# Cardiovascular Fluid Mechanics

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## Lecture – 04
## Rheology of blood

Hello, in today's lecture, we will be discussing about the fluid in the cardiovascular fluid mechanics, which is blood. So, the topic for today is rheology of blood. So, let us look at the definition of this term rheology first.

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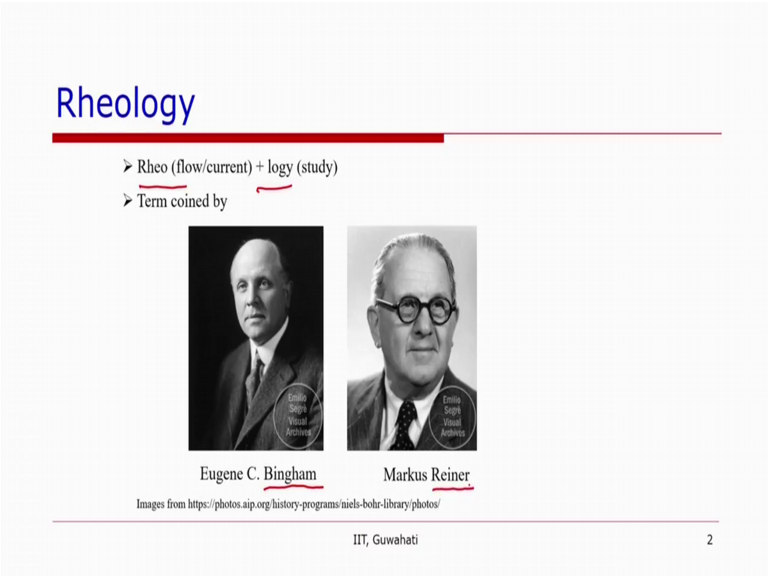


Rheology comes from two terms rheo and logy. So, as you have a would have a studied in a number of terms that logy or logus means study; and rheo, rheo means it is a old Greek term and it means flow or stream or current. So, rheology etymologically or literally means study of flow or study of current ok. The term was coined by two rheologists Prof. Eugene C. Bingham and Prof. Markus Reiner. These two are famous rheologists. If you have done a course on fluid mechanics you might have had a sub topic on non-Newtonian fluids where you would have studied about Bingham plastic fluid. In this lecture also, you will be studying what Bingham plastic fluids are. So, that those Bingham plastic fluids are named after Prof. Bingham.

And Prof. Markus Reiner he has also given a number of rheological models for complex fluids. So, if you do a detailed course on rheology, or if you study in detail about rheo rheology of non-Newtonian fluids, then you will come across about the work of Prof. Markus Reiner ok. So, the rheology literally means the flow of study of flow.

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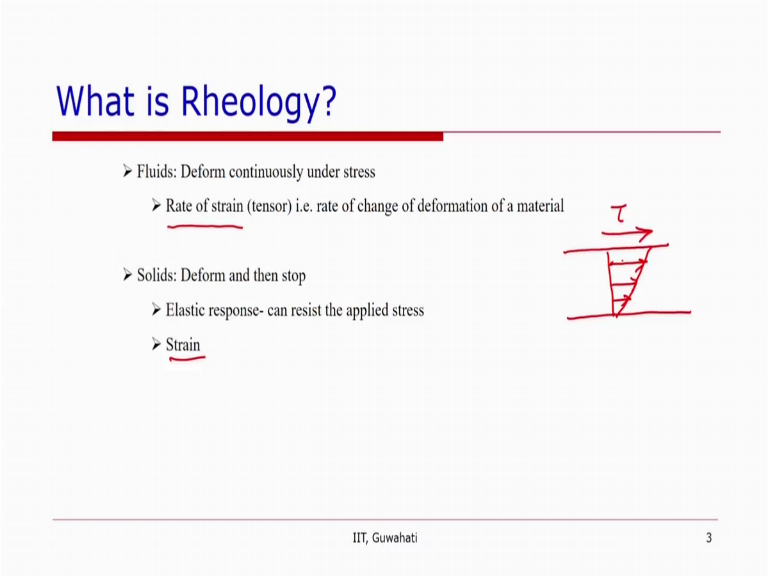


So, to understand rheology when we apply a stress or a force on a fluid, what happens, the fluid starts continuously deforming unlike a solid when we apply a force on a solid then what happens the solid deforms and then and then it is stop after sometime. Now, once the force has been removed from the solid then the solid may retain its position completely or partially. So, if the solid retains or comes back to its position completely, then it is called perfectly elastic solid or if it is partially comes back to its original position then it is called partially elastic solids. And the energy of deformation is stored as elastic energy in the solid, and then that energy is used for the solid to come back to its original position.

So, the solids they deform and then after sometime they stop. So, the change or the deformation the solid is non-dimensionally is termed as strain. Whereas, fluids they deform continuously, so a strain or a term strain which measures only the length or the or the dimension of deformation is not appropriate for fluids because fluids deforms continuously. So, what is used there is the measure of deformation in fluids is rate of a strain which is a tensor and it measures the rate of deformation of the material. So, you might have seen this image in many places when a force or a stress is applied, then the fluid have a velocity and that velocity profile is linear in case of Newtonian fluid.

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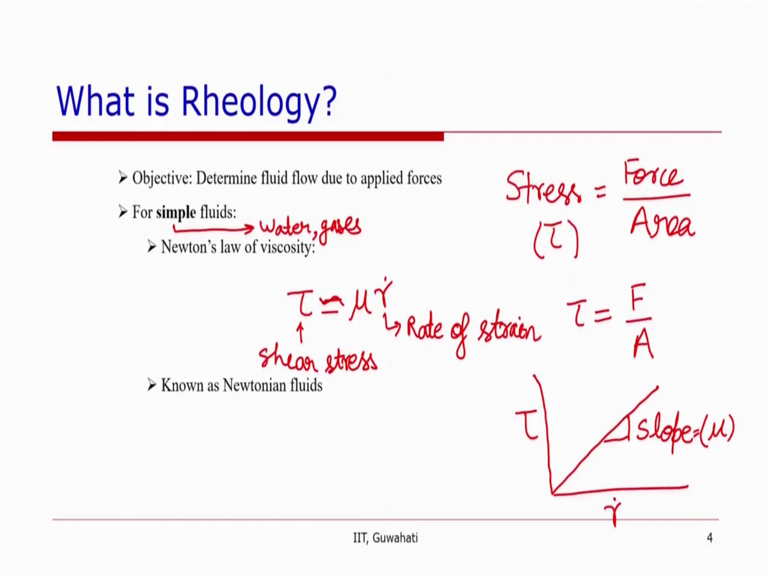


So, we will look at further what rheology is. So, the objective of rheology is that how does the fluid flow respond to the applied stresses or applied forces? As you know that stress is force per unit area. So, the objective of rheology is that how does the fluid stress is generally represented by the term tau? So, you can write this tau is equal to F over A. Now, the objective of rheology is to understand that how does a fluid respond to the applied forces. For simple fluids, for example, simple fluids can be water, gases etcetera which does not have large chain molecules or particles suspended in it, they are generally simple fluids. In such case Newton's law of viscosity is followed by the fluid.

