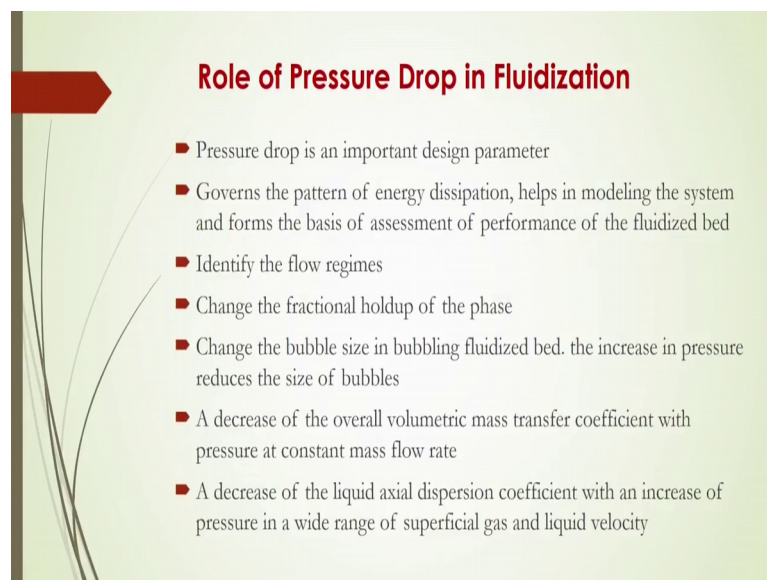


Fluidization Engineering
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Lecture – 08
Frictional pressure drop in fluidized bed; fluid-solid system

Welcome to massive open online course on fluidization engineering, today's lecture will be on frictional pressure drop in fluidization bed for fluid solid system. So, what is the role of pressure drop in fluidization?

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Role of Pressure Drop in Fluidization

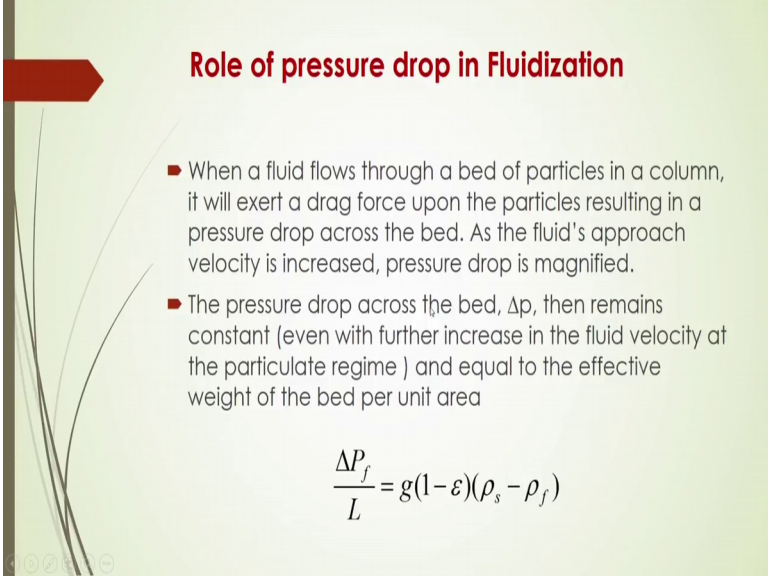
- Pressure drop is an important design parameter
- Governs the pattern of energy dissipation, helps in modeling the system and forms the basis of assessment of performance of the fluidized bed
- Identify the flow regimes
- Change the fractional holdup of the phase
- Change the bubble size in bubbling fluidized bed. the increase in pressure reduces the size of bubbles
- A decrease of the overall volumetric mass transfer coefficient with pressure at constant mass flow rate
- A decrease of the liquid axial dispersion coefficient with an increase of pressure in a wide range of superficial gas and liquid velocity

Of course this pressure drop is an important design parameter, whenever you are going to design any fluidized bed you have to consider this factor of frictional pressure drop inside the bed and of course what should be the energy dissipation inside the fluidized bed that depends on the frictional pressure drop inside the bed. And also it helps in modeling the system and forms the basis of assessment of performance of the fluidized bed. And also this frictional pressure drop will identify the flow regimes, for basically is the, that different pressure drops will give you a different types of flow regime maps. So, change the frictional pressure drop you will get different flow regimes and also based on that, you will see that there will be a change of frictional hold up in the fluidized bed; it is not frictionally fractional hold up in the phase.

Change the bubble size in bubbling fluidized bed, of course the pressure drop has important role in size of the bubbles that are formed in the bubbling fluidized bed, and I in this case if bed pressure drop is increased the bubble size reduces. So, it has important role in size of bubble and because this bubble size of course, will affect the interfacial surface area from which or through which the mass transfer occurs from this bubble phase to the fluidized phase. A decrease of the overall volumetric mass transfer coefficient with pressure at constant mass flow rate of course we will change.

The decrease of the liquid axial dispersion coefficient happens with an increase of pressure in a void range of spherical gas and liquid velocity. So, we have seen that there is there is a role of pressure drop inside the fluidized bed, by which we can get different phenomena of fluorescents by which we can get the energy dispersion inside the fluidized bed. We can say what should be the performance of the fluidized bed based on the a mass transfer and heat transfer also and then what should be the bubble size distribution inside the bed, of course it depends on the pressure drop.

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Role of pressure drop in Fluidization

- When a fluid flows through a bed of particles in a column, it will exert a drag force upon the particles resulting in a pressure drop across the bed. As the fluid's approach velocity is increased, pressure drop is magnified.
- The pressure drop across the bed, Δp , then remains constant (even with further increase in the fluid velocity at the particulate regime) and equal to the effective weight of the bed per unit area

$$\frac{\Delta P_f}{L} = g(1 - \varepsilon)(\rho_s - \rho_f)$$

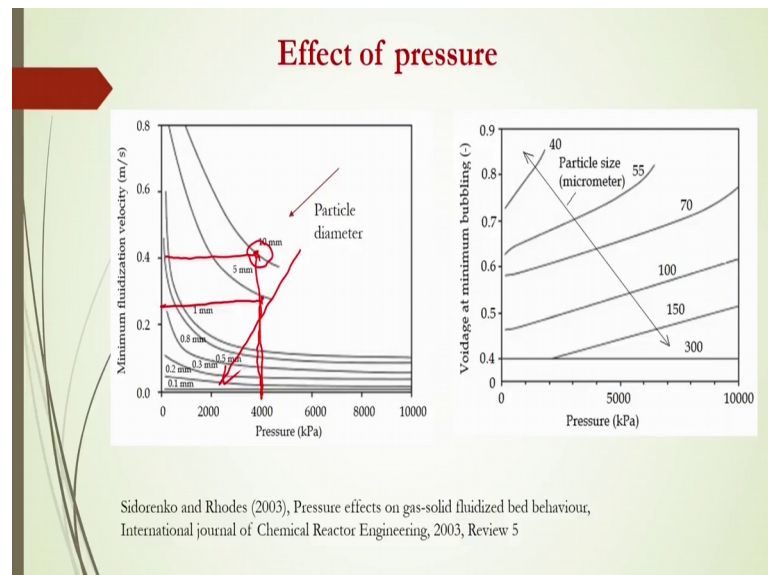
And also we will see that when a fluid flows through a bed of articles in a column of or in a fluidized bed, it will exerted drag force, upon the particles resulting in a pressure drop across the bed.

At the fluid approach velocity is increased the pressure drop is magnified. The pressure drop across the bed it is seen that it will remain constant even with further increase in the

fluid velocity at a particular flow regime. Generally for particulate flow regimes where bubbling is not possible only just through the space of the particle bed, just gaseous flowing upward, in that case it is called the particulate flow regimes. So, in the particular flow regimes the pressure drop remains constant even if you increase the gas velocity or fluid velocity inside the bed. Also this will be equivalent to the effective bed of the bed per unit area in the bed.

So, this is the friction or pressure drop per unit length that is nothing, but the effluent weight or effective weight of the bed for unit area here and you will see here.

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In this figure the effect of pressure is shown, how the minimum fluidization velocity will change with pressure drop, it is seen that minimum fluidization velocity will decrease with increase in pressure drop. Here these are the trends are shown, but also this trend will change if the particle size is changed; if suppose particle size is increased you will see minimum velocity also will increase here at any point uses observed, that here this is the point that at the particle diameter of 10 mm here what should be the minimum fluidization velocity for the particular pressure drop of 4000 here kilo Pascal. Even if you change this particle size to this 5 millimeter, you will see for the same pressure drop you will get lower value of minimum fluidization.

Whereas in this case if you see that if you keep on decreasing the particle size for the same pressure drop, and if you do the fluidization operation and you will see the

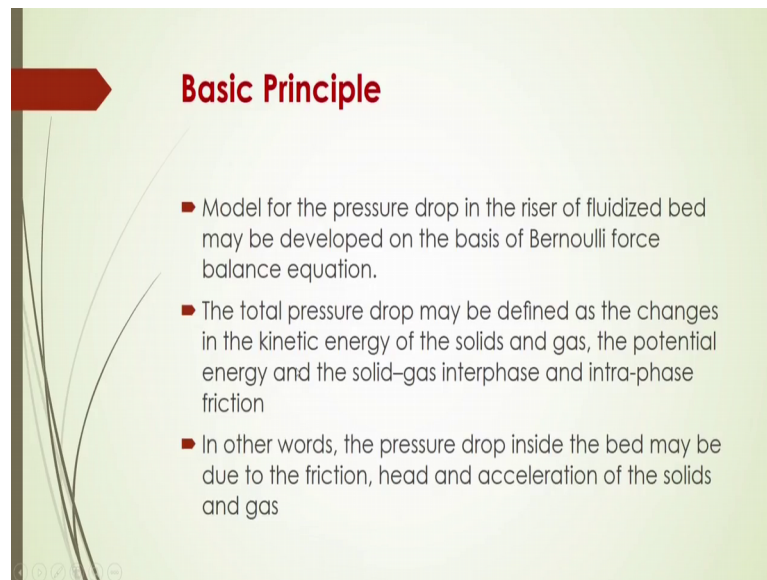
minimum fluidization will decrease. Whereas, at the higher pressure it is seen that at any particular any particular diameter there will be no significant change of minimum fluidization velocity and all but for lower pressure, it is seen that that there will be a significant change of minimum fluidization velocity as per particle size. In this case why for lower a pressure you will see there will be that other factor of course, the velocity of the fluid if you change the velocity accordingly you will see there will be a change of pressure drop.

