Cardiovascular Fluid Mechanics
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Lecture – 07
Viscometers and Rheometers

Hello, until now we have been looking at the fundamentals of rheology, what rheology basically is, the properties of different time independent non-Newtonian fluid and different simple models which can model the non-Newtonian behaviour of complex fluid including that of blood. Now, all these models are need to be validated, the parameters in these models are need to be found out. So, this is done by doing careful experiments to measure the viscosity or apparent viscosity of complex fluids. In this lecture, we will be looking at the, a simple and frequently used viscometers and rheometers specially in the context of blood.

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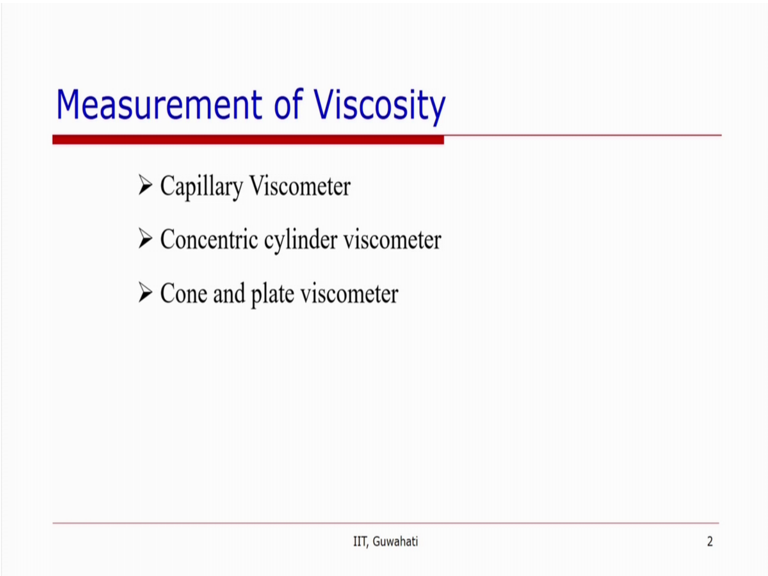

So, the measurement of viscosity or apparent viscosity, we will discuss in this lecture three different viscometers or rheometers. So, there are three viscometers that have been listed here - capillary viscometer, concentric cylinder viscometer and cone and plate viscometer. You might wonder the difference between viscometer and rheometer. So, the viscometer in my opinion is an instrument using which the viscosity of a Newtonian fluid can be measured; whereas, rheometer is a general term by which the rheological characteristics including non-Newtonian viscosity or apparent viscosity which is generally defined as the ratio of shear stress and shear rate is measured

So, the difference between viscometer and rheometer essentially is that viscometer, the term viscometer is used for measurement of the viscosity of Newtonian fluids because their viscosity is constant at any particular temperature; whereas, rheometer is used for non-Newtonian fluids. So, the three viscometers that we have listed here are capillary viscometer, concentric cylinder viscometer and cone and plate viscometer.

Capillary viscometer is it basically works on Hagen-Poiseuille principle that a known amount of liquid is flown, the liquid flows through a small capillary, so that the capillary is small or channel is small, so that the Reynolds number is small flow becomes fully developed and the exit and entrance effect can be neglected. And when the time of the flow of a fluid is measured, and this time of flow of a known volume of fluid is compared with the time required for the flow of a known fluid or the flow of a fluid of known viscosity at the same temperature. And then by comparing the times for the flow of that two fluids, one can define or one can find out the relative viscosity of the fluid. So, it is generally used for Newtonian fluids.

The next viscometer that we will discuss in this lecture is concentric cylinder viscometer in which they are two cylinders which are concentric, they have same axis. And the gap between the cylinder is very small when it is compared with the radius of the either cylinder. One of the cylinder generally it is the outer cylinder is rotated at a known angular speed by doing which one can impart a known amount of shear rate. And the torque on the inner cylinder which is fixed is measured. By doing this, one can calculate what is the shear stress being exerted on the inner cylinder and by comparison and by the ratio of the two shear stress and shear rate one can measure the viscosity of the fluid.

Another very popular and frequently used viscometer or rheometer is cone and plate viscometer in which a cone and plate arrangement is there. Over a flat plate a cone of very high apex angle is rotated. So, again the flow the cone rotates and by this rotatary motion and from the angular velocity the shear rate is defined and the shear stress over the cone for this particular shear rate is measured. And from the ratio of two the viscosity can be found out.

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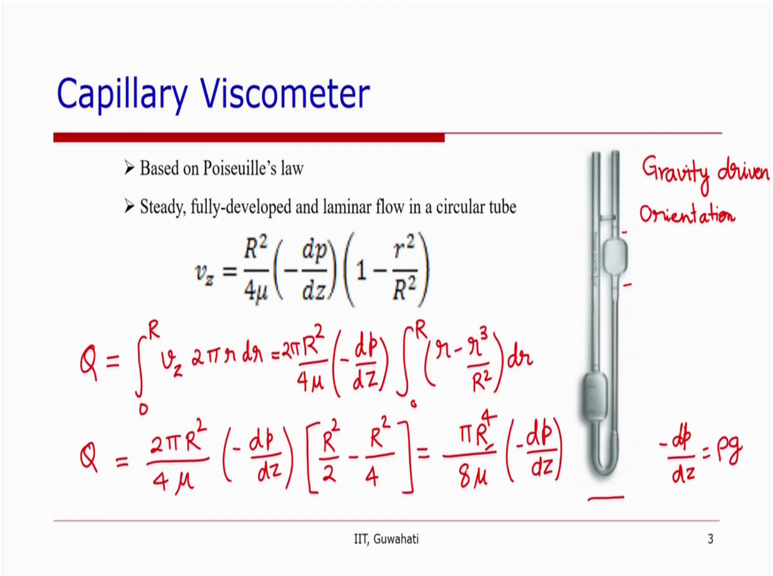

So, we will look at this it a bit in detail now. So, capillary viscometer first. So, this is a simple diagram of a capillary viscometer in which a known amount of liquid it flows through a channel. What is the driving force here? The driving force here is gravity driven. One can also have another or an external pressure difference to drive the flow. When it is gravity driven, then it is very important that the orientation of the channel should be vertical; if not vertical then the angle that it makes from the vertical direction is required, so that if the effective value of gravity can be used in the calculations.

