Multiphase Micro fluidics

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Lecture – 12
Smooth Interface Laminar-Laminar Annular Flow

Hello, in today's lecture, we will be looking at annular flow. In particular we will look at an ideal case of annular flow, where the interface is smooth and the film thickness or the size of the code, or the radius of the code is same throughout the channel. Ok?

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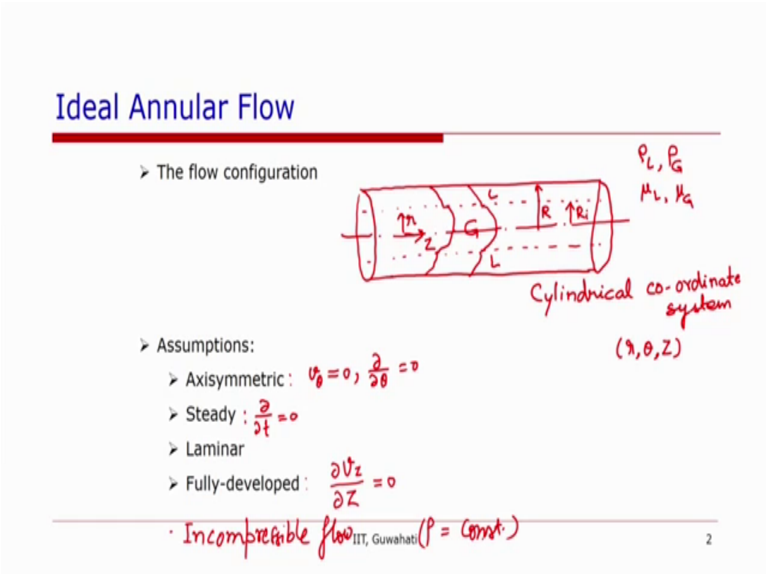

So, let us draw a configuration of an ideal annular flow. We have a channel. Let us say this is a cylindrical channel and in this cylindrical channel, the interface is located somewhere here. So, this is phase 1. Let us consider gas liquid annular flow.

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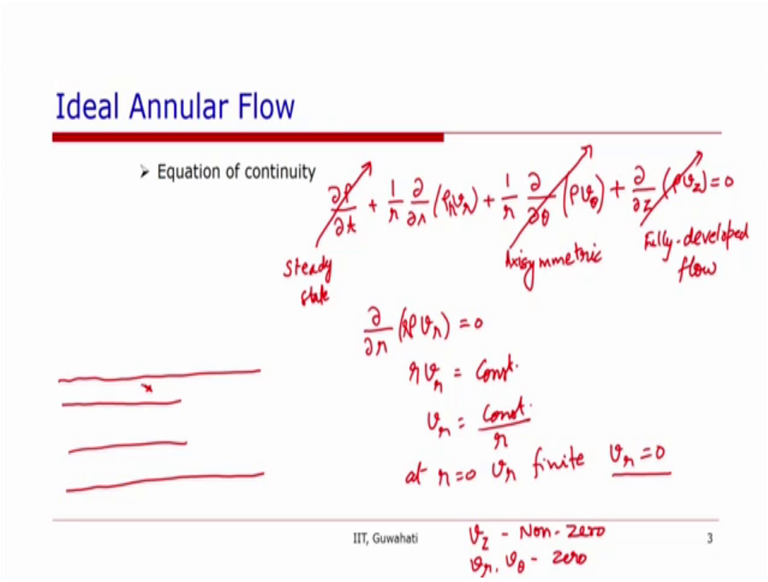

So, gas is at the code and liquid is on the sites. Ok? And the radius of the channel is capital r and the radius of the interface, let us name it as I and we have properties of liquid Rho l and Rho g and mu l and mu g are the density and viscosities of gas and liquid. Now, this configuration, when we look at the flow regimes in gas liquid flow in micro channels, we often encounter annular flow, but this is really the annular flow that we see in gas liquid flow, in micro channels it is really a smooth interface annular flow.

