Cardiovascular Fluid Mechanics
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Lecture – 02
Fluid Mechanics: A Review

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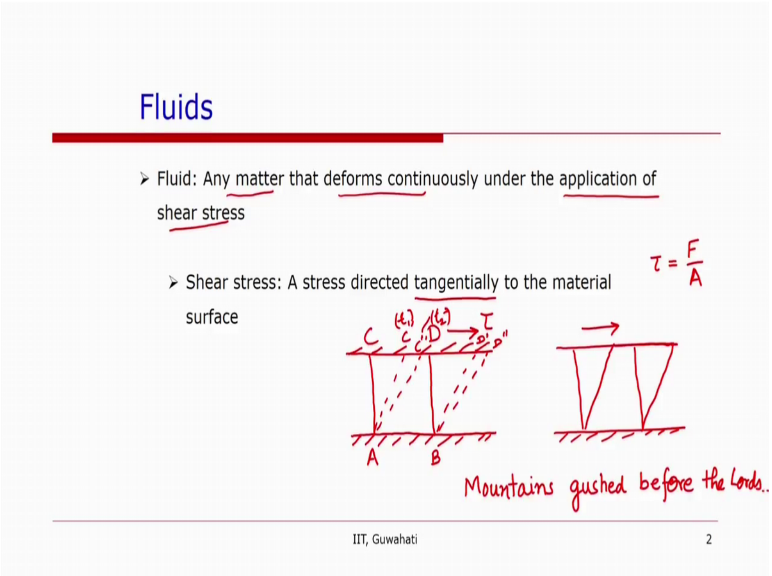

In this lecture, we will review some of the basics of fluid mechanics especially the once that will be required throughout this course. So, before looking into the fluid mechanics, we need to know or we need to remind ourselves what does a fluid mean or what constitutes if fluid or what is the definition of a fluid. So, if you go back to your early childhood, you would have learned definitions of liquid and gases which are both fluids. So, they are the materials you might have learned the definition that the materials which takes the form of the vessel in which they are contain or which takes the shape of the vessel in which they are contained or are called fluids.

The another definition or a more complete definition for the fluids is that any material or any matter that when a shear stress is applied on the fluid, so under the application of a shear stress, if a material deforms continuously then it is known as the fluid. So, for example, let us consider a fluid material that is confined between two plates which are kept parallel. And in this consider a fluid element or a small fluid volume and let us name that as A, B, C and D. Now, if a shear stress is applied on this plate, the shear stress let us remind ourselves that shear stress is a stress that is directed tangentially to the material surface. So, a stress is force per unit area and shear stress is the one that is directed tangentially to the surface.

So, a tangential force is applied on this surface, and because of this the material will deform to a location let us say these locations are C dash and D dash. If you look at this after sometime then the material further will have located to another location C double dash and D double dash. So, because of the application of a shear stress, a shear rate or the material shear, so the material deforms keep deforming continuously. So, that is why at two different time instant, so let us say that C dash is at time t; and C double dash is t plus this is t 1 and this is t 2. So, the material deforms continuously under the application of a shear stress.

Now, if we want to differentiate or we want to remind ourselves that how is fluid different from a solid then under the application of a shear, the solid material will deform. So, there will be a deformation, but it will not deform continuously there will be with the application of force there will be a certain deformation of the fluid, but it will not keep deforming continuously. Whereas, in the fluid that deforms continuously under the application of a shear stress, so the fluid they have the properties of flow or they flow under the application of a driving force, whereas, solids do not.

Now, philosophically speaking or depending on the time scales everything flows. So, there is a famous verse in the bible in the song of Debora that says that the mountain gushed before the Lords. So, that means, that the mountains flowed before the lords, but not before the man. So, what does that mean that means, that even the mountains which are considered to be solid they also flow, but not at the time scale of few hundred or at the time scale of hundred years which is the life of a man. But the god who is considered to live forever in his untimed scale, the mountains flow they change shape so even.

So, every material does flow, but depending on the time scale we may or may not perceive or experience that this material is flowing, anyway. The materials that we are considering at this course because this course is concerned with a cardiovascular fluid mechanics where blood or plasma is the fluid that we are dealing with so the plasma or blood is a fluid in any case.