So, Newton's law of viscosity is a relationship between a stress and strain. So, tau is proportional to strain. And when then equality is brought into it tau is equal to mu this is sorry this is not a strain this is rate of a strain. So, the shear stress is proportional to rate of a strain in case of a Newtonian fluid and we plot it on a graph stress verses strain you will get a linear relationship. And the slope is called viscosity. So, the fluids which follow this simple relationship between stress and strain are called Newtonian fluids. Now, one must remember that this is not coming from fundamental principle, but this is an empirical formula which comes which has come from observation and measurement of a stress and rate of a strain in a number of fluids. So, the relationship between shear stress and shear rate is linear in a Newtonian fluid, and the law is name[d]- is known as Newton's law viscosity.

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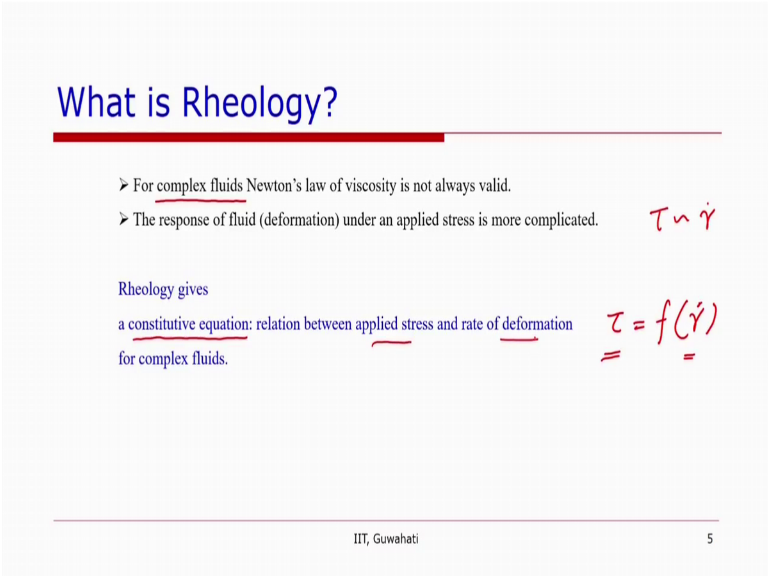


Now, what happens when the fluid is not simple? So, all non simple fluids can be put together under a term which is called complex fluids. So, what can be complex fluids, complex fluid, blood is an example of a complex fluid, because blood has number of particles suspended in it. And blood is further complex because the size of the particles, size of the molecules that are suspended in blood varies. It has long chain molecules such as proteins, anti coglo coagulating agents or the antibodies not antibodies and number of things which are suspended into it in the plasma plus particles red blood cells, white blood cells and platelets which are of the size of few microns. So, a few nanometres to few microns, the size of the molecule suspended in the blood, and they affect the rheology or the rheological behaviour of the blood significantly. So, blood is a complex fluid that is why we are studying this topic rheology ok.

Another example of complex fluids can be any suspension or any colloidal suspension. From chemical engineering perspective all the polymers behave as complex fluids, long chain polymer molecules are suspended in the liquid. So, they are all non-Newtonian or complex fluids. So, the fluids which do not behave or which do not follow Newton's law of viscosity are called non-Newtonian fluids and most of these fluids are complex fluids. So, the response of the fluid in such case is not simply linear that is it is not necessary that tau is proportional to rate of a strain or the shear stress is not directly proportional to the strain rate.

So, in such cases, it is important to understand the rheological behaviour or the a develop a constitutive equation between a stress and rate of a strain and that is the objective of rheology. And such equation, which is obtained from rheology which called Constitutive Equation. And this can be done using experiments which has been done for few last few centuries now. And now it is more and more also being done by molecular dynamic simulations where people are modelling the behaviour of molecules and then finding a constitutive relationship between stress shear stress and rate of strain. So, the objective of rheology is to finally, give a constitutive equation between applied stress and rate of deformation for complex fluids because we know that for simple fluid what is the law?

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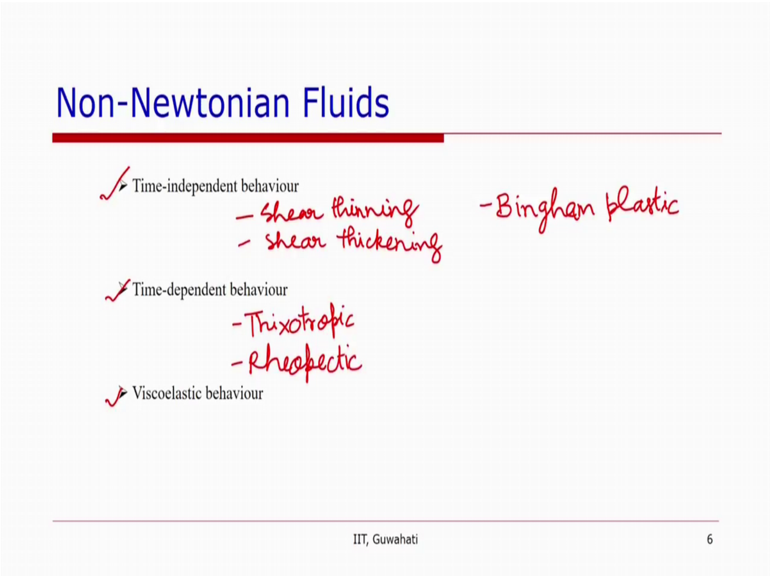


So, based on the behaviour of these complex or non-Newtonian fluids they can be divided into three different categories time independent behaviour, time dependent behaviour and viscoelasticity behaviour. Now, this category or categorization is only for the understanding purpose. So, we can demarcate different behaviours. It is really possible that a fluid a complex fluid shows only one type of behaviour. It may be a the fluid is showing all three kind of behaviour or it can show two different kind of behaviour. So, time dependent behaviour means that the response of the fluid or rheological behaviour of the fluid is independent of time. So, it does not change with time, some examples are shear thinning fluid ok.

