Cardiovascular Fluid Mechanics
Dr. Raghvendra Gupta
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

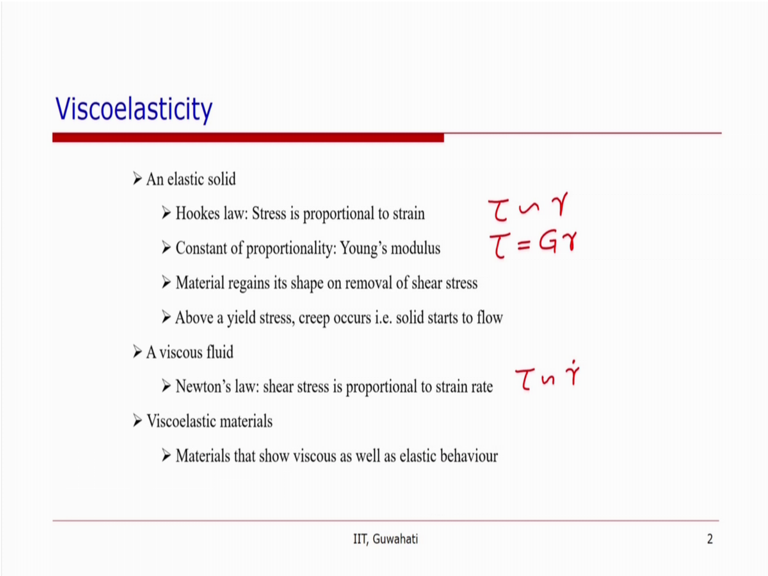
Lecture – 08

Viscoelasticity

So, until now we have looked at the time dependent behaviour time, independent behaviour of blood or the time independent rheological behaviour of the blood. And we learned that under certain conditions, the blood behaves as a Newtonian fluid, but for other conditions the blood behaves as a non-Newtonian fluid. But all those measurements, all those analysis have been done under steady state conditions or when the shear flow was at a steady state.

Now, if the shear is time dependent that means if the flow is unsteady which is the case in the cardiovascular system? As we know that in the cardiovascular system, the flow is pulsatile and it is unsteady, so it changes with time. So, that means to understand the rheological behaviour of blood we need to consider the time dependence of time dependent rheological behaviour of blood. So, as we have discussed while understanding the morphology of blood that the red blood cells, white blood cells, they have viscoelastic behaviour. So, we need to so as a result blood also might show some viscoelastic behaviour. So, in this lecture, we will look at fundamentals of viscoelastic fluids, what is viscoelasticity, and then briefly touch upon the viscoelastic behaviour of the fluid.

(Refer Slide Time: 02:58)



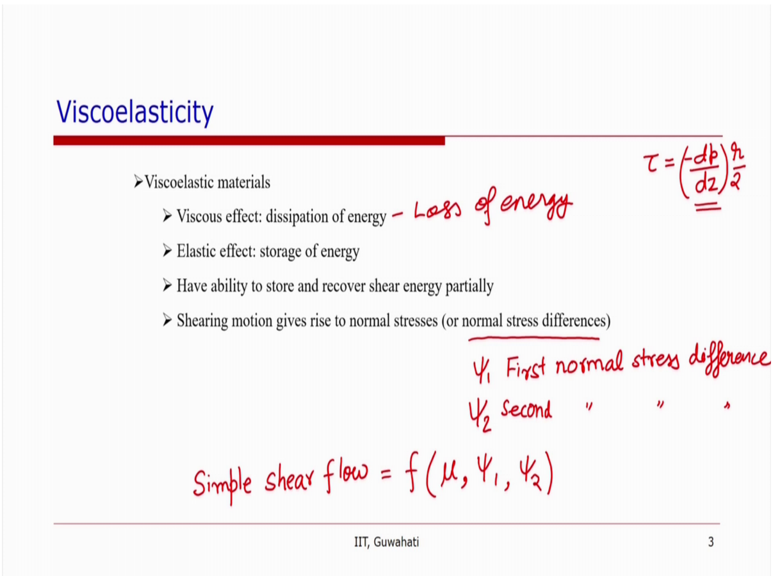
So, let us now consider what is viscoelasticity? So, let us look at the two terms that the term viscoelasticity is made up of; one term is viscosity, and the other term is elasticity. So, elasticity is generally a property of solids you might have studied about elasticity while undergoing a course on solid mechanics, you might have heard the term elastic solid or perfectly elastic solid. So, if we throw a ball on the ground, and it bounces back and comes up to the same height then such a elastic such a solid we call elastic an elastic solid.

It follows - an elastic solid follows Hooke's law. So, what does the Hooke's law say Hooke's law say that the stress which is say tau is proportional to the strain. Note we have been talking about the fluids in this course until now, so we have been saying the relationship between stress and strain rate; whereas, for a solid it is the relationship between a stress and a strain. So, for an elastic solid, the stress is proportional to strain that is what is called Hooke's law, and then when it becomes equality tau is equal to G gamma where G is called young's modulus.

