

Fluidization Engineering
Dr. Subrata K. Majumder
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture – 04
Minimum Fluidization Velocity: fluid-solid System

Welcome to massive open online course on fluidization engineering today's lecture will be on minimum fluidization velocity of fluid solid system, before going to that minimum fluidization velocity, we have know something about terminal velocity of the particle because this terminal velocity of the particle has a immense rule.

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Terminal Velocity of Particle

Terminal velocity of a particle is the velocity at which it falls freely through a fluid.
 At the terminal velocity, the weight of the object is exactly balanced by the upward **buoyancy force** and **drag force**.

$W = F_b + F_D$

Where

W = weight of the particle

F_b = Buoyancy force

F_D = Drag force

Drag is a force acting opposite to the relative motion of any object moving with respect to a surrounding fluid.

$W = \frac{\pi}{6} d_p^3 \rho_s g$

$F_b = \frac{\pi}{6} d_p^3 \rho_f g$

$F_D = C_D \frac{\rho_f}{2} u_t^2 A$ for $Re_p > 500$
 $= 3\pi \mu d_p u_t$ for $Re_p < 0.2$

d_p = diameter of particle,
 u_t = terminal velocity
 ρ_f = density of fluid
 ρ_s = density of solid
 C_D = Drag coefficient
 A = projected area of particle $= \pi/4 d_p^2$

Whether the fluidization occurring or not because beyond this terminal velocity of course, fluidization will start so, what is that terminal velocity of the particle in the fluidized bed or fluidized manner that you have to know. This is basically the velocity at which to the particle falls freely through a fluid and at the terminal velocity the weight of the object is exactly object or particle you can say here is exactly balanced by the upward buoyancy force and of course, it will be balanced by the drag force.

And here see if you consider based the weight of the particle as W and a buoyancy force is denoted by F b here and drag force is represented by F D then this weight of the particle will be balanced by the summation of these 2 courses like F b and F D F b means buoyancy force and F D means drag force what is drag here drag is nothing, but a force

acting opposite to the relative motion of any particle moving with respect to a surrounding fluid.

Now, what is that how to calculate this weight of the particle W this weight of the particle is nothing, but mass into gravitational acceleration how to calculate mass if you know the volume of particle like what will be that volume of particle if you know the effective diameter of the particle d_p then $\frac{\pi}{6} d_p^3$ it will be the volume of the particle and density of the particle is ρ_p then volume into density then it will give you the mass of the particle then mass of the particle into gravitational acceleration g it will give you the weight; that means, that force that will be acting downward that is the under gravity.

F_n , F_b is the buoyancy force what is buoyancy force buoyancy force is the force that is applied on a particle by the fluid what is that actually this buoyancy force is nothing, but how much volume of fluid is displaced by this particle of course, the volume of the fluid will be displaced by the volume of the particle. What will be the fluid that is being displaced by the particle is $\frac{\pi}{6} d_p^3$ and then this mass volume that will be considered for fluid volume by which the buoyancy is acting on the particle.

Now, this volume into density of the fluid density of the fluid then you will get the mass of the fluid then into g this gravitational acceleration. This mass into gravitational acceleration it will give you the buoyancy force here this mass will be the how much mass of the fluid that is displaced by the solid particle and then F_D , F_D there is drag force 2 types of drag 2 drag force you will get sometimes you will see for very laminar region.

That means, a flow is under terminal condition means the fluid is acting on the particle when particle is going downward under it is that is normal gravitation normal gravity that is called a terminal condition; that means, they are the velocity whatever it is it will less than it will be less than 0.2 for Reynolds numbers. Reynolds number if Reynolds number of the particle if it is less than 0.2 then the drag force will be calculated as this by stokes flow that will be equals to $3 \pi \mu d_p U_t$

Under this terminal velocity what will be the drag force this will be $3 \pi \mu d_p U_t$ and this is applied only for Reynolds number if it is less than 0.2 what is that Reynolds number Reynolds number is nothing, but this what will be the inertia force

under this the terminal velocity and it will be inertia force and it will be ratio to the viscous force. Reynolds number is nothing, but the inertia force to the viscous force.

Now, this Reynolds number of particle will be calculated based on this particle diameter here. So, Re_p will be is nothing, but $\rho u d_p$ by μ , now if suppose Reynolds number is greater than 0.2 or it is generally for (Refer Time: 06:19) told that that it will be if it is less greater than 500 then this drag force will be calculated by this here this drag force will be directly related to the kinetic energy of the fluid at which this moving under this terminal velocity.

Now, what will be the a is the projectional area of the fluid particle and U_t is the terminal velocity of the solid particle. $\rho F U_t^2$ that will be is equal to ρa half of ρF into U_t is square this is called kinetic energy by which this solid particle will be moving downward or upward or anyway that is under this terminal velocity.

Now this drag force the drag force is directly related to this kinetic energy projectional area of the particle then if what will be the then const proportionality constant that proportionality constant will be is equal to C_D that is called drag coefficient. So, F_D will be is equal to C_D into half of $\rho F U_t^2$ into A , this will be is equal to drag force. Drag force this is under the condition of Reynolds number is greater than 500.

Now, how to calculate this projectional area if it is fluidical particle of course, this projectional area of the particle will be is equal to that is cross sectional area of the particle; that means, π by 4 into d_p^2 square this is your cross sectional area of the particle if it is fluidical of course, this will be π by 4 into d_p^2 square.

Now, if we if we equalize this force is as per this equation W is equal to F_b plus F_D then we will get here.

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$$W = F_b + F_D$$

$$\frac{\pi}{6} d_p^3 \rho_p g = \frac{\pi}{6} d_p^3 \rho_f g + 3\pi \mu d_p u_t$$

Implies

Terminal velocity (Stokes flow)

$$u_t = \left[\frac{g d_p^2}{18 \mu} (\rho_p - \rho_f) \right]$$

If $Re_p < 0.2$

$$Re_p = \frac{\rho_f u d_p}{\mu}$$

W here if we substitute here for W that will be is equal to pi by 6 d p cube rho p into g and F b this F b. F b will be is equal to this here pi by 6 d p cube rho f into g and then F D this F D is the drag force if we consider it as a under that is stokes flow stokes flow then ah; that means, here if Re p is less than 0.2 then it will be 3 pi mu d p into ut.

Now, if we simplify it or rearrange it then you will get the terminal velocity under this stokes flow that will be U t will be equals to U t will be equals to gd p square by 18 mu into rho p minus rho f. So, here of course, this by this equation you can calculate what should be the terminal velocity under this stokes flow this stokes flow of will considered if Re p less than 0.2 here see one particle this on this particle this is one W force is acting and then what is thus drag force is acting and buoyancy force acting.

This buoyancy force and W; that means, weight of the particle if it is going downward of course, it will be relative velocity will be W minus F b this is called effective weight of the particle or you can say apparent weight of the particles this then this apparent weight of the particle will be balanced by the drag force this drag force the direction of the drag force will be opposite to the particle motion. If you balance say similar way it is nothing, but w minus F b will be is equal to apparent weight.