So, the time dependent behaviour can be shear thinning or shear thickening or another example as I was telling Bingham plastic fluids. So, they show a non-linear kind of or non-Newtonian behaviour, but that does not depend on time, the fluid does not have a memory. Whereas, time dependent fluid the behaviour is time dependent, it depends on the time and the examples are thixotropic fluid and rheopectic fluids. So, it may happen that the shear thinning behaviour a fluid showing a shear thinning behaviour, but it is also time dependent.

In addition to that, the fluids also show viscoelastic behaviour, complex fluid also show viscoelastic behaviour and blood is one example. So, in this case, the fluids have viscous behaviour plus elastic behaviour. So, as we discussed some time back that the elastic behaviour that after the stress has been removed the material regains its original position is elastic behaviour. So, partially the fluid regains its original position. Now, this happens generally if the fluid is suspended with flexible particles for example, blood has red blood cells which are flexible disc shape particles and those particles when the shear stress is removed from them they regain their shape. So, because of that, the fluids also show viscoelastic behaviour some kind of elastic behaviour ok. So, we will look at time independent behaviour in this lecture.

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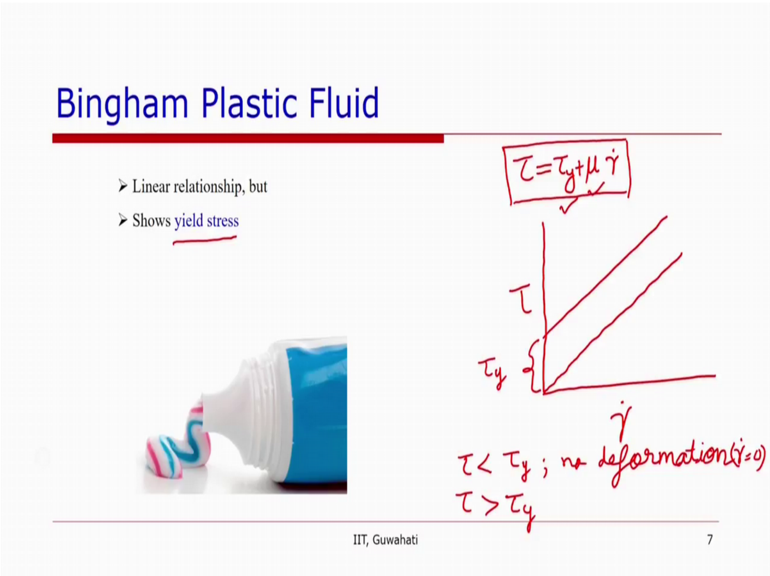


Ah One example is for first fluid that we will be looking at Bingham plastic fluid. So, Bingham plastic fluid, they still have the linear relationship between a stress and rate of a strain. So, we can say that tau is equal to tau y plus mu gamma dot. So, if we plot this on a stress versus rate of a strain plot, then we will get some this kind of behaviour this intercept is known as tau y or yield stress.

So, what does this mean physically, when we apply a stress on a fluid one of the examples of this is toothpaste. So, when we push the toothpaste for sometime the paste does not come out only after a certain or a critical amount of force, the paste starts coming out and that is because the when tau is less than tau y there is no deformation. And when tau is greater than tau y, so as you can see on this graph that gamma dot is equal to 0 for tau is less than tau y. When stress shear stress is less than yield stress then the deformation is 0; and when is greater than yield stress then it follows a behaviour like a Newtonian fluid. So, here we plot a Newtonian fluid with the same viscosity, it will be a line parallel, but passing through the origin. Whereas, in case of a Bingham fluid or Bingham plastic fluid, it will have certain amount of yield stress. And this is again a property that is shown by complex fluid which has particles suspended in it.

So, imagine a fluid which has number of particles which are which have taken a certain kind of shape in the fluid they do not move until a certain amount of forces applied on them. And when the force increases that critical force or when the stress increases beyond that critical stress, they reorient themselves and the fluid behaves as a Bingham plastic fluid. So, the Bingham plastic fluid the property is that they show yield stress. Blood also shows yield stress behaviour, but not exactly Bingham plastic behaviour they have a some non-linear relationship with it which we will show. So, again let me emphasise here that the relationship between a stress and a strain again is a empirical relationship or a model. And in this case there are two parameters in the model tau y which is yield stress, and viscosity of the fluid which is mu ok.

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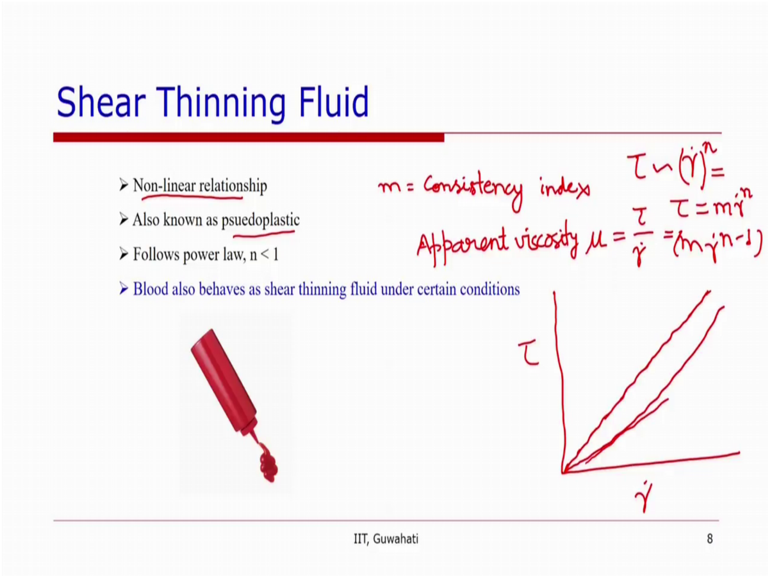


Ah For shear another behaviour is shear thinning fluid. So, as you see here they show non-linear relationship, so that means, tau is not proportional to directly proportional to gamma dot, but it is it follows a power law where n is less than 1. So, we plot it on a stress versus rate of a strain curve, it will still go through origin and this straight line represents a Newtonian fluid, the power law behaviour will be it is also known as pseudo plastic. So, the slope of the curve in this case is less than what for Newtonian fluid. So, the slope of curve will be you can also say this is apparent viscosity which will be defined as tau over gamma dot we introduce a equality constant here. Then tau will be equal to some m gamma dot to the power n, where m is called consistency index.