So, if you change the pressure drop to this lower value accordingly the minimum fluidization also will affect. Now here, in this case is void is at minimum bubbling. Now if you are considering the bubbling flow fluidization regime in this case the fractional void age or you can say fractional gas hold up inside the bed will decrease if you increase the particle size in micrometer.

So, in this case also the void age we will increase with respect to pressure. So, if you increase the pressure the void age will increase, whereas if you increase the particle diameter where does will decrease. The pressure effects them on gas solid fluidized bed behavior, it is seen that we can observe from this fluidization behavior, what should be the minimum fluidization velocity at different pressure, and also what should be void age.

So, all this pre pressure drop at minimum fluidized or other minimum fluidization condition or other (Refer Time: 08:30) done it is required to know, because for design of a fluidized bed in which way in which flow regime it will be acting and based on that you can say what should be the pressure drop from this operation.

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Basic Principle

- Model for the pressure drop in the riser of fluidized bed may be developed on the basis of Bernoulli force balance equation.
- The total pressure drop may be defined as the changes in the kinetic energy of the solids and gas, the potential energy and the solid-gas interphase and intra-phase friction
- In other words, the pressure drop inside the bed may be due to the friction, head and acceleration of the solids and gas

Another important point is that we should know then what should be the basic principle for pressure drop analysis, there are several models to analyze the pressure drop inside the bed.

Now, basic principle is that model for the pressure drop in the riser of fluidized bed may be developed on the basis of Bernoulli force balanced equation. So, as per Bernoulli's equation you will see the total force will be the total kinetic total energy will be constant. So, based on that you can obtain what should be the pressure drop by changing the velocity and also if you change the elevation of this fluidized bed or level of the mixture of the fluid and solid particles inside the bed, what should be the pressure drop also that depends on the level of the bed.

So, what will be the total energy that should remain constant as per Bernoulli's theory? So, based on these fluids balance equation from Bernoulli's equation, the total pressure drop babe obtained or friction or pressure drop may be obtained based on that question. Now the total pressure drop may be defined as the changes in the kinetic energy of the solids and gas, the potential energy and the solid gas inter phase and intra phase friction. So, different types of pressure contribution to the total pressure drop.

Now one is that kinetic energy of the solids and gas, one contribution and the potential energy another contribution and the friction between solid gas or you can say that solid and fluid interface and also the interaction between the particles, because of that there

will be a change of energy and for which the frictional pressure drop or total pressure drop will change. We will discuss as later on; how different contribution will give the total pressure the drop and what should be the percentage of that contribution will be calculating.

So, in other words the pressure drop inside the bed may be due to the friction head and acceleration of the solids and gas. So, pressure drop inside the bed of course, there will be a three component maybe it is those are due to the friction; that means, frictional pressure drop, due to the head that is called static or hydrostatic pressure drop and due to the acceleration of the solids and gas inside the bed that will accelerative pressure drop.

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Mathematical Background

On the basis of a momentum balance on gas–solid suspension for steady state, the pressure drop in the riser can be mathematically expressed as

$$\left(\frac{dp}{dz}\right)_{total} = \left(\frac{dp}{dz}\right)_{head} + \left(\frac{dp}{dz}\right)_{acceleration} + \left(\frac{dp}{dz}\right)_{friction}$$

Static pressure drop is the pressure drop due to the change in the potential energy of gas and solids

$$\left(\frac{dp}{dz}\right)_{head} = [\rho_p(1-\varepsilon) + \rho_f\varepsilon]g$$

P = pressure
Z = length
 ρ_p = solid density
 ρ_f = fluid density
 ε = operating average voidage

What should be the mathematical then expression to analyze this total pressure drop? So, on the basis of momentum balance on gas solid suspension for steady state, the pressure drop in the riser can be mathematically expressed as this is dp by dz , that will be equals to that is total pressure drop which is measured in the fluidized bed operation. So, it should be measured pressure drop this is per unit length that is dp by dz and what is this is the head pressure drop, what should be the hydrostatic pressure drop inside the bed that depends on the level of the fluid mixture inside the bed, that is called hydrostatic pressure drop on that is it is called head pressure drop. And another component is accelerative pressure drop; what does it mean? If suppose there is a change of velocity

inside the bed of the solid particles and gas, there will be a change location wise or you can say region wise or you can say point wise there will be change or velocity.

There will be no uniform velocity inside the bed because, of which there will be some energy formation or energy that is distribution in saw say that is all some acceleration of the solid particles, so due to which there may be a accelerative pressure drop. Now another is very important that frictional pressure drop; this frictional pressure drop means here this friction between fluid and solid friction between fluid and particle, now friction between fluid and wall, friction between solid and solid particles, frictional between a solid and the column wall or bit wall. So, there may be different frictions of course will be there. So, this frictional pressure drop will be due to the friction between phase and wall and between phases.

So, these 3 components are very important to know that how this pressure, how this total pressure is contributed by these different component of this pressure drop; now let us see what is that static pressure drop in the bed, this is static pressure drop is the pressure drop due to the change in potential energy of the gas and solids inside the bed. Now here see this is the a pressure drop equation here, this is the pressure drop due to the change in potential energy of this gas and solids now here this $\rho_p \int (1 - \epsilon) g dz$ this is due to the pressure drop due to head of this solid bed and then $\rho_f \int \epsilon g dz$ this is the pressure drop due to the head of this fluid inside the bed. So, this is the total head pressure, this is nothing but that is that you can say this is the $\rho_m \int dz$.

Here ρ_m is nothing, but the density, effective density inside the bed this effective density depends on the void age inside the bed. Now this is the particle density this is $\rho_p \int (1 - \epsilon) dz$; $1 - \epsilon$ is nothing but what is the solid volume inside the bed and this fraction then total this will be the fraction of the solid volume inside, so the solid volume inside the bed and then $\rho_p \int \epsilon dz$ is nothing, but the actual volume of the fluid inside the bed. So, this will be your mixture volume into an mixture volume and then to density it is called that what is that mass effective mass or you can say effective mass per unit volume or you can say the mixture density inside the bed.

So, mixture density into z this will be your head pressure. Now this is p is nothing pressure z is length here and ρ_p is the solid density, ρ_f is the fluid density, ϵ is nothing, but operating average void inside the bed.

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Accelerative Pressure Drop

The **accelerative pressure drop** is the pressure drop due to the acceleration of particles from the inlet of the riser to the point where the particles attain the maximum velocity in the riser

$$\left(\frac{dp}{dz}\right)_{acc.} = \rho_p (1 - \varepsilon) \frac{du_p}{dz} + \rho_f \varepsilon \frac{u_{sf}}{\varepsilon} \frac{d(u_{sf} / \varepsilon)}{dz}$$

P = pressure
 Z = length
 ρ_p = solid density
 ρ_f = fluid density

ε = operating average voidage
 u_{sf} = superficial fluid velocity
 u_p = particle velocity

When the gas-solid two-phase flow in the riser reaches fully developed state, the acceleration pressure drop, $(dp/dz)_{acc}$ should be zero.

Now, what should be the acceleratory pressure drop; how to calculate this accelerative pressure drop inside the bed? The accelerative pressure drop is the pressure drop due to the acceleration of the particles from the inlet of the riser or you can say in the fluidized bed to the point, where the particles attain the maximum velocity in the riser or in the bed; this riser means nothing but the fluidized bed. So, here accelerative pressure drop can be calculated as here this will be how this particle velocity will change with respect to z , and what should be the here the particle velocity. So, what would be the momentum change here due to the particle velocity change inside the bed, and what should be the momentum change due to the velocity change inside, the velocity of the fluid change inside the bed.

So, this part is nothing, but the momentum change due to the velocity change of the fluid and this is the moment change particle due to the particle velocity inside the bed, from the inlet of the fluidized bed to the point where the particle certain it is maximum velocity. Now it is important to note down here that when the solid gas 2 phase flow in the fluidized bed reaches fully developed state; that means, every where the velocity change will be constant.

So, in that case you can say the accelerative pressure drop should be zero; that means dp by dz of acceleration it will be 0. So, for fully developed state of the fluidized bed the

accelerative pressure drop will not be considered it will be neglected and what should be the frictional pressure drop.

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Frictional Pressure Drop

The **friction pressure drop** is due to three types of frictional forces developed:

- ✓ Frictional force between gas and solid. ✓
- ✓ Frictional force between the solid and the walls of the bed. ✓
- ✓ Frictional force between the gas and the wall of the riser. ✓

$$\left(\frac{dp}{dz}\right)_{\text{friction}} = \left(\frac{dp}{dz}\right)_{\text{fluid-wall}} + \left(\frac{dp}{dz}\right)_{\text{particle-wall}}$$

The friction between the gas and solids may be negligible due to the lesser concentrations of the solids in the riser.