Sometimes in the different reference because they are directly related as the apparent force will be equals to drag force. Apparent force will be calculated as W minus F b what

will be the relative force of weight and buoyancy force, this by this way we can calculate then terminal velocity under this stokes flow that if Reynolds number is less than 0.2

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$$W = F_b + F_D$$

$$\frac{\pi}{6} d_p^3 \rho_p g = \frac{\pi}{6} d_p^3 \rho_f g + C_D \left(\frac{1}{2} \rho_f u_t^2 \right) \left(\frac{\pi}{4} d_p^2 \right)$$

Implies

Terminal velocity (turbulent flow)

$$u_t = \left[\frac{4 g d_p (\rho_p - \rho_f)}{3 C_D \rho_f} \right]^{1/2}$$

If $Re_p > 500$

$$Re_p = \frac{\rho_f u_t d_p}{\mu}$$

Now, if Reynolds number is greater than 500 that is it is not stokes flow; that means, under turbulent flow then of course, you have to substitute the drag force as C_D into half of half of $\rho_f u^2$ into π by 4 into d_p^2 . This is your drag force at turbulent condition; that means, Reynolds number of the particle if it is greater than 500

Now, this see Reynolds number how it is defined here this is density into velocity into particle diameter into by μ means viscosity here, μ is the viscosity ρ_f is the density of the fluid u is the velocity d_p is the particle diameter.

Now, again if you substitute this W as this a weight of the particle as π by 6 into d_p^3 into ρ_p into gravitational acceleration then you will see this will be your volume this will be your density and volume into density will give you the mass into g that is the weight of the particle and this is buoyancy force again the same way this buoyancy force how much volume will be displaced by the particle into it is density it is into gravitational acceleration this will be buoyancy force and drag force under this condition of Re_p is greater than 500.

After rearrangement and simplification you will see this terminal velocity at this terminal turbulent flow will be calculated as here just $4 g d_p$ by $3 C_D$ into ρ_p minus ρ_f

divided by rho F whole to the power half; that means, the square root of this. In this way a turbulent condition you can calculate what should be the terminal velocity of the particle.

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Dimensionless Particle diameter and Terminal velocity

$$d_p^* = d_p \left[\frac{\rho_g (\rho_s - \rho_g) g}{\mu^2} \right]^{\frac{1}{3}} = Ar^{\frac{1}{3}} = \left(\frac{3}{4} C_D Re_p^2 \right)^{\frac{1}{3}}$$

$$u^* = u \left[\frac{\rho_g^2}{\mu (\rho_s - \rho_g) g} \right]^{\frac{1}{3}} = \frac{Re_p}{Ar^{\frac{1}{3}}} = \left(\frac{4}{3} \frac{Re_p}{C_D} \right)^{\frac{1}{3}}$$

$$u_t^* = \left[\frac{18}{(d_p^*)^2} + \frac{2.335 - 1.744 \phi_s}{(d_p^*)^{0.5}} \right]^{-1}, \quad 0.5 < \phi_s < 1$$

$$u_t^* = \left[\frac{18}{(d_p^*)^2} + \frac{0.591}{(d_p^*)^{0.5}} \right]^{-1}, \quad \phi_s = 1$$

And then we will represent this terminal velocity and also particle diameter as a dimensionless form. What will be the dimensionless particle diameter and terminal velocity why we should do this sometimes some references you will see that it is easier to actually represent the flow design and also the flow behavior by this dimensionless number.

We can define this dimensionless number of this d_p that was particle diameter as d_p^* this d_p^* will be into d_p into this ρ_g into ρ_s minus ρ_g into g by μ square here ρ_s is the solid density and ρ_g is the gas density if you are considering the fluid as a gas then you have to say we are you have to you have to consider this gas density as ρ_g whereas, μ is square μ square is also the gas viscosity if it is air then of course, it will be air viscosity. So, d_p will be defined in this way d_p^* this is a dimensionless diameter.

After rearrange rearrangement you can get it here this totally here it will be Archimedes number to the power 1 by 3 here this portion d_p cube into ρ_g into ρ_s minus ρ_g into g by μ square it is called as Archimedes number already we have defined earlier

this Archimedes number. This Archimedes number here it will be 3 by 4 then C D into Re p square to the power 1 by 3 this is one form.

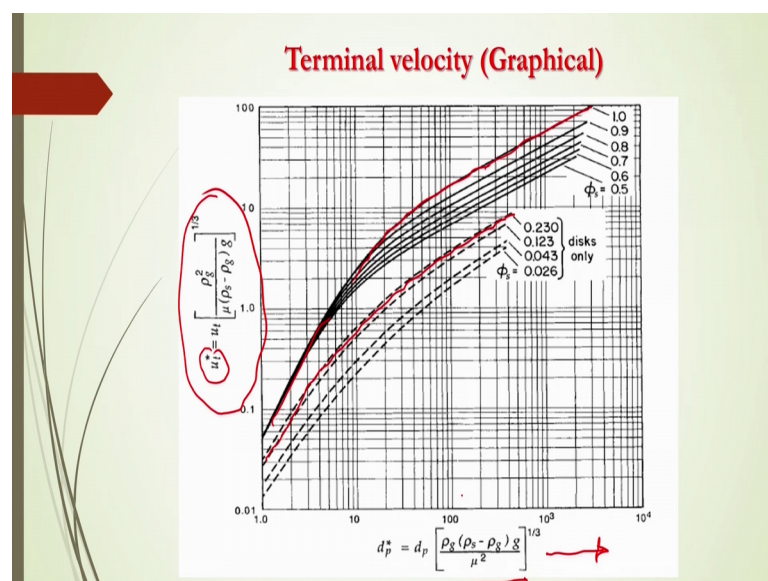
Now, u_{star} u_{star} ; that means, here you can define it as u into some dimensionless number here is ρg square by μ into ρs minus ρg to the power 1 by 3 then it will be Re p by Archimedes number to the power 1 by 3 and that will be is equal to 4 by 3 into Re p by C D to the power 1 by 3.

In this way this u_{star} will be defined what is that $U t_{\text{star}}$ $U t_{\text{star}}$ means what will be the dimensionless number of the terminal velocity this terminal velocity of this dimensionless form will be equals to like here this 18 by d p square plus 2.35 minus 1.74 phi is by d p star to the power 0.5

This terminal velocity you can represent in terms of this dimensionless number of particle diameter as this of course, it will be valid or can be calculated within the range of 0.5 to 1 for sphericity and then here $U t_{\text{star}}$ will be is equal to then u 18 by d p star square plus 0.5 19 by d p star to the power 0.5 to the power minus 1. This will be power if sphericity will be equals to 1 like for a spherical particle.

Here it is important to know that here it is important to know that what will be the terminal velocity at it is dimensionless form what should be the particle diameter at it is dimensionless form. This will be actually used later on for a different aspect.

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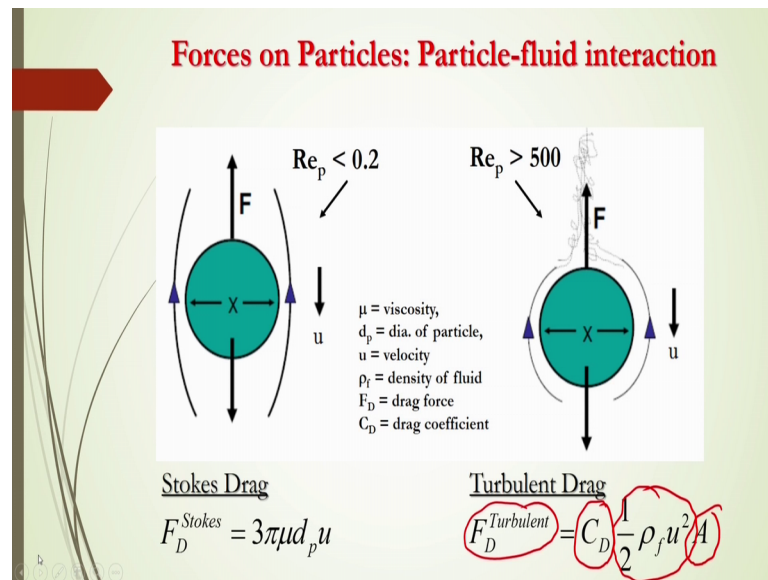
And then terminal velocity this terminal velocity you can represent this terminal velocity graphically also.