So, blood also behaves as shear thinning fluid under certain conditions. And they are a number of complex fluid which show shear thinning behaviour. They have a power law kind of behaviour, and n is less than 1. And this viscosity will be m gamma dot to the power n minus 1 and which is what this slope represents ok, sorry the image shown here is for tomato ketchup which also behaves as a shear thinning fluid.

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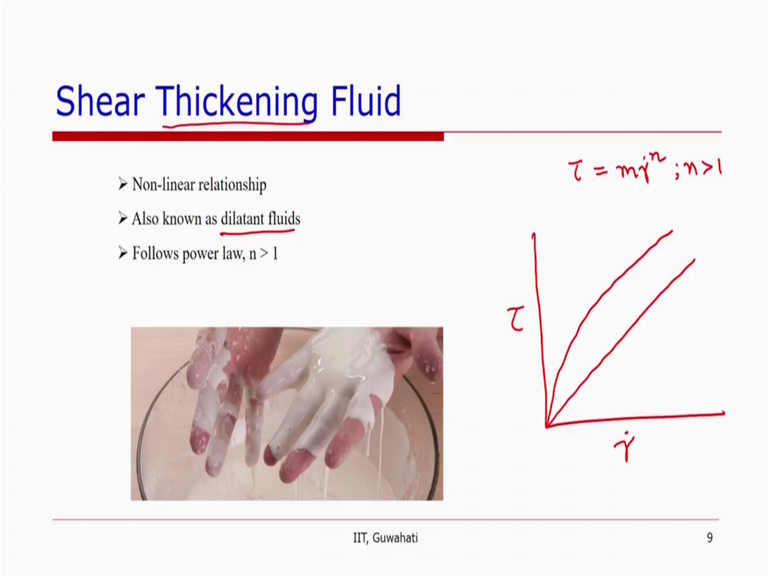


For shear thickening fluid as the name suggest then when the shear is applied or when the shear rate increases the viscosity of the fluid when it decreases then it is called shear thinning fluid. When the rate of shear increases and the viscosity increases, then it is called shear thickening fluid. So, when the fluid thickens because viscosity in general term or in general perception viscosity represents the that how difficult is it for a fluid to flow. So, if the it is difficult for the fluid to flow then we will say that this fluid is thin; if it is difficult to for the fluid to flow then it is thick if it is easy to for the fluid to flow then it is called thin fluid.

So, from that nomenclature, we can understand that shear thinning fluid, if the shear rate when the shear rate is increased and if the viscosity of the fluid is decreased then the that means, the fluid thins then it is called shear thinning fluid or pseudo plastic fluid. Whereas, when the shear is increased or rate of shear is increased and the viscosity of the fluid increases then it is called shear thickening fluid, they are also known as dilatant fluid. They again follow a power law kind of relationship tau is equal to m gamma dot to the power n, but n is greater than 1 ok. So, if we plot it on a stress versus rate of a strain curve, then now this is for Newtonian fluid and this is for the shear thickening or dilatant fluid.

One example is cornstarch, cornstarch and water solution. So, if you dissolve or if you make a solution of cornstarch in water and you apply the shear is a low shear or if you rotate it slowly then you will see that it flows, but if you increase the rate of shear if you rotate it fast then the fluid does not move at all. So, that is an example of shear thickening fluid. Another example of shear thickening fluids is quicksand.

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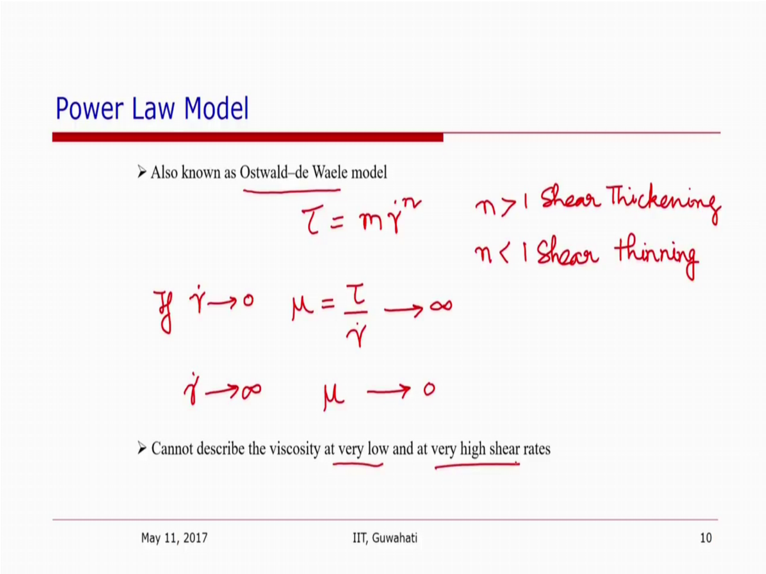


So, combining two we can give the power law model. And in the power law model, we can say again tau is equal to m gamma dot to the power n, where m is the consistency index, and n is greater than 1, then and n is less than 1. So, n is greater than 1, we called shear thickening fluids or shear thickening behaviour shown by the fluid; and is less than 1, and it is called shear thinning fluid. However, this in this model what happens that if shear stress is very low, then the viscosity or apparent viscosity of the fluid which is tau over gamma dot that will be approaching infinity that will be very high viscosity. Similarly, if shear rate is very high then viscosity will be approaching zero both of which are not correct representation of the behaviour often. So, some limiting behaviour is to be applied because this model power law which is given by Ostwald-de Waele which is also known as Ostwald-de Waele model, it cannot describe the viscosity at very low and very high shear rates.

