Multiphase Microfluidics
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Lecture – 11
Liquid Flow Regime & Plug Flow

Hello. So, in today's lecture we will be looking at the flow regimes that occur in liquid-liquid flow in micro channels. And in addition we will also look at the; pressure drop in the plug flow regime or in the plug flow. So, as we have seen for gas-liquid flow there are different flow regimes that occur in gas-liquid flow in micro gm channels.

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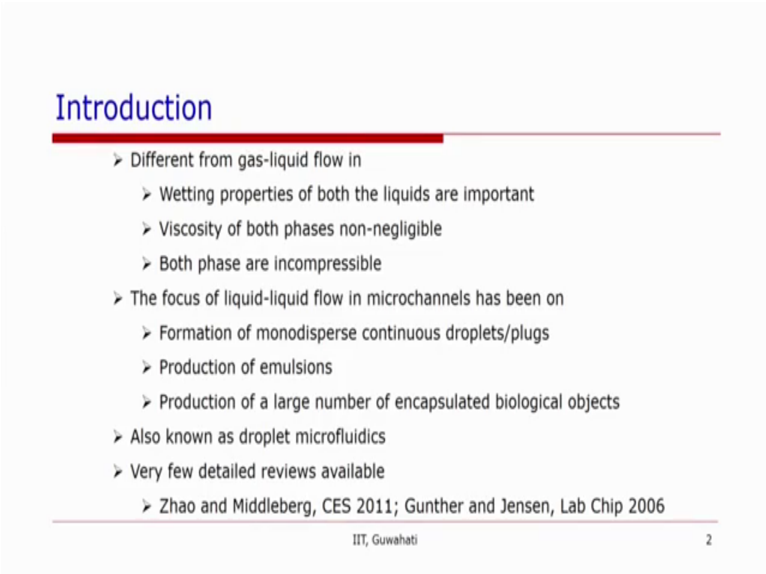

Primarily we have listed as bubble flow Taylor flow or bubbly flow Taylor flow slug annular flow annular flow annular flow and channel flow. Now the objective of the liquid-liquid flow in micro channels is quite different in most of the cases or the application of liquid-liquid flow in micro channels is very different when compared with gas-liquid flow regimes. The primary objective in most of the applications is to generate mono disperse droplets in micro channels or to carry out liquid-liquid reactions in very small size droplets. So, the objective here is to create droplet or plug flow.

So, in most of the cases what has happened that the flow regimes that people have analyzed or people have looked at is limited by the velocity or the flow rates in which one gets a droplet or one gets the plug flow. So, most of the cases what one will have is the occurrence of droplet and plug flow. Now when we compare gas-liquid flow and liquid-liquid flow, then there are few differences that in liquid-liquid flow. We have wetting properties of both the phases. So, there are more chances of formation of plugs; that means, both the phases.

Might be in contact with the channel walls then in the gas-liquid flow the viscosity of the gas phase was negligibly for example; however, in liquid-liquid flow this will not be the case as the viscosities of both the phases will be non negligible and comparable plus unlike gas which is a compressible fluid the both the phases in liquid-liquid flow are incompressible So, as I have said that there are different applications of liquid-liquid flow in micro channels formation of mono dispersed droplet formation of or production of emulsions. And the large number of encapsulated biological objects this is also known as droplet micro fluid people want to have reactions in small droplets which acts like a small slug reactor and can have efficient mixing, because of the re-circulations in the liquid plugs or liquid slugs.

So, they have number of application; in addition say for heat transfer the gas-liquid flow in Taylor flow regime has been used, why? Because the re-circulations internal re circulations in the liquid slug they enhance the mixing and there by heat transfer on Nusselt number; however, in gas-liquid flow the gas bubbles, because they have very low thermal mass very low mcp value the or rho cp value the cp might be comparable with that of the liquid, but the rho is about pre order.

Of magnitude for gases lower than that of liquid whereas, in liquid both the phases will have equivalent thermal mass. So, immediately when we use liquid-liquid flow for heat transfer applications the Nusselt number is bound to increase. So, that is another application that people have been looking into for liquid-liquid flow in micro channels. Now unlike gas-liquid flow, the where a number reviews and a number of papers are available on the flow regimes in micro channels this is not the case. So, so the; what to the best of my knowledge there are two reviews available one is by zhao and middleberg where they have reviewed two phase.

Micro fluidic flows and in that there is a; quite a bit of section devoted on flow regimes in liquid-liquid flow in micro channels. So, most of the discussion that we will have in this is based on that paper and the other paper is by Gunther and Jensen in lab and chip by the name multi phase micro fluidics from which the name of this course is inspired this. So, they have also looked at review of liquid-liquid flows, but very small and short and crisp review about liquid-liquid flow in micro channels ok.

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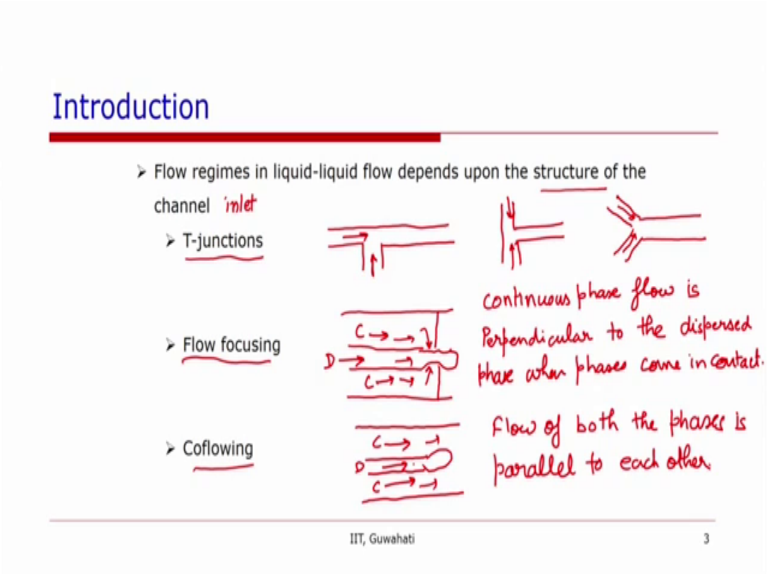

So, one of the things to understand is that liquid-liquid flow in small diameter channels or in micro channels is dependent on the structure of the channel or a structure of the channel inlet so.

Ah gas-liquid flows they are affected by the inlet condition, but not to the extent that is observed in liquid-liquid flow, they are very sensitive to the channel inlet conditions. So, there are number of different devises that have been used for liquid-liquid flow in micro channels three main are: T-junctions, Flow focusing devises and co flowing devices. So, T-junctions one can have T-junctions where the flow is happening like this there are two phases coming from two different streams or another example of T-junctions can be that one phase and that the two phases come head on and. So, the variation of T-junctions is also y junction were people have looked at the effect of the angle between the two streams on the flow regime.

And another is flow focusing devices. So, the difference here is that there is a the dispersed phase come through the center and the continues phase comes from here and before that there is a construction. So, the velocity that it has is normal to the velocity of the dispersed phase flow in the flow focusing devises co flowing devices on the other hand have a similar structure, but in this case the dispersed phase again comes from the center and the continuous phase comes from the sides.