Now, the frictional pressure drop is due to the friction between phases and friction between wall of the fluidized bed; now there are 3 types of frictional force developed generally the frictional force between gas and solid, the frictional force between the solid and the walls of the bed, the frictional force between gas and the wall of the bed.

Now, this tend this frictional pressure drop will be is equal to the frictional pressure drop between fluid and wall and the frictional pressure drop between particle and wall, the friction between gas and solid may be negligible due to the lesser concentration of the solids in the inside the bed. So, here you can neglect the frictional pressure drop between solid and gas, if it is only gas solid operation.

But in case of liquid solid operation you cannot neglect this 1, in that case the concentration of the solid or concentration with the there very important factor. So, for gas for gaseous medium gas solids fluidization you can neglect whereas, liquid solid here you can of course, consider that a frictional pressure drop between the friction between a particle and fluid. So, let us see that frictional pressure drop between particles and wall how it can be calculated.

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Frictional pressure drop between particle and wall

Frictional force per unit volume between solid phase and wall can be calculated by Fanning equation which can be written for gas-solid fluidization bed as

$$\left(\frac{dp}{dz}\right)_{\text{particle-wall}} = \frac{2f_p(1-\varepsilon)\rho_p u_p^2}{d_{\text{bed}}}$$

P = pressure
 f_p = Fanning frict. Factor of particle
 u_p = particle velocity
 ρ_p = particle density
 d_{bed} = bed diameter

There is no universal formula for the calculation of friction factor, f_p . However, several correlations have been proposed by the different investigators.

Now, this frictional force per unit volume between solid phase and wall can be calculated by fanning equation, which can be written for gas solid fluidized bed as like this here, this is the frictional pressure drop between particle and wall that will be is nothing but here 2 into f_p is nothing but friction factor fanning friction factor per particle, this is colliding with the wall and then 1 minus epsilon into rho p into u_p square by d_b .

So, this frictional pressure drop between particle and wall as per fanning equation depends on the particle velocity inside the bed, what should be the bed diameter and what should be the density of the particle and of course there will be the void age inside the bed; now we can say from this equation what should be the frictional pressure drop because of the collide collision or because, of the friction between particle and wall. So, there is no universal formula for the calculation of actually friction factor here f_p ; however, several correlations that you can use or to estimate or to calculate this friction factor that has been proposed by different investigators. So, we will see how to calculate this friction factor for particle here.

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Calculation of Fanning friction factor for particle flow, f_p

$$f_p = 1.025 \times 10^{-2} \frac{(1-\varepsilon)}{\varepsilon^3} \left(\frac{(1-\varepsilon) \text{Re}_t}{\text{Re}_p} \right)^{-1.021} \quad \text{for } \frac{u_{sg}/\varepsilon}{u_t} < 1.5 \quad \text{Yang (1978)}$$

$$f_p = 3.15 \times 10^{-3} \frac{(1-\varepsilon)}{\varepsilon^3} \left(\frac{(1-\varepsilon) \text{Re}_t}{\text{Re}_p} \right)^{-0.979} \quad \text{for } \frac{u_{sg}/\varepsilon}{u_t} > 1.5$$

$$\text{Re}_p = \frac{\rho_f u_r d_p}{\mu_f} \quad u_r = \frac{u_{sf}}{\varepsilon} - u_p \quad \text{Re}_t = \frac{\rho_f u_t d_p}{\mu_f}$$

$$u_t = \left[\frac{g d_p^2}{18 \mu} (\rho_p - \rho_f) \right] \quad C_D = \frac{24}{\text{Re}_p} \left[1 + (8.1716 e^{-4.0655 \phi_s}) \text{Re}_p^{0.0964 + 0.5565 \phi_s} \right]$$

$$+ \frac{73.69 (e^{-5.0748 \phi_s}) \text{Re}_p}{\text{Re}_p + 5.378 e^{6.2122 \phi_s}}, \quad \text{for } \phi_s \neq 1$$

If $0.1 < \text{Re}_p < 500$

Let us see how to calculate the friction factor for particle flow. So, this friction you can calculate from this correlations that is developed by young 1998, based on their experimental work here it is seen that this friction factor depends on the Reynolds number of particle and the Reynolds of the terminal velocity of the particle, of course this factor is you know where that is the epsilon that is what is the void age that is important factor that governs this friction factor inside the bed for the particle; now it is seen that this correlation will be valid only for if the ratio of actual gas velocity to the terminal velocity of the particle if it is less than 1.5.

Whereas for this ratio if it is greater than 1.5 you have to calculate this friction factor from this correlation here, again this correlation is a function of this Reynolds number of particle and Reynolds number of terminal Reynolds number of the particle based on terminal velocity. Now how this Reynolds number of particle is defined is nothing this $\rho_f u_r d_p$ by μ_f , what is this ρ_f ? ρ_f is nothing, but the fluid density, u_r is nothing but the relative velocity of the solid particles relative to the fluid velocity; now here u_r is nothing but here what is this u_{sf} by epsilon what is s for superficial velocity f for fluid either maybe liquid or gas, now for gas solid fluidized bed you have to consider here u_{sg} instead of f it will be g .

So, u_{fg} is nothing but the superficial gas velocity. So, if you divide the superficial velocity by the void age you will get the average gas velocity inside the bed or you can

say actual gas velocity inside the bed; now what should be the actual gas velocity and what is the particle velocity if you just subtract it you will get the relative velocity of the particles, through which at which this particle should be moving upward inside the fluidized bed or particle fluidized with this relative velocity.

Now, this is the what should be the terminal Reynolds number, if this is Re_t is defined as $\frac{\rho_f u_t d_p}{\mu_f}$, in this case this of course the velocity will be considered as terminal velocity, others are same here ρ_f is the fluid density μ_f is the fluid viscosity here. So, f for fluid maybe liquid or gas, if it is gas solid operation then f will be represented by g .

If it is liquid and solid fluidized bed then f will be represented by l and what should be the terminal velocity then to calculate this Reynolds number at terminal velocity; this terminal velocity again you can calculate based on this Stokes equation $\frac{g d_p^2}{18 \mu}$ into ρ_p minus ρ_f and also you can calculate this terminal velocity whenever fluidizing condition if it is if Reynolds number is greater than 0.1, then you have to calculate this terminal velocity from this equation. But they are of course it will be related to this drag coefficient, now this drag coefficient is related to this the Reynolds number that we have already discussed earlier, when a lecture that how to calculate the drag coefficient and how what is that drag coefficient.

So, you can calculate friction factor from this correlation by using this terminal velocity and the relative velocity and what should be the Reynolds number of particle, what should be the Reynolds number at terminal velocity of the fluidized bed.

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Frictional pressure drop between fluid and wall

Frictional force per unit volume between the fluid phase and bed wall is given by Fanning's formula as the following equation for fluidized bed:

$$\left(\frac{dp}{dz}\right)_{\text{fluid-wall}} = 2f_f \frac{\epsilon \rho_f (u_{sf} / \epsilon)^2}{d_{bed}}$$

The fluid friction factor f_f is calculated using empirical Blasius formula

$$f_f = \frac{24}{Re_f}; \quad Re_f < 2300$$

$$f_f = \frac{0.079}{Re_f^{0.25}}; \quad Re_f > 2300$$

$$Re_f = \frac{\rho_f u_{sf} d_{bed}}{\mu_f}$$

Now, what should be the frictional pressure drop between fluid and wall, friction between fluid and wall; now suppose gas solid operation what should be the frictional pressure drop between gas and bed wall fluidized bed wall.

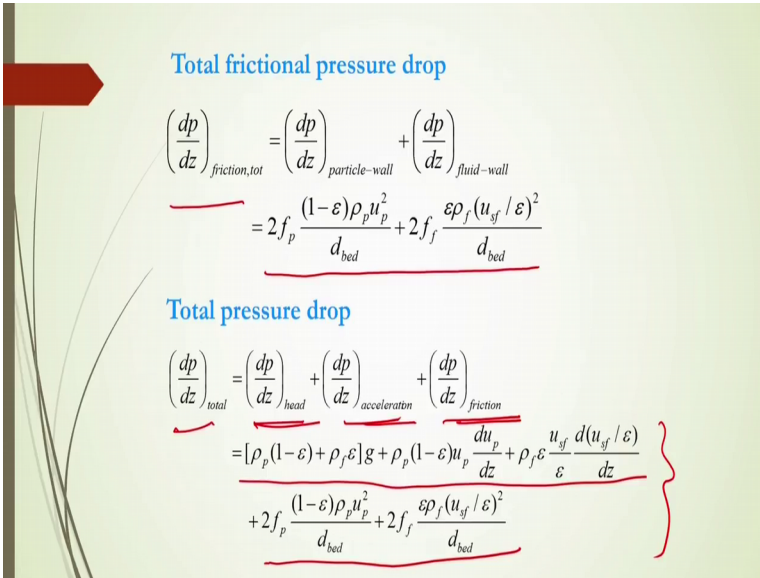
Now, frictional force per unit volume between the fluid phase and the bed wall is given by fanning's formula, as the following equation for fluidized bed as here this fluid wall that is the again as per fanning equation you can calculate what should be the fluid wall frictional pressure drop. So, in this case this friction factor will be represented by f_f , here it is only the between fluid and wall gas and wall or liquid and wall if you are considering gas and gas solid gas and wall.

So, here f_f into $\epsilon \rho_f$ into u_{sf} by ϵ this is your actual a gas velocity or liquid velocity inside the bed. So, here this is the fanning equation from which you can calculate; now for this again you have to calculate the friction factor of fluid flow. So, this friction factor of fluid flow fluid flow you can calculate from the equation, if it is laminar flow you can directly calculate a f_f will be equals to 24 by ref, this ref if it is less than 2300 then you can directly use this equation, this will be equal $\rho_f u_{sf} d_{bed}$ by μ_f μ_f is the viscosity.

