z}{dz} = 0$

*For thermally FD flow*  $\frac{dT}{dz} \neq 0$

$\theta = \frac{T_w - T}{T_w - T_m}$        $\frac{d\theta}{dz} = 0$  for thermally fully-developed flow



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$Nu_H = 4.36$   
 $Nu_T = 3.6$

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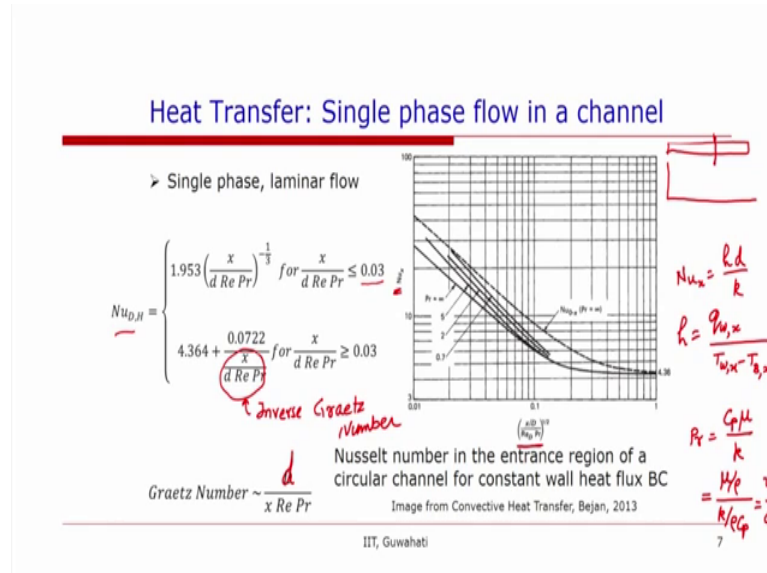
Now, for thermal heat fully developed flow. So, if we remember or if we remind ourselves that hydrodynamic fully developed flow refers to that  $\frac{dv_z}{dz}$  or the velocity in the axial direction. Let us say  $\frac{dv_z}{dz}$  is equal to 0. So, the velocity profile is not changing along the axial direction. So, if we look at the velocities at two different locations  $x_1$  and  $x_2$  in the channel  $x_1$  and  $x_2$ , then we will find that the velocity at the corresponding radial locations are same in both the cases

Whereas if we look at the temperature profiles, here it clearly tells us the temperature has to increase, because it absorbs more and more heat. So, we cannot have that the thermally fully developed flow in analogy with the hydrodynamically fully developed flow, we cannot have  $\frac{dT}{dz}$  is equal to 0; that is not possible for thermally fully developed flow. Whereas, a non dimensional temperature. So, the temperature do increase, but the non dimensional temperature which can be defined as  $T_{wall} - T$



divided by  $T_{\text{wall}} - T_{\text{mean}}$  and  $d \theta / dz$  is equal to 0 for thermally fully developed flow. So, let us remember all this

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Now, what we talked about that, when the flow is has developed, what is the, what are the temperature profiles and how we define the non dimensional temperature and so on and so forth. So, it can also be seen, shown that for laminar fully developed flow the Nusselt number in a channel in for fully developed flow and fully developed heat transfer, the Nusselt number is a constant. So, for  $h$  boundary condition  $Nu_h$  is equal to 4.36 and for constant temperature boundary condition it is about 3.6 around them ok

So, for developing flow as one can, I expect that the heat transfer or the or the heat transfer coefficient or the Nusselt number will be higher, when the flow is developing. And then eventually it becomes constant, it has been plotted the Nusselt number, the local Nusselt number  $Nu_x$ , it has been plotted along the channel length when I say local Nusselt number; that means, it is being calculated along the length of the channel..

So,  $Nu_x$  is  $h d$  over  $k$  and  $h$  is  $Q_w / A_s$ , because this is constant and  $T_{\text{wall}}$  at the  $x$  location minus  $T_{\text{bulk}}$  at the  $x$  location. It does not consider the entire the thing, the in the heat transfer from the inlet to this point, this has been ported only the heat transfer at the cross section. So, that is why it has been termed as  $Nu_x$ .