So, elastic solid what do they do for elastic solid as I said earlier that the material it stores when a stress is applied on the material then it because of that stress the energy that is there the material stores that energy and after the removal the of the stress, it regains it shape. So, that is energy helps in regaining the material its shape. But after a certain yield stress the creep occurs that is the solid material starts flowing, so that is what an elastic behaviour is which is shown by the solids, and it is generally a property of the solids.

Now, a viscous fluid all the fluids that we see show little or more amount of viscous behaviour. So, a viscous fluid as we have studied earlier for a viscous fluid they follow Newton's law viscosity, and the shear stress that is applied on the fluid that is proportional to shear rate. So, material deforms continuously. And what we are concerned here is not the deformation, but the rate of deformation of the material. So, that is a difference that we experience between solids and fluid. In the solids, we are concerned about the deformation of the solid; whereas in the fluids we are concerned about the rate of deformation of the fluid. So, viscoelastic material can say transient from solid to fluid or fluid to solid, and these materials show both solid and fluid behaviour which is they show elastic as well as viscous behaviour.

(Refer Slide Time: 07:09)



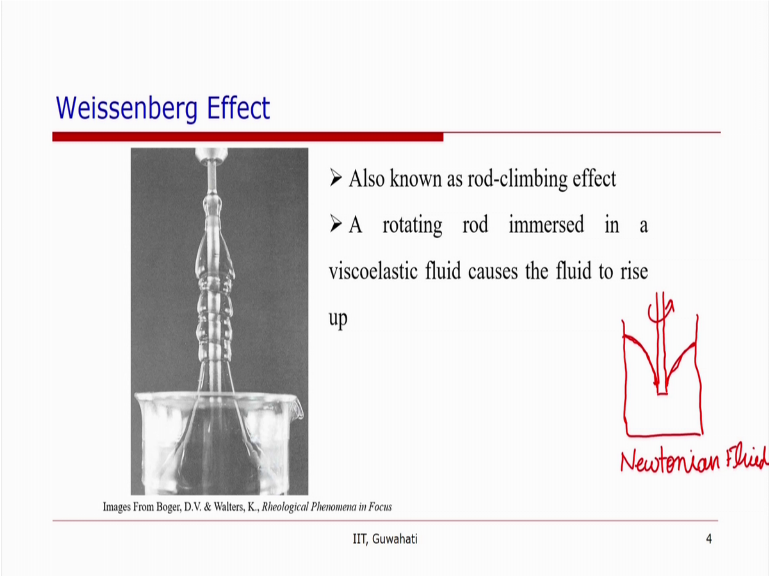
So, because of viscous effects as you know that when there is flow of a viscous fluid in a channel, then there is some pressure loss, and you might remember while discussing flow in a channel we derive this relationship that tau is equal to minus dp by dz into r by 2. So, what I am trying to say here that the dp by dz is the pressure loss because of the viscous systems. So, because of viscous effect or because of the viscosity, there is loss of energy or the dissipation of energy, the energy in is dissipated in overcoming the viscous effects. Whereas, as I said just now that elastic effect that means for a solid, the energy is stored in the fluid; and after the stress is removed, this energy is released, so that is a storage of energy. So, viscous due to viscous effect there is loss of energy, whereas in during the elastic effect the energy is restored. So, because viscoelastic material shows both viscous and elastic effects, so they have the ability like the elastic materials to store and recover the energy, but not completely, partially only. So, they have the ability to store and recover the energy partially and some part of energy is lost.

So, when the stress is applied on it, some part of energy is lost to overcome the viscous effect, and some part of the energy is stored by the fluid. So, depending on where the energy is going more, the fluid will show that kind of behaviour. If the more energy is stored then it will be more like an elastic solid or the viscoelastic material will have more viscoelastic or more elastic behaviour. If more energy is lost then it will more like a viscoelastic or the viscous fluid behaviour.

Another interesting behaviour that is seen in viscoelastic material is that due to the shearing motion, due to the shearing of the fluid, in general for a fluid what we see is that there are shear because of the shearing motion or because of the shear rate when the fluid experiences the shear rate, shear stresses are generated. But in a viscoelastic material in addition to this shear rate, the material also experiences some normal stresses or what we call normal stress differences. And these normal stress difference is because if the material is isotropic that means, if the normal stresses are isotropic that means, if they are same in all the directions then there will be no deformation of the material.