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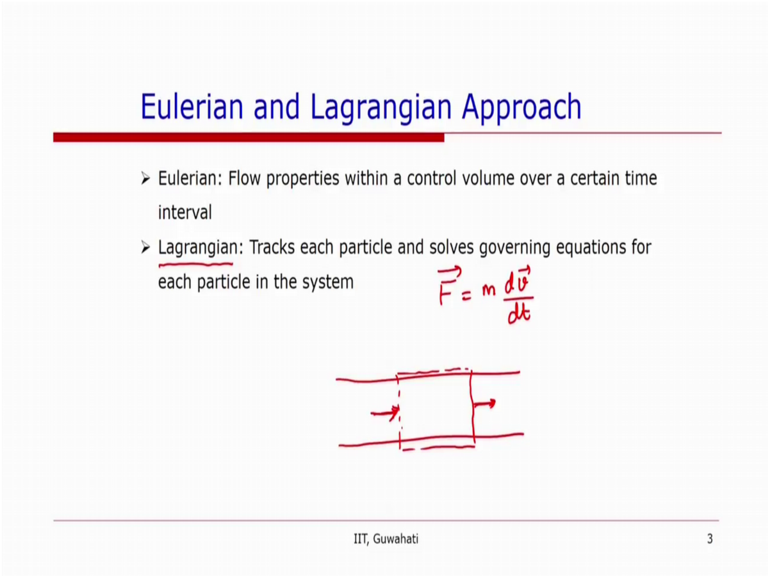

So, another fundamental thing that we would like to revise or we would like to look up on is Euler and Lagrangian approach. So, there can be a different approach to analyse the problems in engineering mechanics. In fluid mechanics, in general two approaches are very popular the first one what we call is Lagrangian approach. So, in the Lagrangian approach, a particular mass of the fluid or particular fluid particles they are tracked and the governing equation for the each particle can be solved by say Newton's second law of motion that F is equal to m d v by d t . So, this is what we call Lagrangian approach.

Or another approach is Eulerian approach. So, in the Eulerian approach, what happens that one considers a control volume and look at the flow that is coming in the control volume and going out the control volume. So, he is concerned or in the Eulerian approach we are concerned with the flow that is there in the control volume. So, the fluid that comes in, fluid goes out and the fluid that is there in the control volume.

So, the flow properties such as density, viscosity and the velocity and pressure are studied in this control volume. And because the fluid flows, so the fluid particles that are there in the control volume at time t may not be there at time t plus delta t. But irrespective of that we look at the fluid particles that are inside the control volume; and we do not follow a the entire follow the same particles for the entire time of study for which the analysis is being made for the entire time period for which the analysis is being made.

So, one example which we generally look at the considering the flow of a boat, and you have a consider that you and your friend, so your friend is going on the boat and crossing the river. So, if he looks at the motion and that is that if you track the motion of the friend in his reference frame, so he is looking at that is if you follow the motion of the friend or motion of a particular boat while crossing the river, then it is the Lagrangian frame of reference or because you are tracking one particular boat.

On the other hand, if you consider a certain area in a river and look at what is coming in and what is going out in that certain area or a certain volume if you consider the depth also then that is Eulerian analysis. So, coming to the cardiovascular fluid mechanics, we will not be looking at in most of the cases the Lagrangian approach or rather we will take the Eulerian approach, so we will consider a control volume and look at the blood coming and going out from this control volume.

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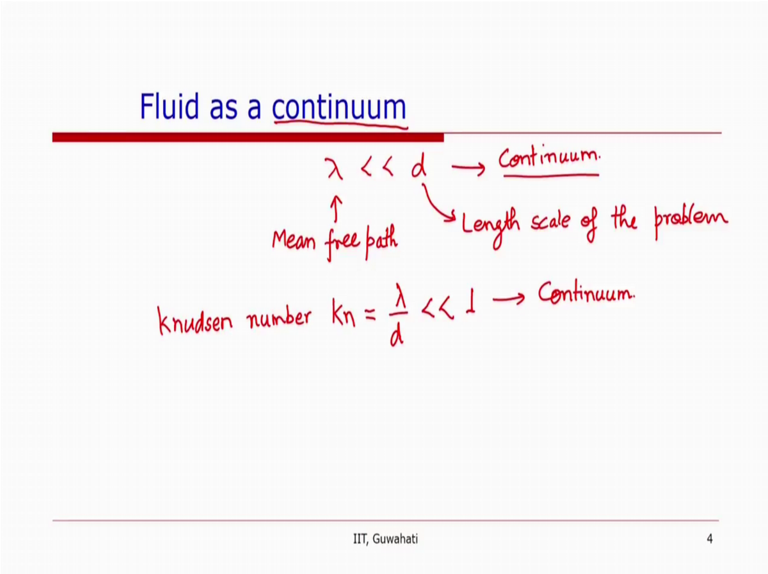