So, the heat transfer coefficient keep decreasing as we move in the axial direction. Notice the non dimensional parameters at the x axis what we have here is,  $x$  over  $d Re Pr$ . So, all of you will know that  $Pr$  is defined as  $C_p \mu$  over  $k$ . Now we can rearrange this as  $\mu$  over  $\rho$  divided by  $k$  over  $\rho C_p$ . So,  $\mu$  over  $\rho$  is known as dynamic viscosity or momentum diffusivity and  $k$  upon  $\rho C_p$  is known as thermal diffusivity..

So, this is basically the ratio of momentum diffusivity and thermal diffusivity in the Prandtl number is the ratio of momentum diffusivity and the thermal diffusivity. So, one gets this the, the non dimensional number that is important in this case is  $x$  over  $Re Pr$  and this Prandtl number is a function of the property of the fluid only, and this decreases there..

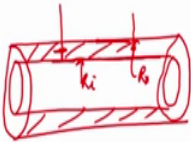
There are different correlations that has been given and one of them is that it the Nusselt number decreases as  $x$  over  $d Re Pr$  of this non dimensional number raised to the power  $1 - 1/3$ , when this is less than 0.03, and then it decreases according to this equation when  $x$  by  $d Re Pr$  is greater than 0.03 and this number inversely. This is written as, we can change it as  $d$  small  $d$  for uniformity

So, the Graetz number is written as  $d$  over  $x Re Pr$ . So, this number or this term is also known as inverse. Graetz gave an analytical solution for a hydrodynamically fully developed and thermally developing flow and it is a series solution. So, to simplify the solution in number of approximation has been presented, and this is one of those. So, what I would like to emphasize here that the developing Nusselt number is a function of inverse Graetz number or Graetz number ok, which we will use later on when we develop the model for Taylor flow heat transfer.

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### Typical Experimental Set-up

- Pulsatile but periodic flow
- Wall boundary condition for temperature
- Developed or simultaneously developing hydrodynamics
- Non-volatile liquid for evaporation to be negligible
- Wall temperature measurement
- Bulk fluid temperature
- Conjugate heat transfer



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So, coming to heat transfer in the Taylor flow regime, the flow in Taylor flow regime is pulsatile. So, the flow changes with time, but it is periodic. So, it repeat itself after a certain time or you can see this periodicity in space as well. So, if you are standing at one place, you will see one bubble coming along and then one slug comes in and then you have another bubble coming along and this will be periodic. So, you can see if you are staying at one place and you can see the, the volume fraction varying with respect to time and this variation maybe periodic

Similarly, if you are looking at in space that one time instant, if you look at you will have number of bubbles at different at regular intervals, the distance between the two bubbles will be same and the shape of the two bubbles will be same. So, the flow is periodic in space as well. Now to do experiments for this for Taylor flow, one need to make sure what boundary condition is being used, and one also need to look at that what kind of heat transfer one is looking at, developing or simultaneously a developing hydrodynamics and heat transfer or they are looking at the heat transfer. The hydrodynamics of the flow has already developed and the and the heat transfer is developing and then it becomes fully developed.

Now, when we do simulation or when we are studying, we can decouple phase change easily. You can say that we are not going to include a phase change model in our computational model saying CFD is, but when this happens experimentally, then there is

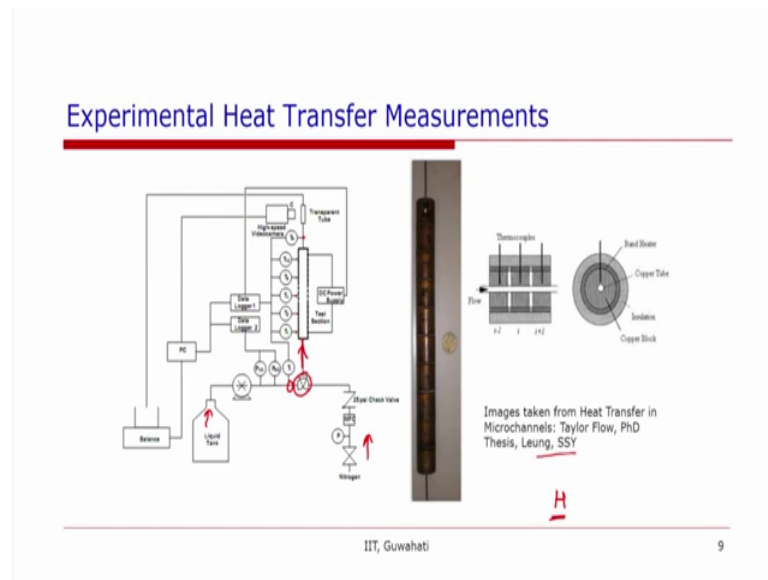
no way to suppress or it is difficult to suppress the heat the phase change, change that might happen, because of evaporation. So, what one can do that, it can be minimized to the maximum extent possible and that can be done. If one uses a non volatile gas liquid combinations or non volatile liquid and the temperature of the fluids is at slightly lower range. So, the volatility or the evaporation rate is negligible or smaller is small.