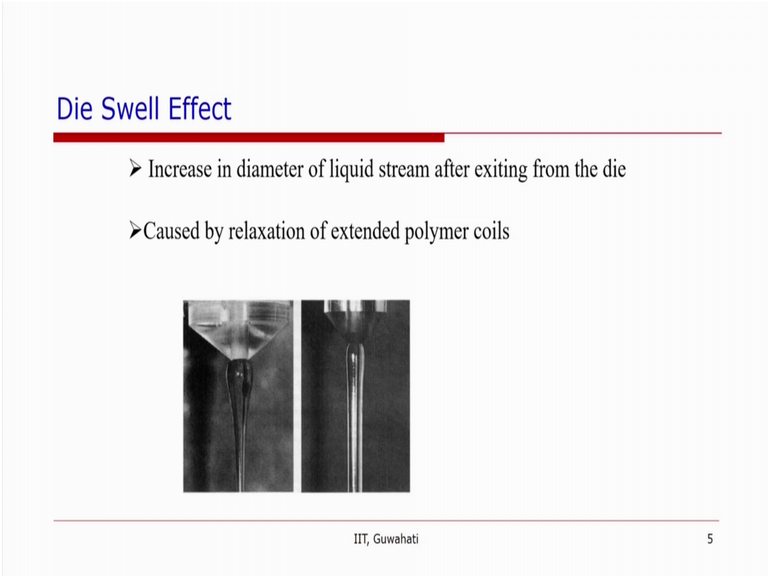
So, for the deformation of the material to happen it is important to have the difference between the normal stresses. So, what is important for deformation is the difference of the normal stresses not the normal stresses itself, so that is why we consider the normal stress differences. And there can be two independent normal stresses which are called first normal stress difference, and second normal stress difference. So, under shear effect, the fluid is characterised by three properties; under simple shear flow a viscoelastic material require three properties, this is called let us say phi 1, this is phi 2. So, simple shear flow and there are three properties for the viscoelastic material that characterises viscosity first normal stress difference and second normal stress difference.

(Refer Slide Time: 12:58)



So, let us look at some of the effects or some of the physical phenomena that as that are peculiar to viscoelastic fluids. So, one such effect is called rod-climbing effect or Weissenberg effect. So, if we take a beaker and a Newtonian fluid into it, and then put a rod in the beaker and then rotate the rod about its axis, then what will happen due to the centrifugal effect the fluid will move away from the rod and reach near the wall. So, after it reaches its steady state, we will see a fluid profile which will look something like this for a Newtonian fluid. The top meniscus will take a shape like this where the level of the fluid is lower near the rod and higher near the walls because of the centrifugal effect. However, if the fluid is viscoelastic what is seen is this that when the rod is rotating and because of that rod motion, the fluid rises up on the rod. And this effect is known as rod-climbing effect or after Weissenberg is known as Weissenberg effect.

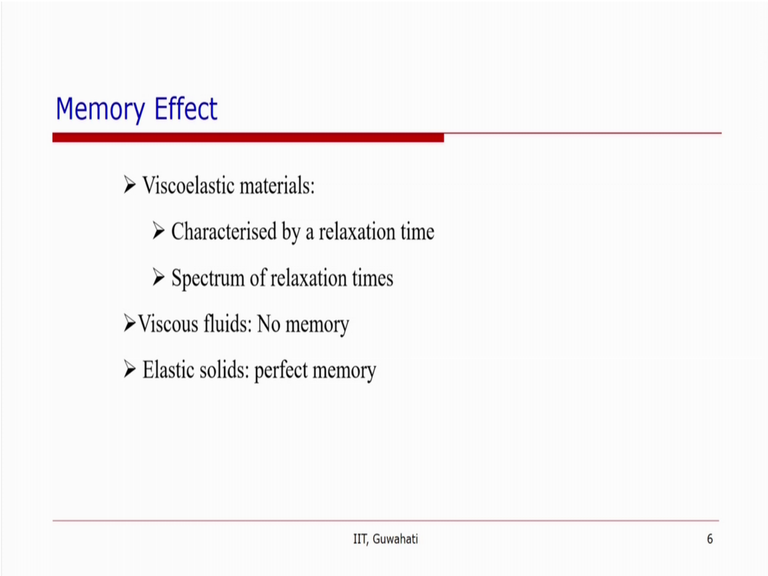
(Refer Slide Time: 14:47)



Another related and similar effect is called die swell effect. So, when a Newtonian fluid comes out of a die or comes out of a say vessel from a small hole, then the fluid stream, the diameter of the fluid stream either remains constant or it starts decreasing when the velocity of the fluid increases. Whereas, for a viscoelastic material, when specially or this phenomena is observed in polymers which are viscoelastic which show viscoelastic behaviour, the diameter of the die increases just after it exists from the die.

This is because there are particles soft particles suspended or large molecular chains in the fluid; and once they come out in the air where the pressure is low, so the molecules polymer molecules, large polymer chains, they relax and the volume increases. So, the diameter of this fluid z that is coming out it increase. So, this effect is known as die swell effect.

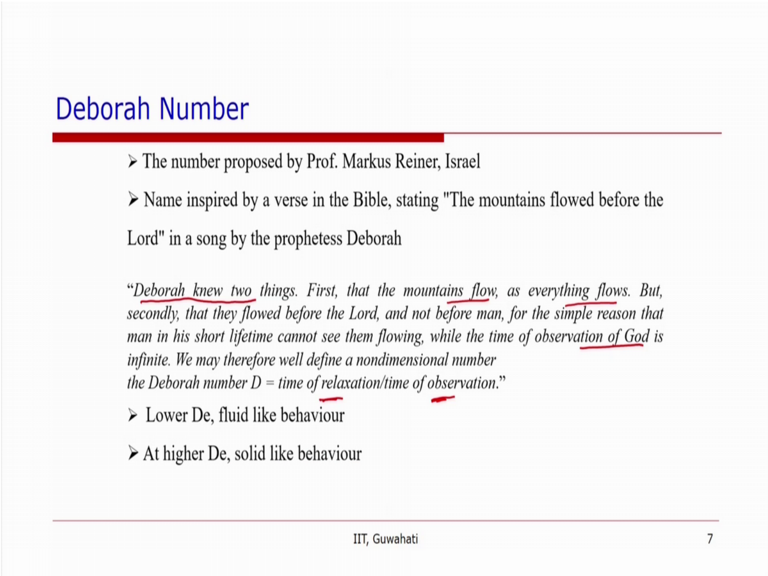
(Refer Slide Time: 16:23)