But, the flow is parallel in this case. So, in here we can say the flow is perpendicular we can say that continues phase flow is perpendicular to the dispersed phase when phases come in contact. So, in this place the flow is parallel, but when they come into contact the flow in the flow focusing device the continuous phase flow become normal to the dispersed phase flow in the co flowing devises the flow is ok.

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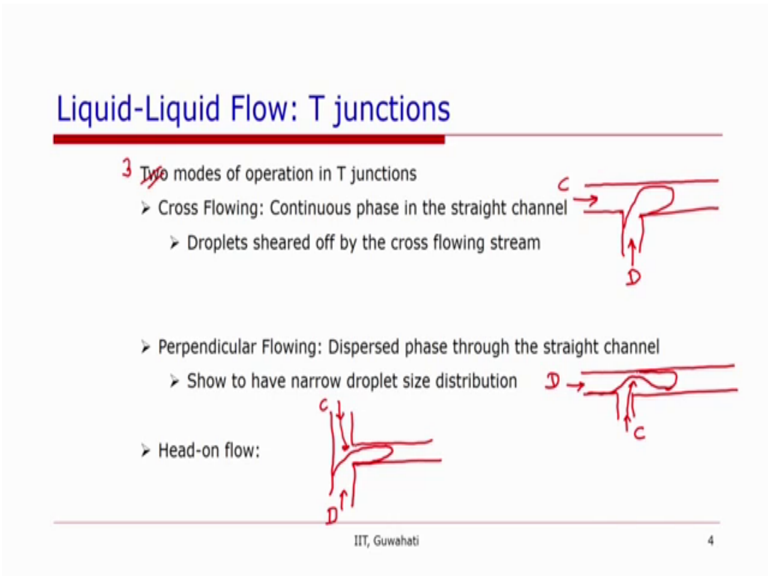

So, now let us look at the flow regimes in these three different configurations. So, in T junctions there are actually three modes of operation in T junctions the first one is what we call cross flowing. So, we have a T junction; in this case the cross flow. So, the continuous phase is coming in the straight channel and the dispersed phase comes from there. So, when the dispersed phase comes from here as the bubble generation or the droplet generation the droplet generation. In this case will be like this and the droplets will be moving from here. So, that is for cross flowing in the perpendicular flow where the channel configuration is same, but the.

Now, the continuous phase comes from the; the branch where as the dispersed phase comes from the straight channel. So, in this case the dispersed phase will be experiencing a force or because of the velocity of the continuous phase, and then it will it will break off into droplets the other configuration, where we have T junction, but in different configuration where we have a T junctions and the liquids.

The two liquids continuously in dispersed phase, they come from two different arms of the channel and then, they come head on depending on the inertias of the two fluids probably one of the fluid will enter earlier than the another and then So, this might be the case for other way round. So, we can have the dispersed phase breaking around the dispersed in such and the probably by changing the flow rates of the two phases and depending on the hydrophobicity of the two liquids one can have one can change the continuous and dispersed phase in such a configuration ok.

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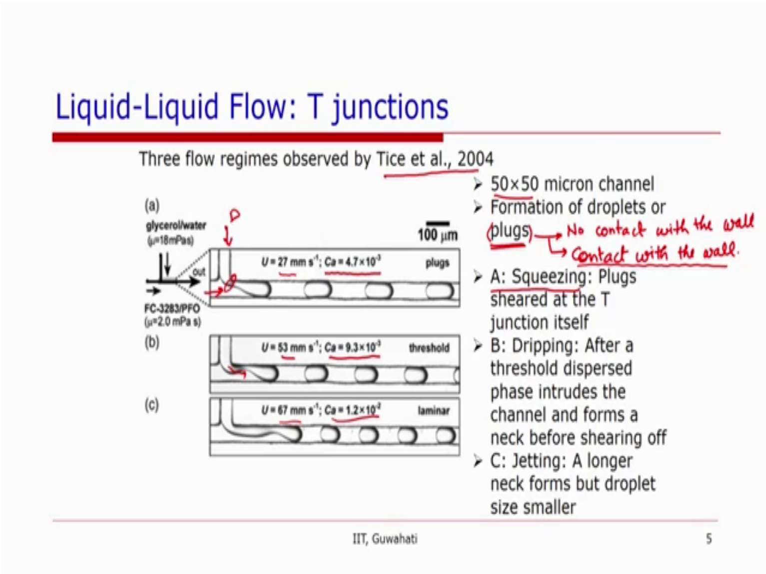

So, now some of the examples from the literature for T junctions. So, in this case what is happening this is from Tice et al., and there is cross flow, where the dispersed phase coming from here and the continuous phase straight through it is glycerol water is the dispersed phase and FC 3283 FO is the is the continuous phase and we can see three different flow regimes and they have named as and the distinction between the these flow regime is coming from the capillary number 3, different values of capillary number which have of course, will varied by changing the velocity of three phases the density and surface tension and the sorry the viscosity and surface tension for same So, the channel size is 50 by 50 micron square channel and one can absorb the formation of droplets or plugs or So, those plugs when they say the plug can be of two types where no.

Contact with the wall and we will in we will be focusing or we will be saying the plug more to the; flow regime where it is in contact with the wall. So, in this case we can say that there is there is a thin film that separates. Now these three different flow regimes; what is different here? That in such cases the break up in the first flow regime which they have named as squeezing the breakup happens just at the T junctions. In the second case which is known as dripping the breakup happens downstream of the T junctions not at the T junctions itself, and then in third case we see a jet of this liquid coming into and then longer neck forms in the droplets forms in we can see that the size of the droplet which reducing has been moved from there to here in the three cases.

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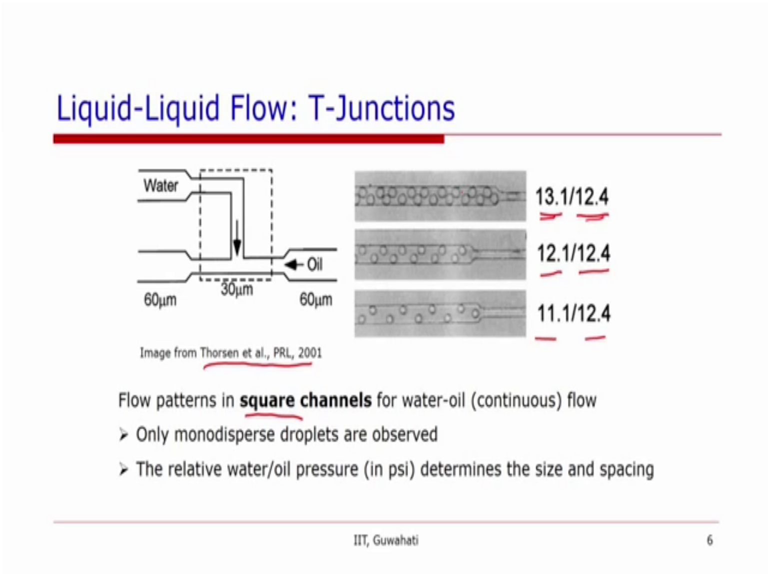

Then another work is in due to Thorson et al., and what they have looked at the flow regimes in a square channel of 60 by 60 and then again it is a cross flow type of configuration they have looked at in the square channels as well as circular channels. So, in the square channel for the; now what they have specified is the pressures for the two phases and these pressure these numbers are in psi. So, water pressure and oil pressure and one has the droplets forming in this case.