Now, if we represent this terminal velocity by this graph shown here the slide see here in the x axis in the x axis the d_p star is denoted as d_p will into $\rho_g \rho_s$ minus ρ_g by μ square g to the power $1/3$, and y axis this is U_t star that is for terminal velocity what should be that U_t into this, by this how this U_t that is terminal velocity will be related to the particle diameter you will see here U_t star if you consider the if you consider here.

See the U_t star will be changing with d_p for different sphericity for different sphericity you will see for sphericity is equal to 1 this line this line will give you the different data of terminal velocity for different d_p that is particle diameter and also for different sphericity will get different terminal velocity and from the graph it is observe that if you decrease the sphericity; that means, if it is deviated from the spherical particle then you will see the terminal velocity how it will be changing with respect to particle diameter of course, the terminal velocity will increase if d_p is increased.

That means particle size if it is increased then terminal velocity will increase of course, if size is increased that very; that means, how fast this particles will fall down freely and for different other shapes like 0.23 you will see this type of terminal velocity, from this graph you can easily calculate you can identify the number of terminal velocity at a different particle of size the different particle diameter.

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Now, let us discuss about the particle fluid interaction now you will see whenever a particle is flowing in a fluidize bed or it is falling in a fluid medium freely then you will see the liquid and particle there will be interaction now there will be some force acting on the particle whenever it will be moving or falling freely.

We have already seen that there are 2 types of forces are acting the particle on the particle like buoyancy force wind drag force we will see buoyancy force already we have discussed and stokes there will be if it is there if the particle is falling freely with the laminar flow; that means, if Reynolds number is less than 0.2 then you will see there will be a force acting on the particle it is called drag force that drag force is a calculated as $3\pi\mu d_p u$.

And whereas, this drag force will be acting whenever it will be moving with Reynolds number is greater than 500 also in that case this drag force is called turbulent drag force this turbulent drag force the will be calculated as C_D into half of $\rho_f u^2 A$, of course, you will see there will be a force acting whenever it will be moving down or upward based on the particle size also and also what will be the velocity on that the depends on.

Now what should be the drag coefficient then from the drag force of course, you will be able to calculate the drag coefficient in the previous slides we have seen that here and the previous slide here you will see this is the drag force and this drag force is related to the

kinetic energy acting on the particle by which the particle is moving and also the projectional area of the particle a and this proportionality constant is called drag coefficient.

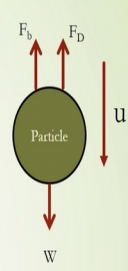
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Drag coefficient (C_D)

$$C_D = \frac{\text{Drag force}}{\text{Area} \times \text{Fluid-stress}} = \frac{F_D}{\pi d_p^2 / 4 \times \rho_f u^2 / 2}$$

$$C_D^{\text{Turbulent}} = \frac{F_{D,\text{turbulent}}}{\pi d_p^2 / 4 \times \rho_f u^2 / 2} = 0.44 \quad \text{If } Re_p > 500$$

$$C_D^{\text{Stokes}} = \frac{F_{D,\text{Stokes}}}{\pi d_p^2 / 4 \times \rho_f u^2 / 2} = \frac{3\pi\mu d_p u}{\pi d_p^2 / 4 \times \rho_f u^2 / 2} = \frac{24}{Re_p} \quad \text{If } Re_p < 0.2$$

$$Re_p = \frac{\rho_f u d_p}{\mu}$$


Now, more precisely you can say this drag force will be defined as this drag force divided by area; that means, here projectional area by fluid stress by fluid stress; that means, here this drag force is represented by F_D and here projectional is the $\pi d_p^2 / 4$ this is d_p is the particle diameter and fluid stress will be represented as by $\rho_f u^2 / 2$. This will be is equal to fluid stress or you can say kinetic energy acting on the particle here.

Now, by this you can calculate from this equation you can calculate what should be the drag coefficient now this drag coefficient for the turbulent flow you will see it will be calculated as what will be the $F_{D,\text{turbulent}}$ and then $\pi d_p^2 / 4$ by 1 by $\rho_f u^2 / 2$ here you will see u^2 will be high. So, that the Reynolds number of the particle will be is greater than 500.

In that condition in that condition it is seen that this drag coefficient will remain almost constant and this constant value will be equal to 0.4. At the turbulent condition the drag coefficients will remains constant this is will be equals to 0.4 whereas, in the stokes condition; that means, if Reynolds number is less than 0.2 then you will see the drag coefficient as per this equation it will be represented by this quantity like this C_D^{Stokes}

will be equals to 24 by Re_p here it will be depending on the Reynolds number of the particle of course, here it will not be remained constant of course, within this range of this Reynolds number that is less than 0.2 if it is suppose 0.1 this will be 0.01.

In that region that is if Reynolds number will decrease then you will see C_D will increase; that means, drag coefficients inversely proportional to the Reynolds number. By this equation you can calculate what should be the drag coefficient in the stokes condition and also turbulent condition.

Drag coefficient you will be able to calculate now different authors. They have actually obtained different drag coefficients from their experimental data and they made the correlations in different way to represent the drag coefficient.

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Other Correlations for Drag Coefficient

Haider and Levenspiel (1989) correlation

$$C_D = \frac{24}{Re_p} \left[1 + (8.1716e^{4.0655\phi_s}) Re_p^{0.0964+0.5565\phi_s} \right] + \frac{73.69(e^{-5.0748\phi_s}) Re_p}{Re_p + 5.378e^{6.2122\phi_s}}, \text{ for } \phi_s \neq 1 \quad Re_p > 0.2$$

For spherical particles this expression reduces to

$$C_D = \frac{24}{Re_p} + 3.3643 Re_p^{0.3471} + \frac{0.4607 Re_p}{Re_p + 2682.5}, \text{ for } \phi_s = 1 \quad Re_p > 0.2$$

And here see Haider and Levenspiel 99 they have proposed a correlation for drag coefficient and they have represented the correlation as the C_D will be equals to 24 by Re_p into 1 plus 8.1716 e to the power minus 4.065 into ϕ_s and into Re_p to the power 0.0964 plus 0.565 ϕ_s . Here this portion not only this is big correlation of course, some other part here this is 73.69 into e to the power these and divided by Re_p plus these.

From this observe from this correlation what to see in that if Re_p is greater than 0.2, we can easily calculate what should be the drag coefficient from this correlation and this correlation from this correlation it is saying that this C_D not only depends on the

Reynolds number it depends only on depends also on the sphericity of the particle see how this sphericity of the particle actually effect on a C D you can calculate it from this correlation.

If you have different sphericity and a different velocity of course, diameter then you can use this equation to calculate the drag coefficient even for spherical particles if suppose ϕ_s is equal to one then very simple here from this you can directly obtain that correlations obtained the value of C D from this correlations are putting the ϕ_s is equal to 1.