So, another important approximation or another important assumption that is made that fluid is a continuum. We all know that each and every material is constituted of molecules and atoms which are particles of very small size. So, there is always some space between those atoms. Now, depending on the length scale, we can say that if lambda is very, very small than d then flow can be considered or the fluid can be considered as continuum. So, we can describe the fluid as a continuous medium or as a continuous field on the properties of the fluid such as pressure and velocity, they can be considered as a field.

So, what is lambda? Lambda is the typical distance between the molecules or the fluid molecules, so the mean free path of the molecules for a fluid is lambda; and d is the length scale of the problem that you are considering. So, there is a non-dimensional number associated with it what is known as Knudsen number. So, K n is equal to lambda by d. Now, if Knudsen number is small than 1, then fluid can be consider as continuum and we can describe this as a continuum.

So, all the discussion that we will have in this course we can assume the fluid to be continuum, because the mean free path is order of few nanometres; and the smallest scale that we will be talking about is of the order of few microns. So, we can safely consider the fluid to be a continuum or the blood to be a continuum.

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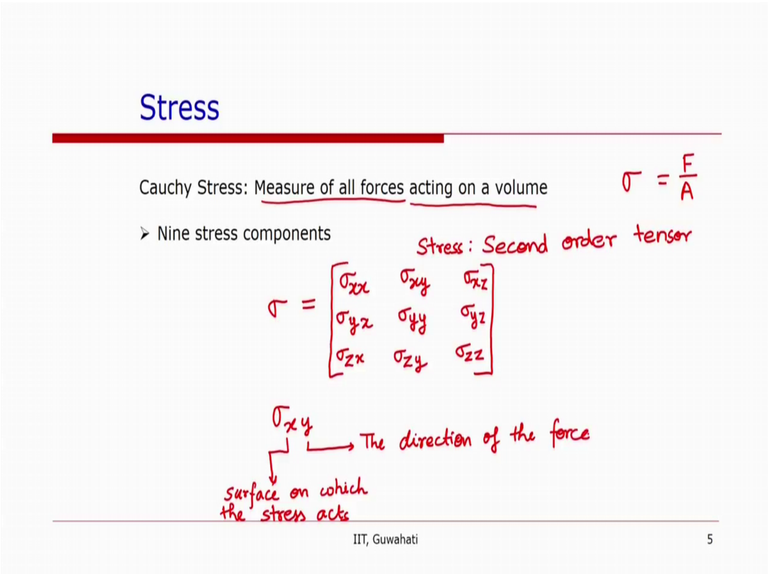

Now, we will continuously deal with the stresses in fluids as we just while defining fluid we say that the under the application of a sheerest stress the fluid which deform continuously or the material that deform continuously as known as fluid. So, we need to also define stress. So, stress is the measure of all forces that is acting on a volume. So, if we define stress as sigma, sigma is equal to F over A - a force per unit area. Now, stresses will have a nine components; it is a second order tensor.

So, we are not going to cover the details of vectors and tensors in this, but it is strongly recommended that you read a bit about the vectors and tensors or any student of fluid mechanics should have a good idea about of vectors and tensors. So, they will have a will write down this as a matrix sigma x x, sigma x y, sigma x z, sigma y x, sigma y y, sigma y z, sigma z x, sigma z y and sigma z z. So, if you look at a typical component of a stress, there are two subscript into it x and y. So, x is x denotes the surface on which the stress acts; and y denotes the direction of the force.

