

Fluidization Engineering
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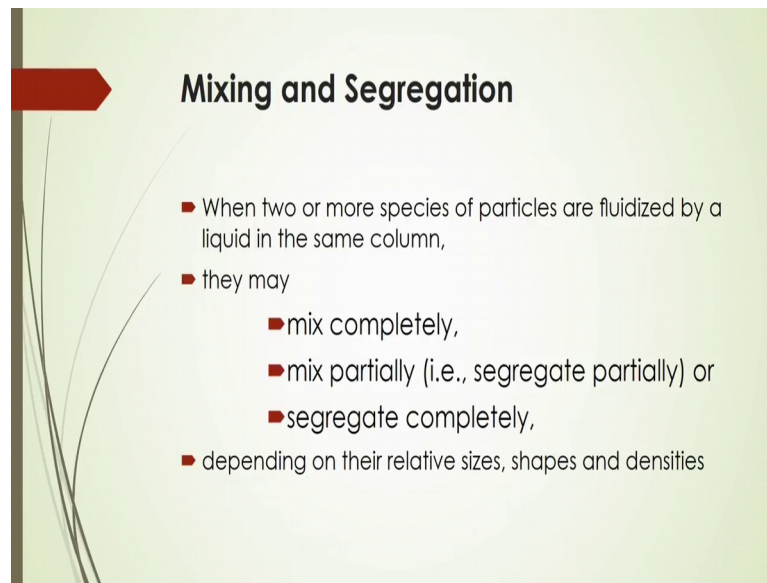
Lecture – 26
Solid mixing and segregation: Liquid- solid fluidized bed

So, welcome to this MOOCs online course on Fluidization Engineering. So, today's lecture will be on solid mixing and segregation in liquid solid fluidized bed we think we have discussed about the, solid mixing and segregation in the gas solid fluidized bed in previous lectures. And, they are we have actually discussed about the, basics of segregation mechanism even how the solids will be mixing they are only for two phase flow of gas and solid system but, what about the solid mixing and segregation, in liquid solid or gas liquid solid systems in the fluidized bed.

So, this lecture will be continued with that continuation of the, solid mixing and segregation mechanism for this liquid and solid system. I think we have already discussed about what is the, mixing and segregation, what is the definition for mixing and segregation they are what is the mixing index what is the segregation index and how this mixing index and segregation index are correlated.

Even, what are the different models for that mixing and segregation, for this gas and solid systems here. Same type of definition of course, will be there ah, but in this case you will see that maybe some some different phenomena will be seen therefore, liquid if instead of gaseous is used in the fluidized bed.

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Mixing and Segregation

- When two or more species of particles are fluidized by a liquid in the same column,
- they may
 - mix completely,
 - mix partially (i.e., segregate partially) or
 - segregate completely,
- depending on their relative sizes, shapes and densities

In that case, you will see that sometimes the liquid properties will affect the segregation and mixing inside the bed because, that gas has lower viscosity. Whereas, if you use the liquid then of course, there will be some difference in phenomena because of this a fluid properties.

In this case, of course there is the same criteria for that whether this solid should be considered as a complete mixing or other solids will mix partially or it will segregate partially or not even if there is a segregation, whether it will be complete segregation or partial segregation. And, also what should be the, mixing and how it can be measured this mixing criteria for the solids in the liquid phase. And, how that mixing and segregation depends on I think we have already discussed about that, that this mixing or segregation that depends on the relative size of the particles and shapes and also the density of the particles.