We generally see the presence of waves on this annular flow, but to understand this, let us consider a case, where for which we can obtain an analytical solution and can get relationship, say for example, between pressure drop and film thickness or interfacial radius and the flow rate of the phases. So, this is the case that we are going to consider in this lecture..

There can be annular flow or what is termed as in general, core annular flow for in the reference of liquid flow. So, they are depending on the viscosities and densities of the two fluids plus hydrophobicity or hydrophilicity of the two liquids. One phase will be at the core and another phase will be these properties. We will determine which phase will be at the core and which phase will be near the wall. Ok?

So, we are looking at this. We will use the terms for rho l and rho g or mu l and mu g; however, the same terms can be used with reference to liquid 1 and liquid 2. Ok? So, we will solve the mass conservation and momentum conservation equations and because this is a cylindrical configuration, so we will prefer cylindrical coordinate system. So, where we have r theta and z? So, z is the direction, r the radial direction and when we say that the flow is Ax symmetric; that means, there is no flow in the angular direction and no gradients in the angular for azimuthally direction.

So; that means, v theta is equal to 0 and del over, del theta is 0. So, this is the cemetery configuration, ax symmetric configuration at steady state; that means, the dou by dou T term is equal to 0 study, means as you will know that steady means the properties, the things, the configuration does not change with time. So, all the variables with respect to time are constant. So, that is why dou by dou T term in the conservation equations will be 0, flow is laminar and we do not need any models for turbulence and we consider fully developed flow here.

So, when the flow is fully developed, we can say that del v z over del z is equal to 0; that means, the velocity profile, whatever the velocity profile be obtained at one axial location, a similar velocity profile will be obtained at another location. So, the corresponding difference delta v will be 0 or delta v z will be 0. So, the flow is fully developed or that is the definition of a fully developed flow. Now, let us write the conservation equations. So, coming to the continuity equation, let us write a continuity equation in the cylindrical coordinates.

So, that is del rho by del T plus 1 over r, del over del r rho v r plus 1 over r, del over del theta rho v theta plus del over del z rho v z is equal to 0. Now, because this is at steady state, so, this term will be 0 and ax symmetric. So, this term will be 0. Now, what we have is del v z over del z is constant, because of fully developed flow. So, this is because the flow is ax symmetric and in this case, this says that fully developed flow. So, del v z over del z is 0 the flow. We consider in this case that the flow of both the phases is incompressible. So, we can add the assumption of incompressible flow. So, rho h is constant in these cases. So, that is why we can neglect this term.

Now, what we have from this, that del over del r of rho v r is equal to 0 or v rho sorry, this is rho r v r, here, rho r v r and; that means, r v r is equal to a constant and v r is equal to constant over r, but v r needs to be finite. So, that r is equal to 0, v r is finite. So, the term v r will be 0; that means, that is obvious from the flow configuration itself, because if the flow is happening in the radial direction; that means, the interface radius will change, because some flow will happen along this direction and the radius interface radius will have to change to accommodate that flow.

So, because we do not have any such acceleration flow, so, the radial component of d will 0. So, the only component of velocity that we have non - 0 is, v z h non – 0 and v r and v theta, they are 0. Ok?.

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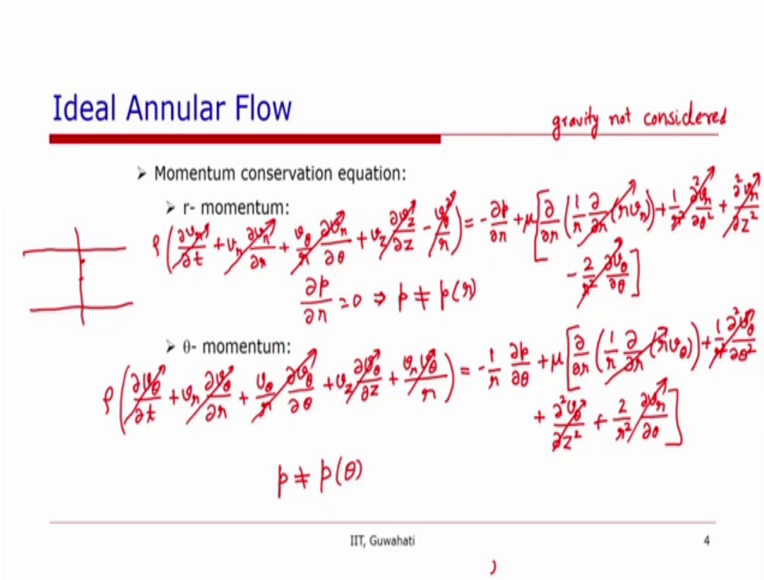