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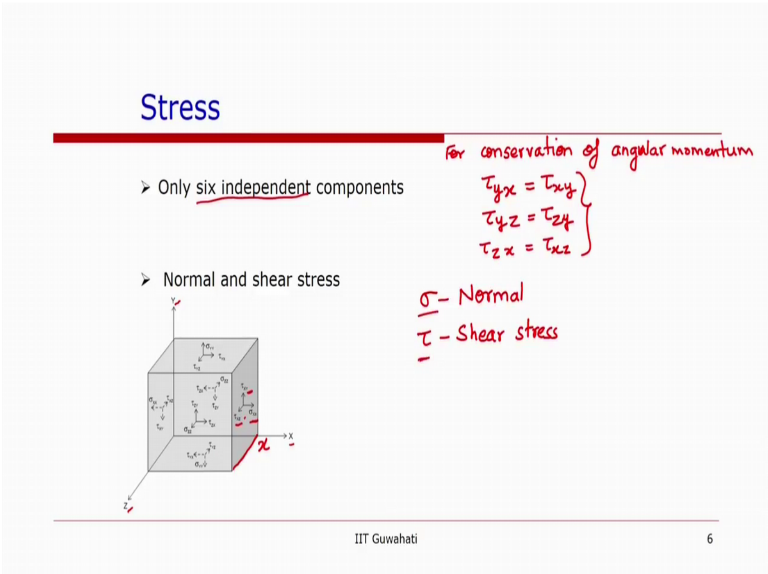

Now, in this representation, where the on a cubic element different components of a stresses are shown. So, it is in Cartesian coordinate system x, y and z. So, let us take one component here on this surface, you can see that sigma x x, so sigma has been represented sigma represents here normal stress, and tau is denoted as shear stress that is the normal convention that you will see in a number of books. So, in this case on this surface which is an x surface having area vector a normal to it.

And the normal stress is sigma x x; and it has on this x surface the force acting in the y direction on the extra setting in the y direction is tau x y. Similarly, the force acting in the z direction is tau x z. So, there are two shear stress components acting on this surface, and one normal stress component. Similarly, on the other x component and you can see the same thing for the other components as well.

Now, for conservation of angular momentum, it is necessary that tau y x is equal to tau x y. Similarly, tau y z is equal to tau z y. And tau z x is equal to tau x z, so that means, that there are only out of a nine components of a stress there are only six stresses that are independent. So, that is just recapitulate that stresses are some of the, or they are the measure of the forces that act on the surface. Now, pressure is a normal stress that acts normal to a surface.

And in the Navier-Stokes equation or in the momentum conservation equation, it is taking out taken out or sometimes in computational fluid dynamic applications when we are looking at the stresses the pressure is clubbed with the normal stresses. So, one need to take this into account or one need to keep this into keep this in mind. So, then the forces they can be on they can be recomposed into normal and the shear forces. So, the normal forces are represented here by sigma, and shear forces are represented by tau.

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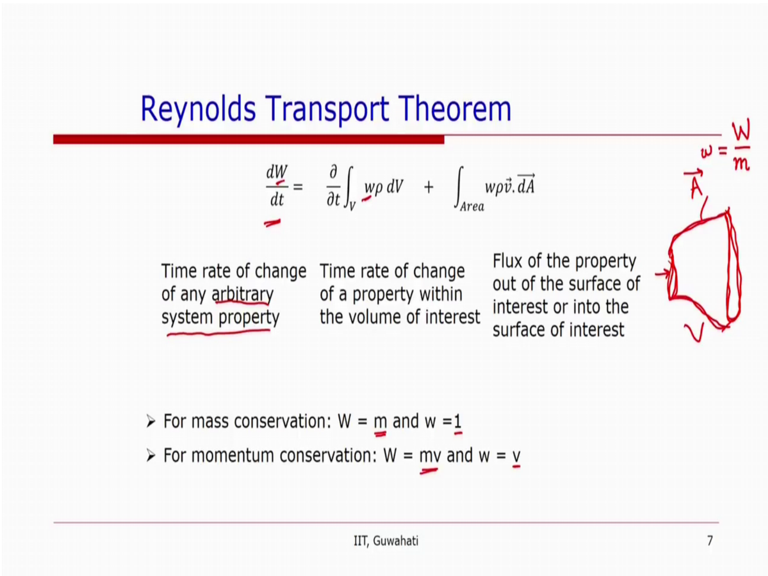