Now,. So, then the question comes when we measure what, what is to be measured. So, it is  $Nu$  is equal to  $h d$  over  $k$ . So, if one have specified a constant wall heat flux, which is generally the case during the experiments, then what is need to be measured is  $T_{wall}$  and  $T_{bulk}$ . So, the  $T_{wall}$  can be easily measured by putting the thermocouples at different locations on the wall or by different wall temperature measurement techniques..

However, it is not easy to measure bulk fluid temperature, because if one put a thermocouple inside the channel, then the flow behavior is going to change, because of the presence of the temperature sensor or the that is put there. Then in most of the cases the temperature in the channel walls. So, if let us say this is the channel, this is the internal wall of the channel and this is the outer wall of the channel at the phase, the channel is thickness of the channel..

So, this is  $R_i$  and this is  $R_o$ . Now the sensors will be put somewhere here on the wall temperature will be measured. So, one need to take into account the heat transfer; that is happening from here to the liquid solid interface. So, in general when one put the temperature sensors here, then one can obtain an analytical solution for the conductive heat transfer in this direction, or if you want to take into account the actual heat transfer also, then one can have a full 3D simulations to look at the effect of conjugate heat transfer ok. So, all these factors needs to be thought about when performing the experiments.

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So, this is a typical experimental setup in which the liquid is coming from a liquid tank, and the gas nitrogen, gas is being supplied and then the inlet flow temperature is being measured. Then they come into a mixing section and the fluid move along the test section, and the temp, wall temperature is being measured at different sections. The wall is being heated A by A power supply..

So, this is a test section that was used by Miss Leone in her experiments at the University of Sydney and then the outlet temperature is measured. They have also arrangement for looking at the and confirming the flow regime in the channel and then it is being brought together, been brought to outside and released to the atmosphere

So, that is a typical experimental setup and what is being measured, they have a H boundary condition. In this case and the wall temperature is being measured at different sections ok. one, I advice to further look into the details of the test section, because this is, there is lot of detail in while designing the test sections, especially the H boundary condition or 2D boundary condition in the, in this blocks ok.

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### Experimental Heat Transfer Measurements

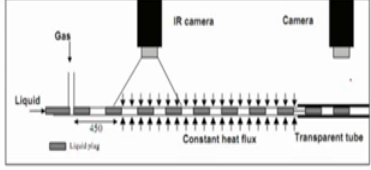


Image taken from Walsh et al., 2009

- Constant wall heat flux by Joule heating
- Local measurement of surface wall temperature
- 1.5 mm tube with wall thickness of 0.25 mm; 0.5 m long heated section

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So, the another test section that has been used by walls at all, it is a similar test section and they have made sure. Here actually they have also made sure that the flow is hydrodynamically fully developed, and then there is heat being applied on it, and this heat is being applied by joule heating, and the local measurement of surface wall temperature is being done and using an IR camera probably..

And then this is different tube cross sections they have taken and wall thickness and the length of the (Refer Time: 39:24) So, then they are also looking at the fluid behavior or the flow regime and the exit

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### Nusselt number for Taylor Flow

- Local Nusselt number
  - Based on local wall and bulk fluid temperature at the cross section
 

$$Nu_x = \frac{h d}{k_c} \rightarrow h = \frac{q_w}{T_{w,x} - T_{b,x}}$$

$q_w \rightarrow$  available from experiments (input)  
(measured)

$$T_{b,x} = \frac{\int_A \rho |u_x| C_p T dA}{\int_A \rho |u_x| C_p dA}$$
  - Not useful for comparison with experimental data
  - Cannot be used for design calculations



