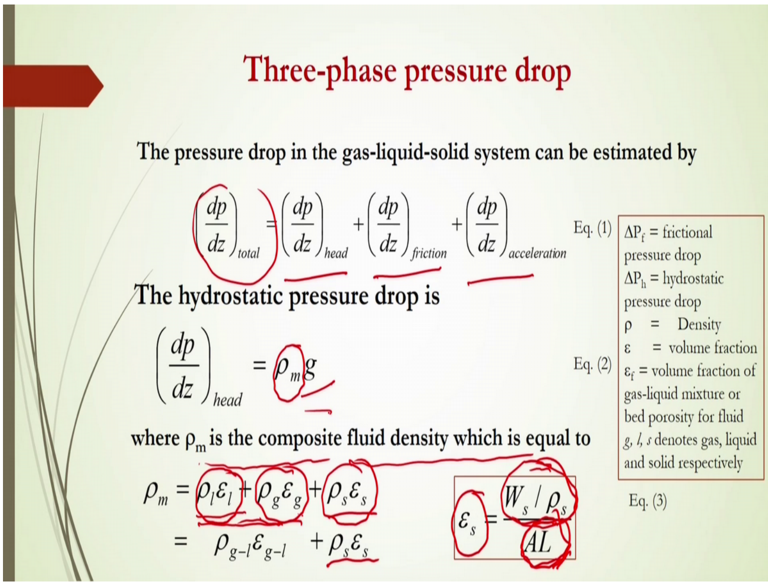
Fluidization Engineering
Dr. Subrata K. Majumder
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture – 09
Frictional pressure drop in fluidized Bed: Gas-liquid-solid system

Welcome to this massive open online course on fluoridation engineering. Today's lecture will be on frictional pressure drop in fluidized bed in gas liquid solid system. So, in earlier lectures we have discussed to the frictional pressure drop in fluidized bed system with gas solid and liquid solid system. So, in this lecture 3 phase system will be discussed whatever the frictional pressure drop, based on the gas liquid solid movement; that will be discussed in this lecture.

(Refer Slide Time: 01:05)



So, we know that that frictional pressure drop in any system have 3 parts. One is, that is frictional due to the liquid solid interaction. Frictional due to the solid wall interaction, and frictional due to the liquid wall interaction. And with this 3-fictional pressure drop of course, that the main contribution of this frictional pressure drop due to this interaction. Other than this interaction 3 phase fluidized system also have other 2 parts that is head pressure drop and acceleration pressure drop.

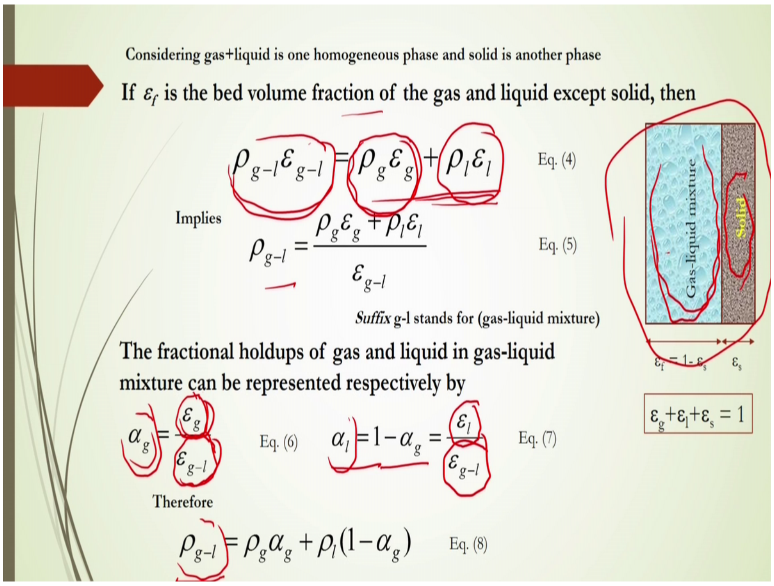
So, total pressure drop in this gas liquid solid system can be estimated by this total pressure drop that will be is equal to pressure drop due to the fluid head, and pressure drop due to the frictional pressure drop that is friction between gas solid friction between liquid solid friction between wall a solid and friction between gas solid and liquid wall. So, there are other questions like acceleration pressure drop whenever the phase will be moving inside the bed with a certain velocity if there is a acceleration of this movement of the liquid; that means, they are velocity gradient will be there or radial direction there will be a change of velocity or fluctuation of this velocity because of which some development of the pressure drop inside the bed. And so, total pressure drop will be equal to head pressure drop frictional pressure drop and acceleration pressure drop.

Now, what is that? Head pressure drop or you can say this can be written as a hydrostatic pressure drop also. Now this hydrostatic pressure drop is nothing but what should be the pressure drop due to the gravity of the mixture of the fluid. Now this gravitational force of the mixture of the fluid this mixture of the fluid will be denoted by rho m here rho m is the mixture of mixture density of the fluid and g is the gravitational acceleration. So, where this rho m is the composite fluid density of 3 phases which is equal to rho m that will be is equal to rho l into epsilon l plus rho g into epsilon g plus rho s into epsilon g. This is nothing but the portion of liquid this is nothing but the portion of gas this is nothing but the portion of solid here. So, this will be the total what will be that mass of the gas liquid solid out of the total volume, though this can be represented by this.

So, this epsilon l epsilon g and epsilon s are the volume fraction of the liquid gas and solid respectively. Now if we consider that only liquid and solid or liquid and gas or solid and gas, if we divide it into 2 parts I by combining either of the way; so here if we consider that this liquid and gas as one phase. So, in that case what should be the density of that phase or what should be the portion of the density for those that would be rho s rho g l; that means, the density of the gas liquid and what will be the volume fraction of that gas liquid mixture there that will be represented by epsilon g l. And here this one this called rho s epsilon s is nothing but the what is that what will be the density of the solid inside the bed.

And here epsilon s you can calculate this epsilon s; that means, fraction of the solid inside the bed will be is equal to just you have to weight the solid of the made before entering before inserting into the bed the water to the solid of the part of the what will be weight of the solid or mass of the solid if you divide it by density of the solid, then you will get what should be the volume of the solid inside the bed. And what will be the total volume? That will be equal to cross sectional area into l, l is the characteristics length this characteristics length will be the height of the gas liquid mixture. So, here this will be your total volume of gas liquid mixture and this is the volume of solid here. So, this volume of solid divided by volume of total gas liquid mixture that will give you the volume fraction of the solid inside the bed. So, from this portion we can calculate from this head pressure by this mixture density of the phases inside the bed.

(Refer Slide Time: 06:24)



Now considering gas and liquid as one homogeneous space and solid is another phase.

So, if suppose considering that epsilon f epsilon f is the bed volume fraction of the gas, and liquid except solid then we can write here this will be your density of the gas and liquid mixture. Here in this figure, if we consider this total bed and if we consider this total bed here of gas liquid solid, and if we divide it into 2 parts of this portion this is, this portion will be as gas liquid mixture and this in this portion only solid. So, this this this portion or this fraction volume fraction will be the present or a gas liquid mixture volume fraction and this will be the solid volume fraction.