Finally, you can get reduces this equation to this equation if we put the ϕ_s is equal to 1 then C D will be is equal to 24 by Re p plus this. From this equation you will be able to calculate what should be the drag coefficient of course, this drag coefficient correlation is limited to the flow if Re p is greater than 0.2 you cannot apply this correlation for stokes condition; that means, Re p and; that means, Reynolds number if it is less than 0.2.

Now, let us see 1 example here is that they calculate the terminal velocity for the sharp irregular sand particles if there is a regular sand particles then what should be the terminal velocity and this of course, this irregular sand particles is allowed to fall in a air medium; that means, to air then what should be the terminal velocity.

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Example

Calculate terminal velocity for the sharp irregular sand particles

$\rho_g = 1.2 \times 10^{-3} \frac{g}{cm^3}$ $\mu = 1.8 \times 10^{-4} \frac{g}{cm.s}$ $d_p = 160 \mu m$ $\rho_s = 2.60 \frac{g}{cm^3}$ $\phi_s = 0.67$

Solution

$$d_p^* = 0.0160 \left[\frac{0.0012(2.6 - 0.0012)980}{(0.00018)^2} \right]^{\frac{1}{3}} = 7.28$$

$$U_t^* = \left[\frac{18}{(7.28)^2} + \frac{2.335 - 1.744 \times 0.67}{(7.28)^{0.5}} \right]^{-1} = 1.2954$$

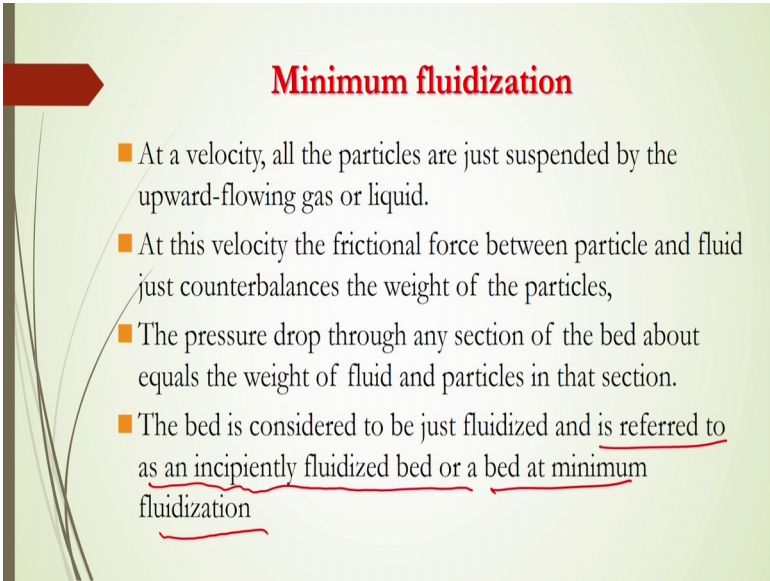
$$U_t = U_t^* \left[\frac{\mu(\rho_s - \rho_g)}{\rho_g^2} \right]^{\frac{1}{3}} = 1.2954 \left[\frac{0.00018(2.6 - 0.0012)980}{(0.0012)^2} \right]^{\frac{1}{3}} = 88 \frac{cm}{s}$$

The density of the air will be taken as 1.2×10^{-3} gram per cc and viscosity is 1.8×10^{-4} this is at standard condition at 20 degree centigrade whereas, this d_p star what is the dimensionless particle diameter is 160 micrometer and density of the solid is given here 2.60 gram per cc a sphericity is taken as 0.67.

Now, if you calculate the d_p star as this here this is not d_p star it will be d_p it will 160 then you will see d_p star will be is equal to 7.28 and U_t star will be is equal to 1.294 and then finally, terminal velocity will be equals to 88 centimeter per second.

You can use the equation that what has already been shown earlier that how to calculate the U_t .

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Minimum fluidization

- At a velocity, all the particles are just suspended by the upward-flowing gas or liquid.
- At this velocity the frictional force between particle and fluid just counterbalances the weight of the particles,
- The pressure drop through any section of the bed about equals the weight of fluid and particles in that section.
- The bed is considered to be just fluidized and is referred to as an incipiently fluidized bed or a bed at minimum fluidization

And then U_t star from U_t star you can calculate U_t as here 88.88 centimeter per second and then we know now what will be the terminal velocity of the particle. Before going to that minimum fluidization condition that is why this terminal velocity is required because minimum fluidization will occur beyond this terminal velocity of the particle.

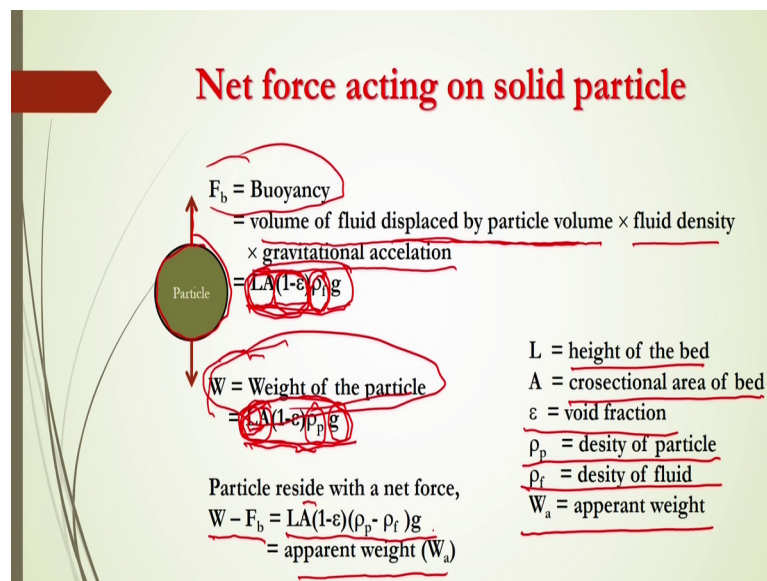
Now, of course, balancing the terminal velocity; that means, here the force that is net force what is the acting downward you have to balance that net force to get the minimum fluidization now this minimum fluidization what is that minimum fluidization; that

means, the velocity at which all the particles are just suspended by upward flowing gas or liquid by balancing the terminal velocity of the particle.

Now, at this velocity you will see the frictional force between particle and fluid just counter balances the weight of the particles then only you will get the minimum fluidization the pressure drop through any section of the bed about equals the weight of the fluid and particles in that section. So, at any section you will see pressure drop to any section of the bed it will be about equal the weight of the fluid and particles in that section.

It is happening only for minimum fluidization condition the bed is considered to be just fluidized and is referred to as an incipiently fluidized bed or bed at minimum fluidization. This minimum fluidization just balancing or counter balancing the weight of the particles.

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And then what should be the net force acting on solid particle at this minimum fluidization condition you will see here on particle this will be to be fluidized now what are the force now 1 is buoyancy forces acting on this particle and that will be calculated that is volume of fluid displaced by displaced by the particle volume into fluid density and then into gravitational isolation you can calculate the buoyancy force

And what are the other forces that will be apparent weight apparent weight of the particle now this apparent weight is nothing, but what will be the actual weight of the particle is weight of the particle is this is $l A (1 - \epsilon) \rho_p g$ what is these this is not a single particle here total amount of particle here what will be the weight of the particle in the fluidized bed.

Now, what will be the l is the length of the fluidized bed and A is the cross sectional area of the bed ϵ is the void fraction void fraction means other than particle what will be the space there and then ρ_p is the density of the particle ρ_F is the density of the fluid and W_A is the apparent weight.