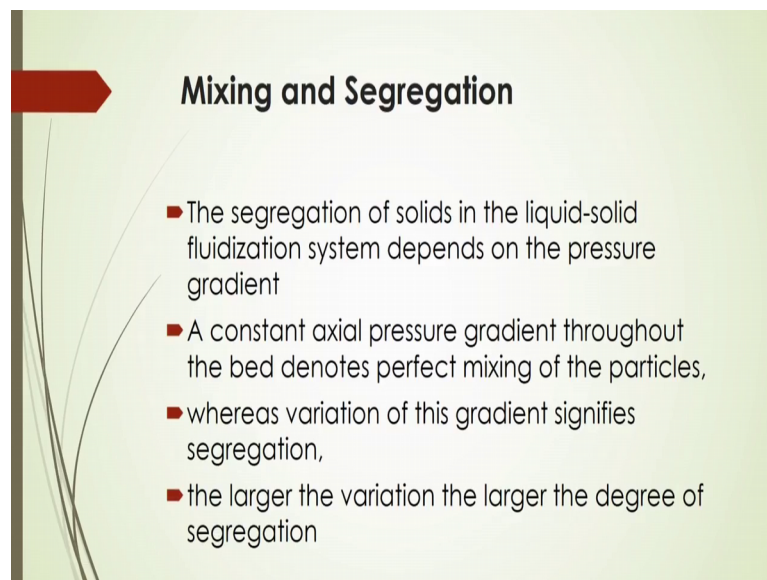
But, you will see that relative density in case of gas and solids there will be very you know that effective in that gas in that case you will see that, when the gas density is very low compared to the solid density. Whereas, in case of liquid that if you are using water or some other liquid of course, it is density will be more than one thousands there, if it is heavier liquid and water also. And, in that case that effective density of that, that is density difference of that solid particle and the liquid will be of course, lower than that density difference of that gas solid system there.

So, based on that also the density affective density difference, that mixing and segregation will be affected. So, in this case of when two or more species of particles that are in fluidized condition then, then you will see that the solid or liquid mixing will be maybe completely or maybe partially or maybe segregation and the complete a manner that depends on the relative size, shapes and densities.

And, the segregation of the solids in the liquid solid fluidization that depends on, of course pressure gradient you will see that, that gas solid system the pressure gradient will be something lower than that the pressure gradient should be more higher in the liquid solid systems because, here frictional resistance to the wall would be more in case of liquid there.

So, segregation of the particles in the liquid solid fluidized systems that how are what would be the extent of pressure gradient they are that depends. And, of course you will see that constant axial pressure gradient throughout the bed that denotes that perfect mixing of the particles.

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Mixing and Segregation

- The segregation of solids in the liquid-solid fluidization system depends on the pressure gradient
- A constant axial pressure gradient throughout the bed denotes perfect mixing of the particles,
- whereas variation of this gradient signifies segregation,
- the larger the variation the larger the degree of segregation

And, whereas the variation of this gradient of signifies there will be a segregation. And, the larger the variation pressure drop or pressure gradient larger the degree of segregation will be in the fluidized bed in case of liquid solid system.

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Segregated Particle Distribution

■ The cumulative fraction of particles larger than $d_p(z)$, all of which are presumed to be located below level z , can be expressed as

$$\frac{m_z}{M} = \frac{\int_0^z (-dp_f / dz) dz}{\int_0^L (-dp_f / dz) dz} = \frac{\int_0^z [1 - f(d_p)] dz}{\int_0^L [1 - f(d_p)] dz} \quad (1)$$

The mass m_z of particles between the distributor and the height z ;
 M is total mass

$$m_z = \frac{\rho_p A}{g(\rho_p - \rho)} \int_0^z \left(-\frac{dp_f}{dz} \right) dz \quad \varepsilon = 1 - \frac{-dp_f / dz}{g(\rho_p - \rho)} \quad M = \frac{\rho_p A}{g(\rho_p - \rho)} \int_0^L \left(-\frac{dp_f}{dz} \right) dz$$

$\varepsilon = f(d_p)$

Now, how this segregated particles, can be distributed and what should be the segregated particle size distribution they are in the fluidized bed and how it can be represented. Generally, a cumulative fraction of the particles that larger than particular size like d_p in that case, all of which are presumed to be located below of level z then it can be expressed as that by this equation here one given in a slide. Here see, this will be is equal to m_z by M here m_z is defined in such a way that is related to the, pressure gradient here small m_z .