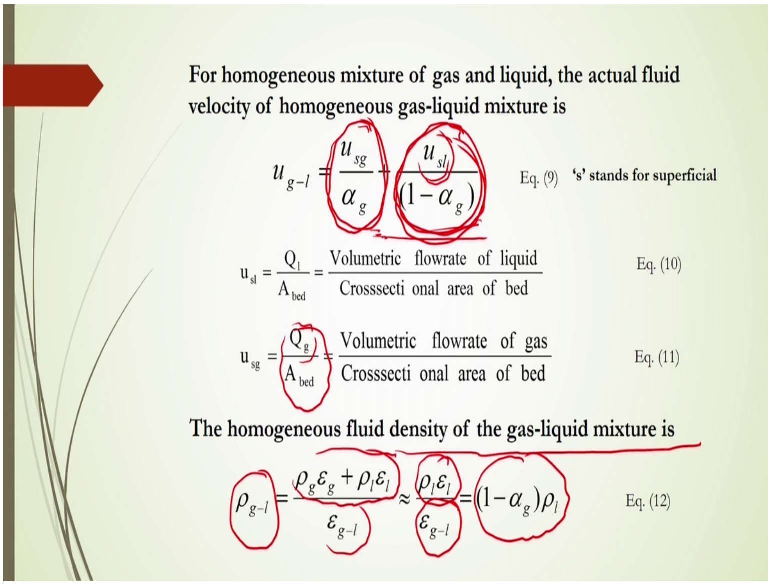
Now so, what should be the effective density in this gas liquid mixture? That will be represented by rho g l into epsilon g l. Rho g l in the density of gas liquid mixture and epsilon g l is the volume fraction of gas liquid mixture. So, this will be ultimately coming as what is that as per that mixture density is here rho z epsilon g; that means, what is the gas portion that is gas density effective gas density and what will be the effective liquid density. Summation of this density will be represented over that what will be the effective gas liquid mixture density.

So, from this equation we can rearrange it as rho g l will be is equal to rho z epsilon g plus rho l epsilon l total divided by epsilon g l. So, here the g l will be represented by only gas liquid mixture. And this f f is nothing but f is here representing the fluid, fluid means here gas liquid mixture here the frictional, then hold saw holdup of gas and liquid in gas liquid mixture can be represented respectively by here. So, in this case alpha z will be equal to epsilon g by epsilon g l. What does it mean? This alpha g is the gas volume fraction inside the bed when we are considering on the gas liquid portion.

So, in the gas liquid portion what should be the volume fraction of gas in gas liquid mixture. So, that will be defined as alpha g. And this is actually mathematically can be expressed as epsilon g divided by epsilon g l. What is this? Epsilon g is the volume fraction of gas out of total volume fraction of the gas liquid mixture inside the bed. And then what should be the then remaining portion in the gas liquid mixture that is liquid? And what should be the liquid volume fraction inside the bed? That will be just 1 minus epsilon g, because here alpha l plus alpha z that will be equal to 1. This is fraction nothing but fractions. So, alpha l is nothing but liquid volume fraction in the gas liquid mixture or the portion of gas liquid solid fluidized bed. That will be is equal to then epsilon l divided by epsilon g l.

So, therefore, we can write this epsilon g l from this equation 5, that rho g l will be equal to then what will be that rho g, what is that? Rho g alpha g rho z alpha z plus rho l into 1 minus alpha z. What is this rho z l? Rho g l is nothing but effective density of the gas liquid mixture inside the total 3 phase fluidized bed.

(Refer Slide Time: 10:12)



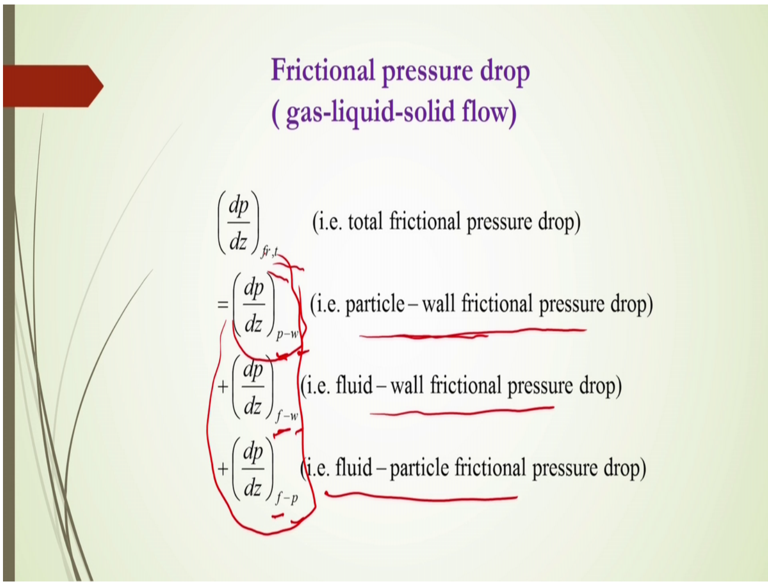
Now, if we consider that homogeneous mixture of gas and liquid, the actual fluid velocity of this homogeneous gas liquid mixture inside the bed will be represented by this u z l. U z l is nothing but the actual fluid velocity of the homogeneous gas liquid mixture. That will be represented what will be the actual gas velocity plus what should be the actual liquid velocity.

Now, what would be the actual gas velocity? How to calculate if you know that epsilon is that is if you know that what should be the velocity of the gas inside the bed, that is represented by that is superficial gas velocity. How to calculate the superficial gas velocity that? What will be the volumetric flow rate of the gas you divided by cross sectional area of the bed, then you will get the superficial gas velocity. And if you divide the superficial gas velocity in this portion of gas liquid mixture by this volume fraction of the gas then you will get the actual gas velocity.

Now, what should we then? Actually, liquid velocity actual liquid velocity in that portion of the gas liquid mixture of these 3-phase fluidized bed will be is equal to what will be the superficial a liquid velocity, and what should be the volume fraction of the liquid inside the bed. If you divide it this superficial liquid velocity by this volume fraction of liquid that will be 1 minus epsilon g, then you will get this actual liquid velocity inside the bed. Now what should be then total or actual fluid velocity of the homogeneous mixture of this gas. And liquid just you have to just sum up these 2-actual gas velocity and liquid velocity.

Now, what is homogeneous fluid density of this gas liquid mixture is you know that you rho z l that will be is nothing but the what will be the portion of the gas and liquid divided by out of total volume of this volume fraction of this gas liquid mixture, then it is simply rho l epsilon l by epsilon g l into 1 minus epsilon rho l. So, this is the homogeneous fluid density that you have already calculated based on the in the previous slides that how to calculate this a density of the gas liquid mixture.

(Refer Slide Time: 12:41)

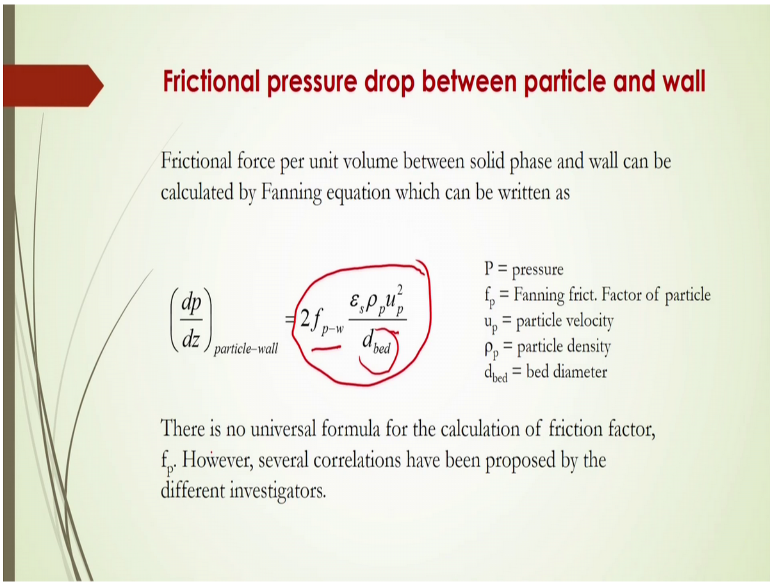


And once we know these mixture density of the gas liquid portion of the total 3 phase fluidized bed, and if you know the density of the solid inside the bed if you know the volume fraction of gas and liquid inside the bed, and also what should be the frictional volume fraction or fractional volume of gas or gas or liquid or solid inside the bed of course, you would be able to calculate what should be the total frictional pressure drop and inside the bed.

Now, this frictional pressure drop portion because this total pressure drop is the is the is that the contribution of frictional pressure drop head pressure drop and accelerative pressure drop. Now let us consider this frictional pressure drop first. So, gas liquid solid flow in that case frictional pressure drop will have these 3 parts. One is the frictional pressure drop due to particle wall friction. And one frictional pressure drop would be fluid wall friction and one pressure drop would be fluid particle friction. There are 3 components of this frictional pressure drop.