Now, if this is weight this is weight means solid weight in the bed this is length of the bed into cross sectional area that will give you the volume of the bed and then $1 - \epsilon$ means what will be the volume of the solid particles and into density of the particle this will give you the mass of the particle and into g this will give you the weight of the particle and buoyancy force of course, it will be what will be the $l A \epsilon \rho_F g$; that means, volume of the bed into $1 - \epsilon$ means this is the solid volume now this solid volume will displace the water volume or fluid volume or gas volume you can say this gas volume if it is in the gaseous medium then into g this will give you the buoyancy force.

And what should be the apparent force apparent force is nothing, but $W - F_b$ $W - F_b$ W means weight and this buoyancy force this is the apparent weight apparent weight is nothing, but here is just you will just subtract this buoyancy force from it is weight then you will get the apparent weight. Apparent weight is nothing, but $l A (1 - \epsilon) \rho_p g - l A \epsilon \rho_F g$.

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At Onset of Fluidization

$$\boxed{\text{Drag force by upward moving gas}} = \boxed{\text{Apparent weight of particles}}$$

Or

$$\boxed{(\text{Pressure drop across bed}) \times (\text{Cross sectional area of bed})} = \boxed{(\text{Volume of bed}) \times (\text{fraction consisting of solids}) \times \text{Specific weight of solids}}$$

$$\boxed{\Delta P_b A = W_a = A L_{mf} (1 - \epsilon_{mf}) (\rho_s - \rho_g) g}$$

And from this apparent weight of the particles of course, you will get the minimum fluidization.

Now, how to get you have to balance this apparent weight of the particles with a drag force that is acting upward acting by upward moving gas. This will be the drag force will be balanced by this apparent weight of the particle, that you can get the minimum fluidization.

Now, how to calculate the drag force by upward moving gas this will be if you know the frictional pressure drop across the bed and if you know the cross sectional area of the bed then drag force will be easily calculated here as drag force as frictional pressure drop across the bed into it is cross sectional area. This will be your drag force here. So, this is total drag force.

What will be the frictional pressure drop that will give you the drag force, this frictional pressure drop into cross sectional area this will be your drag force and what will be the apparent weight that is W that will be is equal to this A into L_{mf} F is the length of the bed at the minimum fluidization condition and this will be is equal to 1 minus ϵ_F into ρ_s minus ρ_g .

How to actually this will give you the volume of the bed at it is minimum today's height into what will be the fraction of solid here 1 minus ϵ_{mf} at this minimum

fluidization condition then it will give you the total volume of the particle inside the bed volume into then what will be the ρ_s minus ρ_g this would be the apparent density or apparent weight from which you can calculate ρ_s minus ρ_g into g .

This already calculated in the previous slides how to obtain this apparent weight. If you balance this; that means, if you equalize these 2 forces of this what is that drag force and the apparent weight you will get the condition of you will get the condition of minimum fluidization.

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By rearranging, one can find for minimum fluidizing conditions that

$$\frac{\Delta P_b}{L_{mf}} = (1 - \epsilon_{mf})(\rho_s - \rho_g)g \quad \text{Eq. (1)}$$

ΔP_b = bed pressure drop
 L_{mf} = min. fluidizing height
 ϵ_{mf} = min. void fraction
 ρ_s = density of solid
 ρ_g = density of gas

At the onset of fluidization, the voidage is a little larger than in a packed bed, actually corresponding to the loosest state of a packed bed of hardly any weight. Thus, ϵ_{mf} may be estimated from random packing data, or, better still, one should measure it experimentally, since this is a relatively simple matter.

Now, how to calculate the frictional pressure drop how to calculate the frictional pressure drop of course, this ΔP_b by L_{mf} is called frictional pressure drop per unit length of the minimum fluidized height that will be is equal to here this will be 1 minus ϵ_{mf} into ρ_s minus this can be obtained just by rearranging this previous equation now here ΔP_b is bed pressure drop L_{mf} is equal to minimum fluidizing height ϵ_{mf} is the minimum void fraction and ρ_s is the density of the solid and ρ_g is the density of the gas.

Now, at this onset of fluidization; that means, minimum fluidization condition the void age is a little larger than in a packed bed condition of course, from this packed bed condition you are just going to suspend the particles to get to the minimum fluidization. What should be the minimum void age to get this minimum fluidization? Here actually

this minimum void age will correspond to the what should be the void age in the packed bed that the minimum condition.

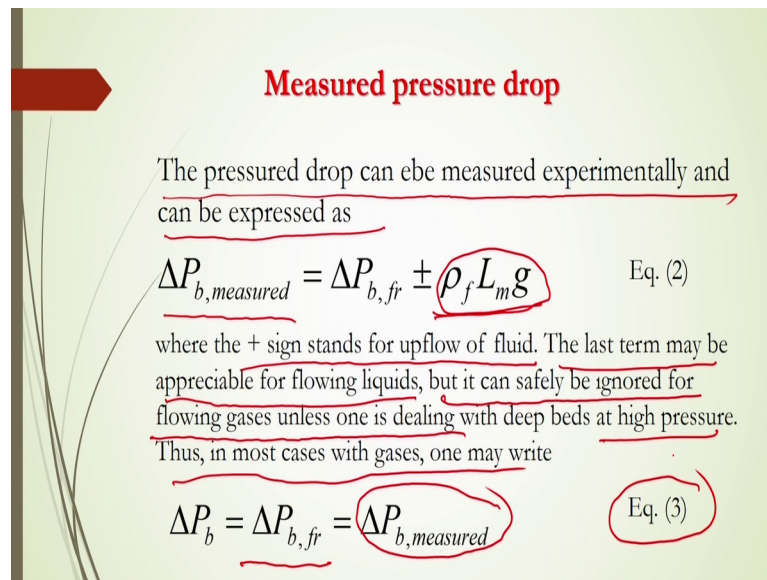
At the onset of the fluidization the void age is the little larger than in a packed bed almost equals to the little bit higher, but you can omit you can neglect also you can directly consider the what should be the porosity or what should be the void age in packed bed condition actually corresponding to the loser state of the packed bed of hardly any weight here that is why this a little larger than in packed bed condition this void age.

Now, thus umf you can estimate from the random packing data or better still 1 should measure it experimentally since this is the relatively simple matter of course, you can calculate to the void age experimentally otherwise you can directly add this minimum condition you just consider what should be the void age at packed bed condition there will be hardly a an error for this if you are taking into consideration of packed bed void age here, but it still for accurate result you can of course, measure the void age at this minimum fluidization condition.

Now, measurement of the pressure drop before going to have this minimum fluidization velocity of course, you have to know frictional pressure drop then drag force you have to calculate already you know that what should be the apparent weight of that then you have to balance these 2 forces then you will get the minimum fluidization velocity.

Now, what should be the frictional pressure drop how to calculate or how to measure also you have to know.

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Measured pressure drop

The pressured drop can ebe measured experimentally and can be expressed as

$$\Delta P_{b,measured} = \Delta P_{b,fr} \pm \rho_f L_m g \quad \text{Eq. (2)}$$

where the + sign stands for upflow of fluid. The last term may be appreciable for flowing liquids, but it can safely be ignored for flowing gases unless one is dealing with deep beds at high pressure. Thus, in most cases with gases, one may write

$$\Delta P_b = \Delta P_{b,fr} = \Delta P_{b,measured} \quad \text{Eq. (3)}$$

Now, the frictional pressure drop can be measured experimentally which can be expressed as here. This measured pressure drop will be is equal to there are 2 parts of the measured pressure drop you can measure it by a manometer or by any electrical electronics device like you know that pressure transducer you can use that to calculate the accurate pressure drop across the column.