So, this m_z by capital M it is this is similarly simply that the ratio of the pressure gradient, up to a certain distance and based on the total distance of course. So, first you have to calculate what should be the pressure gradient at a certain height, if you are considering of that fluidized bed, then up to that distance you have to calculate what should be the pressure gradient. Then, this pressure gradient of course, will be the frictional pressure gradient. And, also you have to calculate what should be the frictional gradient frictional pressure gradient for the whole fluidized medium.

So, in that case what should be the ratio that would be represented by this m_z by M . So, this is actually this pressure gradient frictional pressure gradient, that depends on that particle size distribution of course, they are. So, this can be represented at that if there is up to z location 0 to z integrated then 1 minus $f d$ into $d z$ they are. So, here now 1 minus $f d$ is the vertical size distribution function there, so it will be 1 minus $f d$ into $d z$. And,

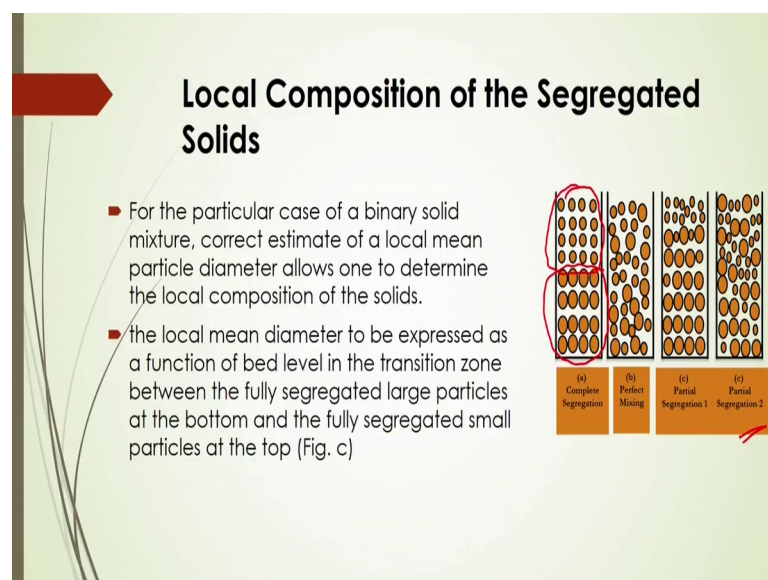
small m_z that can be represented that is actually, defined by this $c_r m_z$ is equal to ρ_p into a divided by gravitational acceleration into ρ_p minus ρ_h . This is the relative density that this particle density to the fluid density they are.

And, integration of this here pressure frictional pressure gradient. So, after that you will be able to calculate what should be the m_z there after knowing all that in pressure gradients there. And, epsilon is one important factor here this epsilon is nothing, but that this will be the porosity or you can see the void fraction of the liquid inside the bed except solids there.

And, this directly it will be related to the frictional pressure drop you will see that, it this can be calculated from the frictional pressure drop here, by this $1 - \frac{d_f}{d_z}$ by d_z by g into of the relative density they are. And, this depends on the, particle size distribution inside the bed. And, capital M should be represented by this again with a pressure gradient based on the, total length of the fluidized bed there.

So, the mass m_z of the particles between the distributor and the height z_M can be calculated and from who is this capital M actually can be represented by the total mass of the solids, inside the bed that can be obtained from that pressure gradient.

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And, again then if you actually know the pressure gradient then what should be the local composition of the segregated solids inside the bed. Now, you will see that for the

particular case of a binary solid mixture in the fluidized bed you will see that, you have to have some local mean particle diameter that will allow one to determine the local composition of the solids.

Now, the local mean diameter to be expressed as a function of a bed level in the transition zone, between the fully segregated large particles at the bottom, and the fully segregated small particles at the top. That, already we have discussed in the gas solid fluidization system that there will be two portions of that fluidized bed one is bottom and the top and the smaller particles will be segregated to the top and the bottom one the coarser particles should be deposited as a segregated solids they are.