However, this problem can be eliminated if the experiments are done for the standard fluid which is used for the calibration and for the fluid of which viscosity is to be measured. If they are done at the same orientation of the viscometer, then the orientation effect can be ruled out. So, the capillary viscometer is based on Poiseuille law. You might remember in the previous lecture we looked at Newtonian fully developed and laminar flow in a circular channel, the flow was steady. So, we derived the following relationship for the velocity profile in the channel. One can then calculate the flow rate by just integrating this 0 to R v z 2 pi r dr. And if we do that we will end up with R square over 4 mu minus dp by dz integral 0 to R r minus r cube by r square dr. We can take 2 pi outside. So, this will give us 2 pi R square over 4 mu minus dp by dz. And if we integrate this that will be and put the limits we will get R square by 2 minus R is to the power 4 divided by 4 R square which will be R square by 4 effectively this will be 4. So, that is equal to sorry, so this is R square.

So, when we change this, this will become R square by 4. And we will get 8 pi R raise to the power 4 divided by sorry pi R to the power 4 divided by 8 mu minus dp by dz which is volumetric flow rate through the capillary. So, you might notice that R is a property of the capillary, and this will remain fixed for a given capillary tube; dp by dz in our case is equal to rho g.

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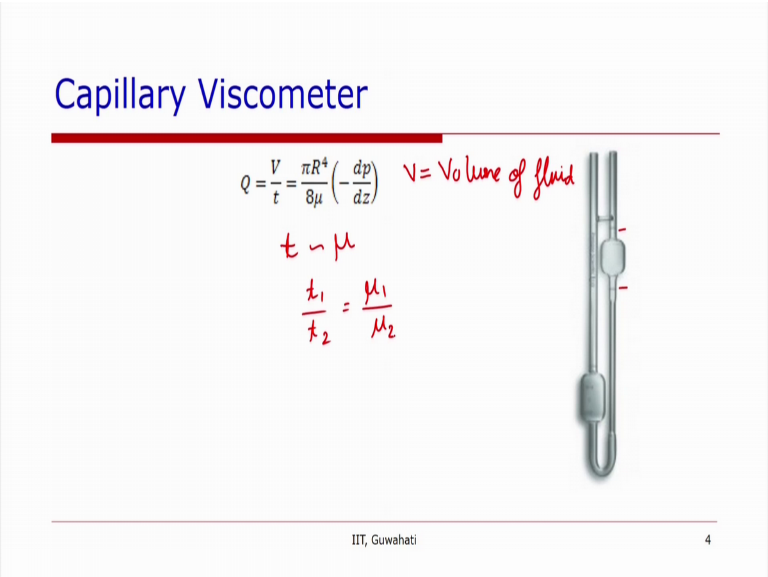

So, one can see that Q which can be written as V over t, where V is equal to volume of fluid generally in a viscometer the volume is fixed between these two points. So, the volume is fixed. Q is equal to V over t pi R to the power 4 by 8 mu minus d p by d z. And so one can see that t is directionally proportional to mu. And if the measurements have been done for a fluid already, then one can just simply use formula t 1 by t 2 is equal to mu 1 by mu 2 to calculate the viscosity of a fluid.

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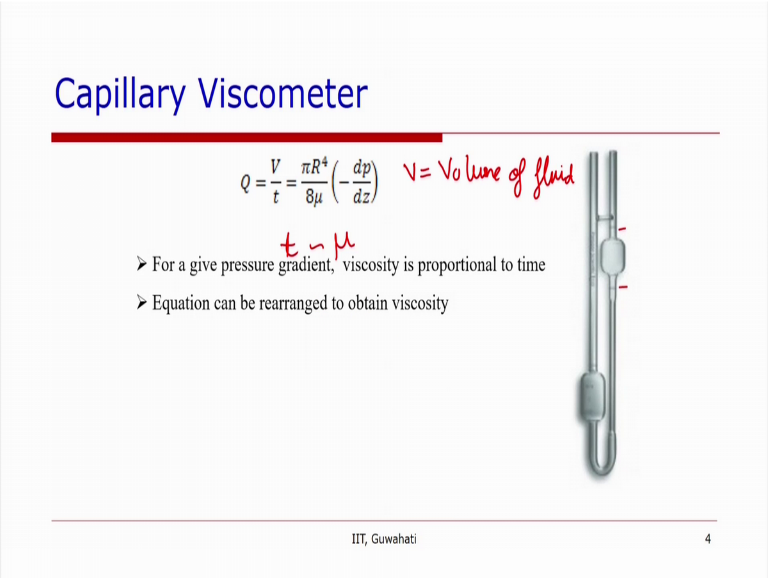

So, this equation can be rearranged to obtain the viscosity.