Another important behaviour is what we call memory effect. So, for a simple Newtonian fluid, the stresses internal stresses are proportional to the instantaneous deformation or a instantaneous rate of deformation. It does not remember what had happened in the past, but for a viscoelastic fluid what is important that it the fluid behaviour or the internal stresses in the fluid are proportional to the deformation history of the fluid and that is understandable because viscoelastic fluids show the viscous behaviour which is like a Newtonian fluid, but also elastic behaviour. So, elastic behaviour that means, it stores energy inside it, and when the viscoelastic fluid stores energy inside it, and the energy is released, so that means, the entire storage history need to be taken into account, so that is why this memory effect is shown by the viscoelastic materials.

So, to characterise this memory effect, a relaxation time, or a spectrum, or a distribution of relaxation time is required to characterise the rheological behaviour of a viscoelastic material. So, on the two extremes, the viscous fluid which does not have any memory, the relaxation time is 0. Whereas, the elastic fluid which has a perfect memory, it will have a finite relaxation time.

(Refer Slide Time: 18:27)



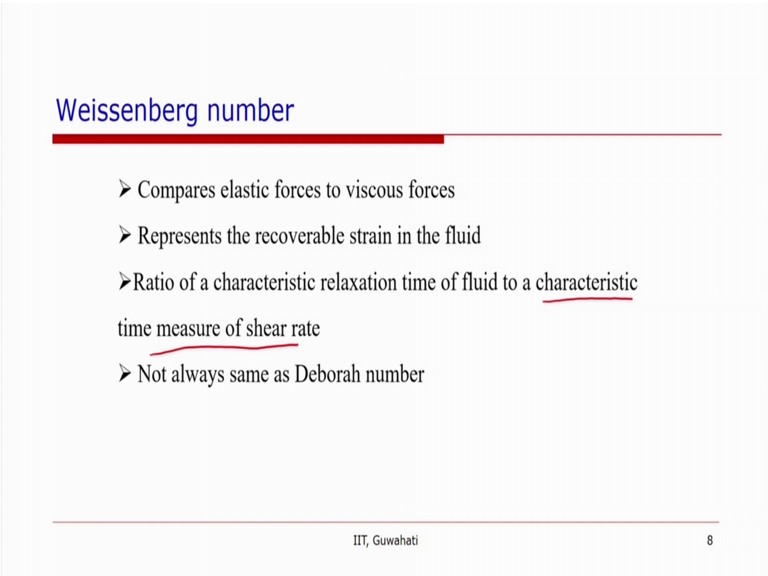
So, let us come to a interesting number which is called Deborah number. And this name is not proposed by or this number is not proposed by Deborah, whereas this has been proposed by Prof. Markus Reiner you might remember that at the start of the Rheology chapter we discussed that Prof. Reiner was one of the professors who gave the term Rheology. So, he also defined, he has also given this number or the nomenclature Deborah number. So, the name is inspired by a verse in the Bible, which says the mountains flowed before the lord and this verse is by prophetess Deborah.

So, let us look at the text from the article by Prof. Reiner in which he introduced the Deborah number. So, what he says that Deborah knew two things first that the mountains flow as everything flows. But, secondly, that they flowed before the Lord, and not before man, for the simple reason that man in his short life time cannot see them flowing, while the time of observation of God is infinite. We may therefore, well define a non-dimensional number the Deborah number which is a ratio of time of relaxation and time of observation.

So, if we try to understand this, what he says that everything flows, it is the time of observation depending on which one can see that the material under observation is flowing or not flowing. So, he takes an example of mountain or the Deborah's description of mountain flows. So, he says because the observation time of a man is about 100 years in which there is a non-appreciable deformation of mountains. Whereas, observation time of God is very large or and in that the deformation of mountains can be observed. So, it depends on the observation time of a particular person or of a particular phenomena or depending on which one can see up the material is flowing or not. So, the Deborah number is defined as time of relaxation and time of observation.