So, this particle wall frictional pressure drop would be represented by this DP by dz at pwp for particle w for wall. So, it will be a particle wall frictional pressure drop, and f for fluid and w for wall. F means fluid here fluid means here, gas and liquid mixture, whereas fluid and particle there will be gas liquid mixture interacting with the particle. In that case what should be the frictional pressure drop whenever there will be interaction between fluid and particle. So, these 3 components of this frictional pressure drop will give you the total frictional pressure drop in the gas liquid solid system in the fluidized bed.

(Refer Slide Time: 14:45)

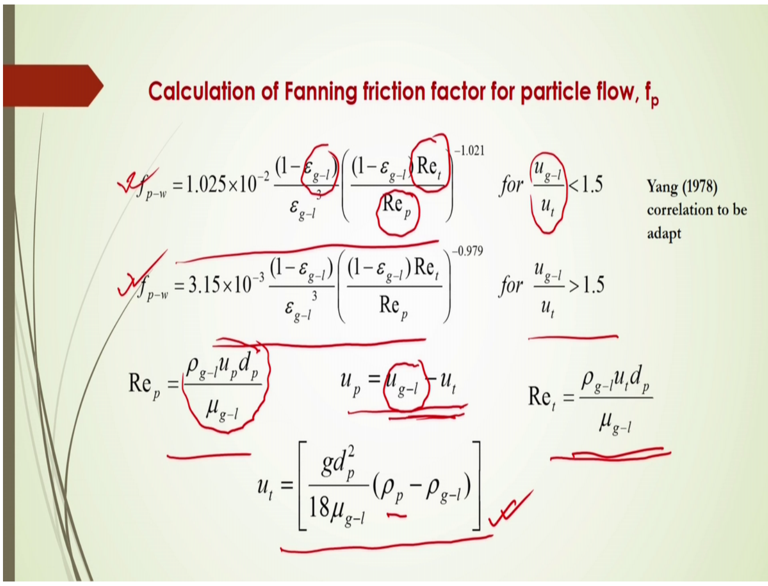


Now, this frictional pressure drop between particle and wall. Let us see how to calculate this frictional pressure drop between particle and wall. So, frictional force per unit volume between solid phase and wall; can be calculated by fanning equation which can be written as delta p by delta z or you can say DP by dz; that means, pressure gradient or per unit length what should be the pressure that will be equal to 2 f p w into epsilon s rho p into u p square divided by diameter of the bed.

So, by this equation you can calculate what with the frictional pressure drop whenever particle wall interaction will be there. So, in this case here c f p w. F pw is representing the friction factor whenever there will be a particle wall interaction. And epsilon s is the volume fraction of solid inside the bed rho p is the density of the particle inside the bed. And u p u p is nothing but the particle velocity inside the bed. And debate is nothing but diameter of the bed. So, as per this fanning equation you can calculate what will be the particle wall a frictional pressure drop. Once you know this particle velocity once you know the friction factor between particle and what wall.

This there is no universal formula actually for the calculation of friction factor. Like this f v w; however, several correlations have been proposed by different investigators. So, from those investigators we can we can calculate what is should be the friction factor. And then finally, if you substitute this friction factor in this equation then you will be able to calculate what should be the particle wall frictional pressure drop.

(Refer Slide Time: 17:01)



Now this calculation of the friction factor for particle flow f p. Now different investigators they got they if they proposed 2 different correlations to calculate or to predict this friction factor of this particle wall interaction.

So, in this case of course, this young 1978 is very important and acceptable correlations, or it is widely accepted because these correlations includes all the operating conditions as well as the particle size, and that is why these correlations can be used to calculate this particle wall friction factor. Now there are 2 conditions of these correlations they have proposed one if this ratio of gas liquid velocity to the terminal velocity of the particle if it is 1.5, then you have to use these correlations. Whereas, if this ratio of gas liquid velocity to this terminal velocity is greater than 1.5 you have to use this second equation of this fanning friction factor.

In this case this friction factor depends on the volume fraction of gas liquid mixture inside the bed, and the terminal velocity of the particle and this what is that the Reynolds number of the particle that is actually because of the what will be the this is the based on the particle velocity and the density of the gas liquid mixture, whereas the terminal velocity is the density of the gas liquid mixture, but it will be the based on the terminal velocity of the particle. So, R e p is nothing but the Reynolds number of the particle which is defined as rho g l u p d p divided by mu g l (Refer Time: 19:09) is the rho is the density of the gas, if you are using gas, if you are using liquid then you have to use of course, this or since it is a gas liquid mixture then you have to use gas liquid mixture density here u p is the particle velocity and d p is the particle diameter, and viscosity of the gas liquid mixture is mu g l. Not here the mixture density of the gas liquid solid inside the bed. Here you have to consider the viscosity of gas liquid mixture.

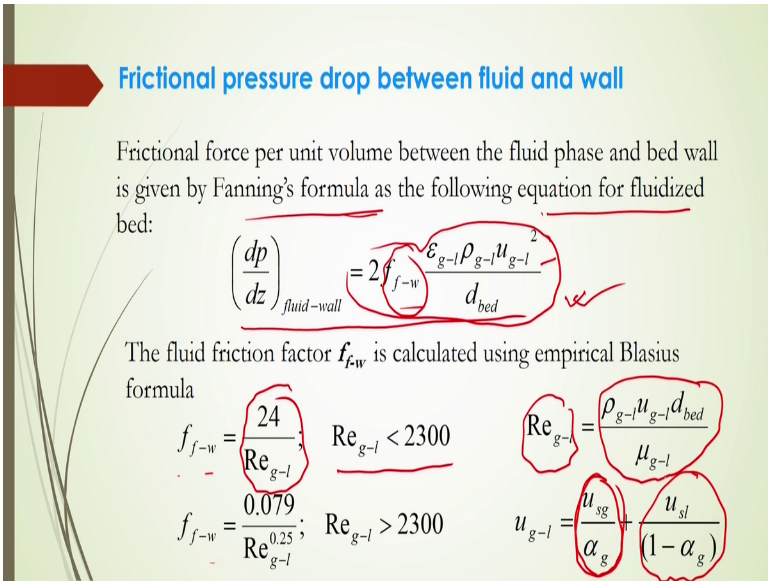
Whereas this u p u p is what whenever particle is fluidized with the certain gas velocity, you will see the particles will move with a certain relative velocity, relative to this terminal velocity of the particle. Now what should be this gas liquid mixture velocity inside the bed? That will be u g l whereas, terminal velocity of this particle is u t. So, terminal velocity generally this it will we move downward whereas, in the fluidized bed with a certain gas liquid mixture velocity it will move upward. So, what should be the effective velocities they are for the particle? It will be the effective upward velocity will be is equal to u g l minus u t the u t is the terminal velocity.

So, u p should be calculated part of the relative velocity of the gas relative to the terminal velocity of the particle. Now what should be the terminal velocity of the particle that already we have calculated if any single particle is suspended in the bed then as per stokes law that terminal velocity can be calculated here by this here g d p square divided by 18 mu gas liquid mixture; that means, viscosity of the gas liquid mixture into what should be the particle density you have to subtract it subtract the gas liquid mixture density to this particle density then you will get this terminal velocity.

So, once you know these terminal velocity and once you know this relative velocity of the particles, you will if you know this Reynolds number of this particle and this turners number based on the terminal velocity. And if you substitute here you will get this friction factor. Of course, before substitution you have to calculate this what will be the ratio of this gas liquid mixture velocity and what should be the terminal velocity. If you are having this ratio less than 1.5 then you have to use this first equation. If you are having this ratio is greater than 1.5 then of course, you have to use this equation.

So, be careful which equation to be suitable for your calculation based on your problem.