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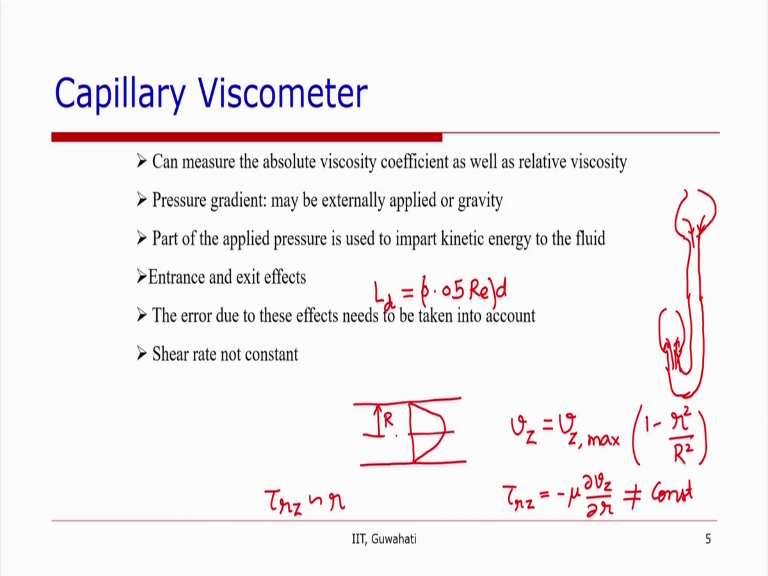

Now as we have seen that we can use capillary viscometer to measure absolute viscosity coefficient as well as relative viscosity. Generally, it is the relative viscosity that is measured using capillary viscometer. The pressure gradient it might be applied externally or gravity can act as a pressure gradient to drive the flow. In simple viscometer which we use in our undergraduate labs or in our day-to-day simple experiments, it is capillary viscometer in which the flow is driven by gravity. However, this instrument has some limitations of the pressure gradient, some of the energy of the fluid because the gravitational energy in our case some part of it will be used to impart the kinetic energy to the fluid.

Then as you might notice that the flow happens from a bulb into a capillary, so in at the entrance, there will be entrance effect; similarly, on the other side, there will be divergence effect or the streamlines are going that way. So, there will be divergence effects. So, these entrance and exit effects are neglected. We have not discussed in the previous class, but one useful relation that you might have or you probably would have studied in your undergraduate fluid mechanics course that for developing flow in a channel, the development length, which is required for the flow to become fully developed is equal to 0.05 Re into diameter of the channel. So, this tells us that we should keep Reynolds number sufficiently small, so that the flow quickly becomes fully developed. So, it is important to take into account all these errors while measuring the viscosity into capillary viscometer.

Another important point here is that as we said earlier that in a capillary, the velocity profile is parabolic. As you might remember, v z is equal to v z max into 1 minus r square by capital R square, where capital R is radius of the channel, and r is any arbitrary radial coordinate. So, we can quickly see that tau r z or the shear stress which is equal to minus mu del v z over del r that will be not a constant, but tau r z is proportional to it is proportional to r. So, the shear rate is not constant. So, if the viscosity of the fluid is shear rate dependent, it varies the shear stress varies with the radius.

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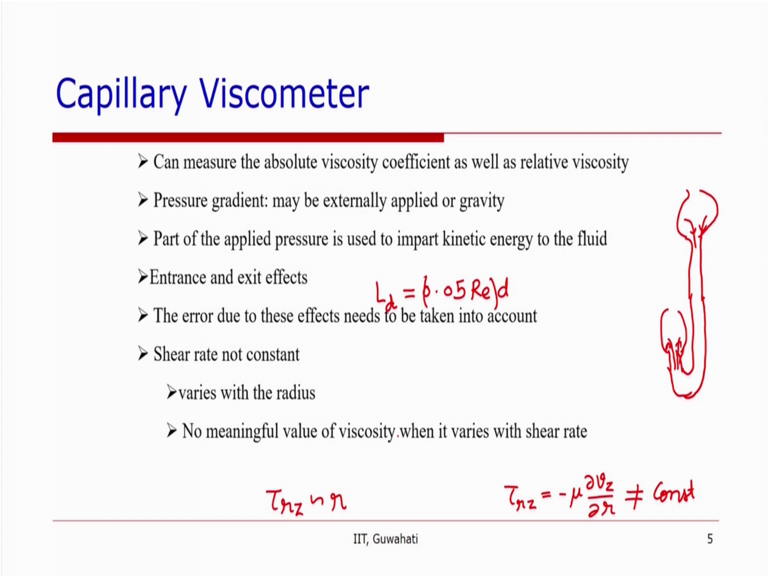

So, if the viscosity of the fluid is dependent on the shear rate, then in such a case it is important that the viscosity whatever viscosity that we measure using capillary viscometer that will not give us the correct value of viscosity. Because viscosity is a function shear rate and shear rate is a function of radius in a capillary viscometer, so the viscosity will be varying across the cross section in the viscometer. So, generally it is useful to have capillary viscometer to measure the viscosity of a non-Newtonian fluid of or a Bingham plastic fluid.

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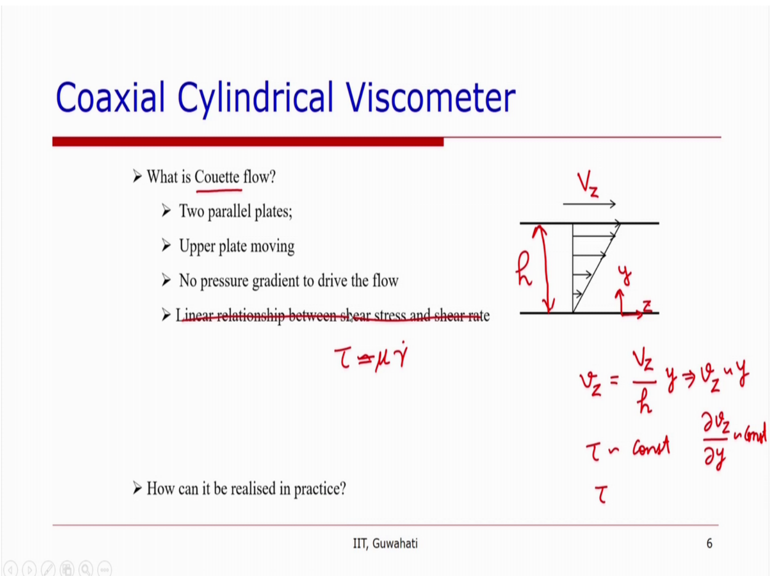