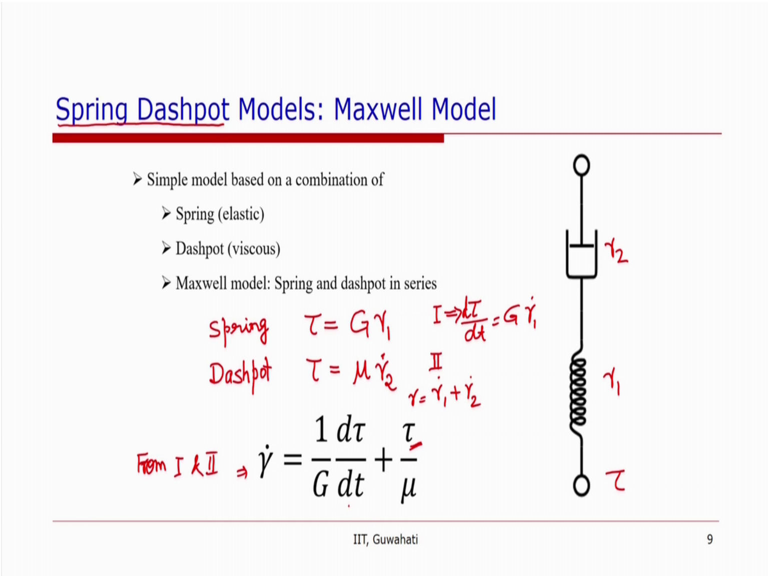
So, if the relaxation time of a material is negligible or small with respect to the time of observation, then one can observe the fluid to be if the relaxation time is very small then one can see the flow happening and it will be fluid like behaviour. Whereas, if the relaxation time is very large as compared to the time of observation as in the case for a human observing the motion of mountains, so it will be solid like behaviour. So, the Deborah number is time of relaxation and time of observation.

(Refer Slide Time: 22:45)



Now, there is another non-dimensional number which has a similar definition. It can also be defined, the number is Weissenberg number and it is defined as the ratio of elastic forces and viscous forces. So, it compares the two effects elastic effect and viscous effect. So, if we say in terms of a strain, the strain that is recovered and the strain that is not recovered. The elastic strain that will be that can be recovered as the viscous cannot be recovered. So, it represent the recoverable strain in the fluid. In terms of the relaxation time, again it is ratio of the characteristic relaxation time to a characteristic time measure of shear rate. So, notice the difference that in the Deborah number, the ratio of relaxation time was with the time of observation, whereas for Weissenberg number it is time measure of shear rate. So, if the time of observation, it is same as time measure of shear rate then the two numbers will be equal, but it is not always necessary that the two times will be equal. So, it is not necessary that the Weissenberg number will be same as Deborah number.

(Refer Slide Time: 24:16)



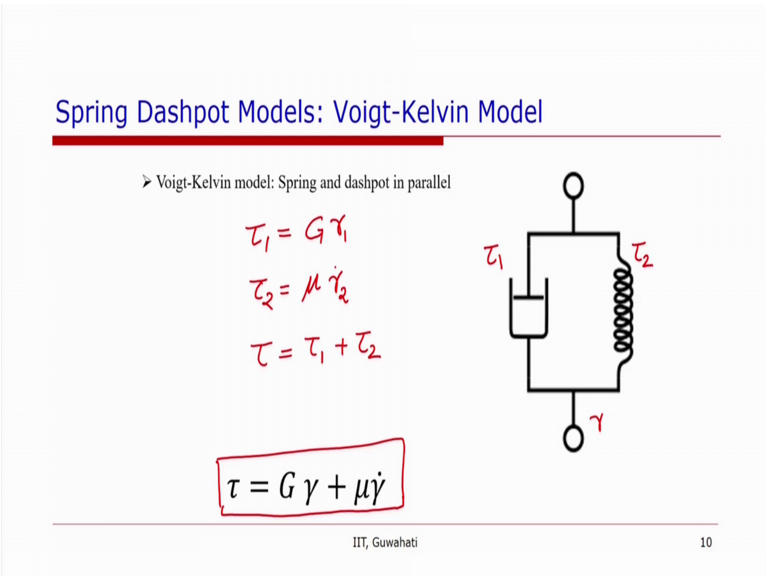
So, until now we have established that the viscoelastic fluids, they have two properties viscous and elastic behaviour. So, if we want to develop a simple model for a viscoelastic fluid for relationship between stress and strain, now because it has a stress strain and a strain rate. So, we might take into account all the things all three things stress, strain and rate of a strain. So, to model develop a simple model, we can consider a fluid which has two components one component which shows viscous behaviour and another component which shows elastic behaviour. So, that is where these spring dashpot models come into picture.

A spring is a elastic component which shows the elastic behaviour, and dashpot is a viscous component which has or which is a viscous damper, so which shows viscous behaviour. So, it is a combination of spring and dashpot two components which shows one component show elastic behaviour another component show viscous behaviour. So, a combination of different components different combinations can be used to model the fluid behaviour viscous the rheological relationship between the viscous and elastic fluids. So, in simple model is called Maxwell model.

In Maxwell model, the spring and dashpot are in series. So, if they are in series; that means, they will have different deformations. So, let us say the deformation in the two are gamma 1 and gamma 2, and they will experience the same stress which is tau. So, we can write for spring tau is equal to G gamma 1, whereas G is the young's modulus for the spring. And for dashpot, we can write tau is equal to mu gamma 2 dot that is the rate of a strain or the shear rate.