(Refer Slide Time: 21:56)



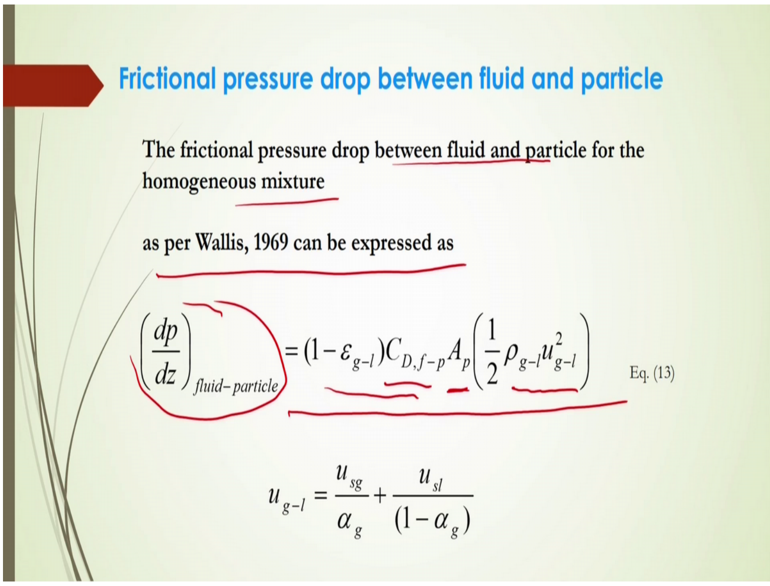
Now, frictional pressure drop between fluid and wall. Now this fluid and one. Now previous 1 is what is that fluid and particle now this is fluid and wall, now frictional force per unit volume between fluid phase and a bed wall is given by again fannings formula if we use this fanning equation again the following equation can be written. Now this d p by d z is equal to fluid wall of friction or pressure drop, this is due to that on the interaction of this fluid and wall.

Now, here it will be again 2 into f f w epsilon g l into rho g l into u g l square divided by d bed. So, from this equation you can calculate what should be the fluid wall frictional. In this scale case f f w is nothing but the friction factor between fluid and wall. And here it depends on this gas liquid mixture density a mixture velocity inside the bed. And of course, by this equation you have to calculate this fluid wall friction factor. Now for this again you have to know what should be the friction factor, whenever there is an interaction between fluid and wall.

So, in this case also you can use whether this flow is this this is laminar flow or turbulent flow based on this also you can calculate directly here. If f f w is 24 by Reynolds number in actually inversely proportional to the Reynolds number. If Reynolds number of gas liquid mixture is less than 2300 you can directly calculate what should be the f f w here by this 24 by Re gl. Where is this Re gl is defined as the mixture density of this gas liquid mixture, and mixture a velocity of the gas liquid mixture what will be the bed diameter and apart from the viscosity of the gas liquid mixture.

So, from this equation here of Reynolds number, you can calculate what with the friction factor between fluid and wall. Similarly, if Reynolds number of this gas liquid mixture is greater than 2300 of course, you can calculate this friction factor from the blasius equation here 0.079 divided by Re g l to the power 0.25. In this case again here you will get this friction factor between fluid and wall at this turbulent condition. But what is that? Gas liquid mixture velocity, that is nothing but the summation of the actual gas and liquid velocity inside the bed. That will be defined as that already we have defined earlier slides that this gas liquid mixture velocity will be is equal to actual gas velocity plus actual liquid velocity here.

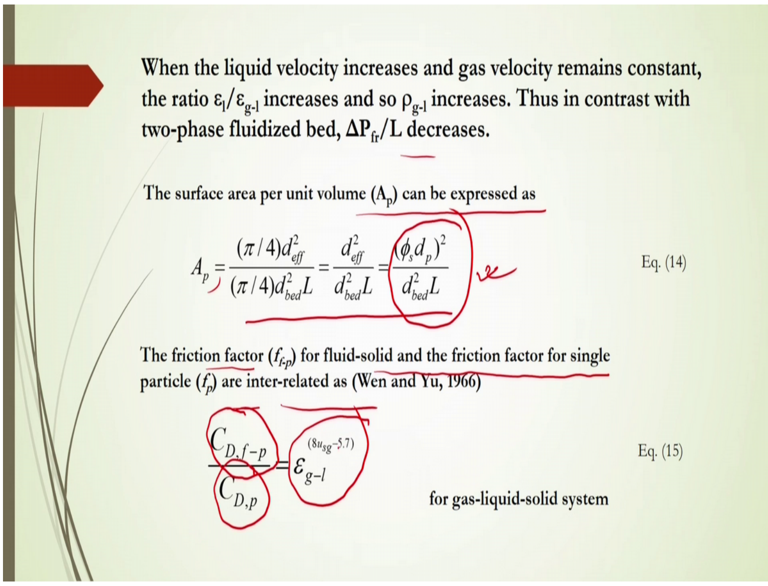
(Refer Slide Time: 25:05)



Now, frictional pressure drop between fluid and now particle, fluid and particle here. Now in this case the friction pressure again you can calculate from the homogeneous mixture as per wallis you can calculate these fluid particle mixture pressure a frictional pressure drop because of this interaction of fluid and particle here like this this will be is equal to 1 minus epsilon gl into cd apap into half of rho gl u g l square. This is as far wall is 1969 model that you can calculate also fluid particle friction of pressure drop because of their interaction. Here cd is nothing but drag coefficient between fluid and particle, and Ap is the projection of cross sectional projection and area of the particle, and what is that u gl u gl is nothing but gas liquid mixture velocity and rho g l is nothing but the gas liquid mixture density.

So, from this equation you will be able to calculate what should be the fluid particle frictional pressure drop.

(Refer Slide Time: 26:16)



Now, when the liquid velocity increases and the gas velocity remains constant, you will see the ratio of epsilon g and epsilon g l; that means, volume fraction of liquid to the volume fraction of gas liquid mixture will increase. And so, in this case gas liquid mixture density will increase. Thus, in contrast to the 2-phase fluidized bed you can say that frictional pressure drop per unit length will decreases. Now the surface area per unit volume here in this case a p can be expressed as here. This what will to be the surface area per unit volume inside the bed for this Ap that will be again required for this wallis equation. Here to calculate this frictional pressure drop between fluid and particle.

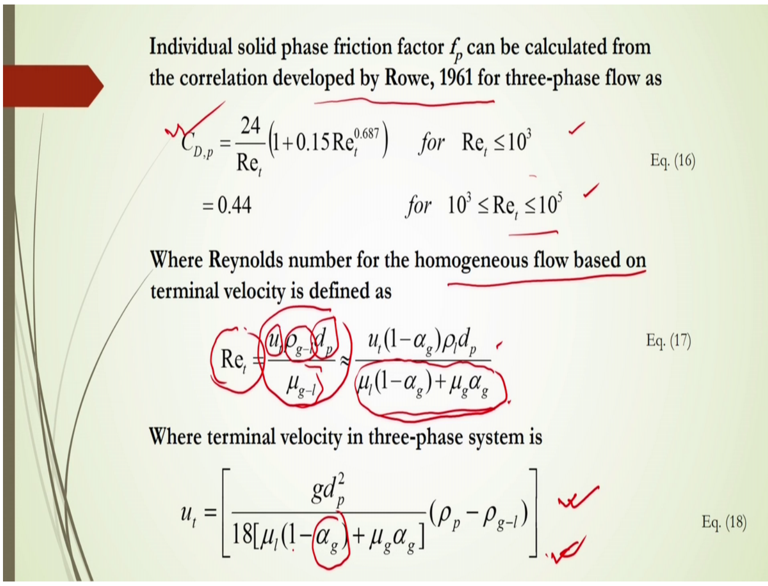
So, Ap should be calculated in this way that would be is equal to phi sdp whole square divided by dv square into L. So, this will be is the surface area per unit volume here.

Now the frictional fact the friction factor this fv for fluid solid and the friction factor for single particle Ap are interrelated. So, in this case is very important to be noted that the friction factor when there will be a fluid and particle interaction, and what should be the friction factor only one single particle is moving in a fluid medium.

So, in that case you can get this friction factor by this equation of these. Rho C D f v by C D DP that will be is a function of epsilon g. So, this is directly related to this friction factor this is this drag coefficient. So, you it is a function of the gas liquid volume fraction inside the bed. And also what should be the gas velocity inside that depends on. So, once you know this C D f P and C D p then you will be able to calculate what should be the C D A p inside the bed. And then once you know this C D A p then what should be the what should be the frictional pressure drop you can calculate by this equation of equation 13.