This measured pressure drop will have 2 parts one is frictional pressure drop another is the hydrostatic pressure drop, now if you consider the gas solid fluidization you will see the hydrostatic pressure of course, this density of the fluid as a gas here it will be very small and also this hydrostatic part is very negligible compared to the frictional parts.

Only here you can say that measured pressure drop you can calculate directly from the frictional pressure drop; that means, frictional pressure drop is equal to the measured pressure drop here you will see an equation 2 it is seen that measure pressure drop is delta p bed for frictional part and also this plus minus rho F into l m into g this rho f l m into g is nothing, but hydrostatic pressure drop this plus sign stands for up flow fluid the last term may be appreciable for flowing liquids, but it can safely be ignored for flowing gases unless one is dealing with the deep bits at high pressure of course, that high pressure you have to consider this hydrostatic pressure also.

Now, in most cases with gashes you can directly obtain the bed pressure drop by considering the frictional pressure drop. This frictional pressure drop of the bed will be equals to the measured pressure drop this is shown in equation number 3.

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At the minimum fluidization, the onset pressure drop where fluidization starts and packed bed condition diminishes, can be estimated from the frictional pressure drop as per **Ergun Equation** as follows

$$\frac{(-\Delta P_{fr})}{L_{mf}} = 50 \frac{(1-\epsilon_{mf})^2}{\epsilon_{mf}^3} \frac{\mu u}{\Phi_s^2 d_p^2} + 1.75 \frac{(1-\epsilon_{mf})}{\epsilon_{mf}^3} \frac{\rho_g u^2}{\Phi_s d_p} \quad \text{Eq. (4)}$$

Viscous *inertia*

Now, at the minimum fluidization of course, the onset pressures drop a fluidization starts and the packed bed condition you will see this diminishes. So, at the minimum fluidization just diminishing the packed bed condition and then this minimum fluidization can be then estimated from the frictional pressure drop now this frictional pressure drop of course, you can measure it experimentally or without measurement also you can consider this frictional pressure drop you can estimate the frictional pressure drop from the Ergun equation.

Ergun equation will give you directly the frictional pressure drop in a fluidization beds here this is that Ergun equation. By the Ergun equation you can calculate the frictional pressure drop per unit length as this here the 2 parts of this Ergun equation 1 is called this is inertia this is inertia and this is called a viscous this is viscous layer and this is inertia layer inertia, this is inertial and this is viscous this is viscous and inertia.

Now, you will see this Ergun equation is giving you the frictional pressure drop per unit length. This is a function of viscosity and velocity and particle diameter even a sphericity. This Ergun equation you can directly use to calculate the frictional pressure drop.

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Equating equations (1) and (4), one can write

$$\frac{(1-\varepsilon_{mf})(\rho_p - \rho_f)g}{\varepsilon_{mf}^3 \Phi_s^2} = 150 \frac{(1-\varepsilon_{mf})^2 \mu u_{mf}}{\Phi_s^2 d_p^2} + 1.75 \frac{(1-\varepsilon_{mf}) \rho_f u_{mf}^2}{\varepsilon_{mf}^3 \Phi_s d_p} \quad \text{Eq. (5)}$$

Rearranging

$$150 \frac{(1-\varepsilon_{mf})}{\varepsilon_{mf}^3 \Phi_s^2} \left(\frac{\rho_f u_{mf} d_p}{\mu} \right) + \frac{1.75}{\varepsilon_{mf}^3 \Phi_s^2} \left(\frac{\rho_f^2 u_{mf}^2 d_p^2}{\mu^2} \right) = \frac{d_p^3 (\rho_p - \rho_f) g}{\mu^2} \quad \text{Eq. (6)}$$

Now, equating this equations 1 and 4, 1 can write here this part this 1 minus epsilon mf into rho F minus rho p minus rho F into g that will be equals to this pressure drop by the Ergun equation, now if you rearrange this equation then you will get this final form of this equation now you can denote this equation big equation in different way also.

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And so eq (6) can be expressed as

$$\frac{1.75}{\varepsilon_{mf}^3 \Phi_s^2} (\text{Re}_{p,mf}^2) + 150 \frac{(1-\varepsilon_{mf})}{\varepsilon_{mf}^3 \Phi_s^2} (\text{Re}_{p,mf}) = \text{Ar} \quad \text{Eq. (7)}$$

Where

$$\text{Ar} = \frac{\rho_f (\rho_p - \rho_f) d_p^3 g}{\mu^2} \quad \text{Archimedes number} \quad \text{Eq. (8)}$$

Or Galileo number

$$\text{Re}_{p,mf} = \frac{\rho_f u_{mf} d_p}{\mu} \quad \text{Reynolds number} \quad \text{Eq. (9)}$$

Solve the quadratic equation for $\text{Re}_{p,mf}$ to get min. fluid. velocity

In order to obtain a value of u_{mf} from Equation (7) we need to know the min. voidage of the bed, ε_{mf} . Taking ε_{mf} as the voidage of the packed bed, we can obtain a crude u_{mf} .

In practice, voidage at the onset of fluidization may be considerably greater than the packed bed voidage

And like this you can simplify this as this equation by 1.75 divided by epsilon M cube into phi s into Reynolds numbers squared plus 150 into 1 minus epsilon F mf that

minimum fluidization condition by $m F \epsilon_{mf}^3$ into ϕ_s^2 into Reynolds number into is equal to Archimedes number.

Now, see this equation becomes the quadratic equation of the Reynolds number at the minimum fluidization condition. This is here see, if we equalize the drag force that is drag force that is calculated from the Ergun equation with the apparent weight of the bed then you are getting the quadratic equation of the Reynolds number at it is minimum fluidization condition.

Where this Ergun number sorry Ergun or this is the not Ergun which is called Archimedes number Archimedes number or sometimes some references they are represented it as Galileo number. This Archimedes number is defined as ρ_f into ρ_p minus ρ_f into d_p^3 into g divided by μ^2 μ means viscosity here and minimum Reynolds number is the $\rho_f u_{mf}$ into d_p by μ .

If you solve this quadratic equation you will get the equation of equation for reynolds number for minimum fluidization condition. Solve this quadratic equation to get the Re_{mf} from this minimum Re_{mf} you will get the minimum velocity. This if you know these Re_{mf} from this equation of 7 and then just after that you can calculate the u_{mf} from this here what should the u_{mf} then u_{mf} will be is equal to u_{mf} from this Reynolds number it will be equals to Re_{mf} it will be is equal to Re_{mf} divided by ρ_f $\rho_f d_p$ here into μ into μ . From this you will be able to calculate what will be the minimum fluidization velocity.