So, as we said that the total strain will be a sum of gamma 1 plus gamma 2. And if we differentiate this with respect to time that means, the total strain rate will be gamma 1 dot plus gamma 2 dot. Now, if we substitute gamma 1 dot and gamma 2 dot from previous equations let us call this equation 1 and equation 2. And if we substitute then we can write from 1 and 2, we get gamma dot is equal to this equation. So, you might notice that because we want to write gamma 1 dot plus gamma 2 dot, so to obtain gamma 1 dot we need to differentiate this. So, we can write del d tau over dt is equal to G gamma 1 dot. And we substitute that here, we will get a relationship between stress the rate of change of a stress and strain rate. So, this model is known as Maxwell model.

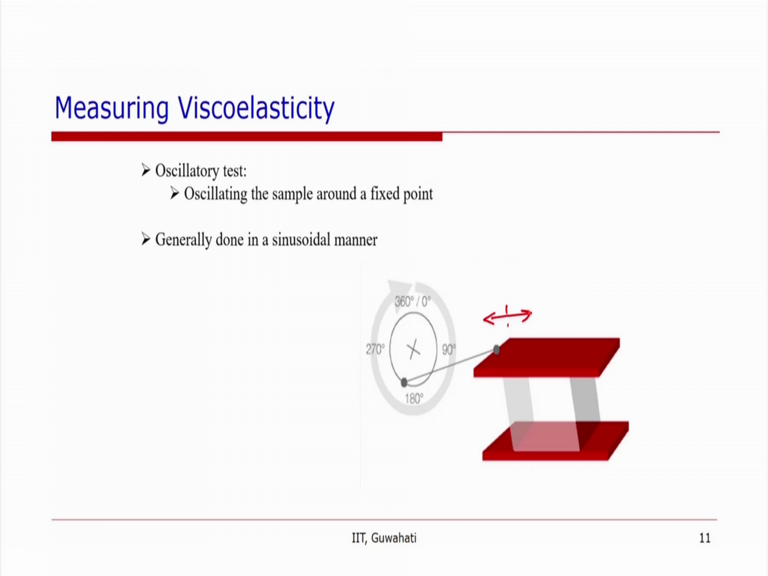
(Refer Slide Time: 28:47)



Another combination which is frequently discussed or which is simple model which parallel combination of spring and dashpot. And in such case the deformation will be same in both the cases, whereas they will experience different stresses let us say tau 1 and tau 2. So, we can write tau 1 is equal to G gamma 1, and tau 2 is equal to mu gamma

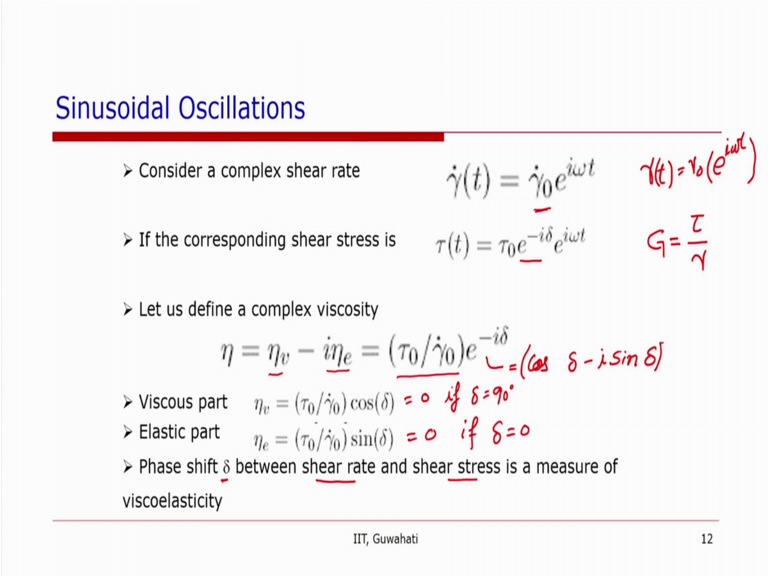
2 dot. And if we add those tau is equal to tau 1 plus tau 2. So, we will have after substitution, tau is equal to G gamma plus mu gamma dot, so that is another model that can be used to model the behaviour of a viscoelastic fluid. So, these are the two simple models. However, people can develop a complex model, which have these spring and dashpot a number of them in series and parallel or in series and parallel combination.

(Refer Slide Time: 30:15)



So, for measuring viscosity what people can do is they have studied simple shear flow of the viscoelastic fluids, and the another behaviour that is or another flow that is often used to measure the behaviour of the viscoelastic fluid is oscillatory test. So, the fluid under goes an oscillatory shearing, so an oscillatory shear rate and the stress is measured or the fluid under goes under a oscillatory shear stress and the shear rate is measured. So, the sample say for example, in a cone and plate viscometer or coaxial cylinder viscometer, the cylinder outer cylinder which is rotating or the cone which is rotating is imparted a say oscillatory motion for example, sinusoidal motion in place of a unidirectional rotation. So, in such case the oscillatory behaviour of the fluid can be studied. Generally, this is done in a sinusoidal manner and the plate can be oscillated say if this flow between two parallel plates, it can be oscillated at in two different directions or about a steady position.