So, f_f at ref is greater than 2300 you can calculate from this Blasius equation, here this 0.079 why ref to the power 0.25. So, from this equation from this 2 correlations Blasius you can calculate what should be the frictional friction factor, fanning friction factor and

then after substituting this friction factor here with other parameters like ρ_f and velocity of the fluid and the diameter of the bed along with the void age, if you know then you can calculate what should be the fluid wall frictional pressure drop.

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Total frictional pressure drop

$$\left(\frac{dp}{dz}\right)_{friction, tot} = \left(\frac{dp}{dz}\right)_{particle-wall} + \left(\frac{dp}{dz}\right)_{fluid-wall}$$

$$= 2f_p \frac{(1-\varepsilon)\rho_p u_p^2}{d_{bed}} + 2f_f \frac{\varepsilon \rho_f (u_{sf}/\varepsilon)^2}{d_{bed}}$$

Total pressure drop

$$\left(\frac{dp}{dz}\right)_{total} = \left(\frac{dp}{dz}\right)_{head} + \left(\frac{dp}{dz}\right)_{acceleration} + \left(\frac{dp}{dz}\right)_{friction}$$

$$= [\rho_p(1-\varepsilon) + \rho_f\varepsilon]g + \rho_p(1-\varepsilon)u_p \frac{du_p}{dz} + \rho_f\varepsilon \frac{u_{sf}}{\varepsilon} \frac{d(u_{sf}/\varepsilon)}{dz} + 2f_p \frac{(1-\varepsilon)\rho_p u_p^2}{d_{bed}} + 2f_f \frac{\varepsilon \rho_f (u_{sf}/\varepsilon)^2}{d_{bed}}$$

Then total frictional pressure drop will be summation of these 2 types of frictional pressure drop, one is particle wall frictional pressure drop another is fluid wall frictional pressure drop; now this particle wall frictional if you substitute the value of particle wall frictional pressure drop and the fluid wall frictional pressure drop then you will get these equation here.

So, total frictional pressure drop will be is equal to $2 f_f$ into 1 minus ε into ρ_p into u_p square divided by d_v plus 2 into f_f into ε into ρ_f into u_{sf} by ε whole square divided by d_{bed} , d_{bed} means diameter of the bed. So, total pressure drop will be is equal to head pressure drop, acceleration pressure drop and friction of pressure drop. So, if you substitute the value or equation for those 3 parts of this head acceleration and friction then finally, you can obtain these equations.

So, from the situation you can say what should be the total pressure drop, but if you know the total pressure drop from the experimental value and this equation is nothing but the theoretical value, if you mess it of course it will be some near about or with the least error what should be the value that you can obtain. So, without doing experimental also you can say what should be the total pressure drop inside the bed, once you know the

particle volume once you know the particle density, once you know the fluid density and other parameter, of course you can calculate from this equation.

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Total pressure drop across the whole length of the riser can be calculated by integrating as the following

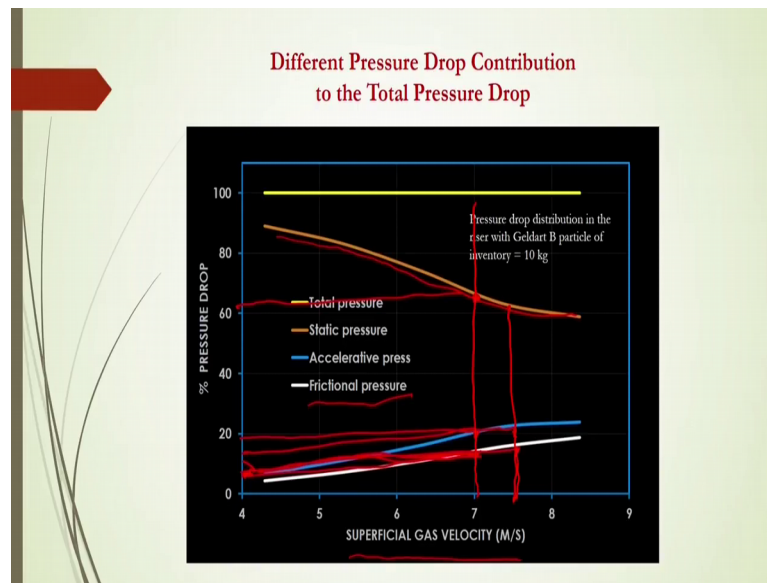
$$\Delta P_{total} = \int_0^L \left([\rho_p(1-\varepsilon) + \rho_f \varepsilon] g + \rho_p(1-\varepsilon) u_p \frac{du_p}{dz} + \rho_f \varepsilon \frac{u_{sf}}{\varepsilon} \frac{d(u_{sf}/\varepsilon)}{dz} + 2f_p \frac{(1-\varepsilon)\rho_p u_p^2}{d_{bed}} + 2f_f \frac{\varepsilon \rho_f (u_{sf}/\varepsilon)^2}{d_{bed}} \right) dz$$

$$= [\rho_p(1-\varepsilon) + \rho_f \varepsilon] g L + \left(\rho_p(1-\varepsilon) u_p^2 + \rho_f \frac{u_{sf}^2}{\varepsilon} \right) + \left(2f_p \frac{(1-\varepsilon)\rho_p u_p^2}{d_{bed}} + 2f_f \frac{\varepsilon \rho_f (u_{sf}/\varepsilon)^2}{d_{bed}} \right) L$$

Now, then total pressure drop what should be the total pressure drop across the full length of the riser, I will be have what you have calculated that will be based on the pressure drop per unit length, that the for whole length of the fluidized bed then you have to integrate that pressure drop from 0 to length of the bed.

This length of the bed actually not the actual length of the bed, it will be the actual length up to which level the gas liquid solid or gas solid or liquid solid level will be there; that means, level of the fluid mixture inside the bed that will be your length here. So, this if you integrate this then you will get this equation, so finally you will get this equation for calculating the total pressure drop for whole length of the fluidized bed. So, it depends on then particle density, it depends on a fluid density, it depends on void days, it depends on particle velocity, it depends on fluid velocity, it depends on diameter of the bed, it depends on density of the fluid; so there are several variables that will affect the pressure drop inside the bed.

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Now, see in this diagram or figure the different pressure drop contribution to the total pressure drop, how it is actually changed this total pressure drop, how it will be changing along with this other contribution; now see if you represent this total pressure drop by this yellow line and then static pressure of by this brown line and then accelerative pressure drop by this green line and then blue line and then frictional pressure drop by white line, then you can see here the percentage contribution of these static accelerative and frictional pressure drop.

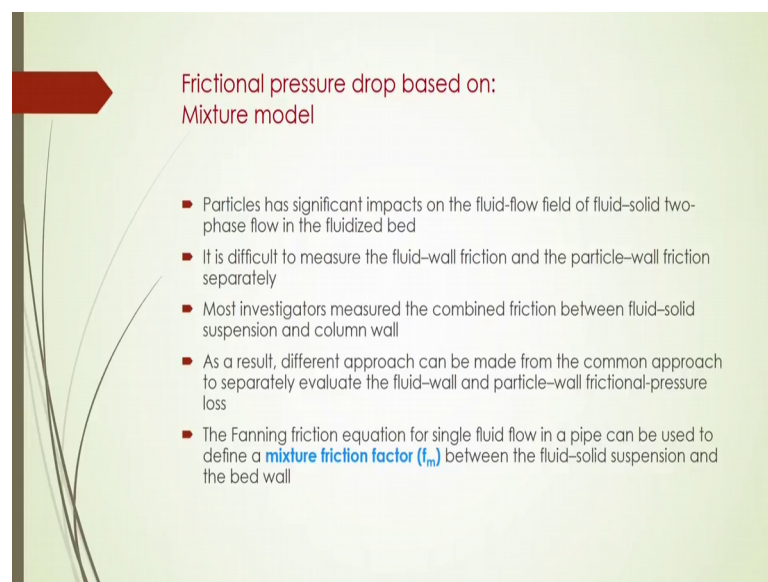
How it is coming now see the frictional pressure drop will increase with respect to the superficial gas velocity and accelerative pressure is also increase with the superficial gas velocity; whereas, the static pressure drop we will decrease with respect to the superficial gas velocity, why if superficial gas velocity increase then of course, void age of course, will increase and that void gas void age will increase; whereas, as the solid load solid head will be increase.

So, over all the effect of the head will decrease with respect to gas velocity, whereas the frictional and accelerative pressure drop will increase with respect to superficial gas velocity, because of the particle interaction and fluid particle interaction, n if you dual pa wall particle interaction for which the frictional pressure drop will increase in that case and total pressure drop will immense. Now if you consider any just line here at a certain superficial gas velocity, what should be the percentage here of this and what will be the

percentage of that is a accelerative pressure drop and what will be the percentage of this total pressure drop you will get here, this the total pressure is 100x here. So, that it is suppose this 1 is like here this case and what should be the value here it is maybe 18 and this is maybe 10 12 13 and remaining 1, of course will be is equal to head pressure drop.

So, in this way you can calculate what will be the percentage contribution. So, here if it is suppose 20 percent and then here it will be what will be the percentage here at this pressure drop, then here it will be of course less than 20 percent; whereas the head pressure would be higher but relative to the previous 1. So, in this way you can calculate what will be the pressure for different parts, then you can calculate from this diagram.