Now, to calculate these CD f P and also C D P, you have to use some correlations from which you can calculate directly what will be the drag coefficient between fluid and particle.

(Refer Slide Time: 29:05)

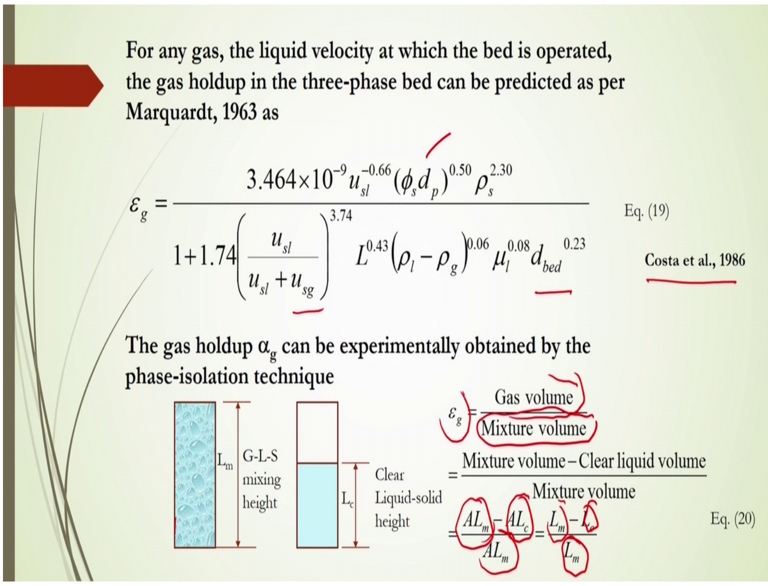


Now, individual solid phase friction factor f p or you can say drag coefficient f p can be calculated from the correlation developed by this row 1961 for 3 phase flow, that will be that can be used this way. And of course, this also will be in a certain range if terminal velocity is less than 10 to the power 3 or you can say one thousands then you have to use this cd as this equation. Whereas, this if from term Reynolds number based on terminal velocity if it is one thousand to 10 to the power of 5, then you have to use this that is turbulent condition this drag coefficient would be almost remain constant that already been discussed in earlier lecture notes. That will be constant as 0.44. A Reynolds number for the homogenous flow based on terminal velocity will be defined as what will be the terminal velocity what will be the density of the gas, liquid mixture what will the density diameter of the particle and this.

So, from this definition of this Reynolds number at it is terminal velocity of the particle you can get directly this equation if you substitute this ut or u t here and then rho z l is a function of only rho l that 1 minus epsilon g into rho l 2 DP. And here the viscosity of the gas liquid mixture you can use this mixture viscosity as mu l into 1 minus alpha g plus mu into alpha g. So, this one is the effective viscosity of the gas liquid mixture inside the bed. And this terminal velocity will be defined as this here, for this 3-phase system here it will be equal to this. In this case density on viscosity will be based on this gas liquid mixture and the particle, what is this? So, based this is as per this equation 18 you can calculate what should be the terminal velocity inside the bed.

So, once you know this terminal velocity of course, for this terminal velocity whenever gas liquid solid system you need to calculate this alpha g. Alpha g is what? This alpha g is nothing but the volume fraction of the gas inside the fluidized bed for the portion of gas liquid mixture. Now how to actually obtain this volume fraction inside the bed? So, you can you can you can obtain you can estimate this volume fraction experimentally just by phase isolation technique inside the bed, suppose at any operating condition what should be the gas liquid solid mixture hide inside the bed?

(Refer Slide Time: 32:05)



If we are we are able to know what should be the mixture hide inside the bed by this if you consider this; we bed and this is the gas liquid mixture gas liquid solid mixture hide inside the bed, and if you suddenly stop this operation and you will see after design gains meant of the gas inside the bed it will be dissolved at the top of the gas and there will be after certain time there will be a clear liquid height inside the bed.

So, from this clear liquid height and the gas liquid solid mixture height inside the bed you can calculate what should be the volume fraction of the gas inside the bed. Now how to do it? What you have to do that you have to first calculate what will be the gas volume inside the bed? And what with the mixture volume inside the bed? Now from this ratio you can calculate what will be alpha epsilon g here now this epsilon g. So, from this gas volume how to calculate this gas volume gas volume is nothing but mixture volume minus clear liquid volume inside this.

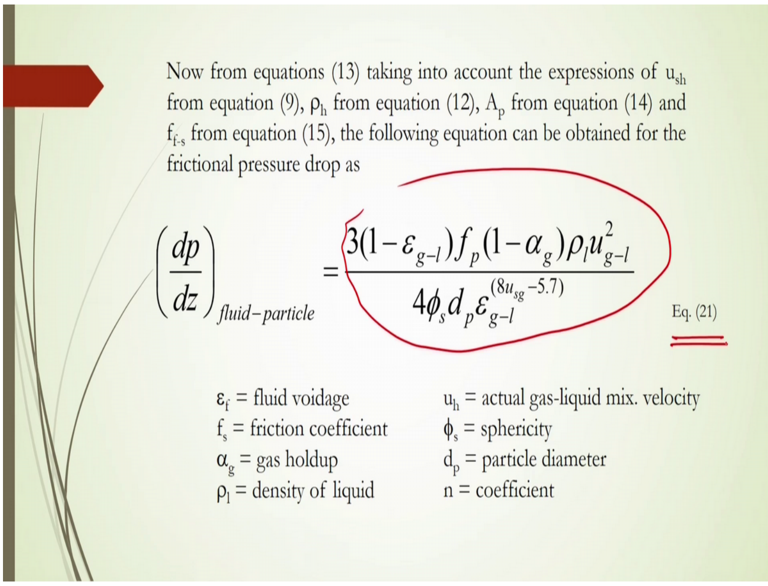
This is the total mixture, gas liquid mixture, volume and this is the only liquid solid mixture volume. So, if you substitute this liquid solid mixture volume from this total gas liquid mixture volume you will be able to calculate what will be the gas inside the bed gas volume inside the bed. Now from this mixture volume you can calculate A into Lm. What is the A? A is the cross-sectional area Lm is the gas liquid mixing height inside the bed gas liquid solid mixing height inside the bed, then this will be your mixture volume.

And this is what A Lc? A is the cross-section area and the Lc is nothing but the clear liquid solid height inside that bed. And this is your total volume.

So, from this if you cancel this a from this numerator and numerator then you will get this Lm minus Lc divided by Lm. From this portion you will be able to calculate what should be the volume fraction of gas inside the bed. Once you know this volume fraction inside the bed. So, out of this total gas liquid mixture volume what will be that portion that will be calculated as alpha g. Now as far as per suggestion of costa et all they have they have developed one correlation as per their experimental work they have developed this volume fraction of the gas inside the bed and they have double one correlation based on the different operating parameters like gas was sliding velocity or can the superficial liquid velocity and gas velocity, and also what will be the diameter of the bed and under density of the solid particle diameter.

So, with those variables so, they have developed this big correlations and from these correlations you can calculate directly what should be the volume fraction of gas inside the bed.