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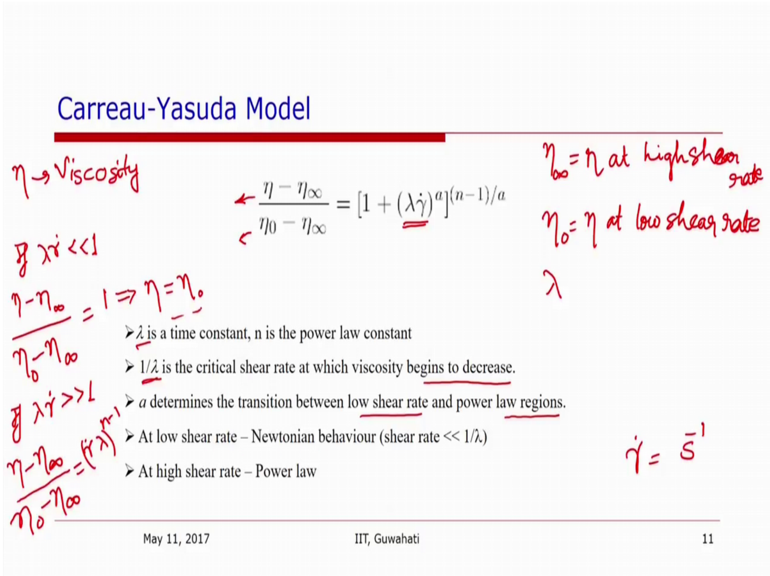


So, number of a corrections to this behaviour has been proposed number of models has been proposed to correct this anomaly in the behaviour of the power law model one of the a examples is Carreau-Yasuda model which is given by this formula. So, you see there that eta is non dimensionalized here. So, eta minus eta infinity where eta infinity viscosity, where eta is in this particular example eta is viscosity; and eta infinity eta at high shear rate viscosity at high shear rate. And eta naught is equal to eta at low shear rate. If you see the left hand side is non-dimensional viscosity on the numerator, you see the difference between the viscosity and the viscosity at high shear; and in the denominator, you see difference in the viscosity at low and high shear rates. And this is equal to 1 plus lambda shear rate lambda into shear rate to the power a total to the power n minus 1 over a. So, another new term here which is lambda?

So, you see here lambda you can easily identify or you can easily see from dimensional analysis that because shear rate has unit of time to the power minus 1 right because shear rate is what non-dimensional deformation per unit time. So, non-dimensional deformation no unit and time second so per unit time is second to the power minus 1. And lambda is a time constant because when we multiply this to be non-dimensionless sorry dimensionless then lambda is a lambda will have unit of time. And it is 1 over lambda is the critical shear rate at which viscosity starts to decrease. What is a, a this power, it determines the transition between low shear rate and when the power law region starts.

So, if you look at if shear rate is low, that means, let us say if lambda gamma dot is very very less than 1, then what will happen eta minus eta infinity divided by eta naught minus eta infinity is equal to 1 because this term will be negligible with respect to 1, so that means, that eta is equal to eta naught. And it is Newtonian behaviour with the viscosity of eta naught. Whereas at high shear rate if lambda gamma dot is very very larger than 1 then you can see that this term will go away or 1 will away or you can neglect 1 with respect to this term, and this will give you eta minus eta infinity is equal to ok. So, this model shows at low shear rate Newtonian behaviour as we saw here and at high shear rate and it shows power law behaviour.

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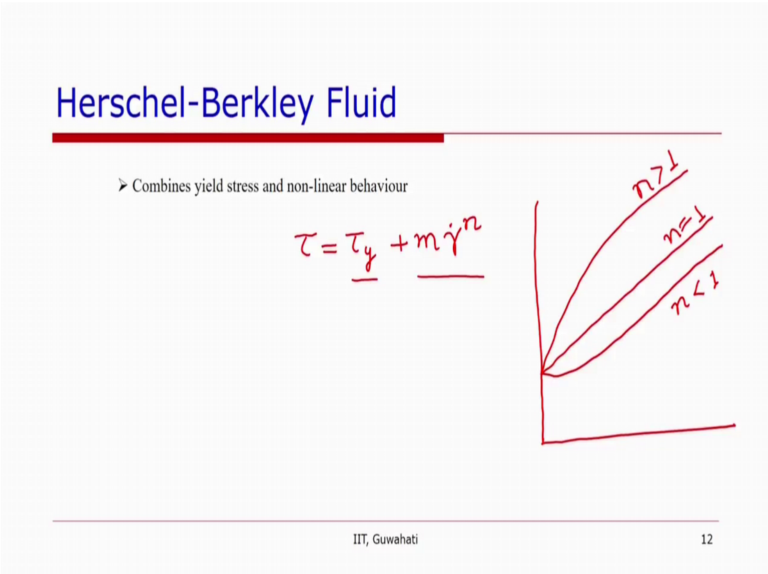


So, what we have done in power law model we have combined the behaviour power law model can represent or can show the behaviour of a power law fluid, it may be shear thinning or shear thickening. In the time independent fluids, we have shown may three kind of behaviour one is yield stress behaviour, another one is shear thickening behaviour, and the third one is shear thickening behaviour. Now, shear thickening and shear thinning behaviour can be combined in power law model and depending on that the power n is more than 1 then it is shear thinning behaviour. And sorry if the power n is greater than 1, then it is shear thickening behaviour; if the power is less than 1 then it is shear thinning behaviour.

Now, we can have a model which can combine the yield stress and the non-linear behaviour together then we can model a fluid which shows power law behaviour as well as yield stress behaviour and that can be done using Herschel-Berkley law or Herschel-Berkley model of for fluids. This is given as tau is equal to tau y plus m shear rate to the power n. So, here we have combined the yield stress as well as power law behaviour of the fluid ok, so that can be given as when n is equal to 1, when n is when n is greater than 1, and when n is less than 1 ok. So, this model is known as Herschel-Berkley fluid. So, if a fluid is showing Bingham plastic behaviour or n power law behaviour, then its behaviour can be given by Herschel-Berkley model.