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Now, frictional pressure drop based on mixture model another model you can calculate the frictional pressure drop, in this case particles has significant impacts on the fluid flow field of fluid solid 2 phase flow in fluidized bed; now it is difficult really to measure the fluid wall friction and the particle wall friction separately. So, in this case most investigators measured the combined friction between fluid solid suspension and column wall, so we can calculate this fluid solids suspension and the column wall.

But it is very difficult to calculate the fluid wall friction and the particle wall friction separately; as a result different approach can be made from the common approach to separately calculate this fluid wall and the particle wall friction pressure loss, the fanning friction equation. Fanning friction equation for single fluid flow in this case can be used,

to define this mixture friction factor calculation between the fluid solid suspension and the bed wall, let us see how to calculate this one.

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Mixture friction factor (f_m) between the gas-solid suspension and the bed wall can be expressed as

$$\left(\frac{dp}{dz}\right)_{\text{friction}} = \frac{2f_m \rho_m u_m^2}{d_{\text{bed}}}$$

$$\rho_m = \rho_f \varepsilon + \rho_p (1 - \varepsilon)$$

$$u_m = \frac{G_m}{\rho_m} = \frac{\varepsilon \rho_f u_{sf} + G_s}{\rho_f \varepsilon + \rho_p (1 - \varepsilon)}$$

$$f_m = \frac{4}{Re_m} \quad Re_m < 2300$$

$$= \frac{0.079}{Re_m^{0.25}}; \quad Re_m > 2300$$

$$Re_m = \frac{\rho_m u_m d_{\text{bed}}}{\mu_f}$$

$$= \frac{(\varepsilon \rho_f u_{sf} + G_s) d_{\text{bed}}}{\mu_f}$$

ρ_m = mixture density
 u_m = mixture velocity
 u_{sf} = sup. fluid velocity
 G_m = mixture flux
 G_s = solid circulation flux
 Suffix m is denoted for mixture of gas and solid

Handwritten notes:
 $Mans$
 $Time Area$
 mg

Again the friction factor based on the fanning equation, here in this case mixture means you have to consider this gas liquid solid as a homogeneous mixture or if it is on the gas solid particles and gas solid homogeneous mixture will be there and if it is liquid solid then you can say the liquid solid homogeneous mixture; now that will be represented by m.

So, as per fanning equation this dp by dz friction will be equal to $2f_m$ into ρ_m into u_m^2 divided by bed diameter. So, in this case f_m is nothing but friction factor by considering the mixture of gas and solid or liquid and solid, in this case ρ_m is the density of the mixture of gas and solid and here u_m is nothing but the mixture velocity of gas and fluid gas and solid or fluid and solid. So, ρ_m is defined as this ρ_m mixture density is defined as ρ_f into ε and ρ_p into $1 - \varepsilon$ what is this ρ_f into ε ρ_f is nothing but density of the fluid either gas or liquid into ε means what is the volume fraction of the fluid inside the bed and $1 - \varepsilon$ is nothing but volume fraction of solid inside the bed into particle density. So, here this is per unit volume what should be the total mass this it is here, then what will be the mixture density you can calculate from this equation.

How to calculate these mixture velocities? Mixture velocity is nothing, but what should be the mass flux of the solid mixture flux, mixture flux of the gas solid inside the bed divided by the density of the mixture; what will be the G_m ? G_m is nothing, but here what should be the mass flux of the fluid and what should be the mass flux of the solid. So, solid mass flux is nothing, but what should be the mass rate of solid per unit cross sectional area, so that will be your mass flux here. So, mass flux is nothing but mass divided by time divided by cross sectional area.

So, this is the mass flux, so here the fluid mass flux is nothing but $\epsilon \rho_f$ into u_{sf} whereas, solid mass flux is denoted by G_s , G_s is nothing but mass of solid into mass of solid into velocity of the solid. So, in this and divided by ρ_m already defined here, this ρ_m will be is equal to $\rho_f \epsilon + \rho_p (1 - \epsilon)$.

So, this way you can calculate this mixture a velocity inside the bed, once you know the mixture velocity inside the bed you just substitute here in this friction fanning equation for friction a null pressure drop, then you can get the final value of this frictional pressure drop by this equation. Now for this f_m is required f_m is nothing but the friction factor for mixture flow this mixture flow frictional (Refer Time: 40:16) mixture flow can be calculated this here 4 by Re_m , whereas Re_m will be is equal to less than 2300 this will be and Re_m will be is equal to greater than 2300 that will be equals to f_m will be equals to 0.079 by Re_m to because 0.25 .

Whereas Re_m will be defined as the mixture density (Refer Time: 41:00) mixture velocity and mixture viscosity. So, mixture viscosity is nothing but the fluid viscosity here because gas and gas viscosity is very negligible compared to the fluid here you can say for liquid solid operation only, this fluid viscosity or liquid viscosity will be considered. So, in this case f_m to be calculate here and then from the fanning equation you can obtained what should be the frictional pressure drop.

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For first fluidization ($Re_m > 2300$)

$$f_m = \frac{0.079 \mu_f^{0.25}}{d_{bed}^{0.25} (\epsilon \rho_f u_{sf} + G_s)^{0.25}}; \quad Re_m > 2300$$

Upon substitution of different parameters, the final equation for frictional pressure drop is

$$\left(\frac{dp}{dz} \right)_{friction} = \frac{0.158 \mu_f^{0.25} (\epsilon \rho_f u_{sf} + G_s)^{1.75}}{d_{bed}^{1.25} [\rho_f \epsilon + \rho_p (1 - \epsilon)]} \quad (\text{for } Re_m > 2300)$$

Now, for first fluidization or that means, the fast it will be is corrected in a first fluidization here Re_m is greater than 2300; that means, under turbulent condition then f_m can be calculated from this equation from this correlation. So, once you know this f_m from this correlation by substituting very various variables, you can directly calculate what should be the frictional pressure drop from this equation.

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Therefore based on the mixture model, the final form of total pressure can be expressed as

$$\Delta P_{total} = \int_0^L \left(\frac{[\rho_p (1 - \epsilon) + \rho_f \epsilon] g + \rho_p (1 - \epsilon) u_p \frac{du_p}{dz} + \rho_f \epsilon \frac{u_{sf}}{\epsilon} \frac{d(u_{sf}/\epsilon)}{dz}}{\frac{2 f_m \rho_m u_m^2}{d_{bed}}} \right) dz$$

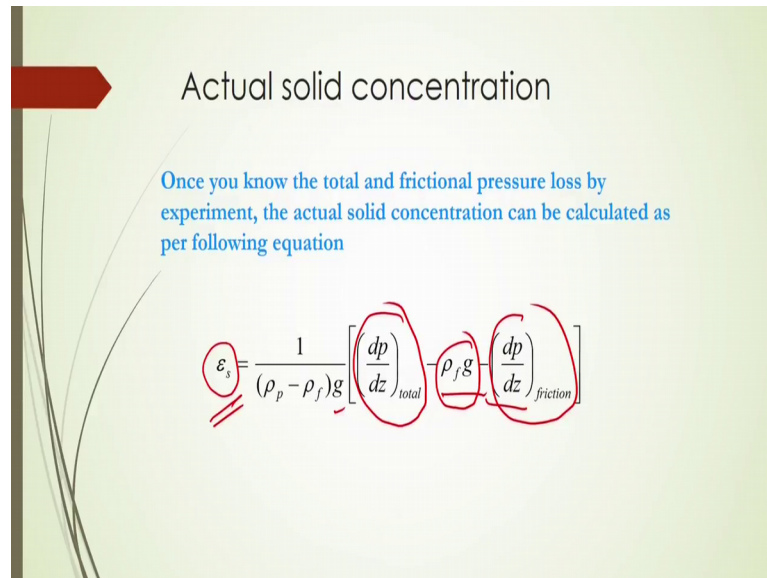
$$= [\rho_p (1 - \epsilon) + \rho_f \epsilon] g L + \left(\rho_p (1 - \epsilon) u_p^2 + \rho_f \frac{u_{sf}^2}{\epsilon} \right)$$

$$+ \left(\frac{0.158 \mu_f^{0.25} (\epsilon \rho_f u_{sf} + G_s)^{1.75}}{d_{bed}^{1.25} [\rho_f \epsilon + \rho_p (1 - \epsilon)]} \right) L$$

Therefore based on the mixture model the final form of total pressure drop can be calculated as like here, by integrating this equation final form of abbreviation you can get

from this equation here. So, this equation will give you what should be the total pressure drop based on the mixture model.

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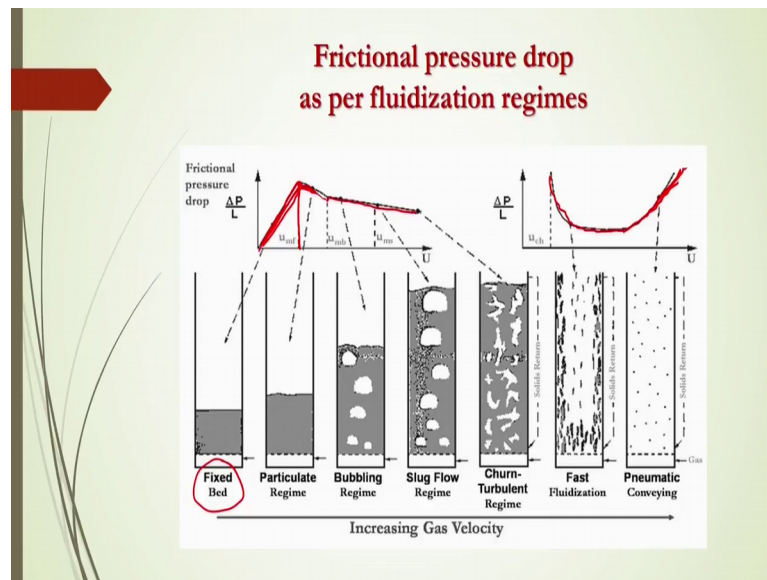
Actual solid concentration

Once you know the total and frictional pressure loss by experiment, the actual solid concentration can be calculated as per following equation

$$\epsilon_s = \frac{1}{(\rho_p - \rho_f)g} \left[\left(\frac{dp}{dz} \right)_{total} - \rho_f g \left(\frac{dp}{dz} \right)_{friction} \right]$$

Now, actual solid concentration how to calculate because, it depends this is the very important point because, you have to anyway calculate what should be the change of solid velocity actual change of solid velocity inside the bed and what should be the void age inside the bed, once you know solid volume fraction inside the bed. So, this can be calculated from the total pressure drop also. So, epsilon s to be calculated here 1 by rho p minus rho f into g and if you know the total pressure drop and if you know the frictional pressure drop and head of course, you can calculate the what should be the solid fraction inside the bed at dynamic condition.