Then another viscometer that we are going to discuss is coaxial cylindrical viscometer. So, in the capillary viscometer, the problem is that the shear rate is proportional to the radius. Whereas, by the arguments that we have made just now we would prefer to have a flow in which shear rate is not constant. So, let us go back to our undergraduate fluid mechanics knowledge, and try to think that which is the flow in which shear rate is constant. So, if you remember the flow between two parallel plates, so two plates infinitely long plates which are parallel to each other there is no pressure gradient in the flow, but the upper plate is moved with a velocity let us say capital V subscript z.

If the upper plate is moved with a velocity V z, there is no pressure gradient to drive the flow then this flow is known as Couette flow. After Couette in such case one can show that the velocity if the distance between the plates is h, and this coordinate is z and this coordinate is y let us say, then in such case the velocity profile V z will be equal to capital V z which is the velocity of the upper plate divided by h into y. So, that means, V z is proportional to y which in terms mean that tau is a constant.

So, in this case in Couette flow the shear rate is constant, so sorry tau is equal to so in this case V z is proportional to y that means, del v z over del y is a constant independent of y. So, tau is also a constant the linear relationship between shear stress and shear rate is not valid it is that shear rate is constant. So, tau is proportional to mu, the tau is equal to mu gamma dot sorry Newtonian fluid. So, how can we realise this in practice?

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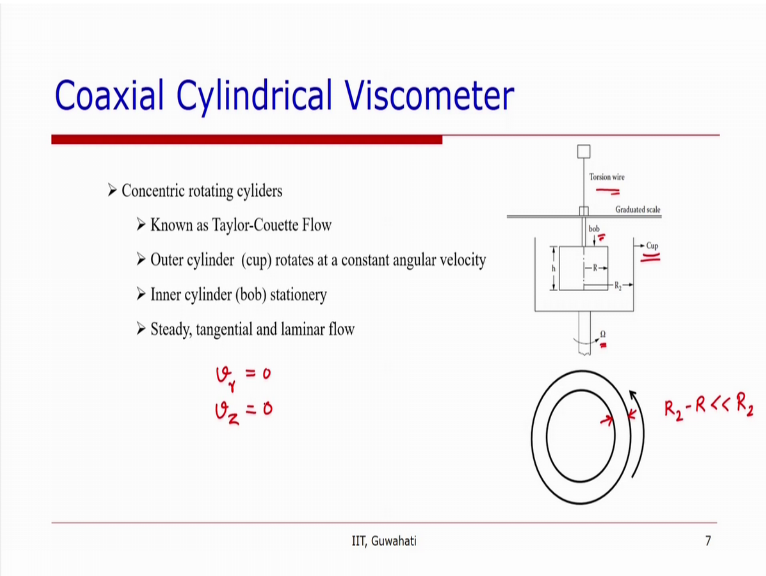

Because it is not easy to have two infinite plates in parallel or the a simpler arrangement is when two infinite cylinders they are moving relative to each other. So, this arrangement is often used for designing a viscometer, and this is also called bop cup viscometers. So, in this case there are two concentric cylinders the outer cylinder is known as cup, and the inner cylinder is known as bob. And the outer cylinder with the help of a shaft is rotated with an angular velocity omega. And because of this fluid motion, the torque that is imparted on the inner fluid is measured by this torsion wire. So, from this one can measure the viscosity.

So, let us look at how this is done. There are two concentric cylinders. The flow between two concentric cylinder is known as Taylor Couette flow. And in this case the gap between the two cylinders R 2 minus R as we see from this figure is very very small from either of course, R 2 or R. So, we assume in this case we make some assumptions to derive the relationship between the torque and the angular velocity and find out the viscosity from there. The flow is steady which is when we do the measurements, then the flow has become steady the flow is tangential. So, there is only V theta component of velocity is there V that means, V r is equal to 0 and V z is equal to 0, and the flow is laminar.

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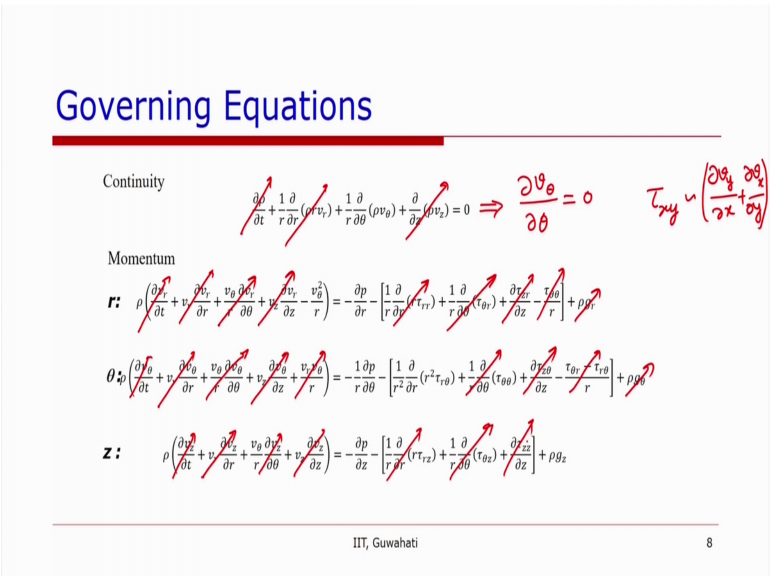

So, let us look at the governing equations conservation of mass momentum in r, theta, z which is cylindrical coordinates.