Now, let us come to the r momentum equation. So, for the cylindrical coordinate, the r momentum equation is rho del v r over del t plus v r, del v r over del r plus v theta over r, del v r over del theta plus v z, del v z over del z minus v theta squared over r is equal to minus del P over del r plus mu, del over del r 1 over r, del over del r of r v r plus 1 over r squared del 2 v r over del theta 2 plus del 2 v r over del z 2 minus 2 over r squared del v theta over del theta.

We do not consider gravity. So, we can say right gravity effect is not considered, if we take horizontal flow, then the flow will not be ax symmetric. If the gravity effect is considered, but remember that in micro channel, where the channel 5 is sufficiently small, the effects of gravity are anyway negligible. So now, we can neglect or we can strike out the terms which are not relevant for this is at steady state. So, this term will be 0, v r is 0. So, this term is 0 v theta is 0. So, this is 0 del v z over del z is 0. So, this is also 0 and v theta is 0. So, this is 0 and because v r is 0, so, this term is 0. This is again 0 because v r and this is because v theta is 0.

So, what we have is del p over del r is equal to 0; that means, v is not a function of r of pressure is same at a particular cross section in the radial direction. So, you take the pressure here and the pressure here, it is independent of the location, but remember for the two phases, what these equations we are writing general equation for phase 1 and phase two, but when we consider the boundary conditions, we will have a place of at the interface. So, you will discuss that, but for a phase inside one phase, pressure is not able to follow. Right?

Now, for theta momentum term, what we have is , rho del v theta over del T plus v r, del v theta over del r plus v theta over R, del v theta over del theta is v z, del v theta over del z plus v r v theta over r is equal to minus 1 over r del p over del theta plus mu del over del r 1 over del over del r r v theta plus 1 over r squared del 2 v theta over del theta 2 plus del 2 v theta over del z 2 plus 2 over r squared del v r over del theta. Ok?.

So, in this again, we neglect the terms. This term is 0 v theta, is 0. So, this term is 0 v theta is 0 v theta is 0. So, all these terms are 0 and again, we will have this is 0, because v r is 0. So, we will have that pressure is not a function of theta so; that means, pressure is, if it is, then it is a function of that only. So, let us look at the z momentum equation. Right?

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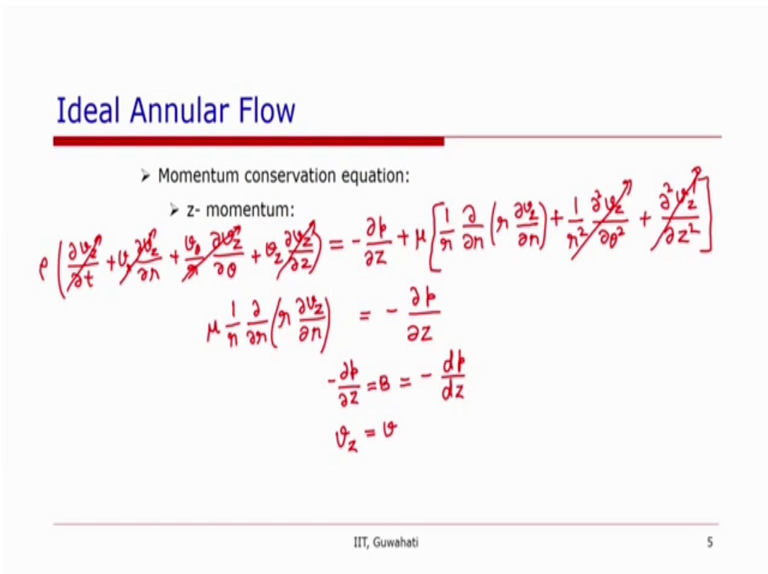