So, depending on the pressure one always get the droplets, but the size of the droplet the distance between the droplets. The distribution of the droplets changes with the pressures; in all the cases the oil pressure is same and the water pressure has been increased. So, as we see that water pressure g increase the droplets are becoming more and more close to each other ok.

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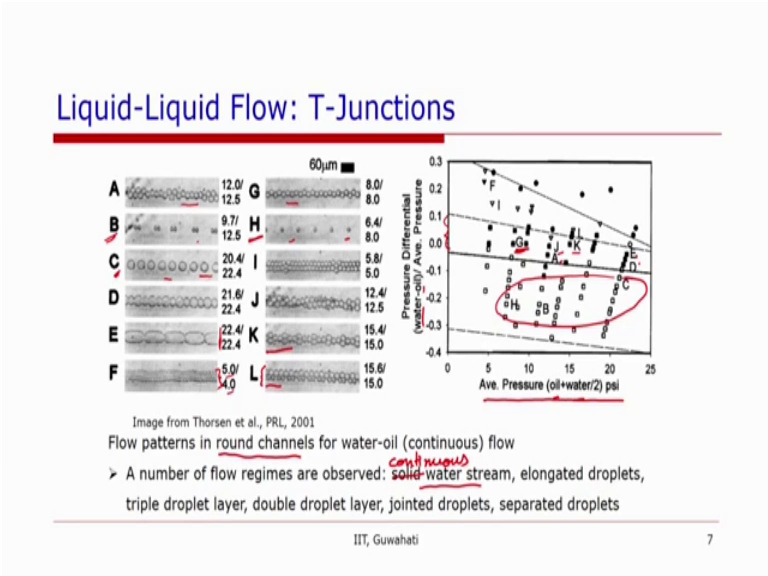

Now, the same configuration, but for a sound or circular channel for water oil continuous flow they have got a very different flow regimes in the circular channel.

And they have drawn a map based on the pressures of the two phases on the x axis, what is drawn is the average pressure in psi. So, oil pressure plus water pressure divided by two whereas, on the y axis the difference between water and oil pressure divided by the average pressure. So, basically the pressure difference here has been plotted. So, when oil pressure is very high when one see or when oil pressure is greater than the water pressure and once only the flow of oil no water when as it is increased then we see three different flow regimes here H, B and C.

So, if we identify this H, B and C and depending on the pressures and the average pressure values one can see that; there is formation of droplets and as the size of or as the pressure is increased. The average pressure is increased size of this droplet which seems to increase, the water pressure in all three cases is am difference of the two pressure which negative so; that means, water pressure is lower than the oil pressure.

Now, as the water pressure is increased and they are almost equal when we see two different configurations depending on the average pressure G, J, A, K, E and D. So, if we look at A and J where they have droplets and they are forming (Refer Time: 19:49) kind of a structure J and A and then K again and then you have sometimes two droplets and interesting a structure that two droplets horizontally followed by two droplets vertically to get that again a very kind of regular structure here and then E, where we have long droplets connected together and then E again the droplets of different size.

So, there are different structures very small change in the pressure can change the flow structure, but what we have essentially is the droplets 1 or 2 droplets and it can be long droplets higher pressure at higher pressure we observe; when they are both of them are equal and they are non droplets, but just a small decrease or slight tilt over water pressure and one see a spherical droplets. So, a number of patterns one can see here that solid water stream or solid streams should not solid, but a continuous stream what you should say.

And then elongated droplets triple droplet layer. So, we can see that higher water pressure where water pressure has increased now in this case we have I and L. So, here we have continuously two droplets with each other in the l configuration in the triple droplets in the F configuration. So, water pressure is higher in this case So, depending on the pressure of the two phases interesting patterns have been formed in this case ok.

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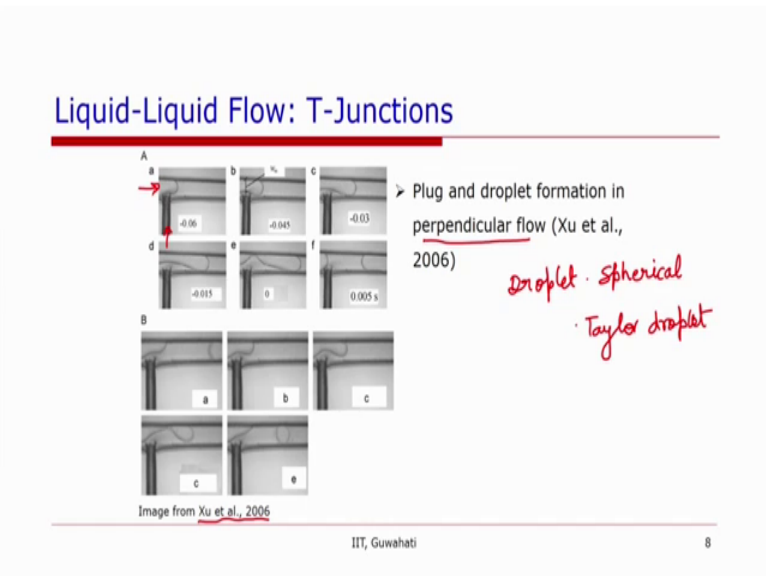

Now, this is an example for perpendicular flow. So, where the dispersed phase is coming from the straight channel and the and the continuous phase.

Is coming from the channel normal to it and then they have different snapshots the image has been taken from Xu et al., has been described in (Refer Time: 22:13) paper. So, one can see the two different configurations the droplet flow or in this case the circular droplet and their; the long droplet. So, we have droplet spherical as well as Taylor droplet which look like which can just say like Taylor bubble.

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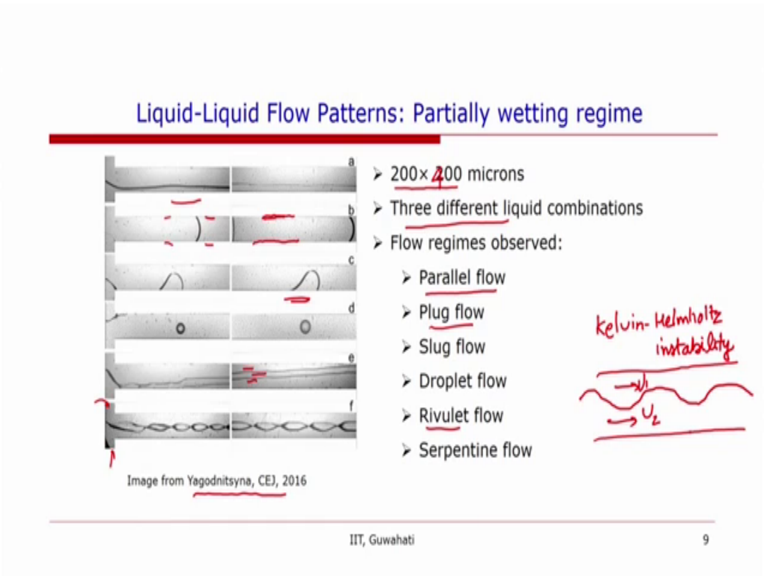

And. So, this is for recently by Yagodnitisyna in chemical engineering journal they have published

Ah their findings and then they have conducted the experiments in 200 by 200 micron channel and use three different liquid combinations kerosene water and different other oils. Now they report different flow regimes they have done over a range of flow rates and eh different liquid. So, they So, that they have observed parallel flow then plug flow. So, this plug flow you can see that there is no self surrounding the wall both phases are in contact with the water and now that is bound to happen when we go at smaller channel size, because at the smaller channel size what is going to happen the contact angle will play a major role?