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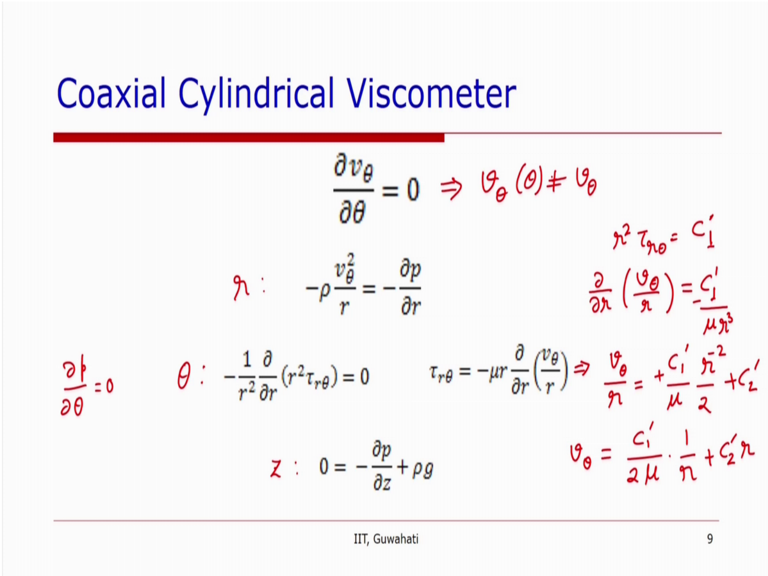

And try to look at different terms so because the flow is steady. So, all time dependent terms will become 0, there is no r. So, all terms containing v r or v z will be 0. So, this term goes to 0, this is 0, this is 0, this is 0, this is 0, because this is del v theta over del theta is equal to 0. And from here we can see that del v theta over del theta is equal to zero; that means v theta is not a function of theta v z is here. So, this is 0, v r is here, so this is 0. V r is here, so this is also 0; v z is here, so this term becomes 0; v z is here, so this term becomes 0.

Now, the gravity is acting in the z direction so these two terms become 0. Tau x y is proportional to del v y over del x plus del v x over del y. So, using this principle, we can say that if v r 0 then tau r r will be 0; tau theta r will be 0, because it is del over del theta del over del z tau z r this is 0. Tau theta theta will be 0, tau theta theta will be 0 tau z theta will be 0; tau theta r will be equal to tau r theta because their stress stands for symmetric, so this term will be 0. And tau r z is zero because v r and v z r 0, v theta and v z, so this will be zero because v z is 0, and v theta over v z is 0; similarly tau z z is equal to 0.

So, finally, we will end up with these terms. From the continuity equation del v theta over del theta is equal to 0. From r momentum equation, we get minus rho v theta square over r is equal to minus del p over del r, so that shows that how does the pressure vary in the radial direction and it is dependent on v theta. V theta is not a function of theta you can see from here. Now, v theta equation, so there we get the del p over del theta is equal to 0; we have not done this in the previous slide. So, we can just note that del p over del theta is equal to 0, because there is no pressure gradient in the angular direction and the flow is happening because of the outer rotating cylinder. And in this we can replace tau r theta with minus mu r del over del r v theta over r.

And the third equation from the z momentum equation is minus del p y del z plus rho g is equal to 0, so that is basically for the hydrostatic pressure or the dependence of the pressure in the z direction. So, if we want to find out the velocity profile, we can substitute this here. Let us look at that we can substitute tau, r, theta. So, we will get r square tau r theta is equal to a constant say c 1 dash. And when we substitute tau r theta is equal to minus mu r and with r it becomes r cube del over del r v theta over r is equal to a constant let us say this so c 1 dash. So, we get this is equal to minus mu r cube, we can now remove this from here. So, we get v theta over r is equal to minus c 1 over mu r raise to the power minus 2 divided by minus 2 plus a constant c 2, so that gives us v theta is equal to c 1. Let us call this c 1 dash 2 mu into 1 over r plus c 2 dash let us say c 2 dash r.

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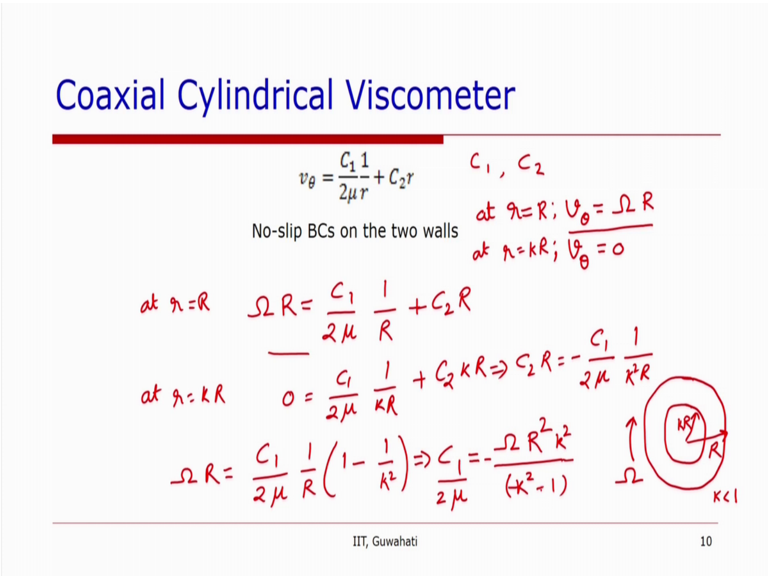

So, if we look at this equation C 1 by 2 mu 1 over r plus C 2 r. Now, we have two constants C 1 and C 2. So, to find out this we need two boundary conditions and these two boundary conditions within will be the no slip boundary conditions on the two walls so; that means at r is equal to capital R, and at r is equal to kappa R. Let us say that the radius of the outer cylinder is R, and the radius of the inner cylinder is kappa times R. So, kappa will be less than 1. So, at r is equal to capital R, which is the outer cylinder, it is rotating with a velocity omega. So, the v theta will be equal to omega into capital R; whereas, v theta is equal to 0 at the inner cylinder which is fixed.