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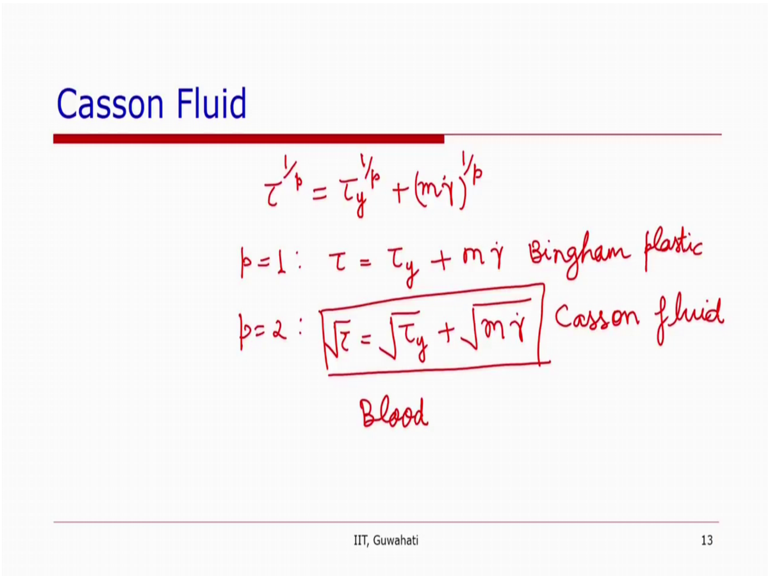


Then another model which is very similar to Herschel-Berkley's model that means, it combines the yield stress as well as power law behaviour is known as Casson fluid model. We look at it in general what it can be shown or it can be said that m is equal tau 1 to the power 1 over m is equal to or you can give this another power let us say tau to the power 1 over p is equal to tau y to the power 1 over p plus m gamma dot to the power one over p. Now, if we say that p is equal to 1, that means, we will get tau is equal to tau y plus m gamma dot which is for Bingham plastic fluid. Whereas, if p is equal to 2 then we will get root tau is equal to root tau y plus root of m gamma dot and this fluid is known as Casson fluid or this model as known as Casson model. And this often represents for the rheological behaviour of blood. So, in this for blood this p is equal to 2 and the fluid model is known as Casson fluid.

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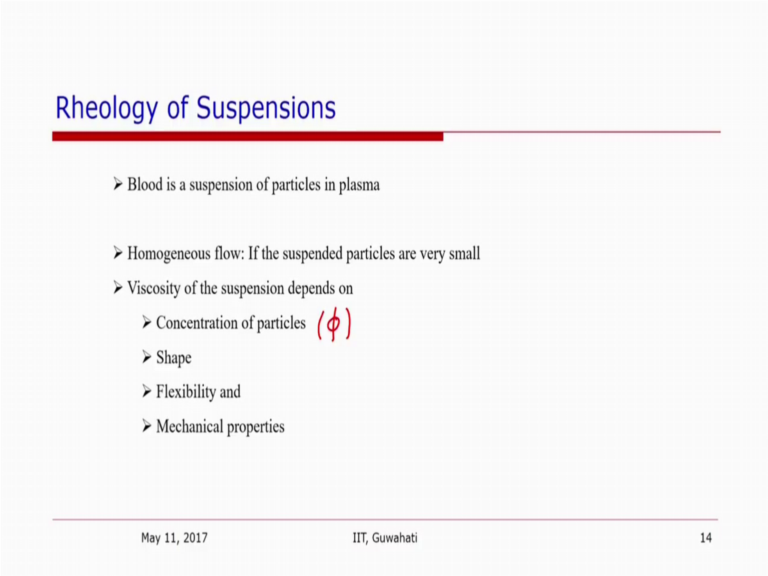


As we said earlier at the start of this lecture that blood is a suspension of different particles in fluid. So, it is important or it is useful to look at some of the a literature that describes the rheology of suspensions or rheology of different particles suspended in a fluid ok. So, blood is a suspension of different kind of particles in plasma. And if the volume fraction of these particles are is small as well as these particles are very small with respect to the length scale of the flow, that means, the say if the flow is happening in a channel in a artery in a tube, and if the particles are very small with respect to the channel dimension, then we can assume it to be homogeneous fluid.

So, homogeneous flow means the viscosity is same in all the dimension and the fluids fluid as well as the particles move with the same velocity ok. And the viscosity of such suspensions will depend on a number of factors which include the concentration or volume fraction of the particle let us say the volume fraction is given by phi. Shape of the particle particles may be spherical, may be ellipsoidal, or it may be disc shape which is the shape of RBCs flexibility of the particles, and the mechanical properties of the particles.

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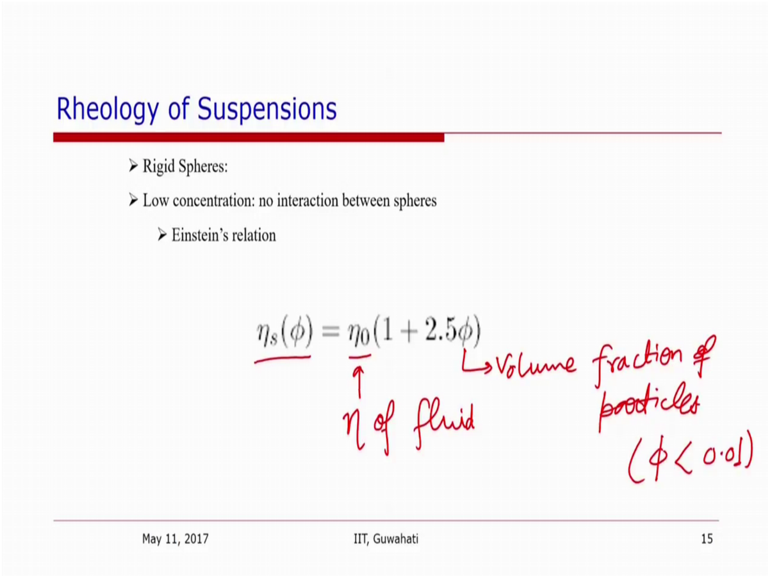


So, if we take a simple case of spheres which are rigid and the concentration of these spheres is very low is so low that we do not need to consider the interaction between the spheres. And for that, Einstein has given a relationship for the viscosity of the susp[ension]- sphere suspended viscosity of the fluid which has low concentration of the sphere suspended in the fluid. And this viscosity is given as eta s which is equal to eta naught. Eta naught is the viscosity of the fluid. And phi is volume fraction of particle. And this is valid for phi is less than 0.01. So, if the concentration of particles is very small then the viscosity of the particles can be given by this relationship. And as you can see from here that the viscosity of the suspension increases then the viscosity of the fluid.