So, in this slides also you will see that from the bottom to top how the solid particles will be segregated you will see that in figure a here there will be complete segregation. It is obviously, actually that, that a smaller particles see how it will be segregated from the coarser particles here, and this smaller particles are separated at the top region of this fluidized bed whereas, this coarser particles will be segregated or deposited in the, lower part of this fluidized bed.

Whereas, if they are the perfect mixing you will see that smaller and coarser particles will be distributed throughout the column and there will be no segregation. In that case, you can say that smaller and coarser particles they are may not be separated in the bottom and top portion and maybe smaller particles will be also at the bottom and final coarser particles also will be at the top portion.

So, there will be a segregation there will be a segregation that depends on the mixing characteristics. Now, if higher mixing or higher flow rate is controlled inside the bed then you will not be able to get the segregation at 100 percent. But, whereas at a certain gas flow rate you will see that there will be complete mixing by which you can say that small and coarser particles will be distributed throughout the column. And then, concentration updates smaller particles and the, coarser particles will be actually equally distributed in such a way that, that you have to maintain certain gas flow rate and pressure distribution there.

And, based on these perfect mixing you will see that the pressure gradient will not be changing along this height of the, fluidized bed. Whereas, for complete segregation you will see that at the bottom part there will be a that means, dense region of the coarser

particles will show you the higher a pressure drop whereas, so that top portion finer particles that means, their porosity will be more higher than in that case that frictional pressure drop will be less compared to the perfect mixing there. Whereas, this partial segregation you will see that some smaller particles very less amount of smaller particles will be actually residing at the bottom of this fluidized bed. Whereas, maximum portion of the coarser particles will be residing in the bottom side I am very small amount of coarser particles will be depositing at the top portion.

So, they are partial segregation that you see that, maybe some concentration of the solid particle very dilute way you can say that it will be a mixed in the lower part with the coarser particles. Whereas, whereas, part partial segregation in that case, you will see in figure c there may be increasing in such way that if you are increasing flow rate and also energy distribution is so, high and also if you change the viscosity of the liquid or higher viscous liquid if you are change using or lower viscous, viscous you using then that case you will see that, there may be a again the mixing some extent mixing of the, coarser and finer particles will be on the fluidized bed. And, then the segregation efficiency or partial segregation will be there some extent they are, relative to the figure c.

So, the this case you will see that very interesting that when about this partial or complete segregation will be there inside the bed then you will see that the local mean diameter that is that, that will be changed across this column and it can be expressed as a function of the bed level in the transition zone between the fully segregated large particles at the bottom and the fully segregated the small particles at the top. So, in that case of course, you have to know how this local composition of this solids will be changing based on the flow rate of the fluidized bed and what extent of this segregation can be obtained by this phenomena.

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Local Composition of the Segregated Solids (contd.)

► **Di Felice et al. (1987)** suggested the following equation to determine the local mean diameter using the axial voidage profiles

$$Ar \varepsilon^{4.8} = [(17.3 Re)^\alpha + (0.336 Re^2)^\alpha]^{1/\alpha}$$

$$\alpha = 2.55 - 2.1 [\tanh(20\varepsilon - 8)]^{0.33}$$

(a) Complete Segregation
(b) Perfect Mixing
(c) Partial Segregation 1
(d) Partial Segregation 2

(2)

$$Ar = d_p^3 (\rho_p - \rho) g / \mu^2$$

$$Re = (d_p U \rho) / \mu$$

And, you will see their of course, to determine the local mean diameter using this axial voidage profile of course, during that complete or partial segregation the local voidage of the liquid there will be change at the top maybe higher voidage will be there whereas, at the bottom maybe lower voidage will be there.