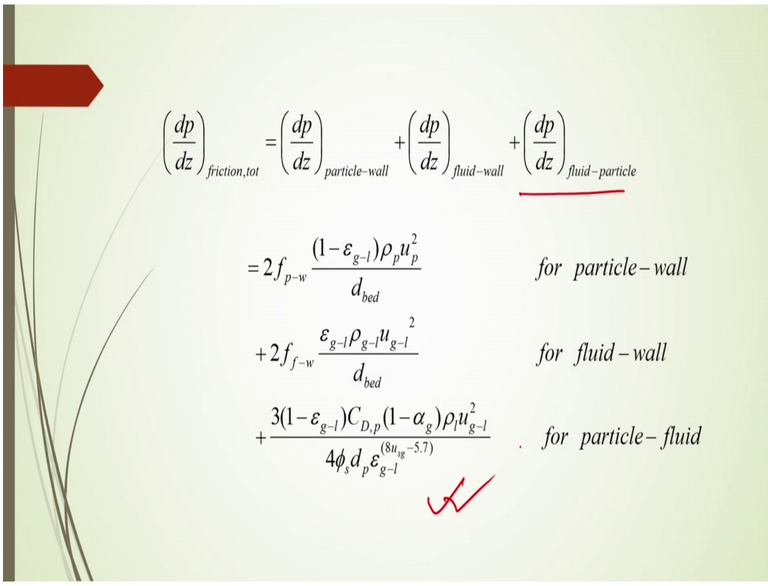
(Refer Slide Time: 35:40)



And now so, how to calculate once you know this volume fraction of gas liquid mixture? And what should be the friction factor of the particle? And if you know the alpha g; that means, volume fraction of gas inside the bed, and what should be the volume fraction of the gas in the gas liquid mixture portion? And if you know the gas liquid mixture velocity inside the bed, from this you just calculate you can calculate what should be the frictional pressure drop whenever there will be a interaction between fluid and particles.

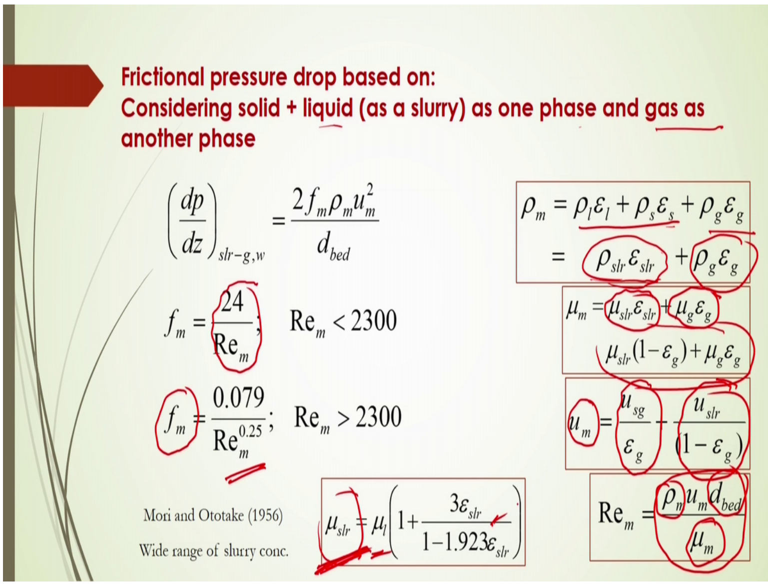
So, from this equation 21 you will be able to calculate a fluid particle frictional pressure drop.

(Refer Slide Time: 36:33)



Now once you know this 3 part of this frictional pressure drop, total frictional pressure drop will be equal to frictional pressure drop because of this particle wall interaction plus fictional pressure drop because of this fluid, and wall interaction and the frictional pressure drop because of fluid and particle interaction. So, then if you just substitute to the respective value of this frictional pressure drop, and then you will be able to calculate will be calculated the total frictional pressure drop.

(Refer Slide Time: 37:09)



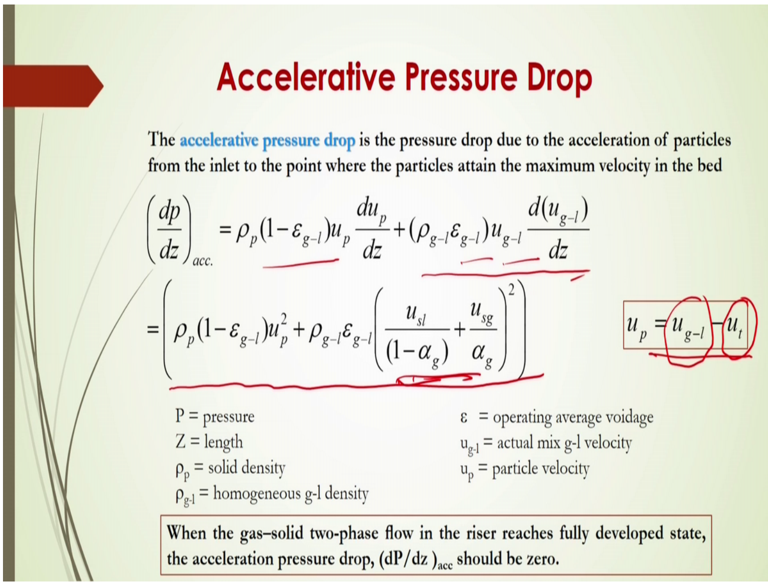
And our frictional pressure drop based on considering solid plus liquid as a slurry as one phase and gas as another place. So, you can calculate the frictional pressure drop on either way. Either you are considering gas and liquid mixture as one phase and solid is another piece. Or you can consider this solid and liquid mixture as a slurry as one phase and gas is another phase. So, you can calculate the frictional pressure drop based on considering solid liquid mixture as a slurry as one phase and gas as another phase. Let us consider here this dp by dz as a slurry gas and wall. So, in this case 2 into fm into rho m into m square please. So, here rho m will be defined as here rho l epsilon l plus rho s epsilon g plus rho g epsilon g.

Now, here this portion will be this portion will be considered as this slurry effective slurry density and this would be the effective gas density. And viscosity will be defined as effective viscosity here as a slurry that is liquid and solid mixture and here it will be the effective gas viscosity. And mixture velocity which will be here this one is actual gas velocity of the gas and actual velocity of the slurry here. And the Reynolds number will be defined as the mixture velocity and mixture density based on this and this mixture viscosity. And then here again this what should be the f m; fm means the friction factor because of this slurry gas mixture. So, in this case slurry gas mixture. So, fm should be calculated as 24 by Re m where Re m should be 2300. Whereas, if the Reynolds number is greater than 2300, then you have to calculate this friction factor of this mixture of gas and slurry this will be 0.07 mined by Re m to the power 0.5.

Remember this here rem is the main factor 2 just to get these to get this mixture of friction factor. Now this rem will be changed because of this viscosity density and the and the velocity of the mixture here. So, viscosity of the slurry you can calculate from this direct relationship either, or you can you can use other viscosity formula for this slurry. Now this slurry viscosity here it is only liquid and solid. You see sometimes this the slurry viscosity will increase or will changed because of the concentration of the solid inside a solid of the slurry. Now if suppose you are the volume fraction is suppose 2 percent you will see the viscosity of the slurry will be something else.

So, what should be the effective viscosity compared to the pure liquid viscosity? So, if you add some solid particles there and make the slurry, then you will see the viscosity of the liquid will change accordingly. Now that depends on that concentration of the slurry, concentration of the slurry concentration of the particle inside the particle of the slurry system. So, this slurry viscosity will be equal to viscosity of the liquid into 1 plus 3 epsilon s l r by 1 my 1.923 epsilon s l r. What is this epsilon slr? Epsilon s l r is nothing but the volume fraction of the solid particles of this slurry.

(Refer Slide Time: 41:20)

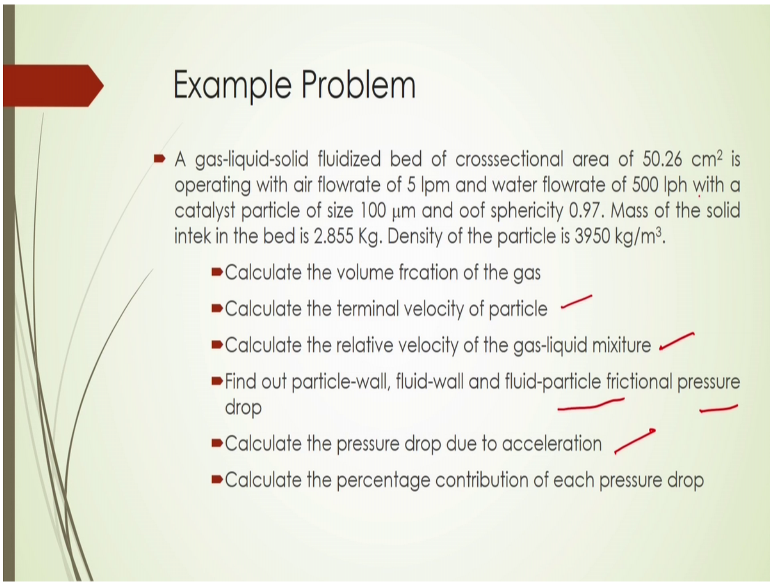


Now, what should be accelerative pressure drop if you are considering that slurry and gas mixture? So, a accelerative pressure drop is the accelerative pressure drop is due to the acceleration of the particles from the inlet to the point where the particles attain the maximum velocity in the bed. So, accelerative pressure drop would be is equal to this rho p into 1 minus epsilon g l into u p u p is the particle velocity into dup by dz. Dup by dz is nothing but for the change of velocity across the that is along the length of the fluidized bed, and then plus what should be the momentum change of this gas liquid a mixture inside the bed.