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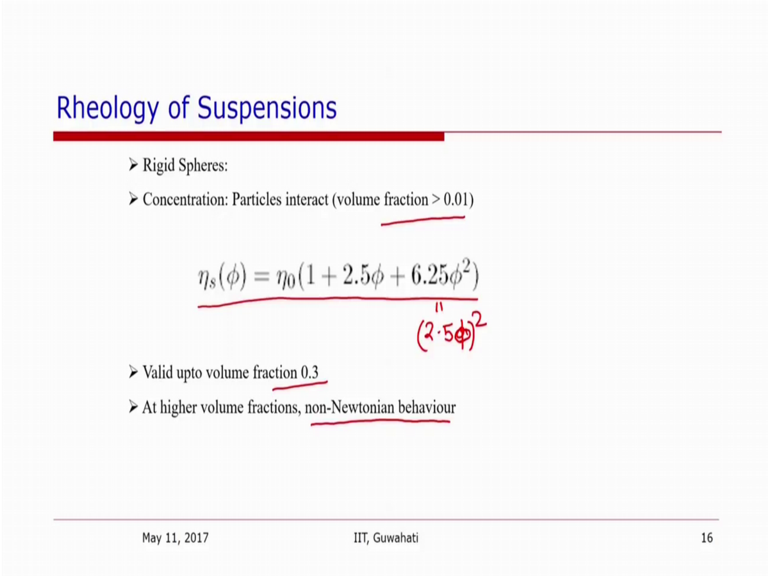


However, if the concentration of the particles is more than 0.01, and in such case particles interact then the viscosity of the particles is further increase which is given by eta is equal to eta naught plus 1 plus 2.5 phi plus 2.5 phi square you might notice that 6.25 is this is just square of 2.5 phi. And this relationship is valid for volume fraction up to 0.3. At higher volume fractions the fluid starts behaving as a non-Newtonian fluid; until volume fraction less than 0.3 phi is less than 0.3 the fluid behaves as a Newtonian fluid when the particles the susp[ended]- particles suspended sufficiently small than the channel dimension.

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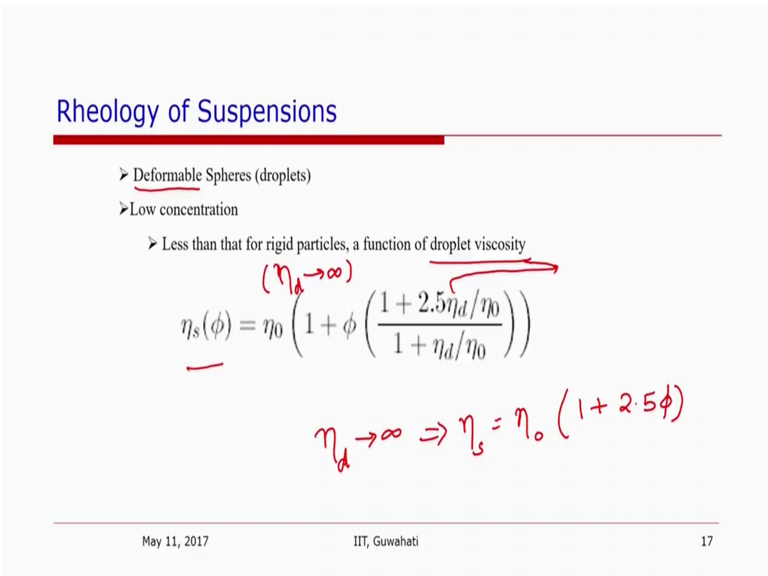

Now, if the particles are sphere, but deformable spheres for examples droplets because droplets will have spherical shape small droplets will have spherical shape because of the surface tension. And if their concentration is low so they show their behaviour or their viscosity can be given by this formula, you might notice where eta d is the viscosity of the droplet ok. When eta d tends to infinity that means, the particles are rigid then they will take in case of eta d is infinity then eta s will be is equal to eta naught into 1 plus 2.5 phi. So, they will come back to the relationship which we saw in the previous slides for rigid spheres ok. So, in such case, when the spheres are deformable, the viscosity is less than for the rigid particles because the particles can respond to the stresses and they can reorient themselves.

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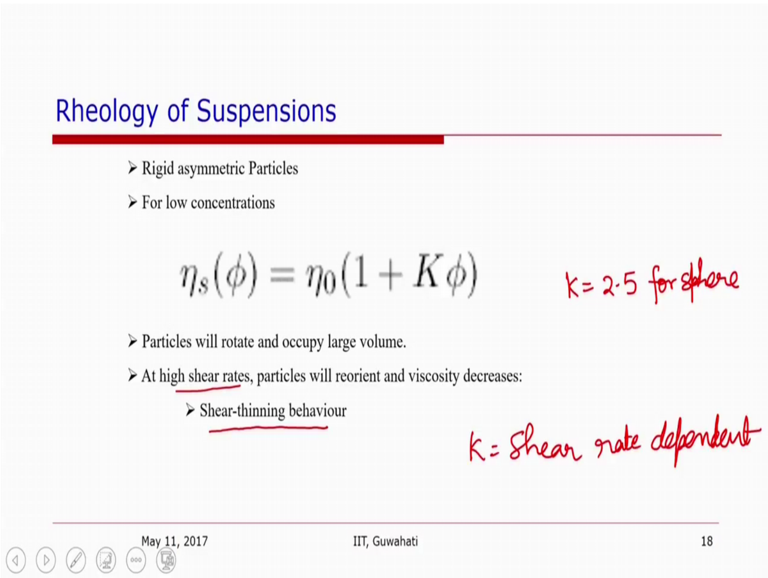


Now, if the particles are asymmetric, that means, they are not symmetric in all the directions which is say non spherical particles and for low concentrations their viscosity is given by eta naught 1 plus k phi. So, this k is equal to 2.5 if it is symmetric particles right, if it is sphere, then k is equal to 2.5. And this k will depend on different fluids the for different kind of particles it will have different number. In this case, what will happen the particle will also rotate and they may occupy larger volume, so the viscosity of the particle will increase? But if the shear rate is high, then particles will tend to orient along the direction of the fluid and the viscosity will decrease. So, they will show a shear thinning behaviour ok. So, at low concentrations you will see this kind of behaviour, but at the same time this value of k will change when the shear rate is increased. So, you can say that k is shear dependent and the particle show shear thinning behaviour for rigid asymmetric particles.