(Refer Slide Time: 31:50)



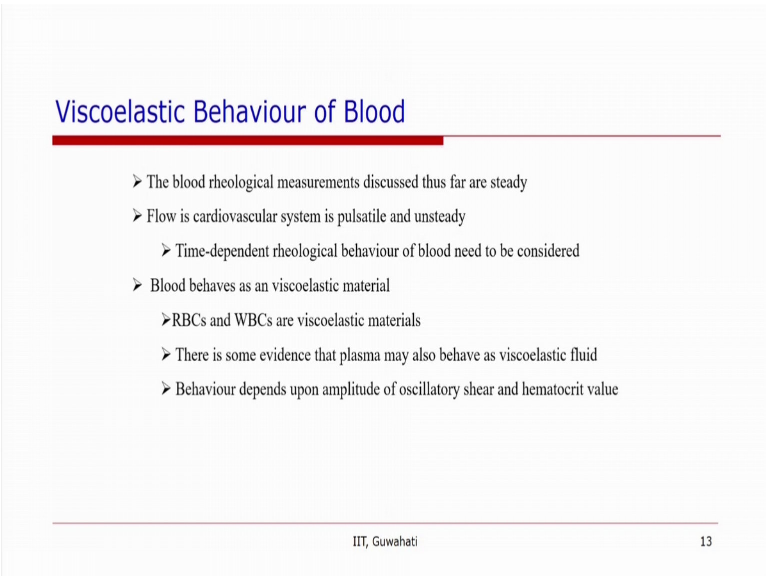
So, if the oscillation is sinusoidal, then it is often convenient to represent the sinusoidal behaviour in terms of a complex number, because it is easier to deal with while multiplying dividing or differentiating one can easily deal with complex numbers. So, let us consider a complex shear rate, which is given as shear rate is equal to a gamma 0 dot e to the power i, i is to represent the complex number omega t and omega is the oscillation frequency. And from that if the corresponding shear stress is tau is equal to tau naught e to the power minus i delta e to the power i omega t. And this minus i delta has been introduced to take into account of the fact that the shear rate and the shear stress are not necessarily will not generally be in phase or will be only under certain cases that they will be in phase, so that means, in that case delta will be 0.

So, if we divide this complex shear stress by the complex shear rate, we obtain a complex viscosity. And this viscosity will have two components. So, we can write this as tau naught over gamma dot naught into e to the power minus i delta. And if we write this in terms of the two components real and imaginary component, then the real component shows the viscous behaviour and the imaginary component shows the elastic behaviour. The viscous part is e to the power minus i delta what we know is this is equal to e to the power i theta is cos theta plus i sin theta. So, e to the minus i delta is cos delta minus i sin delta, this is viscous part. And the elastic part will be sin delta, and this delta is the phase shift between shear rate and shear stress and this is what measures the viscoelasticity. So, this is the phase shift between the two is the viscoelastic behaviour.

So, if delta is 0, then this term will be 0. If delta is equal to 0, and so that means, it will be, if delta is 0 then it is viscous fluid. This term will be 0, if the delta is 90 degree. And in between the fluid will behave as a viscoelastic fluid. This analysis can also be done in a different manner if one consider gamma is equal to or gamma t is equal to gamma naught e to the power i omega t, then one can define a complex modulus which is the ratio of stress verses strain and then it will also have components. So, one can represent either in terms of the modulus or module and one can also represent the behaviour in terms of complex viscosity.

And one can even find out the relationship between the complex modulus and the complex viscosity of for two different components. So, if the modulus is complex modulus then in that case the elastic component is generally known as storage loss and the viscous component the elastic component is known as the storage modulus, and the viscous component is known as the loss modulus.

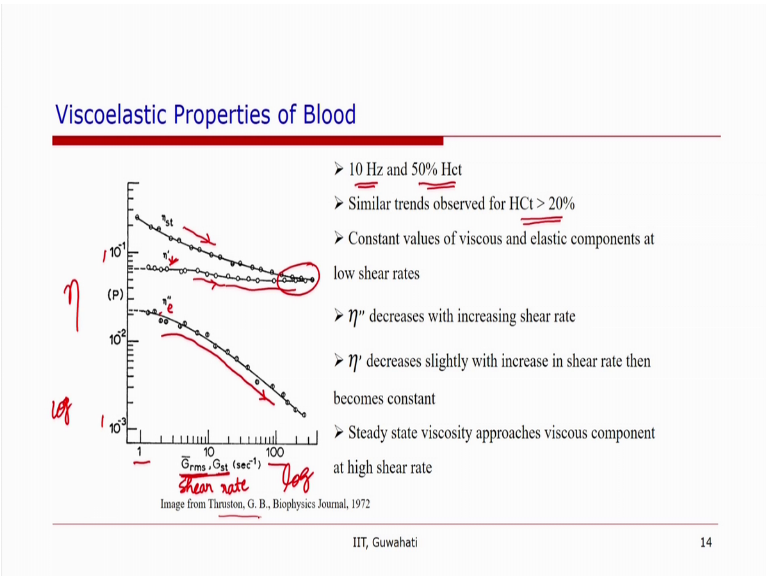
(Refer Slide Time: 37:00)



So, let us briefly look at the viscous or viscoelastic behaviour of blood. As we have said that the all the measurements that we have looked into the about the rheology of blood has been steady. And the flow in the cardiovascular system is pulsatile, it changed with time and is unsteady. So, it is important to consider the time-dependent rheological behaviour of the blood. So, it has been shown, it has been observed first time by professor G. B. Thurston that the blood behaves as a viscoelastic materiel because the red blood cells and white blood cells they are viscoelastic component of the fluid. And on the application of the stresses the RBCs they undergo deformation and once the stresses are removed they come back to their shape, so the blood as a whole also show the viscoelastic behaviour.