So, you might notice here that we have applied no slip boundary condition, but the velocity at the wall is not essentially 0 in this case. So, if one substitute these values here, then one will get at r is equal to capital R, v theta is equal to omega R is equal to C 1 by 2 mu 1 over capital R plus C 2 capital R. And at r is equal to kappa R the velocity is 0 C 1 2 mu 1 over kappa R plus C 2 kappa R.

Now, this gives us a relationship between C 1 and C 2. So, we can say that C 2 R is equal to minus C 1 by 2 mu 1 over kappa square R. And if we substitute this in this equation then we will end up with omega R is equal to C 1 by 2 mu 1 over R this is 1 minus C 2 R will be C 2 r is minus C 1 2 mu 1 over R into 1 over kappa square. So, this is 1 over kappa square is there. So, this gives us C 1 over 2 mu is equal to omega R square divided by you can take kappa square into kappa square minus 1. So, C 1 by 2 mu is equal to omega R square kappa square divided by kappa square minus 1. You might notice that kappa is less than 1. So, this term is going to be negative. So, one might want to write this in this form.

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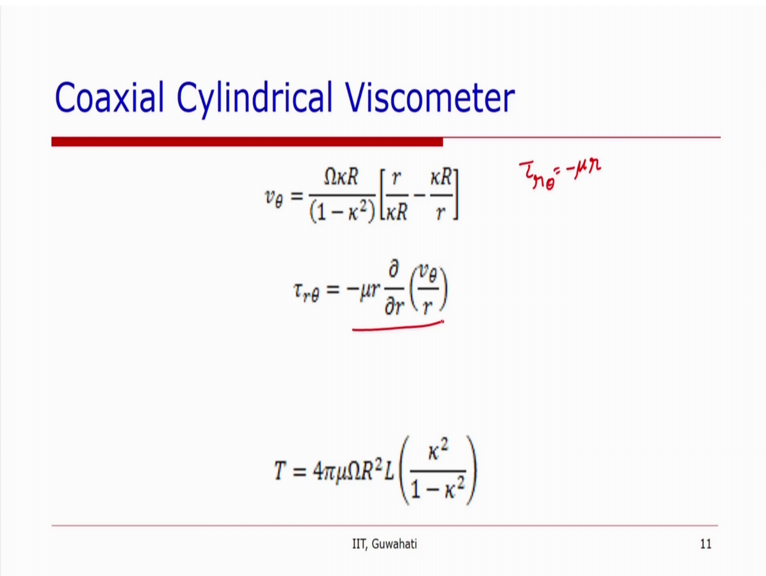

So, let us look at after substitution, one will get the velocity in this form. And from that we can get what is tau, tau r theta is equal to minus mu r del over del r v theta over r.

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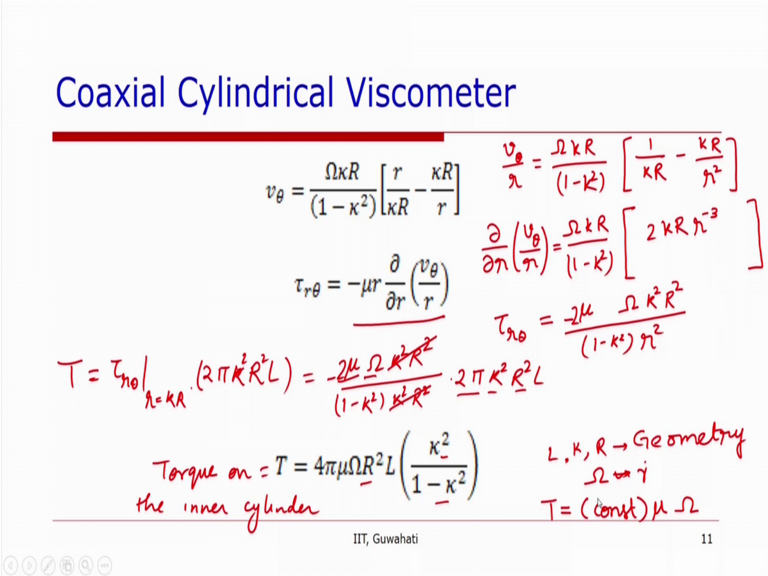

So, we can calculate v theta over r is equal to omega kappa capital R over 1 minus kappa square 1 over kappa R minus kappa R over r square. Now, if we want to differentiate it then del over del r over of v theta over r is equal to omega kappa R over 1 minus kappa square this is a constant. So, this becomes 0 minus kappa R r to the power minus 3 multiplied by minus 2. So, we take all this into account we can make this as plus, this also can go and this two can come here. And tau r theta will be equal to minus mu r omega kappa square R square divided by 1 minus kappa square into r cube.