So, based on this axial voidage profile Di Felice et al. 1987 they have suggested one actually empirical equations, for calculating this what is that local mean axial voidage by knowing that local mean diameter by this segregation phenomena. So, as per their model they have given this equation 2 here and it is expressed as in terms of Archimedes number and the Reynolds number. So, Archimedes into epsilon to the power 4.8 that should be is equal to 17.3 into Re to the power alpha, and plus 0.336 Re to the power 2 here.

Re means Reynolds number and Ar means Archimedes number. So, this Archimedes number is defined as here particle diameter cube into effective density into fluid density to gravitational oscillation by viscosity square. Where, alpha is one important empirical constants that can be obtained from the experimental data, and this is correlated to that porosity again.

So, alpha should be is equal to 2.55 minus 2.1 into tan h into 20 into epsilon minus 8 to the power 0.33 whole cube. So, in this case very interesting that this correlation coefficient here will be some extent that depends on the experimental data, how this

experimental data are fitting with these empirical equations there. Anyway, they have suggested I think after comparing with different models and they have given this finalized model there and, this constants here alpha that depends on this again this porosity.

So, once you substitute this alpha here, then porosity equation will be represented by this equation 2 and solving this non-linear equation of porosity by changing the gas velocity changing the fluid density and the, liquid density they are and with other variables, then you will be able to calculate what should be the porosity at a different level.

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Local Composition of the Segregated Solids (contd.)

- According to **Epstein et al., (1981)** Irrespective of the degree of segregation or mixing, however, the overall bed expands as if it is simply the sum of the N individual species, each acting independently of the other so that

$$1 - \varepsilon = \left[\sum_{i=1}^N \frac{C_i / C_t}{1 - \varepsilon_i} \right]^{-1} \quad (3)$$

C_i = local or overall volumetric concentration in liquid of particle species i , m^3/m^3

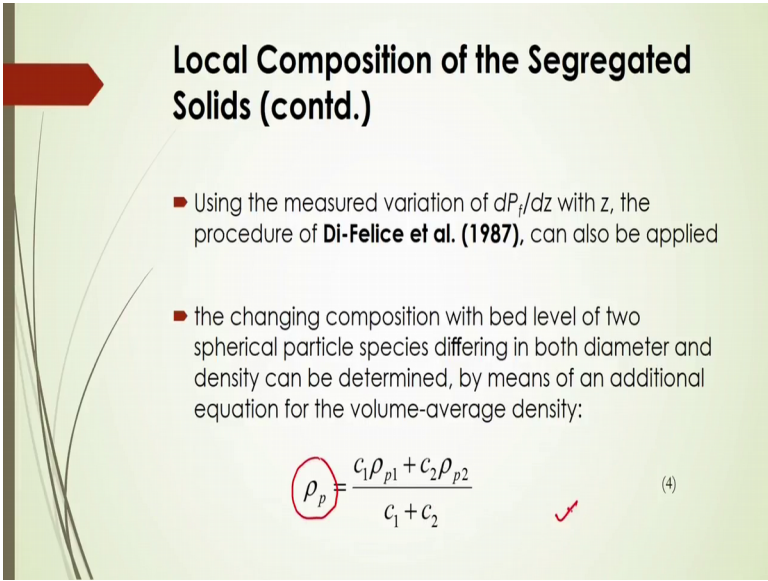
- where ε_i is the voidage when species i is fluidized alone at the same superficial liquid velocity as the mixture, and C_i/C_t is the volume fraction of fluid-free solids which is species i .

And, local composition of the segregated solids also there of course, that will be varying along with this axis of this fluidized bed. So, degree of segregation or mixing irrespective of that, the overall bed expands as it is simply the sum of the number of individual species, each acting independently of the other. So, Epstein et al., 1981 so, irrespective of this degree of segregation or mixing the overall bed expands how much bed expands based on the, voidage inside the bed that depends on the number of individual species. And, then is species will be acting independently of the other.