And. So, the and the and the hydrophilicity and hydrophobicity of the channel will also play a role. So, what it can have a plug flow there they have termed the based flow regime and slug flow which is different from the notion that they have about slug flow, you also notice the configuration that they have is had on flow that two flow regimes comes from the two branches of T-junctions. And then they match at the centre this the channel size is 200 by 400 micron. And these two channels are 200 by 200 microns and they also get rivulet flow.

So, eh one do not get a continuous phase and where as one has a small stream of one phase and then the on the other side you have the other phases. So, this is from the wall. So, this is called the rivulet flow and then they have an interesting flow pattern which is called serpentine flow So, the configuration in 3 d; that they have described in that which the there are very structured or very regular continuous ways on the two phases which one might relate with Kelvin Helmholtz instability.

So, Kelvin Helmholtz instability in generally seen when there are two different fluid streams with velocities different velocities U 1 and U 2. Originally, it was observed for at large derived for the two fluid streams which are flowing parallely and the two steams are envied when the flow streams are envied there is no profile in the two and then miles looked at the effect of the velocity profiles on the instabilities. So, the instability causes the; the way from the interface which was originally i have which initially had a smooth interface and then it was perturbed ok.

So, this looks very much like the serpentine flow; what they describe? This might be caused by Kelvin Helmholtz instability its might be playing a role there

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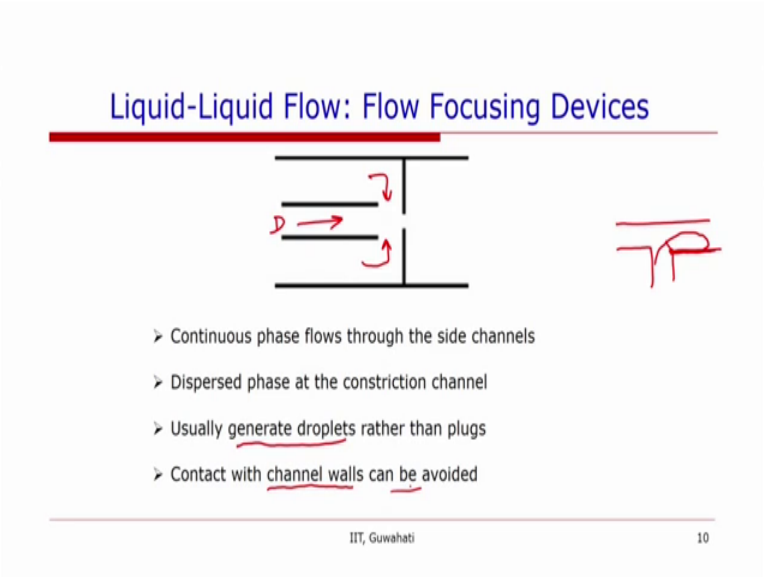

So, then coming to the next configuration the flow focusing device in this case the disperse phase as we said the disperse phase come from the center and the continuous phase comes from the two sides of the channel.

And in this case what is observed that? It generally generates droplets rather than plugs. So, in the T-junctions we generally see that one of the phases for example, in this case the cross through one of the phases in contact with the wall. So, the wall contact angle is bound to play a role in the generation of the droplets or plugs whereas, in this case the separation happens either here or afterwards. So, this is the contact with the channel walls can be avoided in this case and the.

Co flow devices also.

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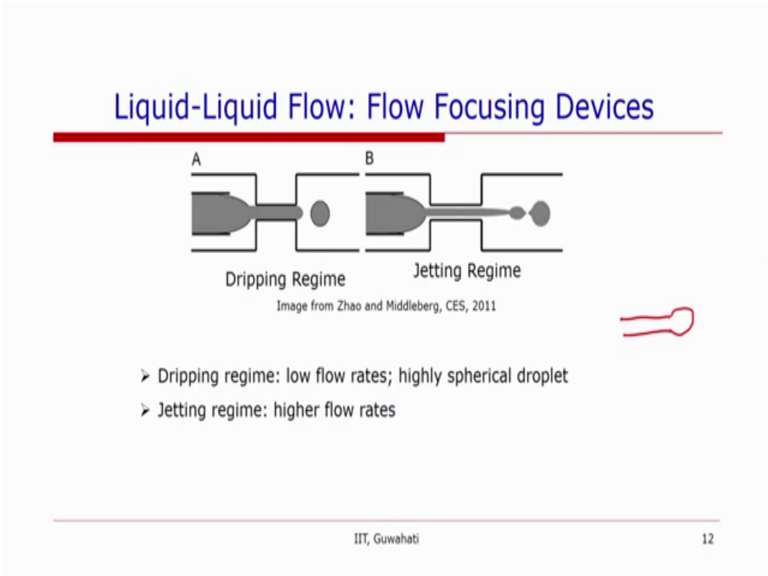

So, one conserve two flow regimes have been observed, in this one is called dripping flow regime which is observed at low flow rate. Another one is called jetting regime which is observed at higher flow rate.

So, in the dripping flow regime one observes the; at low flow rates or at the low velocities the droplets of uniform size mono disperse droplets are being generated. When the flow rate becomes higher then there are droplets of different sizes inertia has become higher and inertia might play a role when these droplets break off from the jet So, in the first case of one can get regular mono disperse droplets which are spherical in say

Now, this kind of a structure where a continuous jet is being broken into droplets as we have seen a.

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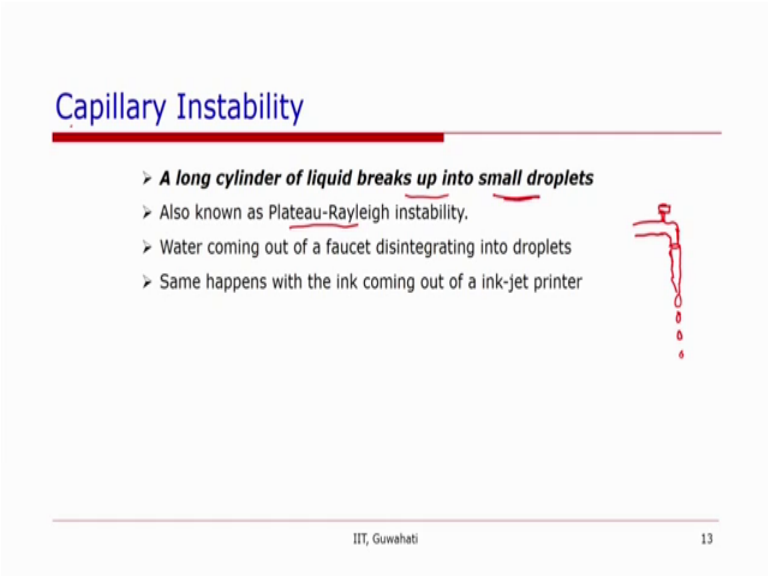

We can a related with the say for example, we have all of us have taps are in our kitchens in our bathrooms and then we see that there is faucet coming out of the tap, and then it disintegrates into a smaller droplets and that generally reactivated what we call capillary instability or Plateau-Rayleigh lateau instability. So, in this case what the instability says that a long cylinder a static long cylinder of liquid it breaks up into small droplets and then the lambda at which this breaks up is equal to the 2 pi r of the of the jet.