When we write the z momentum equation Rho del v z over del t plus v r, del v z over del r plus v theta over r, del v z over del theta plus v z del v z over del z is equal to minus del p over del z plus mu 1 over r, del over del r, del v z over del r plus 1 over r squared del 2 v z over del theta 2 plus del 2 v z over del z 2.

I have been very careful while writing these equations; I would request you to recheck these equations from a standard book. The book from which I have taken the equations is transport phenomena by Bird Stewart and light foot from the appendix B, you can see all those equations are in the chapters where the Navier Stokes equation have been derived. So, again del v z over del t, because it is at a steady state and this term will be 0, because v r is 0, this term will be 0 because v theta is equal to 0 and this term is 0, because del v z over del z is 0.

Now, this is 0 because gradient in theta direction are 0 and this is 0 because flow is fully developed. So, del v z over del z. It is minus del v by del z is equal to mu 1 over r, del over del r, del z over del r. So, we will, for clarity or for convenience, we will assume that minus del p by del z is equal to b. Note, that this can be written. Now as minus dp by dz, because it does not depend, the pressure depends only on z. So, it is equal to the partial derivative is equal to the total derivative and the v z, we will write as v only, now onwards because for our convenience. Ok?

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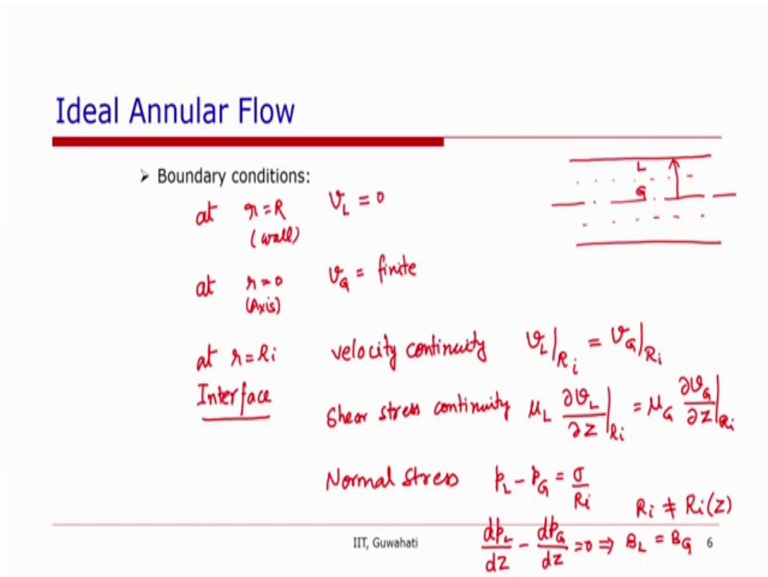

So, we can write this equation later on for both the phases. Let us look at the boundary condition. So, the configuration we look at again. So, this is our interface in the centre. So, at r is equal to capital R, which is general wall. We will have v of liquid, because this is liquid phase and this is gas phase. So, v L is equal to 0 at r is equal to 0, which is at the axes. We will have v gas to be finite. So, if you use this condition or then, we will also have an r is equal to R, i which is at the interface..

There are two boundary conditions, the velocity continuity. So, v liquid at R, i is equal to v gas at R i and we will also have shear stress continuity, mu l, del v L over del z and R i is equal to mu g, del v G over del z R i. We will also have a boundary condition for normal stress and that will be a jump in pressure. So, we will have p l minus p G is equal to sigma, kappa and kappa is 1 over R i in this case. So, we can have sigma over R i, but R i is constant throughout. So, we can say that d p L over d z minus d p G over d v z is equal to 0, so; that means, because R i is not a function of z.