$Nu_{ov} = Nu_{uc}$

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So, while looking at the Taylor flow behavior. Now we have two phases gas and liquid or liquid phase. So, one need to define a Nusselt number or one need to revisit the definition of Nusselt numbers. So, we can define a local Nusselt number  $Nu$  is equal to  $h d$  over  $k$ , and because it is always the continuous phase, which is in contact with the wall. So,  $k$  can be  $k$  of the continuous phase,  $h$  is the heat transfer coefficient and  $d$  is the diameter of the channel. Now  $h$  is defined as  $q_{wall}$  over  $T_{wall}$  minus  $T_{bulk}$

So, we will be defining these definitions for  $h$  boundary condition. Most of the discussion that we will have in this lecture will be based around the constant wall heat flux boundary condition. So, this  $q_{wall}$  is available from experiment and it is an input  $T_{wall}$  is measured. Now generally  $T_{bulk}$  is a problem ok. So, what we are looking at here is the local definition of Nusselt number. So,  $Nu_x$ , everything else is going to be same and everywhere on the cross section  $q_{wall}$  and the temperature of the wall is at that particular location. Now we also need to define  $T_{bulk}$ .

So,  $T_{bulk}$ , we remember the definition of bulk temperature, then it can be defined integral at the area  $\rho u_x$ , if the area or if the axial direction is  $x$  then  $\rho u_x C_p T$  at the particular location  $dA$  divided by integral over the area  $\rho u_x C_p dA$ . Now because the there can be backflow in the liquid. So, if we take in do not take into account, only the magnitude of the axial velocity then the terms will cancel out, and we

will have a wrong definition or sometimes a very small bulk temperature which does not represent the bulk temperature correctly.

So, we have put that the absolute value of the axial velocity is being taken into account here. So, that is our local bulk fluid temperature; however, this is good, if you want to do computations and we can get the local Nusselt numbers at different locations in the channel, but if we want to compare this with the experimental data, we do not have, we might have with the techniques available. Now we can get the wall temperature locally, but it will not be possible to get the bulk temperature with the same definition. So, this is generally not used for the design calculation.

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### Nusselt number for Taylor Flow

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- Nusselt number for a **unit cell**
  - Thermal conductivity
    - Gas or liquid phase?
  - Wall heat flux
    - Area weighted average
  - Wall temperature
    - Area weighted average
  - Bulk mean temperature

$$Nu_{uc} = \frac{h d}{k_c} \quad h = \frac{q_w}{T_w - T_B}$$

$$T_w = \frac{\int_{A_w} T_w dA}{\int_{A_w} dA} = \frac{\int_x^{x+L_{uc}} T_w 2\pi r dx}{\int_x^{x+L_{uc}} 2\pi r dx}$$

$$T_B = \frac{\int_{uc} \rho u_x |G| T dV}{\int_{uc} \rho u_x |G| dV}$$

$$= \frac{\int_x^{x+L_{uc}} \theta_x dx}{L_{uc}}$$



Unit Cell

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Now, So, for design calculations we also need to have overall Nusselt number, now because Taylor flow is periodic. So, if we consider the flow, when the heat transfer also have become periodic, then we can analyze flow in one unit cell which consists of one bubble or droplet and the slug around it. And once we have obtained this we can use the same Nusselt number for the entire channel..

So, it is easier to understand the behavior in one unit cell, which consists of one bubble or droplet which is the dispersed phase and two halves of the liquid slug on the two sides of the unit cell ok. So, when we define the Nusselt numbers for a unit cell, we need to see  $h$  for unit cell is equal to  $h d$  over  $k$  will be  $k$  liquid, no change there  $d$  will be the channel diameter  $h$  is equal to  $q_{wall}$  over  $T_{wall}$  minus  $T_{bulk}$



So, now remember this is for the entire unit cell which consists of the bubble or droplet, and two halves of the liquid cells. So, the thermal conductivity as we discussed already; that it is always the continuous phase or the liquid phase, which is in contact with the wall. So, the conductivity of the continuous phase will be taken into account the wall heat flux. If we have the any boundary condition which is not constant volume flux and we can get area, average area weighted average of the heat flux and the wall temperature that will be measured and for calculations..