So, that will be equal to rho g l epsilon g l into u g l into d u g l by d z. And u p is nothing but the relative velocity of the particle; that will be calculated just subtracting the terminal velocity from the gas liquid mixture velocity inside the bed. So, finally, you can get after rearrangement and substitution of this value here you can get rho p into 1 minus epsilon g into u p square plus rho g l epsilon g l into u sl into 1 minus epsilon g plus u s g by epsilon alpha g to the whole square. So, this is the actually mixture velocity of gas liquid in inside the bed. Now in this case you will see when gas liquid 2 phase flow in the fluidized bed in this riser section, where which that gas liquid solid mixture is rising up the reaches fully developed state, then accelerating pressure drop will be neglected. Or it should be become 0, because they are there will be no change of velocity inside the bed at this fully developed flow.

So, because of ways you have to neglect this accelerative pressure drop. Now let us do an example for this gas liquid mixture pressure drop. So, once you know these 3 component of this pressure drop and summation and substitute the value you will get this frictional pressure drop in 3 phase fluidized bed also.

(Refer Slide Time: 43:53)



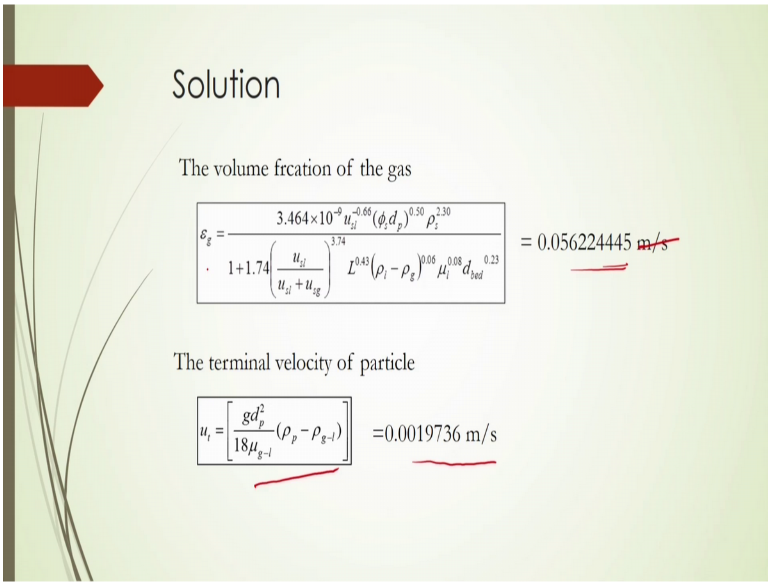
Now, if we consider the gas liquid solid fluidized bed of cross sectional area of 50.26-centimeter square. And it is operating with air flow rate of 5 l pm that is liter per minute and water flow rate of 500 litre per hour with a catalyst particle of size 100 micrometer. And of a sphericity of 0.96 e then mass of the solid intake in the bed is considered as 2.855 kg and density of the particle is given as 3950 kg per meter cube.

Under this condition, you have to calculate what should be the volume fraction of the gas, calculate the terminal velocity of the particle, calculate the relative velocity of the gas liquid mixture. How to calculate the particle wall, part fluid wall, and a fluid particle frictional pressure drop? And also calculate what with a pressure drop due to the acceleration, what will be the percentage contribution of the frictional pressure drop to the total pressure drop or to be the percentage contribution of the head pressure drop to this total pressure drop what will be the percentage contribution of the accelerative pressure drop to the total pressure drop?

Now, you have to calculate based on the adhere equation. Now the volume fraction of the gas you can directly calculate from these correlations. Now this epsilon g is equal to you can do you have to substitute what will be the different variables here in this case. So, if you substitute the different variables like usl; that means, a superficial liquid velocity you can calculate the superficial liquid velocity from this the flow rate of the liquid here if it is 500 lph this is liter per hour you have to convert this 500 lp h to the meter cube per second.

Then if you know this volumetric flow rate of this meter q per second you have to divide these by these cross-sectional area of the bed. How to calculate the cross-sectional area? That is given already otherwise if diameter is given you have to calculate what should be the cross-sectional area. Now if you know this cross-sectional area and a volumetric flow rate of the liquid simply you can calculate what will a superficial liquid velocity inside the bed.

(Refer Slide Time: 46:17)



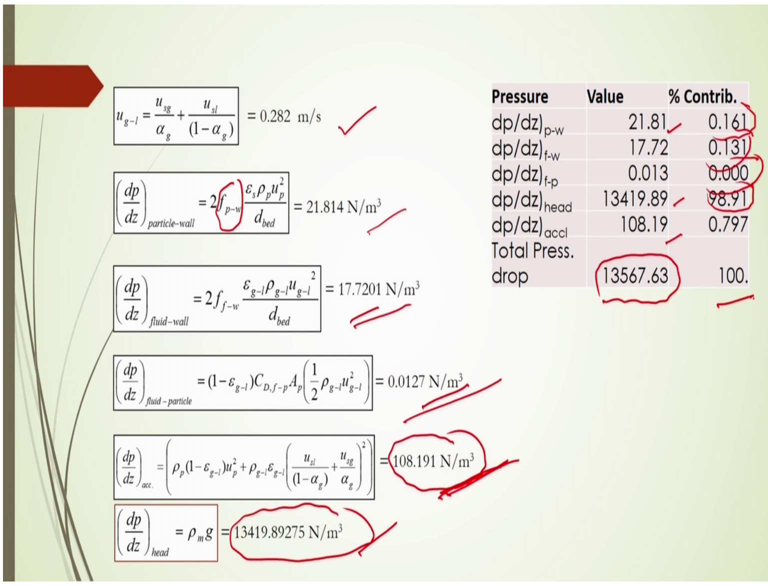
And 5 is this is sphericity it is given to as per this particle shape it is given 0.97, and then DP the particle size also. You can calculate the particle size or you have to know by particle size analyzer working with the particle size if it is given then you have to use directly this particle size here. And rho s the density of the particle is given as per your problem is 3950. And what is the usl, you know that usg usg also, you can calculate this usg from the volumetric flow rate of the gas. And if you divide this volumetric flow rate of the gas by the cross-sectional area of the bed, then you can calculate what will be the superficial gas velocity inside the bed. Then this is usg and then l the length of the bed this is the effective length of the bed height; that means, gas liquid mixture height. If it is they are certain height this is maybe they are certain height, then you have to calculate what will be the epsilon g. And rho l minus rho g rho l is the density of the liquid rho g the density of the gas to substitute they are gas here you can use that air as a gas liquid mixture that is or it is given that some other gases then you have to use this gas density of that other gases.

And viscosity of the liquid and diameter of the bed. Diameter of the bed if it is given the cross sectional you can calculate the diameter of the bed by just what will the cross-sectional area by pi by 4 d bd square from which you can calculate the bed. Now once you know this data then you can directly substitute here and indirectly gate what should be the epsilon g; that means, epsilon g is nothing but the volume fraction of the gas inside the bed. That is here 5 percent of (Refer Time: 48:13) in volume fraction. So, column fraction of the bed this will be volume fraction 0.05. For this terminal velocity of the particle how to calculate this terminal velocity of the particle then as per strokes equation you can calculate this terminal velocity gdp square by 18 mu g l into rho p minus rho gl.