So, the same instability capillary instability might be playing a role in this regime, but it will also have the effect of viscosity effect of inertia playing a role on this disintegration the similar phenomena also happens in an ink jet printer; where one need to have a droplets generating of course, there are or active actuators also which control the droplets generation in the ink jet printers ok.

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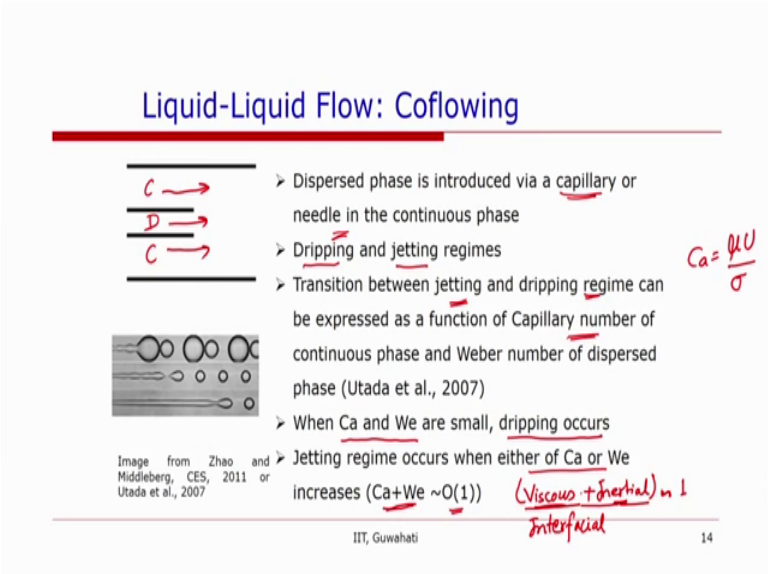

Now, next is coflowing device.

So, as we discussed earlier in the coflowing device the flow happens in the same direction both the continuous and the disperse phases and So, in this case the disperse phase is introduced by a capillary or by a needle in the continuous phase and like flow focusing devices one observed dripping and jetting regimes. In this case also and there are different one can see that sometimes one can droplets of two different sizes at the droplets are of mono dispersed droplets. So, this transition between jetting and dripping regime they have attributed to the capillary number which is basically inter play of mu U over sigma. So, the discussed forces and the intra phased forces.

So, in all the cases in fact, the T-junctions coflowing and flow focusing devices and any other focusing devices the or any other mixing devices for liquid-liquid flow gas-liquid flow the capillary number is the measure factor the interplay of surface tension or the interfacial and the viscous forces in the age and important factor. So, they have suggested then when capillary and Weber number are small.

So, when velocity is small the dripping occurs as we have seen for the flow focusing devices and then jetting occurs, when either of that increases and then they have given a criteria when capillary plus Weber number is of the order of one then one observes the, then when observes the jetting regime say if we look at this the capillary number is the ratio of viscous effects divided by interfacial effects and Weber number is ratio of inertial effects and interfacial effects. So, when this becomes order of one.

So, that what they suggest that when the viscous forces and inertial forces they are equivalent to the interfacial forces than the jetting regime takes over in the smaller regime one would have or at the lower velocities the inertia will be negligible. So, and that interfacial forces will be dominating then over the viscosity forces.

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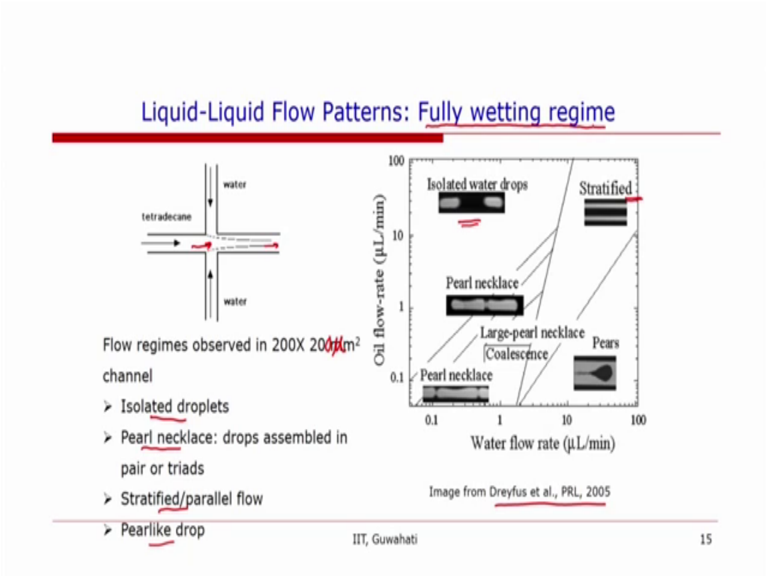

Then So, we have what we have looked at is the different flow patterns that may occur or that has been observed in the literature for flow flux (Refer Time: 33:38) or coflowing devices or T-junctions. Now there is another work by Dreyfus et al., where they have used a flow focusing device a configuration like this the tetradecane and water tetradecane is the disperse phase which is coming from the center.

And water from the two sides and it is in 200 by 200 this is micron 200 by 200 micron channel. And they observed different flow regimes isolated droplets pearl necklaces stratified or parallel flow and pearlike drop. So, we have isolated droplet seen here stratified flow which is as there the what is termed as parallel flow that one phase is coming at the centre and the other two phase is on the sides of it and then with they have pearl necklace type that; there are two droplets which are connected by smaller set share then they also have pear shape droplet different interesting shapes and this has been done when the channel is wetted fully wetted.

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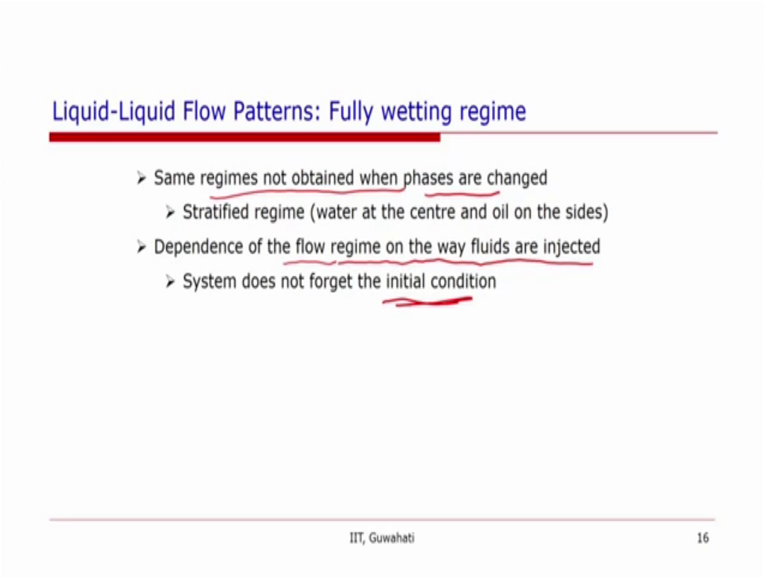

Now, the other case when they have had a that they observed that if the phases are interchanged, then the same regimes are not observed and then they say that it will depend then the system of course, this we have already said that they said that the from the flow regimes it depends on the way fluids are injected. So, the system does not forget its initial condition. So, it is difficult for the system to forget its initial condition probably because the viscosity of the fluid is high.