If we want to get the, because here what we want is, the area weighted average. So, on the wall temperature in the entire location. So, we can do it integral area of the wall  $T_{wall} dA$  divided by integral area of the wall  $dA$ . So, remember this age for a circular channel, it will be integral  $x$  location to  $x$  plus  $l$ , which let us say this  $l_{uc}$  and this is  $T_{wall} 2\pi r dx$  divided by integral  $x$   $2\pi r dx$  on  $T_{wall}$ ..

This will have only  $2\pi r dx$ . So,  $2\pi$  will cancel out and we can get the area weighted average. Now the bulk mean temperature  $T_{bulk}$  will have the same definition, but now the integral will be over the entire unit cell. So, we will have integral  $u c$ . Remember that is that in this case also we have 2 phases. So, we will have the gas phase and the liquid phase, depending on and what location what phases present..

So, you will have a integral  $\rho u x C_p T dV$  divided by integral  $\rho u x C_p dV$ . So, that will give us the bulk mean temperature. Now, one can see and confirm for himself that this bulk temperature will not be same, if one sums up the temperature at different axial locations or it will not be simply integral  $T B x dx$ , that will not be the case. So, one need to be careful when defining this.

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### Nusselt number for Taylor Flow

- Nusselt number estimation in experiments
  - Thermal conductivity
    - Gas or liquid phase? ✓
    - Variation with temperature ✓
  - Wall heat flux/ wall temperature
    - One of them is often constant
    - Often constant wall heat flux BC
    - Wall temperature is measured along the axis

$\dot{m} C_p (T_{b,x} - T_{in}) = \dot{Q}_{in} \frac{x}{L}$

$T_{b,x} = ?$

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Now, so in the experiment the Nusselt number estimation is done, based the axial lengths. So, if this is the channel from where it is being heated, the channel wall is being heated and the total heat flux to this is, let us say  $Q_{in}$ . So, we can write a heat balance at a location  $x$  from inlet. So, it can be written as  $m \dot{c}_p$  and  $m \cdot C_p$  is for 2 phase and  $T_{b,x}$  or bulk temperature at  $x$  location minus  $T_{in}$ , where  $T_{in}$  is the temperature at this location is equal to  $Q_{in} \cdot x / L$ .

So, this is the heat flow rate and if this the  $Q_{in}$  is for the entire length, and then what we are taking the how much heat has gone in this that will be equal to the heat absorbed by the liquid or the gas liquid flow. So, that will be  $m \dot{c}_p$   $m \cdot C_p$  will have to take into account that, there are 2 phases present in it. So,  $T_{b,x}$  minus  $T_{in}$  and. Of course, the thermal conductivity will be the gas and liquid phase and one can use and see the variation of  $k$  in the temperature range between the minimum and maximum temperature, and judiciously use an average value

The wall heat flux is one of them, either the wall heat flux or wall temperature is constant of 10. It has been observed that the experiments are done with the constant wall heat flux boundary condition, and the wall temperature is measured along the axis, and this is the way by which we can calculate  $T_{b,x}$ , as one can see that one will know what is the heat being input it to the system and what location one wants to measure the

temperature, and the flow rate and the properties of the two fluids will be known. So, one can calculate what is the bulk temperature at the location

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The slide is titled "Nusselt number for Taylor Flow" in blue text. Below the title is a red horizontal line. The content is a bulleted list of methods for Nusselt number estimation in experiments. The first bullet is "Nusselt number estimation in experiments", which has a sub-bullet "Bulk mean temperature". Under "Bulk mean temperature", there are three sub-bullets: "Thermocouples placed in channel cross-section", "Laser induced fluorescence", and "Heat balance". The last two sub-bullets are underlined. At the bottom of the slide, there is a red horizontal line, followed by the text "IIT, Guwahati" on the left and the number "14" on the right.