So, that this one minus epsilon; that means, here the, what should be the volume of the solids inside the bed the depends on this local or overall volumetric concentration in the liquid of that particle species i . So, if particle species concentration of i is C_i , then it will be C_i by C_t and divided by 1 minus epsilon, and if you are summing up all these here

for the n number of species inside the bed for different particles. And, then you will be able to calculate here what would be the overall bed expansion based on this concentration of the liquid of particle species i there. And, epsilon i is the voidage when species i is the fluidized is fluidized alone at the, same superficial liquid velocity as the mixer and C i by C t, that is given in equation number 3 here is the volume fraction of the fluid free solids, who is a is for the species i.

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Local Composition of the Segregated Solids (contd.)

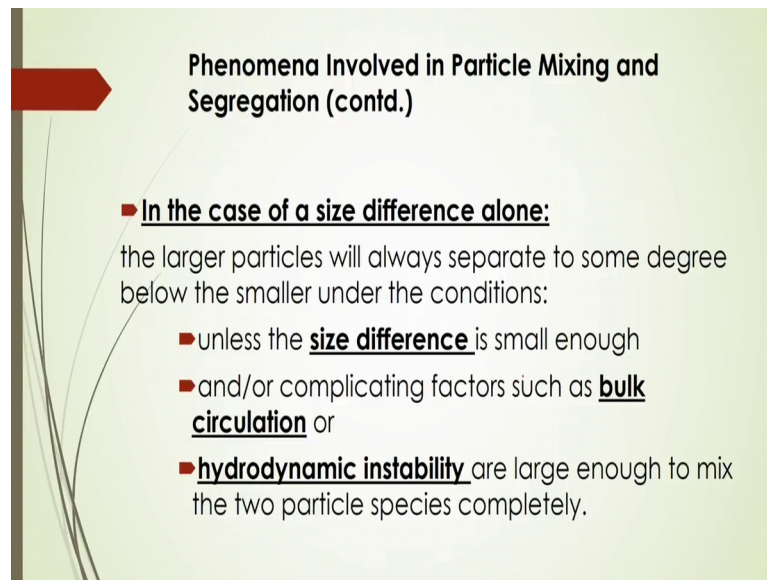
- Using the measured variation of dP_f/dz with z , the procedure of **Di-Felice et al. (1987)**, can also be applied
- the changing composition with bed level of two spherical particle species differing in both diameter and density can be determined, by means of an additional equation for the volume-average density:

$$\rho_p = \frac{c_1 \rho_{p1} + c_2 \rho_{p2}}{c_1 + c_2} \quad (4)$$

Now, using the measured variation of this frictional pressure gradient with height of the column, the procedure of this method a procedure of the that given by that Di Felice et al. 1987, can also be applied here by changing the composition with bed level up to a spherical particle species. And, that species will have of course, the different in both diameter and density.

So, if the spherical particle species would differ and their density also that differ, then you can calculate that, the a density of that particles by means of an additional equation of the volume average density there. So, this average density of that particle they are inside the bed that will be is equal to $c_1 \rho_{p1}$, plus $c_2 \rho_{p2}$ by c_1 plus c_2 there. So, here you can calculate this average volume average density here, as like this by equation 4.