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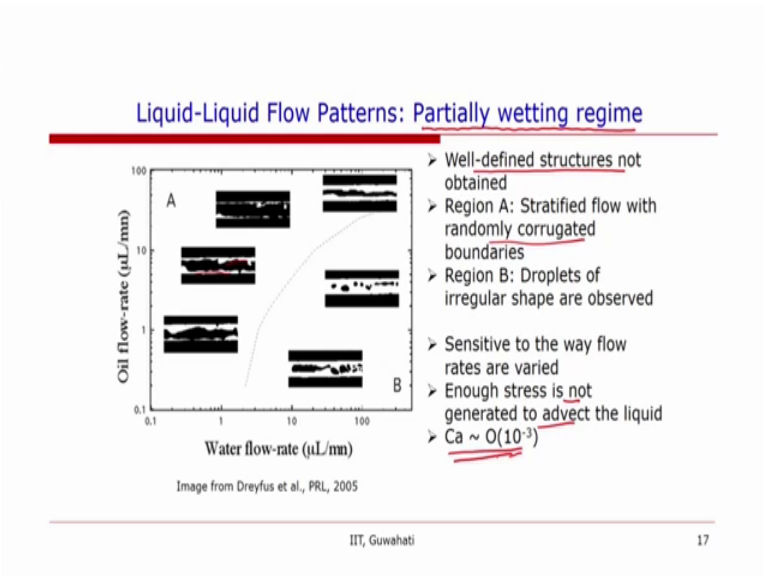

So, what they have seen in the previous case that was for fully wetting regime. So, it was hydrophilic. Now it is partially wetting. So, hydrophobic channels and then they see that they have observed of that well defined structure as was observed in the previous case are not observed they have observed some stratified flow, but not smooth one some corrugations are there on the interface and then they have droplets of irregular shapes, as one can see here they are observed. So, basically if it sensitive to the way flow rates are varied and enough stress, what they have suggest that enough stress is not generated.

So, that the fluid advects the capillary number in these cases was of the order of 10 to the power minus 3. So, a; what they have I mean, what the objective of this? that the hydrophobicity of the channel can play an important role on determining what flow regimes are being generated everything has being the same

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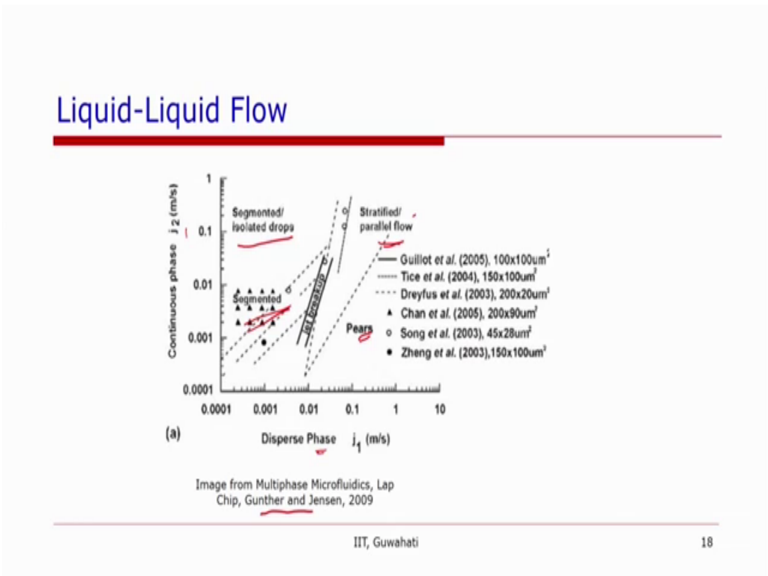

So, Gunther and Jensen in their work they have put together the flow regimes from a different researches in different channel sizes. And they have observed that the segmented isolated drops or the segmented which is Taylor kind of flow or stratified or parallel flow and pears like safe F, G have been observed and this flow regime if they have drawn in for disperse phase and continuous phase as the coordinates of the graph.

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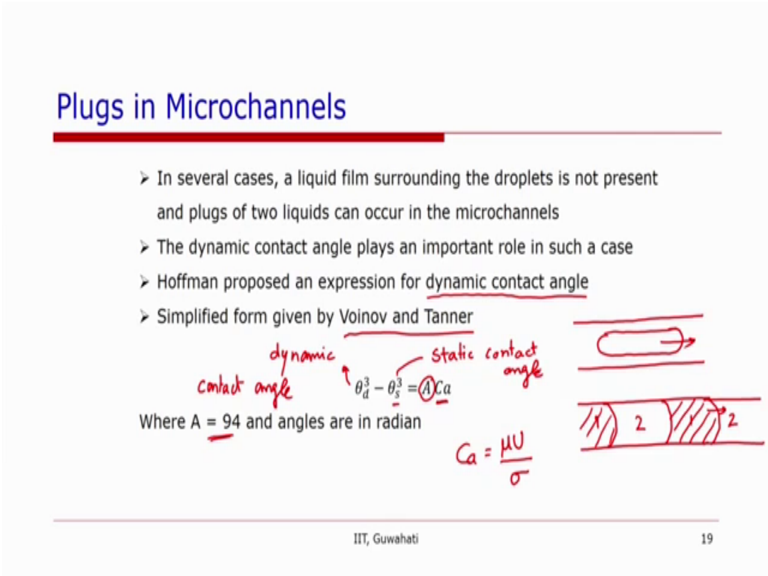

Now, the next topic in this.

So, as we have just seen that depending on the hydrophobicity hydrophilicity of the channel one can have that both the phases are in contact with the channel wall. So, our general notion of slug flow is that there is a long droplet like, Taylor bubble shape they are moving in the channel, but at lower channel diameters one may observe or one as on has observed some of the researches may observed that, there are plugs of different phases.

So, you can have phase one and then another plug of phase 2 and then plug of phase three sorry plug of again phase 1 and then plug of phase 2. So, we can color it differently to distinguish the two phases and in such a case the contract angle phase going to play a in an important role and, because the interface is moving. So, the contact angle is not going to be static, but it will be a dynamic contact angle.

So, Hoffman proposed an expression for dynamic contact angle, but which was simplified by Voinov and Tanner; which says that the dynamic contact angle can be given as a function of static contact angle and capillary number and a constant a which has a value of 94, if the angles are in radian. So, C a we already know is mu U over sigma and theta d is dynamic contact angle and theta s is static contact angle.