Now, if you one want to find out the torque, where T is torque on the inner cylinder. So, T is equal to force into distance. So, this force is tau r theta at the inner cylinder. So, tau r theta at r is equal to kappa R into the radial or the circumferential not the circumferential, but the lateral area of the cylinder 2 pi kappa R L which is the area multiplied by the radius which is kappa R. So, this becomes minus mu this is this can be reduce to r square. So, let us just get rid of this. So, minus mu omega kappa square r square divided by 1 minus kappa square kappa square R square into 2 pi kappa square R square L. So, kappa square kappa square cancels out. So, the torque that we obtain is the two is missing here. So, we substitute this two, so 2 mu. So, this is 4 pi mu omega R square kappa square divided by 1 minus kappa square into lL, so that is the torque on the inner cylinder.

Now, if you see kappa and R are from the geometry of the cylindrical arrangement. Omega is the velocity, which imparts the shear rate, which is proportional to the shear rate. And L is the length of the cylinder. So, again this is also a geometrical parameter. So, we can say that tau torque is a constant times mu into omega. So, by measuring torque and omega, one can find out the viscosity of the fluid that is there in between the two cylinders.

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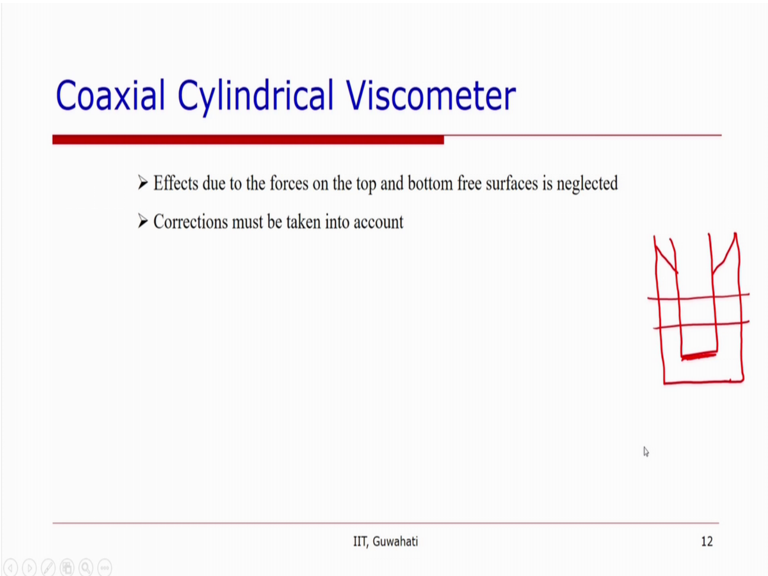

So, again we have made some assumptions here, because this relationship has been derived for the flow of fluid between two infinite cylinder, but in practice what will we have is the outer cylinder in which there is an inner cylinder. So, there will be the effect of top surface plus there will be end effect at the top because at the top because of the mean viscous for a non viscoelastic fluid, the fluid surface will look something like this. So, the surface area covered by the outer and inner cylinders will be different. So, we might need to consider the relationship might be valid somewhere in between. So, such corrections need to be taken into account when using coaxial cylindrical viscometers.

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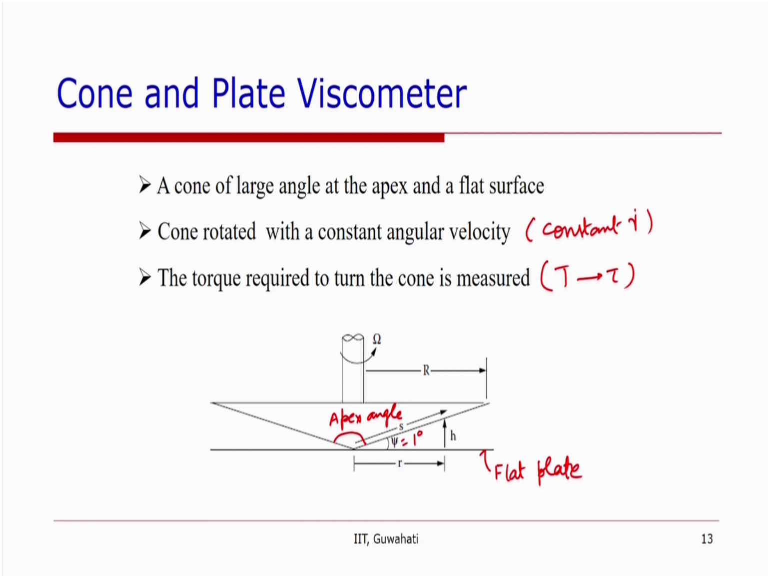

So, the another and the third important viscometer is cone and plate viscometer in which there is a flat plate and over which a cone which has very large angle which is this is called apex angle. And this apex angle is very small, this angle psi sometimes as low as one degree. So, the cone of very large apex angle and its flat surface this is the arrangement. And the cone is rotated with a constant angular velocity so which gives us a constant shear rate, and the torque require to turn the cone is measured. So, from the torque, one get tau principle it is very similar to coaxial flat plate arrangement.