**Nusselt number for Taylor Flow**

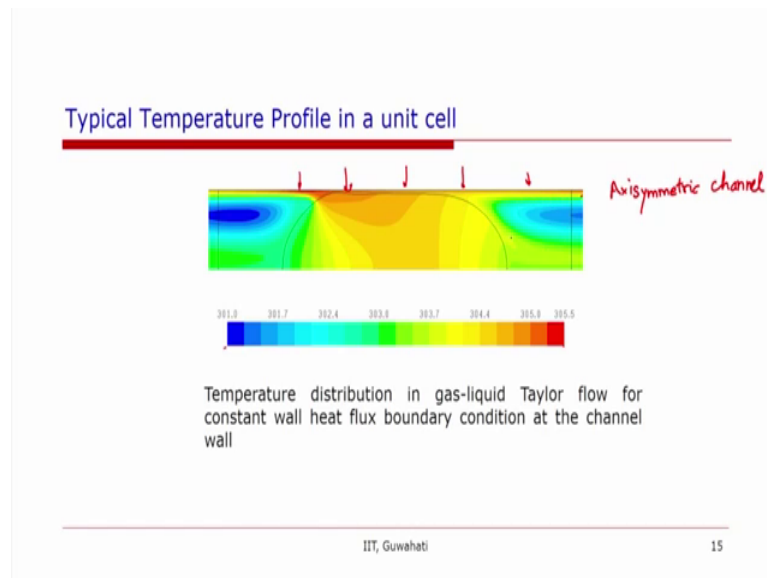
- Nusselt number estimation in experiments
  - Bulk mean temperature
    - Thermocouples placed in channel cross-section
    - Laser induced fluorescence
    - Heat balance

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Now, this is how it is been. Now what we have described just and the bulk mean temperature can be measured from heat bala can be obtained by heat balance, but it is difficult to measure it; however, recently it the technique using laser induced fluorescence, where the particles which are following the flow faithfully, and they are quoted by a fluorescent material, which is temperature sense, which gives temperature sensitive fluorescence..

So, the efforts are being made to measure the local temperature in the fluid. Efforts are also made to put the thermocouple inside the channel, which may affect the flows slightly and then look at the temperature field in the, and the channel and obtain the mean, bulk mean temperature from there ok.

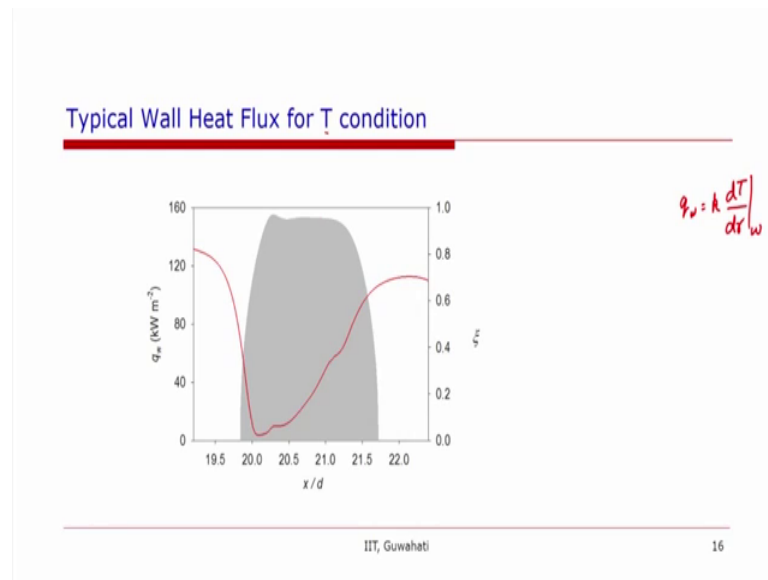
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So, let us look at some of the results, this is a typical temperature profile in a unit cell. So, why you see here from the bubble, because an axisymmetric channel, then this is from a 2D computational, where this is done in an axisymmetric channel. The black line shows the boundary of the bubble, the other bubble will be on the bottom half and you can see the temperature, there is a constant wall heat flux boundary condition