Now, before we discuss the two fundamental conservation principles based on which we can calculate the unknown or the unknown quantities in fluid mechanics, the pressure and velocities. Let us look at the general conservation principle which is known as Reynolds transport theorem we are not going to derive this in this short course, but it is strongly recommended that till you follow any standard fluid mechanics textbook and look at the derivation of Reynolds transport theorem. So, what does Reynolds transport theorem states let us say W, W is any property of the system any arbitrary property of the system and it says that the time rate of change of any such system property. So, the rate of change of time, so how this property changed with time that is equal to the time rate of change of a property within the volume of the interest, so how does this property. Now, you have two w here one is capital W and one is small w.

So, capital W is the arbitrary system property or the any extensive property of the system and small W is the intensive property of the system. So, small w is capital W per unit mass or specific properties, so that is del over del t. So, that is time rate of change of a property within this volume of interest. So, you can consider any control volume, let us say the volume v and the boundary of this is represented by this area. So, this is A and A is area is a vector, you must remember that area is a vector and its direction is outward normal to the surface now of which area you are considering. So, the rate of change of any arbitrary system property is equal to time rate of change of a property within the volume of interest plus the flux of the property.

So, the flux that is coming in flux of the property out of the surface of interest or into the surface of interest so, the total integral of the flux that is coming in or going out would that integral combine. So that says the rate of change of any arbitrary property in a control volume will be equal to that rate can be effected because of two factors. One is that the rate of the property changes within the volume itself or the property is brought in the system or it goes out of the system, so that makes quite sense say in general, but it is also important to describe this mathematically and understand this mathematically.

So, if we consider that this property W is mass of the system, then the small w the intensive property become 1. And if we consider the property as momentum of the system then w becomes the velocity. So, based on this one can derive the mass conservation using a Reynolds transport theorem, one can derive the mass conservation equation and momentum conservation equation. And we might have one of this assignment problem, but we are not going to derive it here.

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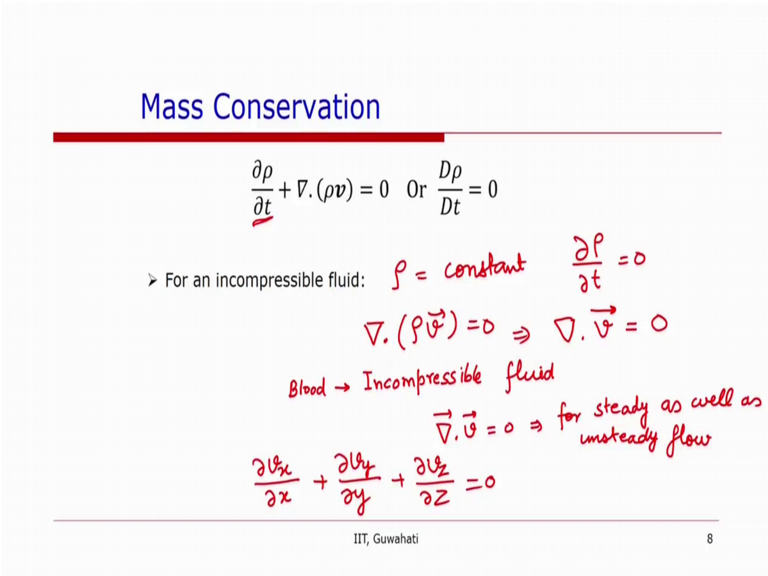

So, the mass conservation principle states that del rho over del t plus del dot rho v is equal to 0. For an incompressible fluid, so if you look at this del rho over del t is rate of change of density. So, for an incompressible fluid the density is constant that means del rho over del t is equal to 0. So, we have what we have is del dot rho v is equal to 0 because rho is constant. So, we can write that del dot v is equal to 0 for an incompressible fluid. Now, when blood is an incompressible fluid generally at the atmospheric temperature all the liquids can be treated as incompressible fluid. So, most of the time in this course; we will using only this as a mass conservation fluid.