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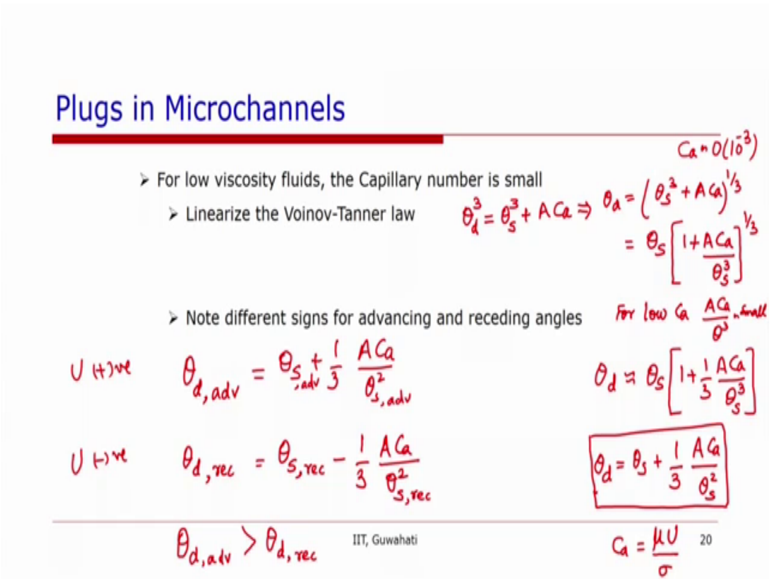

Now, this law which is theta d cube minus theta s cube is equal to a C a, if Ca is small which it generally the case of the order of 10 to the power minus 3; especially for low viscosity fluids then we can linearize it.

So, one we can write is that theta d cube is equal to theta s cube plus a C a or that can be written as theta d is equal to theta s cube plus a C a raise to the power 1 by 3; then we can write this as theta s into 1 plus a C a over theta s cube raise to the power 1 by 3. Now we can approximate for low C a we can suggest that a C a over theta s cube will be small. So, we can write this as theta s into 1 plus 1 by 3 a C a over theta s cube or we can have theta d is equal to theta s plus 1 by 3 a C a over theta s squared. So, that is the linearized partitions of now one need to take into account the sign of the velocity and as we have seen or as we know that C a is equal to mu u over sigma.

So, for advancing contact angle one will have a theta d for the advancing case and U is positive. So, one will have theta s plus 1 by 3 a C a over theta s square we can put this in terms of theta s advancing. So, this is static advancing contact angle and this is dynamic advancing contact angle. Similarly, when U is negative for the receiving case one will have theta r or theta d receding is equal to theta as receding minus 1 by 3 a C a over theta s receding squared. So, immediately one can see that theta d advancing is greater than theta d receding from this expression is of course, theta d as advancing is greater than theta s receiving sufficiently that is ok.

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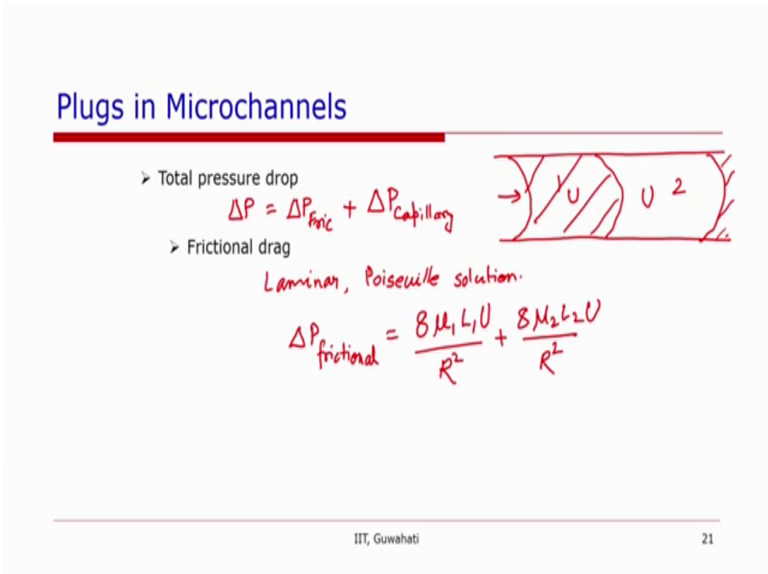

So, based on this these correlations now, we will try to look at the pressure drop when we have plugs in Microchannels.

So, if we have a picture of the plug that we just described that was considered one in itself we have the plugs of two phases 1 and 2. So, this total pressure drop will be the sum of frictional pressure drop plus the pressure drop caused by the capillary force or the capillary action near the wall or by the contact angles. So, the frictional pressure drop is if we assume the flow to the laminar and use Poiseuille solution then we have delta P friction is equal to 8 mu 1 L 1 by R square plus 8 mu 2 L 2 by R square. So, then you will also have U where U is the velocity of the plug and the both the plugs will have to have the same velocity that steady state.

So, that is the frictional pressure drop now let us turn our attention to the capillary pressure drop.

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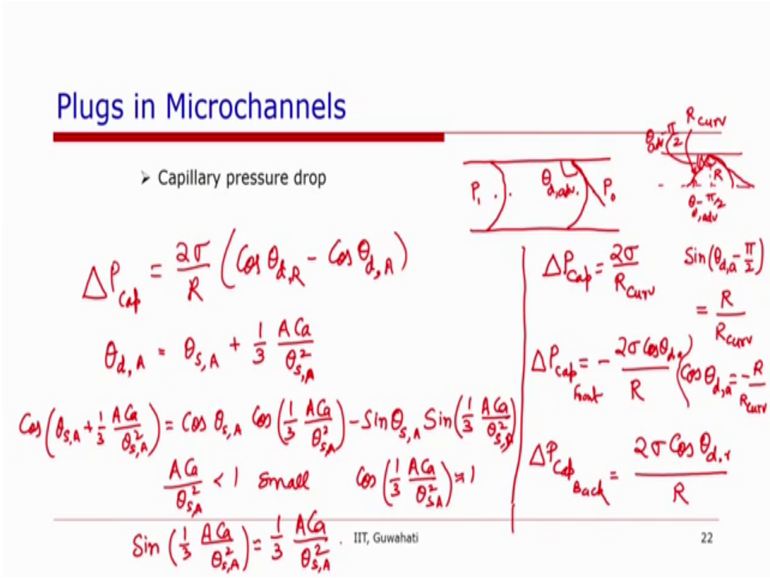

So, for the capillary pressure drop if we analyze one plug then we will have the interfacial pressure drop say that the two junctions P 0 and P 0. So, what we will have? The pressure difference and the front is equal to 2 sigma over R curvature and if we say that this is theta advancing contact angle.

Then we can draw a; geometrical configuration this is the radius of the channel the angle being theta and the radius of curvature is this and the term is R curvature and this angle is 90 degree. So, the angle here is theta advancing minus 5 by 2 and this angle will then be theta d advancing minus 5 by 2. So, what we can write is that sign theta d A minus 5 by 2 is equal to R over R curvature. So, that we give us minus So, we will have this as cost theta d advancing is equal to minus R by R curvature. So, we can substitute that from here that delta P cap is equal to minus 2 sigma cos theta d advancing divided by R similarly. So, that is at the front and the similar expression for we written for can be written for the back.