So, that we can say the gradients in the both the phases are same. So, we can have B L is equal to B G from here.

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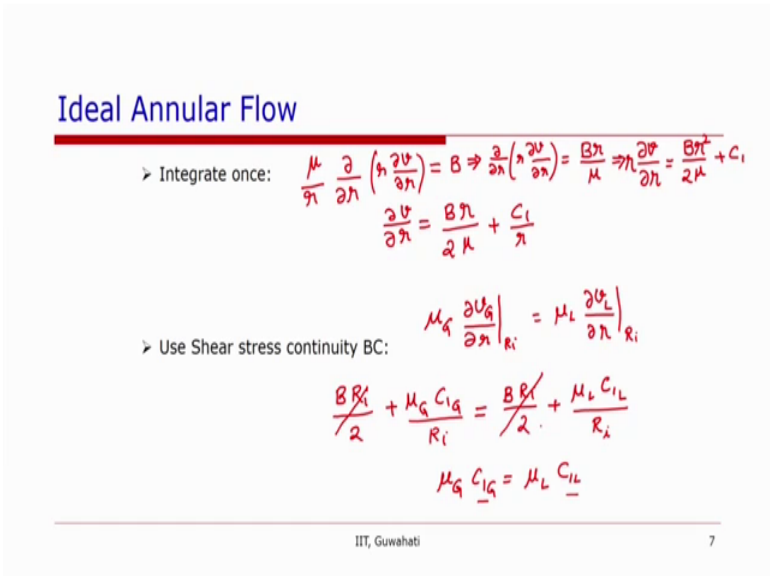

Now, the equation, if we look at this equation again, mu over m r del over del r are mu over r, del over del r, del v over del r is equal to minus del p by del z or d p by d z, which we have named as B now. So, we will have, when we integrate this equation, we will get del r, r del v r over del r is equal to B R over mu which will give us Del, sorry, this is not v r, del v over del r is equal to this, will be r and this will be equal to B R square over 4 mu.

So, we can write del v over del is g plus. A constant of integration, let us say, this at C 1. So, del v over del r is equal to B R over 4 mu plus C 1 by r and now, we can write the same equation for gas and liquid phases, using the shear stress continuity boundary condition at the interface, we will have mu gas del v g over del at R, i is equal to mu l, del v l over del r at R i. So, if we substitute here, we will have four constants, basically when we integrate this fellow.

So, we will have this term, written for two, sorry, term T is constants as C 1 l and C 2 l for two different things. So, when we multiply by Mu, we can have B R i over 4 and mu will cancel out plus mu G C 1 G divided by R i, similarly we can write B R i by 4 plus mu L C 1 L by R i. So, B R i by 4 B R i by 4 will cancel out and what we will have as the boundary condition, that mu G C 1 G is equal to or from this boundary condition, we will get mu G C 1 G is equal to mu L C 1 L. Ok?

So, we have a relationship between the two constants for two phases, C 1 G C L. Ok? Now, we can integrate this equation.

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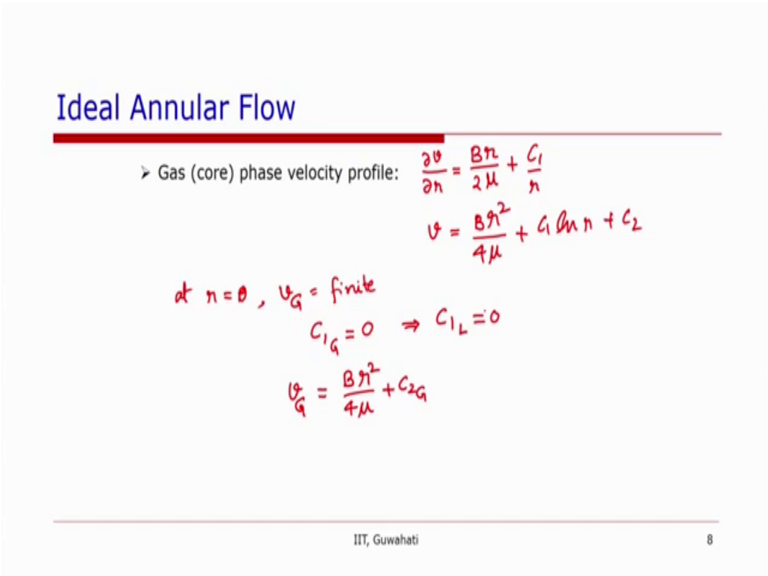