Recently, there is some literature which says that under some conditions plasma may also behave as viscoelastic fluid and that can be attributed to the proteins that are suspended in the plasma. So, these proteins, so because of the presence of these proteins a plasma may behave as a viscoelastic fluid. And this behaviour under oscillatory flow conditions, it will depend on the shear rate and the hematocrit value which is if you remember that hematocrit is the percentage of red blood cells. So, the fraction of a red blood cells which is primarily responsible for the viscoelastic behaviour of the blood their volume fraction that determines the viscoelastic behaviour of the fluid and the shear rate as we have seen that the blood viscosity is the shear dependent viscosity.

(Refer Slide Time: 39:16)

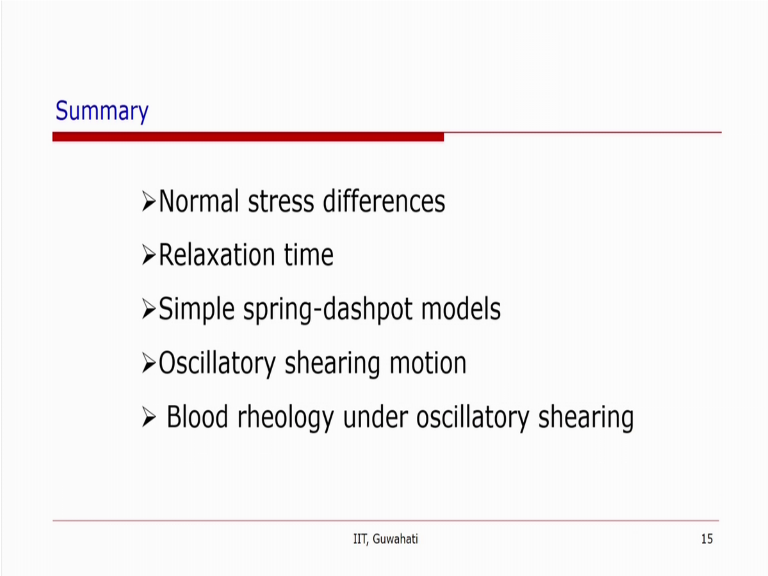


So, what this graph shows is the viscosity and these can be different viscosities as a function of shear rate. So, the shear rate is the rms value of the oscillatory shear and under this oscillatory shear what has been plotted is the viscous that is real, and imaginary that is elastic component of viscosity. So, this is eta we can say eta v and this is eta elastic. And the third graph shows the viscosity under steady shear conditions, so that is why on the x-axis you also have the velocity gradient or the shear rate for a steady state conditions. These measurements were done at a frequency of 10 hertz and for 50 percent hematocrit value, but it has been suggested that similar trends are observed for hematocrit value or the RBC fraction more than 20 percent.

So, what you can see from this graphs that at low values of shear about 2 per second, the complex viscosity both the components the elastic component and the viscous component, they are almost independent of shear at very low shear rate about 2 per second or about 1 per second at that magnitude. When the shear rate is increased, then the elastic component decreases continuously. You might notice that the scale here on the x and y-axis is log scale. So, this is a log-log plot. The elastic component which is imaginary component of a complex viscosity, it decreases continuously with the increase in shear rate. Whereas, the viscous component of viscosity it decreases slightly there is just slight decrease in the viscosity and then it becomes constant and it remains constant.

We have already seen this behaviour for the steady state viscosity which decreases continuously up to 100, when the shear rate is up to 100 per second and above that the velocity is constant which we call shear thinning behaviour. So, what is interesting to know that at high shear rates above 100 per second, the complex the real component of the complex viscosity or the viscosity under oscillatory shearing is same as the steady state viscosity at high shear rates. So, these are some of the observations for the viscoelastic properties of the blood by Prof. Thurston who looked at this phenomena in 70s.

(Refer Slide Time: 42:53)



So, in summary what we can say that viscoelastic fluids in simple shear flow apart from the viscosity, we also need two normal stress differences. First normal stress difference and second normal stress difference to characterise the rheological behaviour of a viscoelastic fluid. The viscoelastic fluid of course they show viscous as well as elastic behaviour. The another important characteristic of viscoelastic fluid is relaxation time. And these relaxation time in the non-dimensional form is defined in terms of Deborah number and Weissenberg number. The simple behaviour or the rheological behaviour of a viscoelastic fluid the simplest possible model are spring-dashpot model in which a elastic compound is spring and a dashpot component or a viscous component a viscous temper or dashpot is considered. And this is called spring-dashpot model. And we have also looked at the oscillatory shearing motion and the complex viscosity of the blood briefly.