So, one can have that delta p cap and back of the plug is equal to 2 sigma cos theta d receding divided by R and that will be the minus sign will be because of the concavities. So, we can have the total pressure drop delta p at cap, because of the capillary force is the sum of the tool. So, we will have 2 sigma over R cos theta dynamic contact angle receding minus cos theta and may contact angle which is for advancing So, now we can recall what we had for theta dynamic for advancing we had an expression that theta static advancing plus 1 by 3 a C a over S A square as we take cos of this say P substitute this here then we will have cos theta S A plus 1 by 3 a C a over theta S A square. So, that will be cos a plus b. So, cos a cos b minus sin a sin b.

So, cos theta S A cos 1 by 3 a C a over theta S A squared minus sign theta S A into sin of 1 by 3 a C a over theta S A square. So, if we have assume in when while (Refer Time: 52:10) assume this that a C a over theta S A squared is small. So, we can say that cos 1 by 3 a C a over theta S A squared is about 1 cos 0. We know that it is 1 and at low value sin theta S equal to theta. So, we can say that sin of 1 by 3 a C a over theta S A square that is equal to about 1 by 3 a C a over theta S A square. So, we can write that as theta from here.

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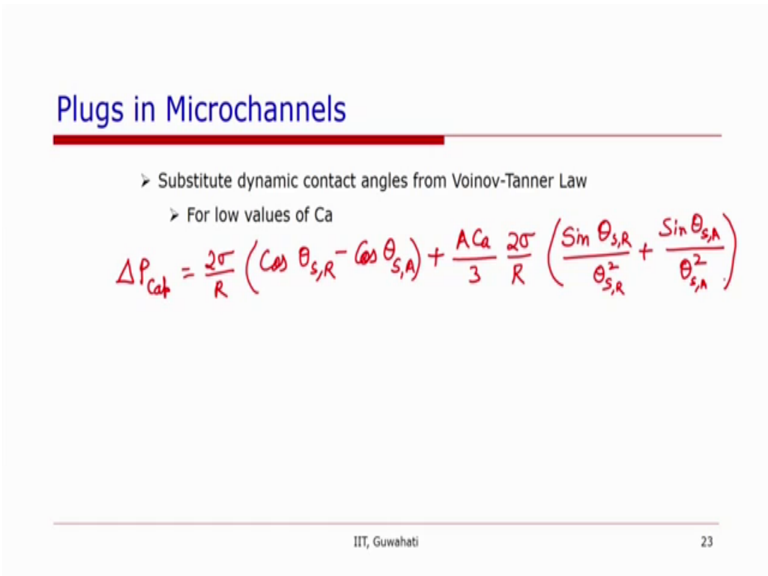

We can write that delta P capillary is equal to 2 sigma over R the same substitution can be done for the receiving angle with the minus sign there and we will get 2 sigma over R cos theta S R.

So, static angle receding minus cos theta static angle advancing plus a C a by 3, 2 sigma over R into sin theta S R divided by theta S R square plus sin theta S A divided by theta S A square.

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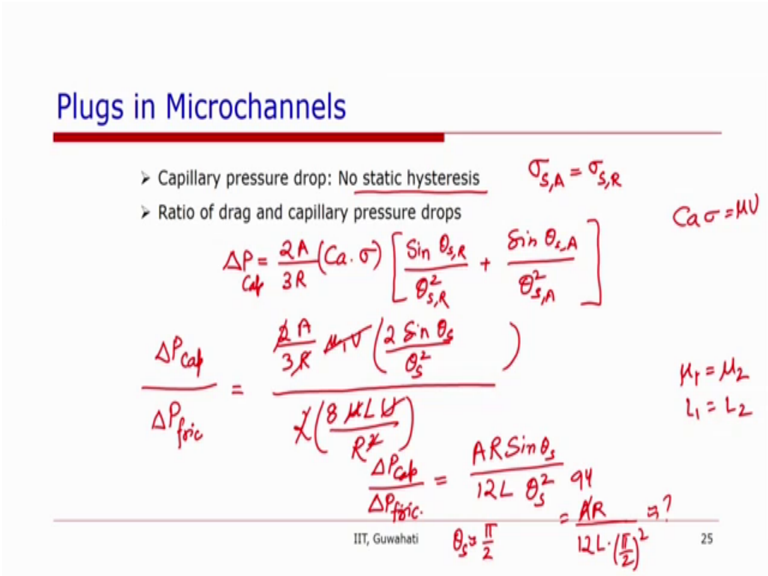

And now what we can have is if there is static no static hysteresis. So, ah; that means, that theta S A is equal to theta S R. So, the first term in this will go away this term may be 0. So, what we will have as delta P is equal to 2 a over 3 R C a into sigma into sin theta S R over theta S R squared plus sin theta S A divided by theta S A square. Now one can have this the ratio of just to compare the two force phase of the delta P capillary divided by delta P the frictional drag. So, this C a into sigma you can see that C as in sigma is mu into U. So, and we assume that mu 1 is equal to mu 2 or simplicity and L 1 is equal to L 2. So, we can write this as the frictional pressure drop and will be eight mu L U by R square multiplied by 2.

And this will be 2 a over 3 R mu 1 U into sin theta S R by theta S R square plus sin theta S A divided by theta S A square and if theta S R and S R a equal. So, one will have this substituted by 2 sin theta S divided by theta S square. So, mu 1 and mu 2 are same. So, we will have this mu to be cancelled out 1 of R will cancel out and we will have this as 8 into 2. So, this two into two will canceled out and we will have 4 into 3, 12 L theta S square at top we will have a R into sin theta S. So, that will beta ratio of delta P capillary over delta P friction. So, one can see from here that if L is smaller than delta P capillary can be significant.

So, if theta S is equal to say pi by 2, then this number will be a R by 12 L pi by 2 square and then one can where a is 94. So, one can estimate what this number is and one will have that these forces are quite equal and the ratio of L by R. Similarly one can have an magnitude when considering when there is static hysteresis. So, the idea here is that one that how to analyze the effect of the wall contact angles by the simple of wine of tanner model and then appreciate the fact that at low are the capillary force might be important when compared with the frictional forces and that is what we see in smaller channels ok.

So, in summary in todays lecture what we have looked at the different flow regimes that has been observed in the literature in different in led configurations T-junctions configuration. Flow focusing devices coflowing devices people have observed the parallel flow mainly the two flow regimes that has been observed at droplet flow and the plug flow or Taylor droplet flow and then people have also observed the parallel flow or what they call is stratified flow and then pearled flow pearl necklace flow and the. So, we charge generally the droplets which are interconnected with each other with bridges around them. So, different interesting observation of are there then we have looked at the plug flow in micro channels where we analyze the pressure drop when there is no film that separates the droplets from the channel wall and the pressure drop.

In such cases and appreciating the effect of capillary pressure drop. Now it has been observed in some cases that at low channel diameters people also get similar kind of plug flow in gas-liquid flow also. So, the same can be same analysis can be applied for gas-liquid flow plus when one have say triangular channel or rectangular channel, then the Taylor droplet may not be it may not have a film everywhere there might be a contact angle or there might be a contact with the wall of the channel at certain places especially at the centre of the channel. So, in such cases also one need to take into account capillary forces when analyzing the pressure droplets and channels. So, that is all from this lecture.

Thank you.