So, we will have del v over del r is equal to B r over 4 mu plus C 1 by r, when we integrate, we get v is equal to, sorry, this is, this will be B r by 2 Mu. So, when we integrate this further, we will have B r squared over 4 mu plus C 1 l n r plus C 2 and now, we can write the same equation for gas phase and liquid phase.

So, if we use the boundary condition for gas phase at the axis at r is equal to 0 v G is finite or defined. So, then because l n 0 is not defined for C 1 G has to be 0 so; that means, we have B G is equal to B r square by 4 mu plus C 2 G. Ok fine! So, now, we need to find out the constant C 2 G. Ok?

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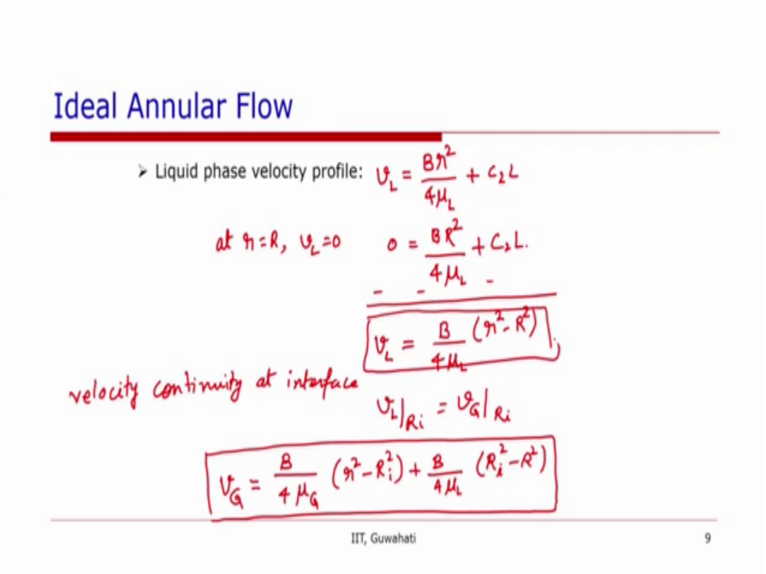

For liquid phase again, we can write the same velocity profile. So, if C 1 G is equal to 0, this will make C 1 L is equal to 0 too. So, we will have v L is equal to B r square over 4 mu L plus C 2 L and at r is equal to capital R, that is at the wall, we have a v L is equal to 0. So, the substitute 0 is equal to B capital R square by 4 mu L plus C 2 L.

So, by subtracting these two equations, we can get the velocity profile or we can get rid of C 2 L v L is equal to B over 4 mu L small r square minus capital R square. Now, to obtain v G, we will apply velocity continuity at the interface; that means, v L and R i is equal to v G at R i and we substitute that, we will get the liquid velocity, we have already got.

So, the gas velocity, we will get after substitution and manipulating the algebra, we will get v G is equal to B over 4 mu gas small r square minus capital minus capital R i square plus B by 4 mu L into R i square minus r square. So, I request you to go through this step and do the required algebra. So, we have got the velocity profile in the gas phase and velocity profile in the liquid phase for a smooth interface annular flow.