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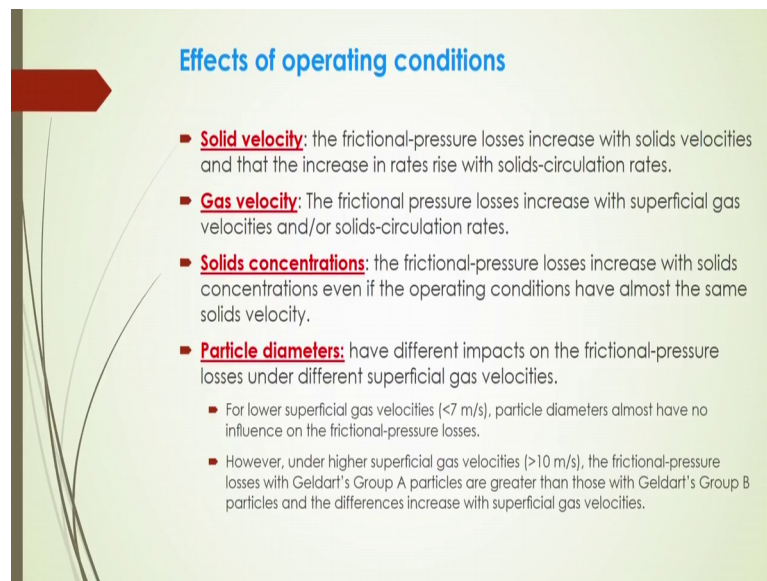
Now, see in this figure it is seen that how the frictional pressure drop will change based on the flow regime of the fluidized bed, now in the fixed bed of course this frictional pressure drop will be as per organ equation it will be directly related to the fluid velocity up to up to a certain point; that means, here where the just fluidization starts up to that this frictional pressure drop is just related here, this is where fixed bed condition. Whereas, beyond this fixed bed condition.

If you increase the gas velocity or fluid velocity how this frictional pressure drop if changing for particular fluidized bed, it is almost unchanged or very that is small change of this pressure drop here almost remain constant; whereas, if you increase little bit that gas velocity of fluid velocity that means, there bubbling flow rate fluidization regime these frictional pressure drop will reduced.

Again from this bubbling fluidized bed the fluid frictional pressure drop if will gradually decrease with respect to the fluid velocity; now it will be up to joint turbulent condition, but beyond this joint turbulent condition first fluidization it is seen that frictional pressure drop will decrease and again it will be remain constant and after certain height you will see that for neurotic condition it will be higher pressure drop, in that case the dilute solid particle there very small amount of particle interaction of and particle wall interaction will be there and maximum part will come now the due to the high velocity of the fluid and the friction between wall.

So, this from this figure how actually frictional pressure drop will change with respect to fluidized map it can be easily understood and it is seen that frictional pressure drop of course, will be going to reduce or going to decrease with the gas velocity or liquid velocity whereas, for pneumatic condition it will be reverse order it will increase with respect to gas velocity.

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Effects of operating conditions

- **Solid velocity:** the frictional-pressure losses increase with solids velocities and that the increase in rates rise with solids-circulation rates.
- **Gas velocity:** The frictional pressure losses increase with superficial gas velocities and/or solids-circulation rates.
- **Solids concentrations:** the frictional-pressure losses increase with solids concentrations even if the operating conditions have almost the same solids velocity.
- **Particle diameters:** have different impacts on the frictional-pressure losses under different superficial gas velocities.
 - For lower superficial gas velocities (<7 m/s), particle diameters almost have no influence on the frictional-pressure losses.
 - However, under higher superficial gas velocities (>10 m/s), the frictional-pressure losses with Geldart's Group A particles are greater than those with Geldart's Group B particles and the differences increase with superficial gas velocities.

Now, effects of operating conditions the frictional pressure drop, we will see if you increase the solid velocity it is seen that frictional pressure loss increases and that the increase in rates rise with solid circulation rates; there will be a solid circulation and because of which if you increase the solid circulation rate inside the bed, you will see the frictional pressure drop will be there because solid interaction and solid wall interaction will be more higher.

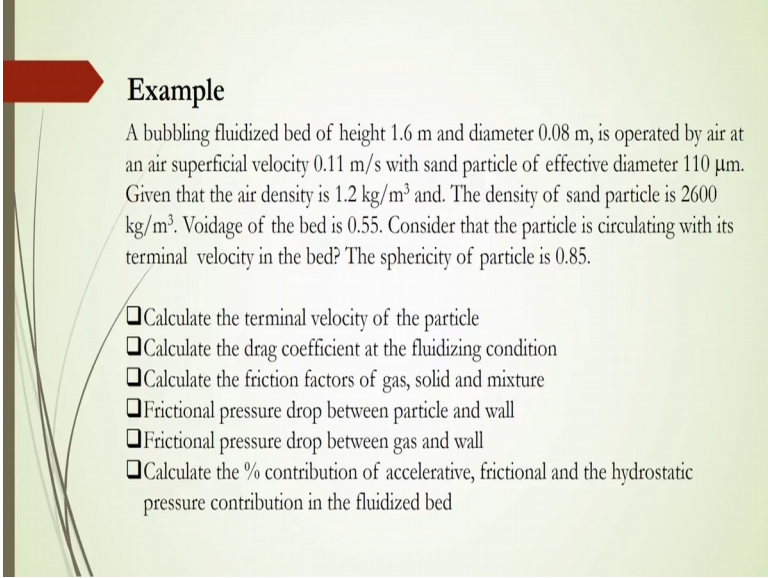
And if you increase the gas velocity of course the frictional pressure losses or pressure drop increases with superficial gas velocity or solid as circulation rate, solids concentrations of course the frictional pressure drop increases with solid concentration. If you increase the solid concentration for final particles, then you will see there will be viscosity increases and in that case it is in that frictional pressure drops increases, because of increase of viscosity inside the bed.

Now, particle diameters also has an effect in the frictional pressure drop and it has the different impact on the frictional pressure losses under different also superficial gas

velocity, the reason that it is seen that for lower superficial gas velocities if it is less than 7 meter per second, the particle diameter almost have no influence on the frictional pressure losses.

Whereas under higher superficial gas velocities if it is suppose greater than 10 meter per second, the frictional pressure loss with geldarts a particles it is seen that it will be greater than those with geldarts b particles and the difference increases with superficial gas velocity.

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Example

A bubbling fluidized bed of height 1.6 m and diameter 0.08 m, is operated by air at an air superficial velocity 0.11 m/s with sand particle of effective diameter 110 μm . Given that the air density is 1.2 kg/m³ and. The density of sand particle is 2600 kg/m³. Voidage of the bed is 0.55. Consider that the particle is circulating with its terminal velocity in the bed? The sphericity of particle is 0.85.

- ☐ Calculate the terminal velocity of the particle
- ☐ Calculate the drag coefficient at the fluidizing condition
- ☐ Calculate the friction factors of gas, solid and mixture
- ☐ Frictional pressure drop between particle and wall
- ☐ Frictional pressure drop between gas and wall
- ☐ Calculate the % contribution of accelerative, frictional and the hydrostatic pressure contribution in the fluidized bed

Now, let us seen an example how to calculate this different part of frictional pressure drop, how to calculate the terminal velocity, how to calculate the drag coefficient of the fluidized bed, how to calculate the friction factor of gas solid and mixture and how to calculate he frictional pressure drop between particle and wall how to calculate the frictional pressure drop between gas and wall, how to calculate the what should be the percentage concentration of the different parts of the total pressure drop inside the bed.

Let us see, let us consider bubbling fluidized bed whose height is 1.6 meter height and diameter is 0.08 meter and it is operated by air; that means, air solid at and air superficial velocity is 0.11 meter per second and the particle is sand particle whose effective diameter is 110 micro meter, given that the air density is 1.2 kg per meter cube and the density of the sand particle is 2600 kg per meter cube, void age of the bed is given 0.55;

now consider that the particle is circulating with its terminal velocity in the bed, the sphericity of the particle is 0.85.

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Calculation of Terminal velocity

$$u_t = \left[\frac{g d_p^2}{18 \mu} (\rho_p - \rho_g) \right] = 0.0952 \text{ m/s}$$

$$u_r = \frac{u_{sg}}{\varepsilon} - u_p = 0.11 / 0.55 - 0.3 = 0.105 \text{ m/s}$$

$$\text{Re}_t = \frac{\rho_g u_t d_p}{\mu_g} = 0.0698 < 0.1$$

$$\text{Re}_p = \frac{\rho_g u_r d_p}{\mu_g} = 0.0768$$

Now here what should be the different part of this, first of all calculate the terminal velocity based on the Stokes law you know d_p particle diameter, you know the viscosity of the fluid that is air here you know the density of the particle you know the density of the gas.