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Voidage at Minimum Fluidization

Wen and Yu (1966) found for many systems:

$$\Phi_s \epsilon_{mf}^3 \cong \frac{1}{14}$$

This correlation can be used for minimum voidage

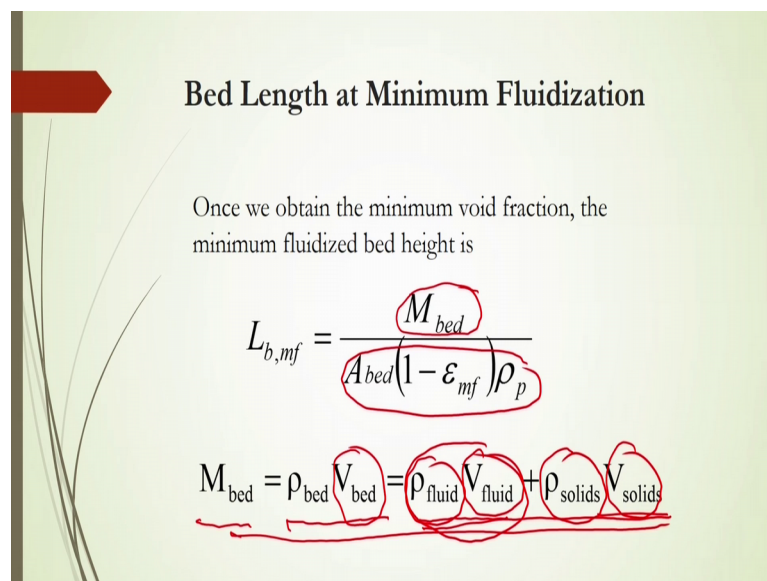
ϵ_{mf} depends on the shape of the particles. For spherical particles ϵ_{mf} is usually 0.4 – 0.45.

Type of particle	Particle size			
	60 μm	100 μm	200 μm	400 μm
	Void fraction, ϵ_{mf}			
Sharp particle ($\Phi_s \cong 0.67$)	0.60	0.58	0.53	0.49
Round Particle ($\Phi_s \cong 0.86$)	0.53	0.48	0.43	0.42
Anthracite coal ($\Phi_s \cong 0.63$)	0.61	0.60	0.56	0.52

And then what should be the void age that minimum fluidization of course, to get to get the to get the minimum fluidization velocity you have to know the minimum porosity minimum porosity minimum porosity is represented by epsilon mf this epsilon mf how to calculate this epsilon mf this epsilon mf you can calculate it from or you can consider from the packed bed condition otherwise you can directly consider this equation which is given by Wen and Yu 96 that is phi s into epsilon mf cube will be equals to 1 by 14 from which is you can calculate for from the minimum porosity here or minimum void age from this equation.

Epsilon mf depends on the shape of the particle for spherical particles epsilon mf is usually 0.4 to 0.4 5 and you will see the if the sharp particle is there with the sphericity of 0.67 for 60 micron particle the sphere void fraction will be is equal to 0.60 whereas, for anthracite coal of sphericity 0.63 you will get the void fraction of about 0.5 to at 400 micrometer in size. This void fraction depends on the particle size and also it is shape.

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Bed Length at Minimum Fluidization

Once we obtain the minimum void fraction, the minimum fluidized bed height is

$$L_{b,mf} = \frac{M_{bed}}{A_{bed}(1 - \epsilon_{mf})\rho_p}$$

$$M_{bed} = \rho_{bed} V_{bed} = \rho_{fluid} V_{fluid} + \rho_{solids} V_{solids}$$

Now, once you know the minimum voidage of the minimum fluidized bed then you will be able to calculate what will the length of the minimum fluidized bed this length of the minimum fluidized bed can be calculated from the mass of the bed mass of the bed what will be the mass of the bed this will be the what will be the solid particle and also what will be the volume of the fluid what will be the volume of the solid then you have to add

these now how to calculate here see mass of the bed will be calculated as density of the bed into volume of the bed.

Now, you can calculate it from this now if you consider only the mass of the fluid then it will be density of the fluid into a volume of the fluid and what should be the mass of the solid mass of the solid will be equals to density of the solid into volume of the solids. From this you will be able to calculate what should be the what should be the mass of the bed now once you know the mass of the bed if you divide this mass of the bed by the what is that by this area into density of the particle then you will get this here this minimum fluidized height.

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Special Case

In the special cases of very small particle, the first term of RHS of equation (7), can be neglected. In this case inertial effect is negligible compared to other effect.

Hence at $Re_{p,mf} < 20$

$$u_{mf} = \frac{d_p^2 (\rho_s - \rho_g) g \epsilon_{mf}^3 \Phi_s^2}{150 \mu (1 - \epsilon_{mf})}$$

Otherwise at $Re_{p,mf} > 1000$ for very large particle, 1st term is dominating and hence

$$u_{mf} = \sqrt{\frac{d_p^2 (\rho_s - \rho_g) g}{1.75 \rho_g} \epsilon_{mf}^3 \Phi_s}$$

You need the value of ϵ_{mf}, Φ_s along with other variables

Now, special cases in the special cases of means whether it is very small particles or not of course, you will see in the special cases of very small particle the first term of the right hand side of equation 7 equation 7; that means, see our equation 7 uses see here this equation 7 what is the quadratic equation of the Reynolds number to get the minimum fluidization you will see that the first term of this right hand side of this equation 7 can be neglected for the very small particles.

In this case inertial effect is negligible compared to the other effect. If Reynolds number is less than 20 then you can directly calculate the minimum fluidization from these by neglecting the right hand side part of the equation 7 whereas, for very large particle whereas, for very large particle this Reynolds number if it is greater than 1000 first one

will be dominating and hence you can calculate the minimum fluidization velocity from this equation.

So, in this case you need of course, the value of epsilon mf and the a sphericity.

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Special Case

If you do not know the value of ϵ_{mf} , Φ_s you can still estimate u_{mf} for a bed of irregular particles as follows: First, rewrite Eq. (7)

$$\frac{1.75}{\epsilon_{mf}^3 \Phi_s} Re_{p,mf}^2 + 150 \frac{(1-\epsilon_{mf})}{\epsilon_{mf}^3 \Phi_s^2} Re_{p,mf} = Ar$$

as

$$K_1 Re_{p,mf}^2 + K_2 Re_{p,mf} = Ar$$

$$Re_{p,mf} = \frac{-K_2 \pm \sqrt{K_2^2 + 4K_1(Ar)}}{2K_1}$$

where

$$K_1 = \frac{1.75}{\epsilon_{mf}^3 \Phi_s} \quad \text{and} \quad K_2 = 150 \frac{(1-\epsilon_{mf})}{\epsilon_{mf}^3 \Phi_s^2}$$

K_1 and K_2 stayed nearly constant for different kinds of particles over a wide range of conditions ($Re = 0.001$ to 4000)

Investigators	K_1	K_2
Wen and Yu (1966)	24.51	1651.96
Richardson (1971)	27.40	1408.22
Saxena and Vogel (1977)	17.51	886.16
Babu et al. (1978)	15.36	777.27
Grace (1982)	24.51	1333.33
Chitester et al. (1984)	20.24	1161.94

And then if you do not know the value of epsilon mf or sphericity you can still estimate the minimum velocity for a bed of irregular particles as follows in this case you have to first write the rewrite the equation 7 as this here this equation 7 and in this case as I considering or denoting by here k 1 and k 2 this k 1 is defined as here 1.75 by epsilon mf cube into phi s and k 2 will be 1 fifty into 1 minus epsilon mf divided by this epsilon mf q into phi s cube phi s square.

Here this equation this equation 7 is represented in terms of k 1 and k 2 these 2 parameters. From these you can calculate what will the minimum fluidization condition for Reynolds number of these here it will be as a function of k 1 and k 2 and also it is Archimedes number. From this you can calculate the minimum fluidization velocity.