Now, when it comes that the flow is not steady even then the density does not change with time. So, del dot v is equal to 0 is good for steady as well as unsteady flow, because rho is anyway constant with time. Now, if you want to expand this, then del is you can write del v x over del x plus del v y over del y plus del v z over del z is equal to 0. For a one-dimensional equation you will have the equation as del v x over del x is equal to 0 and so on so forth so that is mass conservation equation.

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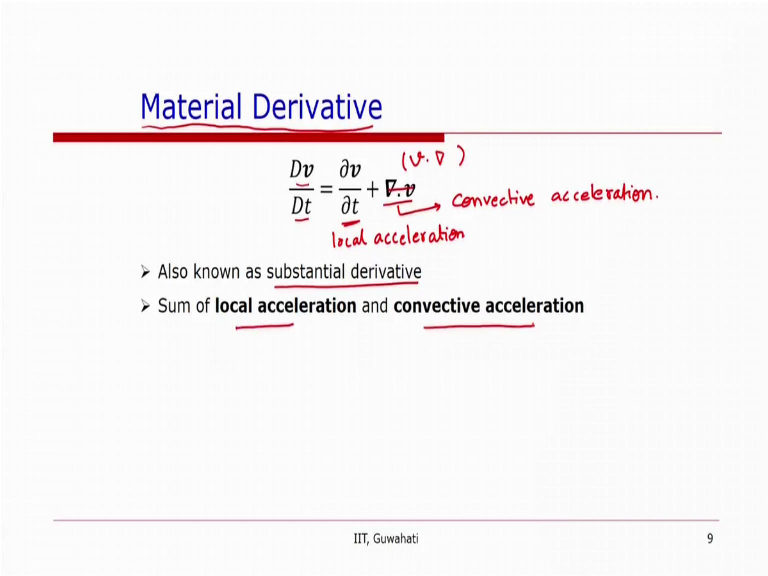

Now, before we go to describe a momentum conservation equation I would like to bring your attention to this term what is called material derivative or it is also known as substantial derivative. So, that has two terms into here into it that d v over d t capital D over D t is known as material date derivative or substantial derivative. So, the first term del v over del t is known as local acceleration and del dot v is actually it should have been a v dot del.

So, that is called so far the in partial differential equation. If you have any property that depends on time as well as Cartesian coordinate, then it is a derivative the total derivative will be a with respect to del v over del t with respect to time as well as with respect to the Cartesian coordinates. So, considering the local acceleration and convective acceleration one gets the material derivative.

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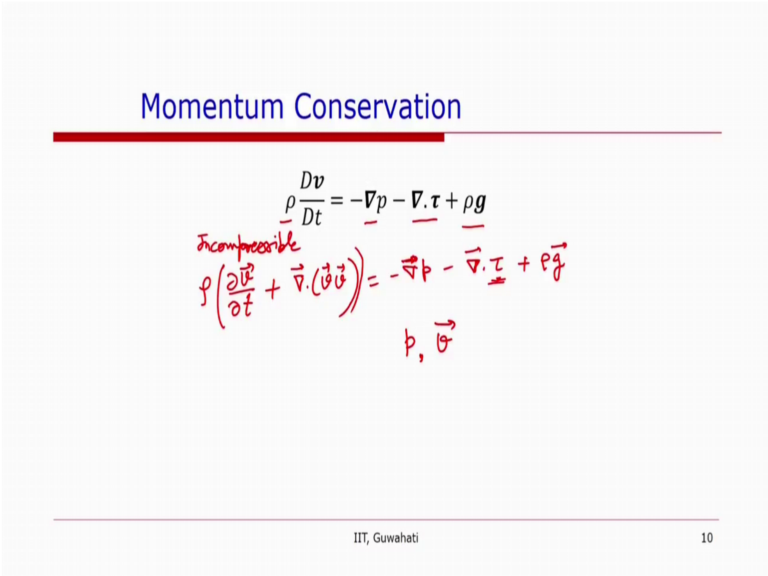

So, the momentum conservation can be written as rho D v by D t is equal to minus del p minus del dot tau plus rho z rho g. Now, this is we can expand. So, this when we write rho out of it, then this means that flow is incompressible, we can write that rho del v over del t plus del dot v v in the vector form is equal to minus del p minus del dot tau plus rho g if the gravity is considered in the system. So, you can see that all the terms are vector in this. And it is quite clear from here that this v is vector, g is a vector. The gradient of pressure is a scalar, but the gradient of pressure will be a vector, but del dot tau is a second order tensor and its del dot product with del will result in first order tensor; that means, it is also a vector. Similarly, v v, it is dyadic product and it will be a second order tensor, but there product will also be a first order tensor. So, it is a vector.