So, how to calculate this terminal is let us substitute this value you will get this terminal velocity is 0.0952 meter per second, whereas what should be the relative velocity of the particle inside the bed; now this is your superficial gas velocity it is given here 0.11 meter per second the porosity or void age is given 0.55, this is your actual gas velocity inside the bed and this is the particle velocity; here this particle velocity will be considered as the terminal velocity inside the bed, that means beyond this terminal it will be fluidized.

So, what would be the minimum this is the particle velocity is the terminal velocity beyond which it will be moving up. So, this will be your and because of that it will be circulating and so this will be your relative velocity of the gas inside the bed. Now what should be the Reynolds number based on terminal velocity, you just substitute the ρ_g that is density of the gassed in terminal velocity of the particle diameter and viscosity of the gas then you will get this is your Reynolds number this is less than 0.1.

So, it is the Stokes region of course then terminal velocity should be this 1 and if Re_p is equal to 0.0768, again this is based on the gas velocity relative velocity of the gas here and then what should be the Reynolds number of the particle.

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Calculation of drag coefficient, and particle friction factor

$$C_D = \frac{24}{Re_p} = 312.313 \quad \text{Since } Re_p \leq 0.1$$

$$f_p = 3.15 \times 10^{-3} \frac{(1-\varepsilon)}{\varepsilon^3} \left(\frac{(1-\varepsilon)Re_t}{Re_p} \right)^{-0.979} \quad \text{for } \frac{u_g}{u_t} > 1.5$$

$$= 0.0205$$

Once you know this terminal velocity relative velocity terminal Reynolds number and the particle Reynolds number, then you substitute here what should be the C_D value; C_D is nothing but 24 by Re_p because, the Stokes region of course this will be equal to 24 by a Re_p this is nothing but 312.313. So, this is your direct coefficient and what would be the particle friction factor, this particle friction factor ap will be is equal to what this is 3.15 into 10 to the power, there are 2 equations for calculating this particle. Friction factor of course this is the first one, if it is suppose this is the ratio of this gas velocity and terminal velocity is greater than 1.5 then only you have to calculate this one.

We have calculated that this U_{sg} by ε by u_t it is coming is greater than 1.5, so you have to use this correlation. So, once you put the value of this Re_t Re_p and ε here, then of course you will get this value of particle friction inside the bed as 0.0205.

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$$Re_g = \frac{\rho_g u_{sg} d_{bed}}{\mu_g} = 58.67$$

$$f_g = \frac{4}{Re_g} = 0.0682$$

$$Re_m = \frac{\rho_m u_m d_{bed}}{\mu_g} = \frac{(\epsilon \rho_g u_{sg} + G_s) d_{bed}}{\mu_g} = 1.10 \times 10^5$$

$$f_m = \frac{0.079}{Re_m^{0.25}} = 0.0043$$

Then what should be the Reynolds number for gas shear it will be ρ_g , here it will be based on the gas velocity superficial as gas velocity inside the bed and the bed diameter instead of particle diameter and the viscosity of the gas it is coming 58.67, of course it is laminar flow then you have to calculate f_g as this 4 by Re_g is equal to 0.0682, again here this Re_m as per mixture model; that Re_m to be calculated as ρ_m into u_m into d of bed that is diameter of the bed divided by the viscosity of gas.

So, once you substitute this value you will get this Reynolds number will be equals to 1.10×10^5 and based on these at this Reynolds number what should be the friction factor of the mixture, this will come here as per this since it is turbulent region then we will get this value of 0.0043.

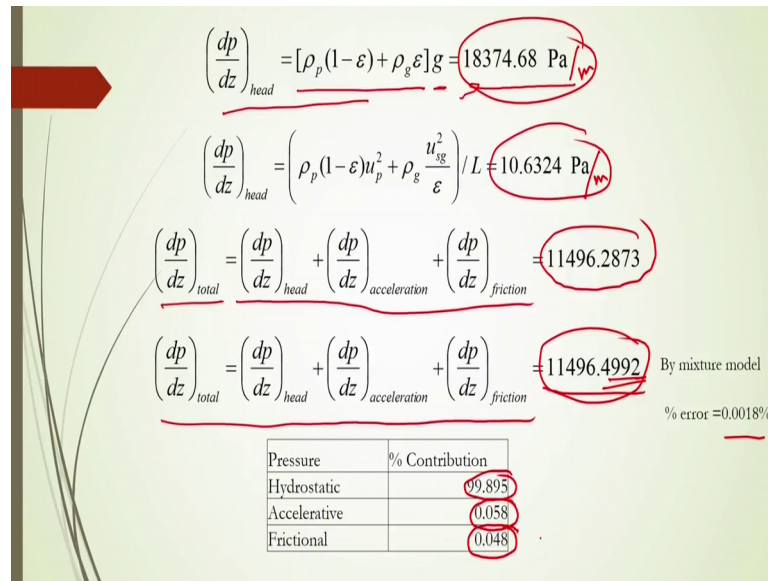
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$$\begin{aligned} \left(\frac{dp}{dz}\right)_{\text{particle-wall}} &= 2f_p \frac{(1-\varepsilon)\rho_p u_p^2}{d_{\text{bed}}} = 5.422 \quad \text{N/m}^3 \\ \left(\frac{dp}{dz}\right)_{\text{gas-wall}} &= 2f_g \frac{\varepsilon\rho_g (u_{\text{sg}}/\varepsilon)^2}{d_{\text{bed}}} = 0.0450 \\ \left(\frac{dp}{dz}\right)_{\text{friction}} &= \left(\frac{dp}{dz}\right)_{\text{particle-wall}} + \left(\frac{dp}{dz}\right)_{\text{gas-wall}} \\ &= 2f_p \frac{(1-\varepsilon)\rho_p u_p^2}{d_{\text{bed}}} + 2f_g \frac{\varepsilon\rho_g (u_{\text{sg}}/\varepsilon)^2}{d_{\text{bed}}} = 5.4674 \\ \left(\frac{dp}{dz}\right)_{\text{friction / mixture model}} &= 2f_m \frac{\rho_m u_m^2}{d_{\text{bed}}} = 5.794 \end{aligned}$$

Now, once you know this value of this different part of this frictional pressure drop, like dp by dz particle wall will be is equal to $2 f_p$ is equal to. So, you know this f_p you have calculated you know ρ_p you know u_p you know bed diameter. So, finally it will come like this 5.422 and then here it will be of course Newton per meter cube and then gas wall frictional pressure drop will be coming like this, after substitution of this value and friction total, frictional pressure drop will be is equal to than frictional pressure drop of particle wall and frictional pressure drop of gas wall, then you will get this total value of this here as 5.4674.

And then the frictional pressure drop based on mixture model, it will come as $2 f_m$ into ρ_m into u_m square by diameter of the bed, here ρ_m you know that ρ_m you know u_m you know and f_m you know after substitution you can calculate it will come as 5.794. Whereas, based on individual interaction to the wall of the bed, then you will see the total frictional pressure drop is coming 5.4674, whereas it is for mixture model it is coming 5.794 almost same here there will be little error for this.

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$$\left(\frac{dp}{dz}\right)_{head} = [\rho_p(1-\varepsilon) + \rho_g \varepsilon]g = 18374.68 \text{ Pa/m}$$

$$\left(\frac{dp}{dz}\right)_{head} = \left(\rho_p(1-\varepsilon)u_p^2 + \rho_g \frac{u_{sg}^2}{\varepsilon}\right)/L = 10.6324 \text{ Pa/m}$$

$$\left(\frac{dp}{dz}\right)_{total} = \left(\frac{dp}{dz}\right)_{head} + \left(\frac{dp}{dz}\right)_{acceleration} + \left(\frac{dp}{dz}\right)_{friction} = 11496.2873$$

$$\left(\frac{dp}{dz}\right)_{total} = \left(\frac{dp}{dz}\right)_{head} + \left(\frac{dp}{dz}\right)_{acceleration} + \left(\frac{dp}{dz}\right)_{friction} = 11496.4992 \text{ By mixture model}$$

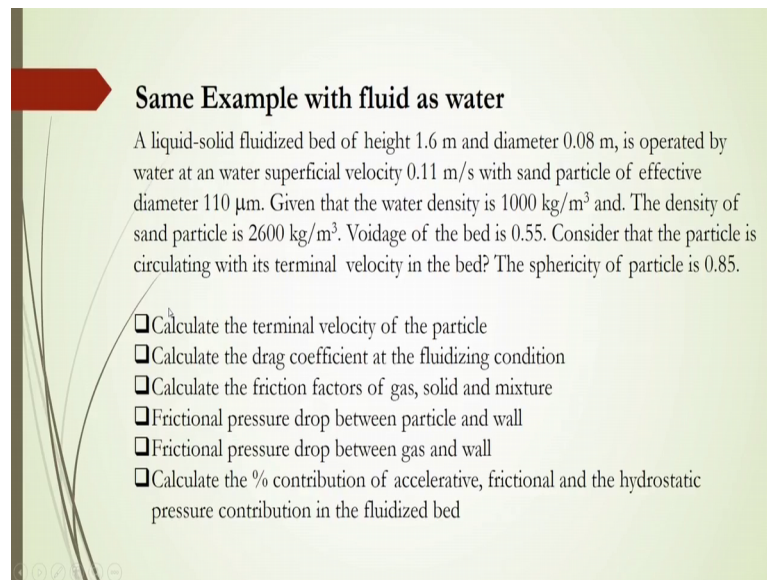
% error = 0.0018%

Pressure	% Contribution
Hydrostatic	99.895
Accelerative	0.058
Frictional	0.048

So, we can consider the mixture model also. And here head; head will be is nothing but here mixture density into gravitational osculation it is coming like this, it will be Pascal per meter and here dp by dz head it will be is equal to this 10.634 Pascal per meter, and then total frictional pressure drop here it will be is equal to this 11496.2873 just summation of this 3 component of this total pressure drop.