Now, k 1 and k 2 stayed nearly constant for different kinds of particles over a wide range of condition; that means, if r e is within the range of 0.01 to 4000 then you will get the k 1 and k 2 within this range now different investigators they got the k 1 and k 2 value here see Wen and Yu they have represented this k 1 and k 2 has 24.51.

These are actually experimental data they have obtained these from the experiment and experiment of course, this whatever this k_1 and k_2 is coming from this equation it depends on this ϵ_{mf} and ϕ_s , with different minimum This void fraction depends on the particle size and of course, with different sphericity they have done different experiments and from their experiment they got this k_1 and k_2 .

Now, using these values for k_1 and k_2 then the minimum fluidization velocity can be obtained as follows here see for force particle; that means, see particle size if it is greater than 1000 micrometer.

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Using the values for K_1 and K_2 , the minimum fluidization velocity can be obtained as follows:

For coarse particles ($d_p > 100 \mu m$)

$$Re_{p,mf} = [(28.7)^2 + 0.0494 Ar]^{1/2} - 28.7$$

$K_1 = 20.24$ and $K_2 = 1161.94$
Chitester et al. (1984)

For fine particles ($d_p < 100 \mu m$)

$$Re_{p,mf} = [(33.7)^2 + 0.0408 Ar]^{1/2} - 33.7$$

$K_1 = 24.51$ and $K_2 = 651.96$
Wen and Yu (1966)

Investigators	K_1	K_2
Wen and Yu (1966)	24.51	1651.96
Richardson (1971)	27.40	1408.22
Saxena and Vogel (1977)	17.51	886.16
Babu et al. (1978)	15.36	777.27
Grace (1982)	24.51	1333.33
Chitester et al. (1984)	20.24	1161.94

Since u_{mf} is the most important measurement needed for design, it has been the focus of a tremendous amount of experimentation under a wide variety of conditions

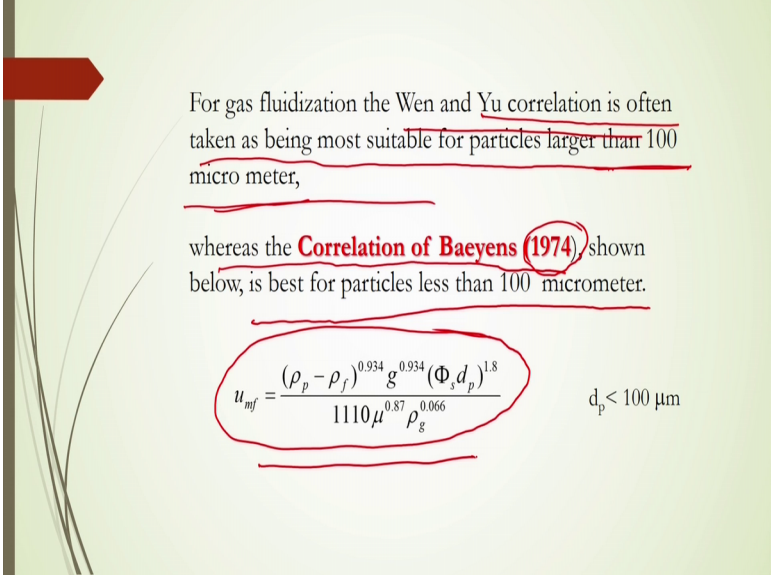
Then you can calculate the minimum fluidization velocity from this equation here in this case k_1 and k_2 are taken from this Chitester et al 94 they got this and it will be k_1 will be equal to 20.24 and k_2 will be equals to 1161.94.

Whereas for fine particles see particle size is less than 100 micrometer then you have to calculate the minimum fluidization from this equation in this case k_1 will be is equal to 24.51 little bit higher than the earlier one whereas, this k_2 will be is equal to again this k_2 value is higher than this earlier one it will be 651.96 this has been given by Wen and Yu.

Different investigators they are getting different results for different sizes particles here if particle size increase or decrease then of course, accordingly this k_1 and k_2 will be

increasing or decreasing in this case is a minimum fluidization velocity is the most important measurement you need for design the and you need to focus of a tremendous amount of experimental condition and data for wide variety of condition for calculating the minimum fluidization velocity.

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For gas fluidization the Wen and Yu correlation is often taken as being most suitable for particles larger than 100 micro meter,

whereas the **Correlation of Baeyens (1974)** shown below, is best for particles less than 100 micrometer.

$$u_{mf} = \frac{(\rho_p - \rho_f)^{0.934} g^{0.934} (\Phi_s d_p)^{1.8}}{1110 \mu^{0.87} \rho_g^{0.066}} \quad d_p < 100 \mu\text{m}$$

Now, for gas fluidization the Wen and Yu correlation is often taken as being most suitable for particle larger than 100 micrometer. Wen and Yu correlation is very suitable if it is if particle size is greater than 100 micrometer whereas, the correlation of baeyens 19 74 he has proposed in this case will be the best for particle less than hundred micrometers. This will be your correlation of baeyens that you can directly use for calculating the minimum fluidization velocity if your particle is being used or you are using particle which is size less than 100 micrometer 100 micrometers.

This will be your suitable correlation to calculate the minimum pressure there are other different investigators. They have calculated or they have developed with the different correlations for minimum fluidization velocities.

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Other Correlations	
Baeyens and Geldart	$Ar = 1823 Re_{mf}^{1.07} + 21.27 Re_{mf}^2$
Leva	$U_{mf} = \frac{7.169 \times 10^{-4} d_p^{1.42} (\rho_p - \rho_g)^{0.94} g}{\rho_g^{0.888} \mu_g^{0.88}}$
Goroshko <i>et al.</i>	$U_{mf} = \frac{\mu_g}{\rho_g d_p} \left(\frac{Ar}{1400 + 5.2 \sqrt{Ar}} \right)$
Bena	$U_{mf} = \frac{\mu_g}{\rho_g d_p} \left(\frac{1.38 \times 10^{-3} Ar}{(Ar + 19)^{0.11}} \right)$
Rowe and Henwood	$U_{mf} = \frac{8.1 \times 10^{-3} d_p^2 (\rho_p - \rho_g) g}{\mu_g}$
Richardson and Da St. Jeromino	$U_{mf} = \frac{\mu_g}{\rho_g d_p} (\sqrt{25.7^2 + 0.0365 Ar} - 25.7)$

These are some correlations that are given here in the slides that you can follow for calculating the minimum fluidization velocity.

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Factors that affect on the minimum fluidization velocity	
<ul style="list-style-type: none"> Geometric variables <ul style="list-style-type: none"> Bed diameter bed height particle size distributor hole diameter 	<ul style="list-style-type: none"> Variables as Physical properties <ul style="list-style-type: none"> Fluid density fluid viscosity fluid surface tension slurry concentration
<ul style="list-style-type: none"> Thermodynamic conditions <ul style="list-style-type: none"> Pressure Temperature 	

Now, what are the factors that affect the minimum fluidization velocity you can see some geometric variables some variables as the physical properties some are thermodynamic conditions now geometric variables like bed diameter bed height particle size distributor hole diameter through which the gas is distributed or liquid is distributed now fluid density fluid viscosity fluid surface tension slurry concentration all are the factors that

affect on the minimum fluidization velocity even some thermodynamic condition at higher pressure of course, you will see the minimum fluidization velocity will be higher whereas, temperature at lower temperature it will be the higher fluidization minimum fluidization velocity. These are the factors who is affect the minimum fluidization velocity next class will be the one example will be given to you for a minimum fluidization.

Thanks for all today.