Now, in this so we have looked at the two conservation equation mass conservation and momentum conservation equation. And in general what we will be concerned with that we need to know the pressure field or the pressure distribution and the velocity distribution. And actually if we want to know only velocity distribution that to know velocity distribution, we need to know pressure distribution or other way around.

So, this is the momentum conservation equation. Now, in this what we need is we have two unknown pressure and velocity. So, if we look at pressure one unknown and velocity as a vector so two unknowns. And we have two equations one is a mass conservation equation, and another is momentum conservation equation. So, in principle using these conservation equations and the appropriate boundary condition, one can solve for the fluids, but we have a unknown here or we have a new term here which is tau shear stress. So, before we solve these equations, we need to close tau or we need to find a relationship for tau in terms of velocities which is called constitutive equation and that comes from rheology or and one of the most common example for many fluids is what is known as Newton's law of viscosity.

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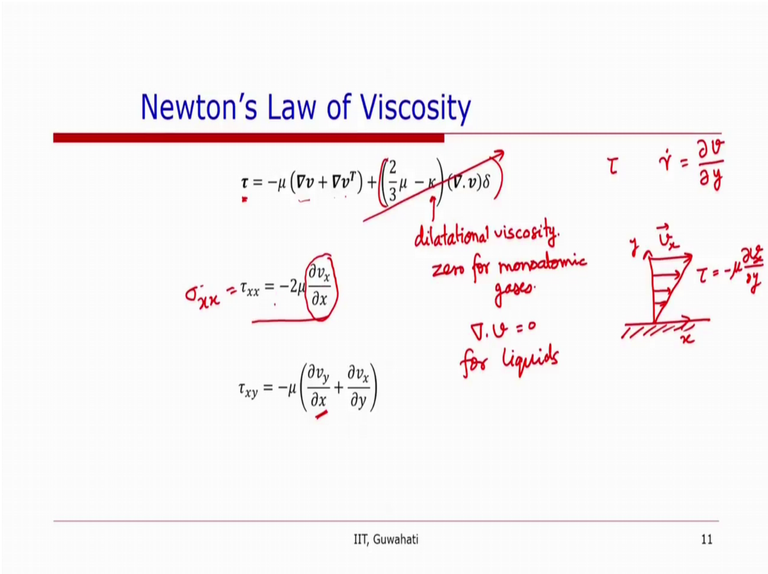

So, Newton's law of viscosity relates tau and shear rate which is sometimes represented as gamma dot or one can say that for one-dimensional flow del v over del y. So, if over a surface, there is a flow then the shear stress tau can be written as minus mu del v this is x direction and this is y direction del v x over del y. Now, if we want to generalise this. So, the generalised Newton's law of viscosity is given as tau or the stress tensor is equal to minus mu del v plus the transpose of velocity gradient plus 2 by 3 mu minus kappa del dot v del.

Now, mu is the dynamic viscosity of the fluid, and kappa is called dilatational viscosity which is often zero for monatomic gases. And we have just seen that del dot v is equal to 0 for liquids in general which because the liquid is incompressible fluid. So, this term is often zero in the fluid mechanics and at least the problems that we are going to consider for the flow of liquids because the blood is a liquid. So, this term is going to be 0. So, looking at the say some components of stresses tau x x or if you want to see say this in the normal stress term, then it can be sigma x x is equal to minus 2 mu del v x over del x and tau x y is equal to minus mu del v x over del v y over del x plus del v x over del y.

What I would like to bring your attention here that the viscous stresses can be normal stress also; it is a different matter because the del v x over del x term is often smaller than the del v y over del x term. So, the gradient in the same direction are smaller than in the transfers direction, so that is why tau x x or del the normal stresses components are often neglected in our analysis as we will see in the later classes.