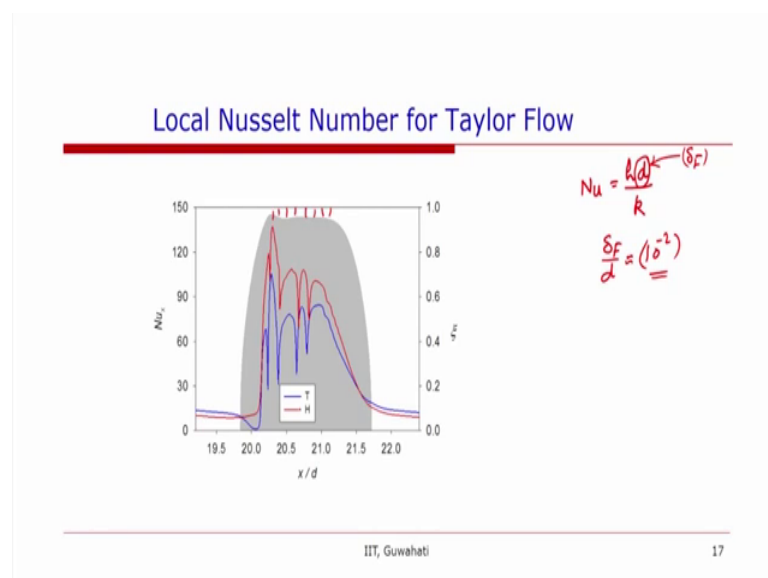
So, the heat flux is, heat is entering from the wall and the fluid as hot as you can see from this temperature scale, which varies from 301 to 305.5. So, the temperature is hot, hot is the yellow color and the red color represents the hot fluid and then this hot fluid is being brought towards the centre. So, the heat is being brought by the fluid using convective convection and then, the mixing of the fluid is happening, the temperature of the bubble seems to be very high, but remember that very small heat, very little amount of heat is required to increase the temperature of the bubble. So, this is a typical temperature profile and this also. So, just clearly the presence of internal recirculations in the liquid flux

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Now, this is a typical wall heat flux for  $T_w$  for the constant wall temperature boundary condition. So, you can see that the wall heat flux is high in the liquid slug zone, and it is small in this region, because that that will be  $q_w$  is equal to  $k \frac{dT}{dr}$  at the wall. So, the temperature difference will be small in this region now. So, the heat flux will be small ok.

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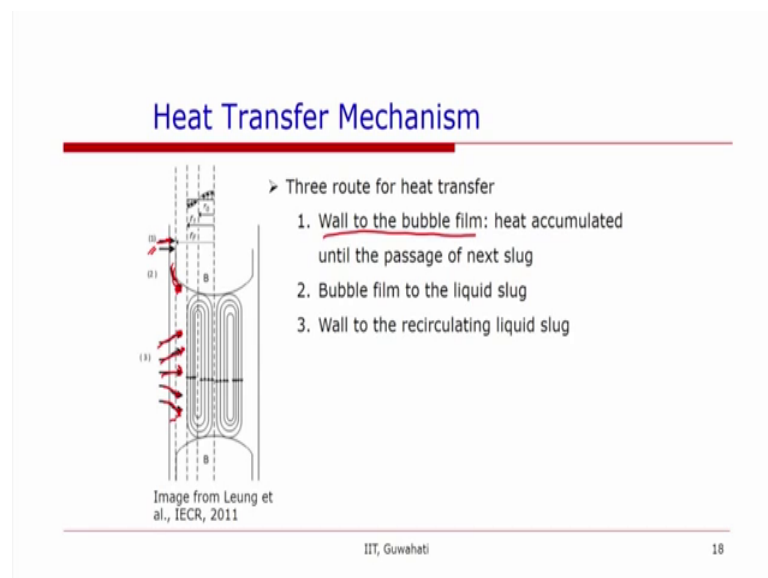


So, then Nusselt numbers and these are local Nusselt numbers. So, you can see that one fine or one octave very high Nusselt numbers in the film region. So, let me remind you at

the outset that  $Nu$  that has been defined is equal to  $h d$  over  $k$ , where we have taken this length scale and diameter; however, if one look at the heat transfer is being, is because of the film only..

So, the actual length scale that should be used here, is  $\delta F$  and if one take that into account this will be very small, because  $\delta F$  over the  $h$  or the order of, it turn 10 to the power minus 2 or smaller. So, the Nusselt numbers are not high. In this case this is just because of the length scale being used here and the Nusselt numbers in this case is about the from one can see from here on the y axis the Nusselt number is about 10. So, this is significantly high when you compare with liquid on the flow in the channel. So, both the boundary conditions.

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So, now let us look at a bit about the heat transfer mechanism. So, we have the bubbles and the internal requisition in the liquid slug, and the heat can find different routes to go from the wall in the liquid, which is, which has the high heat carrying capacity. So, the one route is that the heat goes from wall enters from the wall, and it goes into the bubble fill or the film that is surrounding the bubble, because the heat carrying capacity of this bubble is very small

So, it cannot carry much of the heat. So, it waits until the next slug comes in and then it supplies this into the slug. So, that is first route. Now the second route is that the bubble film, it passes on the heat to the liquid slug. So, and thirdly that wall to the recirculating

liquid slug zones of the heat and that enters from the slug region that passes the heat from wireless film to the liquid slug directory..