So, it will be is equal to 0.001973 meter per second. And then what should be the mixture of gas liquid velocity? Then if you know these u s g and if you know this usl, then you can directly calculate what will be the mixture velocity of gas liquid. We said here if alpha g you can you have to convert this alpha g, from this epsilon g this epsilon g is that gas liquid in the total friction of this is volume fraction of gas in the total gas liquid mixture.

Now, what should be the alpha g? Alpha g is nothing but epsilon g divided by epsilon that will be your alpha g. So, alpha g you have to calculate and if you substitute this alpha g here you will be able to you will able to calculate.

(Refer Slide Time: 49:24)



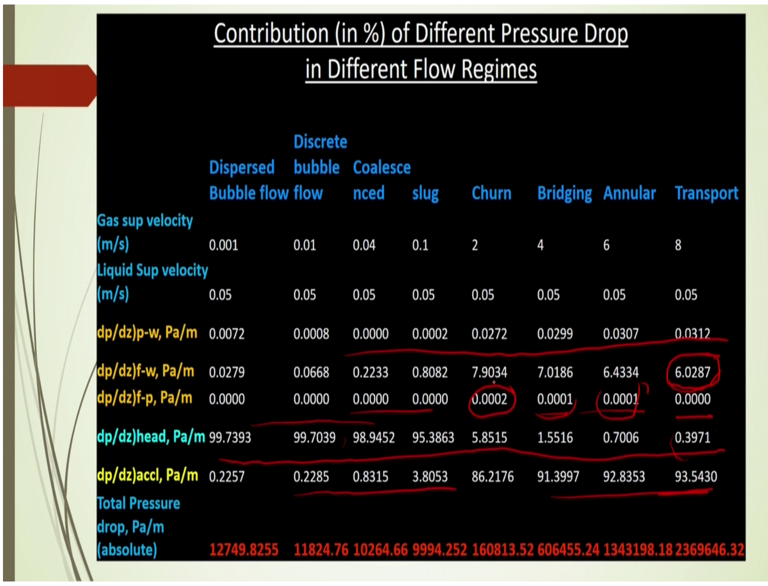
What will be the mixture gas liquid mixture velocity? And then substitute this and also you have to calculate this f f p w again by this equation by fanning equations, and then what should be this? This if you wants to calculate this by substituting there are different data, then you will be able to calculate this fluid wall frictional pressure drop, fluid particle frictional pressure drop, accelerative pressure drop and this head pressure drop.

Now, see from these this particle wall pressure drop is 21.814; however, this fluid wall wall is 17.721, this is lesser then this particle wall. Whereas, fluid particle is very negligible 0.0127, it is very small compared to the other pressure drop. Whereas, head pressure drop is maximum amount here and accelerative pressure drop is also here important factor. Because here in this case and the fluid may not be reaching to the fully developed flow. So, in that case you will get some accelerative pressure drop.

Now, in this case you will see that different portion of this 21.814 is for particle wall 17.72 is for fluid wall and 0.013 for fluid particle and 13419 is for head pressure drop and the accelerative pressure. Total is this total pressure drop is summation of this 3 component of this frictional pressure drop. And then contribution you will see if you get this contribution here as the percentage here particle wall contribution. And this here fluid wall and this fluid particle contribution is very less this negligible. Whereas, the head pressure is drop is the maximum amount of contribution by this hydrostatic pressure drop.

So, in this way you can calculate what will be the total pressure drop if you know these different variables only this gas velocity liquid velocity frictional pressure drop and also what is the friction factor, and what should be the velocity of the particle what will be the terminal velocity of what will be the other parameters if you know then you can directly calculate this.

(Refer Slide Time: 52:00)


Contribution (in %) of Different Pressure Drop in Different Flow Regimes

	Dispersed Bubble flow	Discrete bubble flow	Coalescenced	slug	Churn	Bridging	Annular	Transport
Gas sup velocity (m/s)	0.001	0.01	0.04	0.1	2	4	6	8
Liquid Sup velocity (m/s)	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
dp/dz)p-w, Pa/m	0.0072	0.0008	0.0000	0.0002	0.0272	0.0299	0.0307	0.0312
dp/dz)f-w, Pa/m	0.0279	0.0668	0.2233	0.8082	7.9034	7.0186	6.4334	6.0287
dp/dz)f-p, Pa/m	0.0000	0.0000	0.0000	0.0000	0.0002	0.0001	0.0001	0.0000
dp/dz)head, Pa/m	99.7393	99.7039	98.9452	95.3863	5.8515	1.5516	0.7006	0.3971
dp/dz)accl, Pa/m	0.2257	0.2285	0.8315	3.8053	86.2176	91.3997	92.8353	93.5430
Total Pressure drop, Pa/m (absolute)	12749.8255	11824.76	10264.66	9994.252	160813.52	606455.24	1343198.18	2369646.32

Now see here this different contribution in different pressure drop in different flow regimes you see here, different flow regimes we have calculated here, different flow regimes how it will be they are.

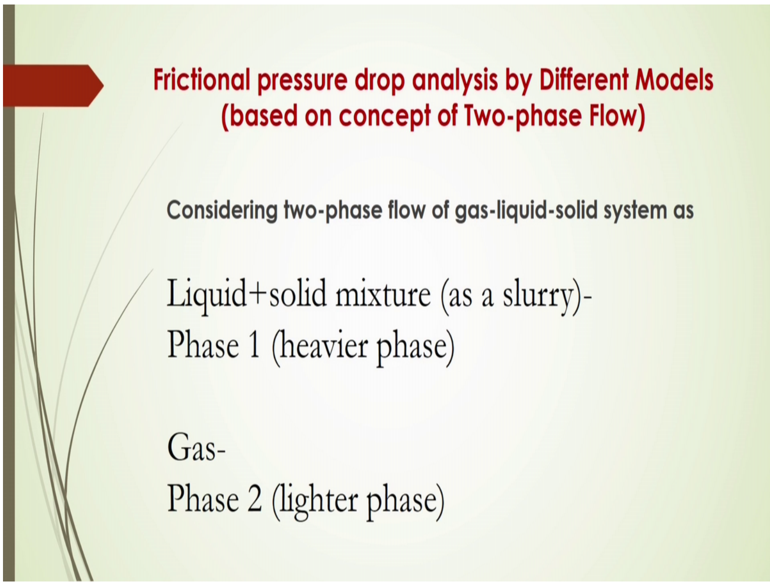
So, dispersed block flow here this is gas superficial velocity here 0.013 and liquid superficial velocity this and in this dispersed bubble flow, and this discrete bubble flow. And if you if you are considering coalescence here this slug churn turbulent bridging annular and transport then you will see a different contribution of this frictional pressure drop at different operating condition here.

So, in this case you will see that here very interesting that here in transport cases here this hydrostatic pressure drop is negligible compared to this what is that fluid and wall pressure drop. So, in the transport regime that case hydrostatic pressure drop will be less. In that case will see the dilute phase the part fluid and particle will be more interaction there. And whereas, in bubble in dispersed or discrete bubble flow regimes you will see that there will be the main contribution of this head pressure.

Now, this head pressure will be gradually decreasing with the different flow regimes, from dispersed this bubble flow to the transport. Whereas this head pressure accelerative pressure drop is also increasing, increasing, increasing from this dispersed bubble through the transport flow. And other contribution of this frictional pressure drop it is as per this here you will see again this particle wall contribution will be increasing with the transport from the dispersed bubble flow. And here a fluid particle in this case disposed to this bridging and annular flow this bridging and annular flow there will be a certain contribution whereas, this transport and other there will be a very negligible contribution in this case.

But churn turbulent the highest contribution for this fluid particle frictional pressure drop.

(Refer Slide Time: 54:43)



Now let us see this frictional pressure drop analysis by different models. Frictional pressure drop analysis with the different models can be done. In this case liquid and solid mixture as a slurry, you have to consider one phase, and that will be heavier pace, and gas phase took the lighter phase. Let us consider in the next class what should be these analysis of frictional pressure drop by a different models. So, in the next class we will be discussing that. So, that is for all todays this lecture.

Thank you.