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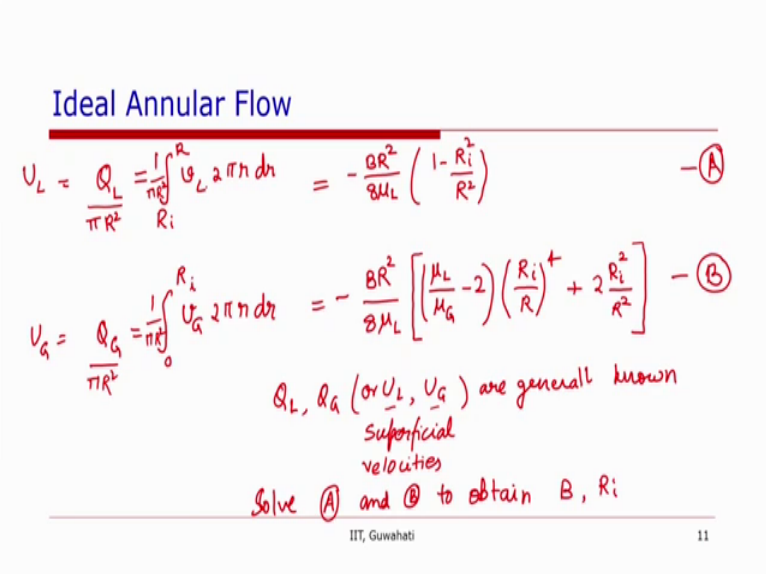

Now, once we have got that, then, we can get the by integration, we can get the Q L and Q G. So, Q L will be integrating v G 2 phi r d r from R i to R Q G will be equal to 0 to R i, sorry, this is v L and v Q G, will be equal to v G 2 phi r t are integrated from 0 2 R i. Now, from this we can also find out the superficial velocities. So, when we do the substitution and the superficial velocity of the liquid phase will be Q L over Pi r squared..

So, 1 over Pi r square and U G is equal to Q G over Pi r square. So, 1 over Pi r square and by substituting the values and do the necessary integration, we will get U G is equal to minus B r square over 8 mu L into mu L over mu G minus 2 into R i by R raised to the power 4 plus 2 R i square over R squared.

Similarly, U L can be written as minus B R the square over 8, mu L into 1 minus R i squared over R square. So, now, in general for any annular flow or for any experimental conditions, for in any experiment, what we will know is the gas and liquid flow rates, if we want to obtain the velocity profile; the problem for us for the velocity profiles are that, they are functions of B, which is pressure gradient and R i which is interfacial radius. Now, we do not know both of these. So generally, what we have Q L and Q G or U L U G, these are the superficial velocities.

So, you can just note that U L U G are superficial velocity are generally known for a flow. So, by knowing this we have two equations and we can solve an equation, let us say, A and equation B to obtain B, which is pressure gradient and R i.

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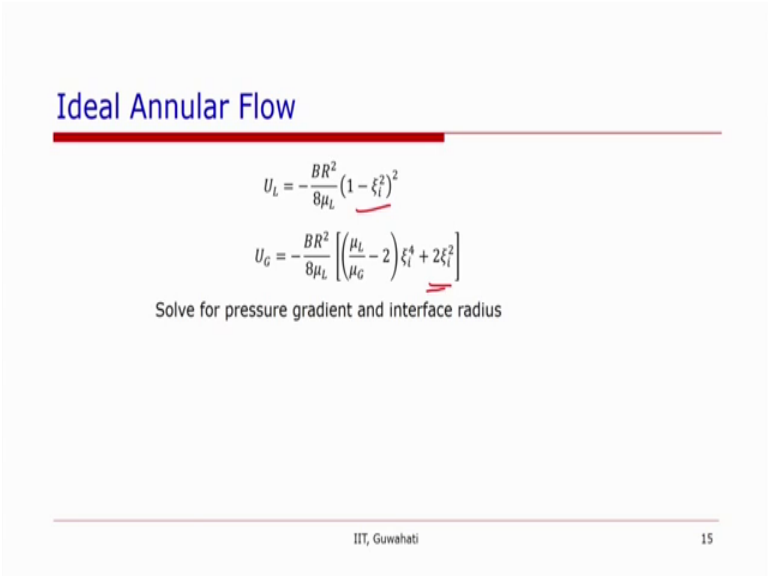

So, we can look at this in the ideal annular flow, these are the U L and U G, the two phase flow rates and the superficial velocities, which we have solved, just solved for and now, when we solve for pressure gradient and interfacial radius.