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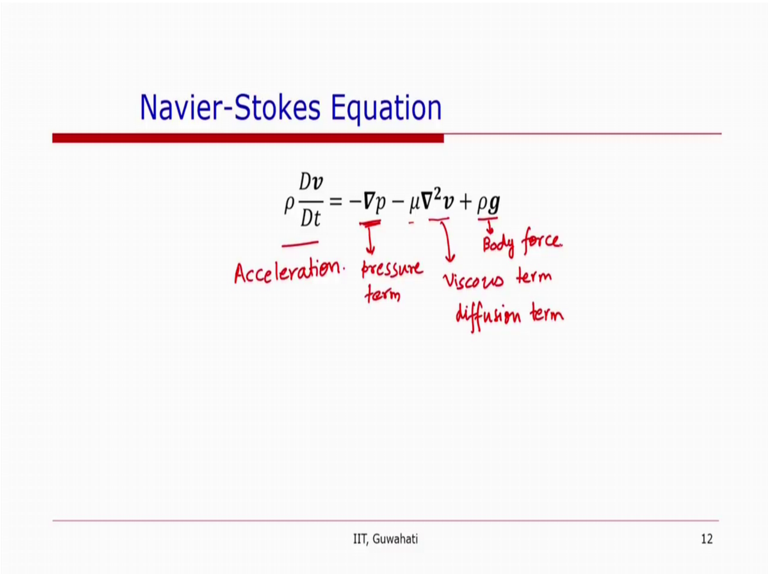

So, if we substitute this Newton's law of viscosity, and then we will end up with rho d v by d t minus del p minus mu del 2 v plus rho g here. So, this is Navier-Stokes equation. And the first term is known as the acceleration term, and it has combination as we have just for the definition of material derivative. It is the combination of local acceleration which is with respect to time and the convective acceleration. And del p is the the pressure gradient or you can say it is a pressure term.

This is called viscous term. And as we know that viscosity is also diffusivity or it is momentum diffusivity because of the viscous properties of the fluid the momentum is diffused in the fluids or between two fluids layer. So, it is also in general called it can be known as diffusion term and any other forces whereas the gravity is the general body force, gravity is the usual body force, but one can also have other body forces here. So, this is the body force.

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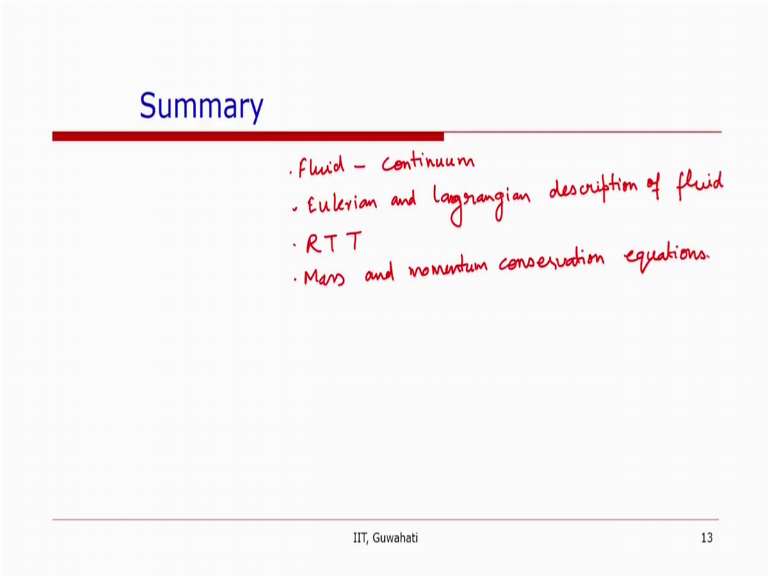

So, in summary what we have looked at today is a some basics of fluid mechanics which are prerequisites or which are require to understand; the problems that we solve later in the course for the fluid mechanics course. We will also look at some basics of solid mechanics in a small lecture. So, what we have looked at today is that what is a fluid? And fluid can be treated as continuum then we have looked at very briefly Eulerian and Lagrangian description of fluid, then we have also looked at the Reynolds transport theorem and mass and momentum conservation equations.

So, thank you.