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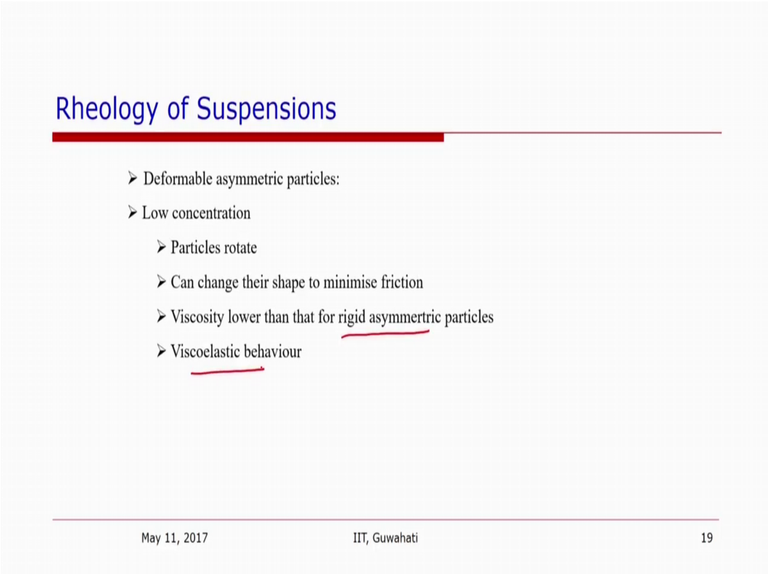


If the particles are deformable and asymmetric and the particles will rotate, they can change their shape, so that the friction is minimised. And viscosity for such case will be lower than for rigid asymmetric particles and because the particles can respond to the fluid. And if the shear rate is or if the shearing is removed then they can regain their shapes. So, they will also show viscoelastic behaviour ok.

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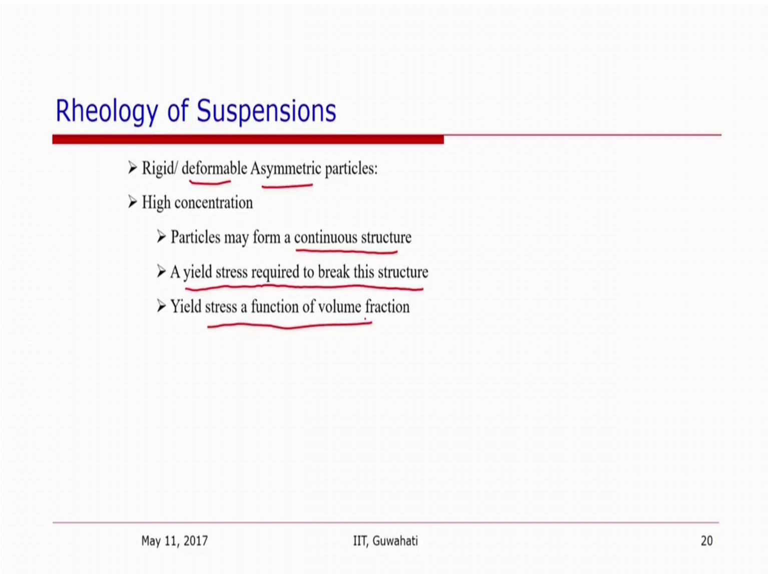

So, for rigid all deformable asymmetric particles, but when the concentration is high particles may form a continuous structure they will show a yield stress behaviour, so that when this structure is broken down and this yield stress is a function of volume fraction. So, blood has number of red blood cells or the volume fraction of red blood cell there is about 40 to 45 percent. So, they show this kind of behaviour which have deformable asymmetric particles RBCs are asymmetric particles, and they form a continuous structure and yield stress behaviour is shown by them, and this yield stress will be a function of the volume fraction of RBCs.

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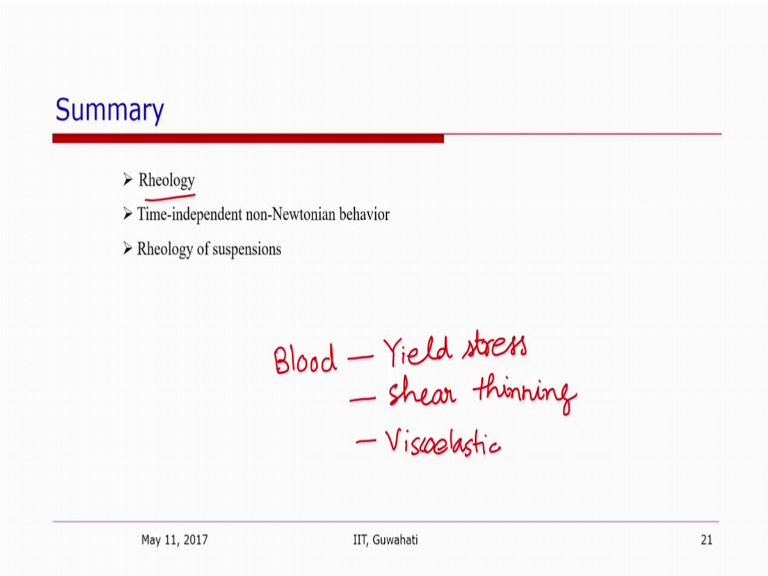


So, in summary, in this lecture what we have looked at it is the, what is rheology what does the term rheology mean, and what can be the different kind of behaviours shown by fluids. We looked at time dependent or time independent behaviour and viscoelastic, we categorized into three categories at the same time we said that any fluid can show only one or two or three kind of behaviour. Ah Then we looked at some of the time independent behaviours Bingham plastic, that means, the fluid which show yield stress, shear thinning where with the increase of the shear rate, the viscosity of the fluid decreases shear thickening where with the introduction of high shear rate introduction of the shear rate, the viscosity of the fluid increases. And we also looked at briefly the rheology of suspensions.

We looked at different rheological models or different models for non-Newtonian fluids and we said that for blood it shows yield stress. It also shows shear thinning behaviour. And because it is a suspension of particles and these particles are viscoelastic. So, it they also shows viscoelastic behaviour. So, it turns out that with all the components of the blood the behaviour of the blood changes. So, it is important to understand that what does the blood constitute and the properties of those different particles. So, in the next section, we will look at the morphology of the blood briefly ok.

Thank you.