And as per mixture model this total frictional pressure drop is coming here 11496.4992, there is error is only 0.0018 percent. Now what should be the percentage contribution this is the total pressure drop, and these are the contribution of different part of this frictional pressure drop, it is seen that hydrostatic part is the maximum one that is 99.89 percent. So, whereas accelerative portion is very low 0.058 and frictional pressure drop is also very small. So, compared to the hydrostatic pressure it is coming very low, these are this accelerative frictional pressure drop. Now this frictional pressure drop once you know put the total measured pressure drop, also you can calculate the frictional pressure drop by subtracting this accelerative and head pressure drop.

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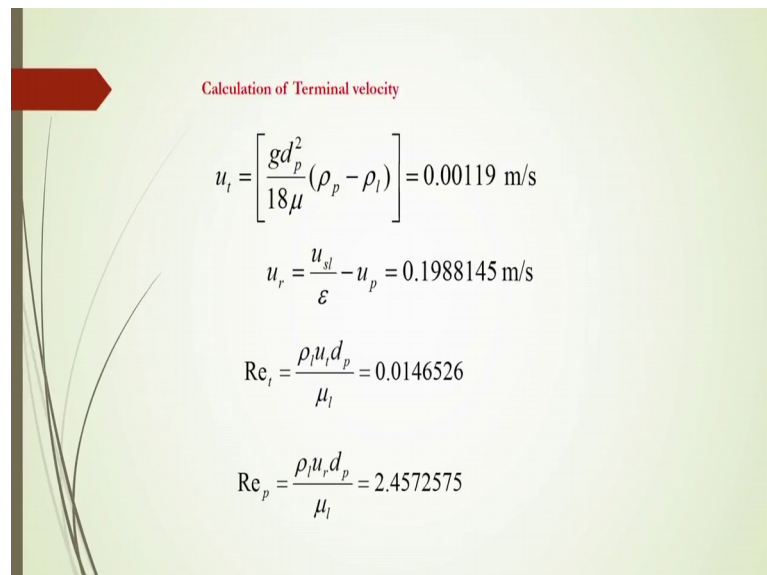
Same Example with fluid as water

A liquid-solid fluidized bed of height 1.6 m and diameter 0.08 m, is operated by water at an water superficial velocity 0.11 m/s with sand particle of effective diameter 110 μm . Given that the water density is 1000 kg/m^3 and. The density of sand particle is 2600 kg/m^3 . Voidage of the bed is 0.55. Consider that the particle is circulating with its terminal velocity in the bed? The sphericity of particle is 0.85.

- ☐ Calculate the terminal velocity of the particle
- ☐ Calculate the drag coefficient at the fluidizing condition
- ☐ Calculate the friction factors of gas, solid and mixture
- ☐ Frictional pressure drop between particle and wall
- ☐ Frictional pressure drop between gas and wall
- ☐ Calculate the % contribution of accelerative, frictional and the hydrostatic pressure contribution in the fluidized bed

Now, same example with fluid as water, let us consider this water instead of gas, now the same here the height of the bed is this and what is that diameter of the bed is 0.08 and water density only will change and others are remain same. So, what should be the different component of this calculation.

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Calculation of Terminal velocity

$$u_t = \left[\frac{gd_p^2}{18\mu} (\rho_p - \rho_l) \right] = 0.00119 \text{ m/s}$$
$$u_r = \frac{u_{sl}}{\varepsilon} - u_p = 0.1988145 \text{ m/s}$$
$$\text{Re}_t = \frac{\rho_l u_t d_p}{\mu_l} = 0.0146526$$
$$\text{Re}_p = \frac{\rho_l u_r d_p}{\mu_l} = 2.4572575$$

Terminal velocity will come here, and relative velocity Reynolds number based on terminal velocity, Reynolds number based on particle diameter it is coming like this.

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Calculation of drag coefficient, and particle friction factor

$$C_D = \frac{24}{Re_p} \left[1 + (8.1716e^{-4.0655\phi_s}) Re_p^{0.0964+0.5565\phi_s} \right] + \frac{73.69(e^{-5.0748\phi_s}) Re_p}{Re_p + 5.378e^{6.2122\phi_s}}, \text{ for } \phi_s \neq 1$$

Since $0.1 < Re_p < 500$

$$= 13.97$$

$$f_p = 3.15 \times 10^{-3} \frac{(1-\varepsilon)}{\varepsilon^3} \left(\frac{(1-\varepsilon) Re_t}{Re_p} \right)^{-0.979} \text{ for } \frac{u_{sl}/\varepsilon}{u_t} = 168.70 > 1.5$$

$$= 2.8038891$$

And then what should be the drag coefficient is coming 13.97 within this range of Reynolds number of particle here and f_p again here this is ratio is coming greater than 1.5. So, it will be 2.80 as per that variable change of density, and the viscosity and others thing as per liquid just water.

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$$Re_l = \frac{\rho_l u_{sl} d_{bed}}{\mu_l} = 988.76 \quad f_l = \frac{4}{Re_l} = 0.0040455$$

$$Re_m = \frac{\rho_m u_m d_{bed}}{\mu_l} = \frac{(\varepsilon \rho_l u_{sl} + G_s) d_{bed}}{\mu_l} = 571.52$$

$$f_m = \frac{24}{Re_m} = 0.04199$$

So, again Reynolds number for liquid Reynolds number for this friction factor for liquid as per fanning equation and then what is the Reynolds number for mixture, it will be coming as 571.52 again the friction factor based on mixture model it is coming this one.

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$$\begin{aligned} \left(\frac{dp}{dz}\right)_{\text{particle-wall}} &= 2f_p \frac{(1-\varepsilon)\rho_p u_p^2}{d_{\text{bed}}} = 0.1152 \\ \left(\frac{dp}{dz}\right)_{\text{liquid-wall}} &= 2f_l \frac{\varepsilon\rho_l(u_{sl}/\varepsilon)^2}{d_{\text{bed}}} = 2.225 \\ \left(\frac{dp}{dz}\right)_{\text{friction}} &= \left(\frac{dp}{dz}\right)_{\text{particle-wall}} + \left(\frac{dp}{dz}\right)_{\text{liquid-wall}} \\ &= 2f_p \frac{(1-\varepsilon)\rho_p u_p^2}{d_{\text{bed}}} + 2f_l \frac{\varepsilon\rho_l(u_{sl}/\varepsilon)^2}{d_{\text{bed}}} = 2.3403 \\ \left(\frac{dp}{dz}\right)_{\text{friction / mixture model}} &= \frac{2f_m \rho_m u_m^2}{d_{\text{bed}}} = 2.468 \end{aligned}$$

And then individual component this frictional pressure drop then it will come 0.1152 and also for liquid wallet it is coming 2.225 friction, then total frictional pressure drop it will coming 2.340, just summation of these two; and then as per mixture model it is coming 2.468 it is also almost same.

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$$\begin{aligned} \left(\frac{dp}{dz}\right)_{\text{head}} &= [\rho_p(1-\varepsilon) + \rho_g \varepsilon]g = 16873.20 \text{ Pa} \\ \left(\frac{dp}{dz}\right)_{\text{accl}} &= \left(\rho_p(1-\varepsilon)u_p^2 + \rho_l \frac{u_{sl}^2}{\varepsilon}\right)/L = 13.751 \text{ Pa} \\ \left(\frac{dp}{dz}\right)_{\text{total}} &= \left(\frac{dp}{dz}\right)_{\text{head}} + \left(\frac{dp}{dz}\right)_{\text{acceleration}} + \left(\frac{dp}{dz}\right)_{\text{friction}} = 6889.291 \\ \left(\frac{dp}{dz}\right)_{\text{total}} &= \left(\frac{dp}{dz}\right)_{\text{head}} + \left(\frac{dp}{dz}\right)_{\text{acceleration}} + \left(\frac{dp}{dz}\right)_{\text{friction}} = 16889.419 \text{ By mixture model} \\ &\quad \% \text{ error} = 0.00075\% \end{aligned}$$

Pressure	% Contribution
Hydrostatic	99.904
Accelerative	0.081
Frictional	0.015

So, after substituting this value you will see that head component is coming like this, and the accelerative component is coming and this total then pressure drop will be coming like this.

Whereas this here frictional pressure drop is coming this total pressure drop where based on mixture model it is coming this one. So, error is again very negligible, but here also you will see that hydrostatic pressure drop will be high, irrelative to this accelerative and frictional pressure drop.

Now you will see if you increase the density of the fluid of course this frictional part will be higher, the frictional part will be higher and accelerative part also in this case the relative to the gas here. But a frictional pressure drop let us see what should be the frictional pressure drop as earlier a portion contribution here.

So, frictional this is contribution is 0.048, whereas in this paper liquid 0.01. So, it is less in case of liquid, whereas the in case of gas there will be higher. So, by this example you can say that the how the frictional pressure drop will change, what will be the terminal velocity of the drag coefficient, how the total pressure drop is contributed to the different components then a portion can calculate, what will be the total pressure drop inside the bed by this concept.

So, next class or next lecture we will discuss again with that gas liquid solid system, how to calculate the frictional pressure drop, total pressure drop all those things and you can go through this portion through your text book, but some portions you will not get different, but you can get it from this only concept it will be useful for you so.

Thank you for this today's part.