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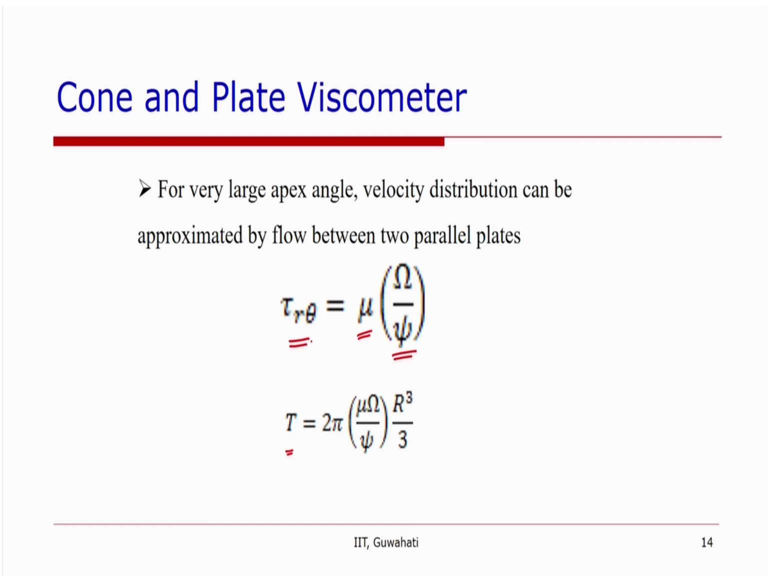

For very large apex angle, one can find out the velocity distribution that and from that the torque and one get that torque is proportional to gamma, and the viscosity is the proportionality constant. So, one can measure the velocity here. And from that one can again calculate the torque. So, this is shear stress and the torque.

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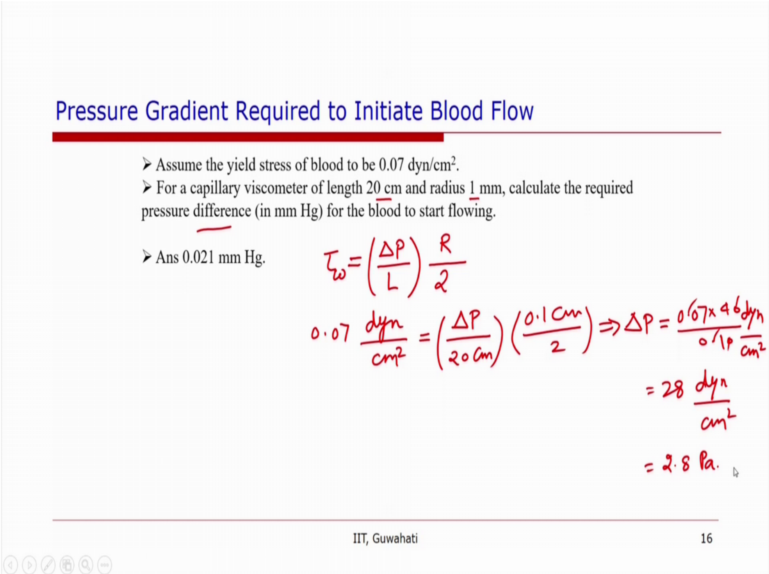

So, let us do a simple question and example. Let assume that it is a capillary viscometer and the yield stress of the blood is about 0.07 dyn per centimetre square. There is a capillary viscometer which has a length of 20 centimetre and the radius is 1 mm. And what we need is we need to calculate the required pressure difference for the blood to start flowing. So, if we remember the relationship between shear stress and pressure we can write this pressure gradient in terms of pressure difference. So, we can write let us say this as delta p which is the pressure difference and L r by 2.

So, if we take this shear stress on the wall because that is when the fluid it is start flowing and in this case r will be equal to the capital R of the channel. So, in this the shear stress is 0.07 dyn per centimetre square and that is equal to delta p which is pressure difference. L is 20 centimetre, R is the radius of channel so let us write this also in centimetre, so that is we get everything in CGS units. And this gives us delta p is equal to 0.07 into 40 divided by 0.10 dyn per centimetre square, so that will be 28 dyn per centimetre square or 2.8 Pascal. One can then change it to rho c using rho Hg, one can then change it to mm Hg and the final answer will be 0.021 mm of Hg.

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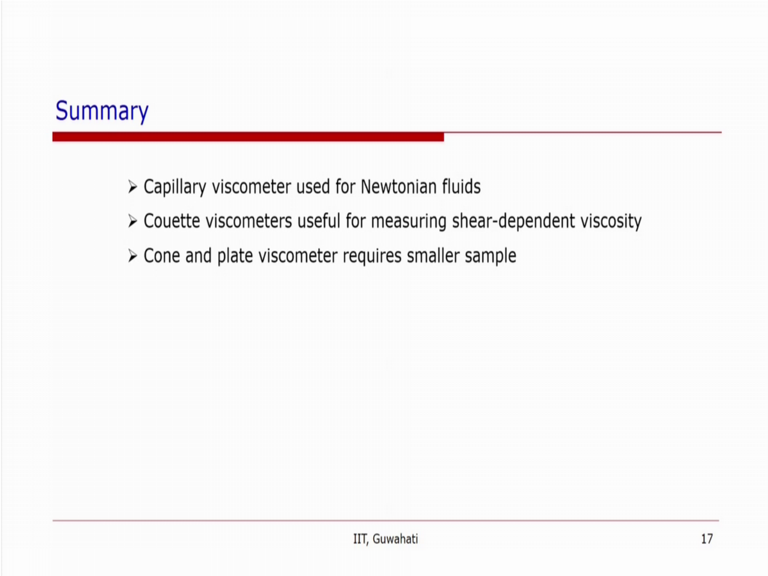

So, in summary, in this lecture, we have looked at the capillary viscometer the coaxial or Couette flow viscometer, and cone and plate viscometer. The capillary viscometer is very simple easy to use, and it can be used very accurately for measuring the viscosity of Newtonian fluids, but it is not useful for measuring the viscosity of non-Newtonian fluids because of the region that the shear stress varies with radius. Then Couette viscometer they can be useful for measuring shear-dependent viscosity, because the shear rate is constant in the Couette viscometers. Finally, the cone and plate viscometer; cone and plate viscometer are most popular or most accurate among the three and they require small amount of samples. So, for measuring the viscosity of the samples, which are costly they are used.