So, one need to remember or the general picture is that, because the heat enters in the film, but the bubbles, if it is gas liquid flow then the bubbles do not carry much of the heat and this heat is taken over by the liquid slug which comes behind. If it is a gas liquid or if it is a liquid, liquid flow then this heat will be redistributed between the two phases. So, we will look at, when we develop a heat transfer model, we will look at this correlation from the ground.

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## Heat Transfer Correlations

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➤ Oliver and Wright (1964)

➤ Modification to Gratez-Leveque Solution for thermally developing flow

$$Nu = 1.615 \left( Re Pr \frac{D}{L} \right)^{1/3} \left( \frac{1.2}{(1-\beta)^{0.36}} - \frac{0.2}{1-\beta} \right) \left( \frac{\mu_B}{\mu_W} \right)^{0.14}$$


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So, let us look at some of the correlations that have been presented in the literature for predicting the Nusselt number in Taylor flow regime. So, this Nusselt number is defined as  $Nu$  is equal to 1.615. And now you can remember the inverse Gratez number which is  $Re Pr$  channel diameter by  $L$  raised to the power 1 by 3

So, this what they have done is modified the Gratez Leveque solution, which is for thermally developing flow, and in this solution they have included the effect of bulk viscosity, its like (Refer Time: 60:35) equation for single phase turbulent fully developed flow, and then they have introduced the effect of homogenous write fraction in terms of beta. So, that has been modified to obtain the Nusselt number.

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## Heat Transfer Correlations

- Hughmark (1965)
- Modification to **Gratez-Leveque Solution** for thermally developing flow

$$\underline{Nu\sqrt{1-\beta}} = 1.75 \left( Re Pr \frac{D}{L} \right)^{1/3} \left( \frac{\mu_B}{\mu_W} \right)^{0.14}$$


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Another correlation is similar modification by Gratez. So, they had a simpler modification that this is the slug Nusselt number and from that one can just calculate the thermally developing flow. So, basically they said that the heat transfer in the slug region is thermally developing flow and that can predict the Nusselt number in for the Taylor flow ok.

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## Heat Transfer Correlations

- Kreutzer et al. (2001)
- Correlation for Nusselt number in the slug region

$$Nu_{slug} = 20 \left[ 1 + 0.003 \left( \frac{L_{slug}}{Re Pr D} \right)^{-0.7} \right]$$


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Kreutzer, they did some experiments, they wanted to do. I mean most of their work was based around mass transfer in Taylor flow regime, and to develop a correlation for mass



transfer they did a simulation for heat transfer. And from there they look at the Nusselt number in the slug region or a cavity which was that they have done a simulation, where the cavity in which the these are two walls and the these two walls are moving

So, the fluid will eventually develop recirculation inside it. So, they calculated the Nusselt number in these cavities and found this correlation for the slug Nusselt number.

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The slide is titled "Heat Transfer Correlations" in blue text. Below the title is a red horizontal line. The content includes a citation "Walsh et al. (2001)" and a sub-point "Correlation for Nusselt number in the slug region". The equation for the Nusselt number is displayed as 
$$Nu = (1 - \beta) \left[ Nu_{sp} + 25 \left( \frac{L_g}{D} \right)^{-0.5} \right]$$
. At the bottom of the slide, there is a red horizontal line, and the text "IIT, Guwahati" and the number "22" are visible.

Walsh et al, they developed another correlation in the slug region and that was a Nusselt number into homogenous wide is equal to homogenous wide fraction single phase, fully developed Nusselt number depending on the boundary condition plus 25 ns by the raised to the power minus half. So, again the effect of developing it

So, we will stop here. So, in summary what we have looked at in this lecture is Taylor flow in microchannels, the heat transfer in this flow regime. Before that we looked at the basics of heat transfer in particular, the definition of Nusselt number definition of heat transfer coefficient and definitions of bulk mean temperature.

So, by looking at these definitions, we have also seen later on that when we looked at the some correlations that has been that have been developed for Taylor flow heat transfer. They are based on the developing flow in for liquid only flow in a channel. So, in the next lecture we will look at the heat transfer in the Taylor flow regime at and a general correlation that can be developed for heat transfer in the Taylor flow regime

Thank you.