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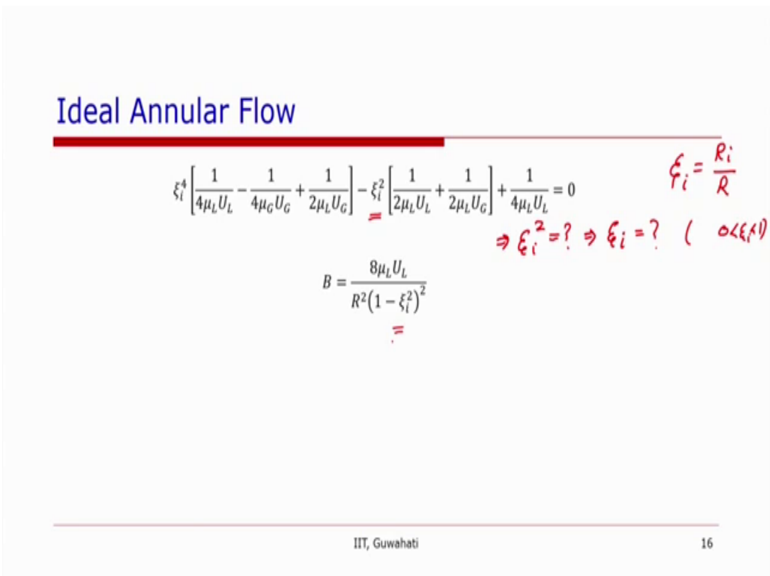

So, we can rearrange these two equations to in such a manner that it becomes a quadratic equation, in sigma i R square, sigma i square where sigma i is equal to R i by capital R. Ok?

Sorry, this is not sigma, this is phi and so, phi I is equal to R i by R. So, once we solve that, we can obtain what is phi i square and from that, we can obtain in the roots of it. Generally, with my experience, I have observed that one root is always greater than 1. So, we know that is unphysical. So, we will have to choose phi i with the condition that phi i is between 0 and 1. So, we can get what is phi i and when we substitute back in the pressure gradient, we will get the pressure gradient.

So, basically by solving what we have done today, is solve the ideal annular flow equation starting with a basic Navier Stokes equation, we have neglected the terms the relevant terms in the R theta and z system of the equations and in the continuity equation and finally, after integrating we got the velocity profile, which had 4 constants; two constants for the liquid phase and two constants for the gas phase..

And then, we had four boundary conditions, one is at the axis, that the velocity gradient should be finite and L N, it should be defined at the axes. Then, we had two boundary conditions at the interface, the interface continuity and shear stress continuity and one boundary condition at the wall that the liquid velocity at the wall is 0. So, by solving that we could obtain the velocity profiles in terms of the pressure gradient and the interfacial radius. Once, we have done that, then we integrate it and obtained the flow rates of the two phases or from the flow rates, we can also obtain the superficial velocities and from solve, by solving these two equations, we could get the pressure gradient and R i or interfacial radius..

So that we can get the velocity profile for any given gas and liquid phase flow rates. Now, this ideal annular flow, there is lot of literature on the stability analysis non - linear and linear stability analysis of the annular flow, this is subject to a different interfacial instabilities.

So, if you remember the capillary instability, which we discussed in the previous class. So, in the capillary instability A z disintegrates into smaller droplets. So, following that, the annular flow is at lower velocities, it becomes Slug Flow or Taylor Bubble Flow and there are generally, what we see in practice that, there are always waves present at the interface and that is because of probably at the Kelvin Helmholtz instability, which is that, when there are two parallel liquids flowing. There are waves generated or there are small perturbations, they grow up to become wave to make the interface maybe ok, Thank you.