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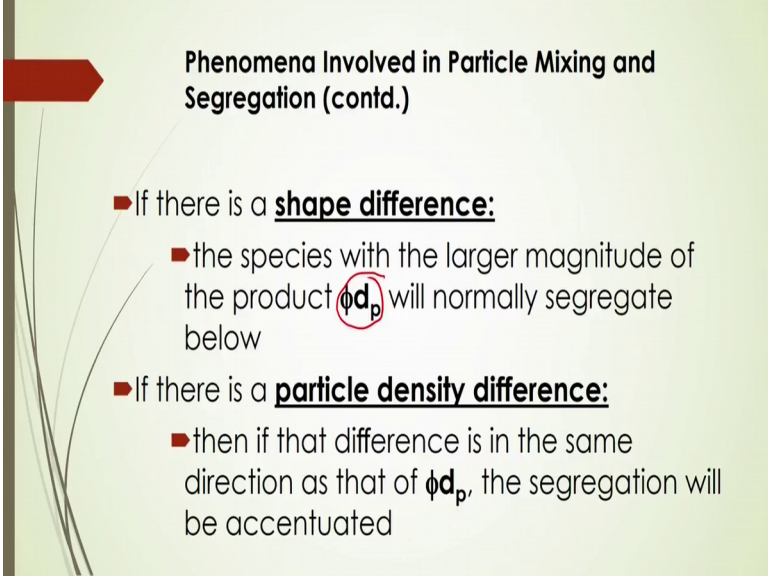
Phenomena Involved in Particle Mixing and Segregation (contd.)

- **In the case of a size difference alone:**
the larger particles will always separate to some degree below the smaller under the conditions:
 - unless the **size difference** is small enough
 - and/or complicating factors such as **bulk circulation** or
 - **hydrodynamic instability** are large enough to mix the two particle species completely.

Now, what will be the phenomena involved in particle mixing and segregation there. In the case, of a size difference alone the larger particles will always a separate to some degree below the smaller under these following conditions there. So, their size difference one condition and so, bulk circulation or hydrodynamic instability. So, the larger particle separate to some degree below the smaller, unless the size difference is small enough.

And, also it is seen that, that complicating factors such as bulk circulation also will govern the, degree of segregation based on the size difference there. Also there will be of course the, hydrodynamic instability may be flow pattern inside the bed will change and because of which that the segregation pattern and also the mixing will also be changed, because of that hydrodynamic instability. And, hydrodynamic instability are large enough to mix the two a particle species completely there in the bed.

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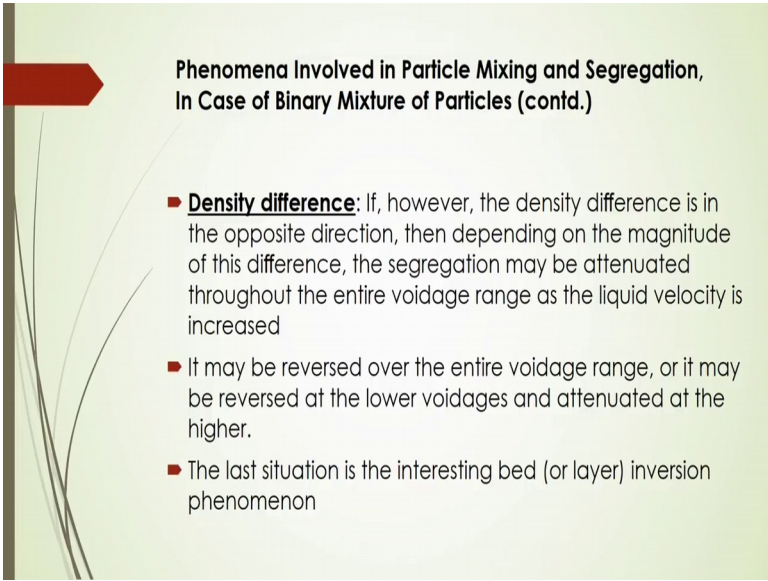


Phenomena Involved in Particle Mixing and Segregation (contd.)

- If there is a **shape difference**:
 - the species with the larger magnitude of the product ϕd_p will normally segregate below
- If there is a **particle density difference**:
 - then if that difference is in the same direction as that of ϕd_p , the segregation will be accentuated

Now, if there is a shape difference what will happen? Now, if the shape differences of this particle sometimes that different shape above the particles will be there inside the bed. Now, the species with the larger magnitude of the product like here ϕd_p ϕ is the sphericity and d_p is the particle diameter, will generally segregate below. And, if there is a particle density difference you will see then the difference is in the same direction as that of ϕd_p in the previous case and the segregation will be accentuated.

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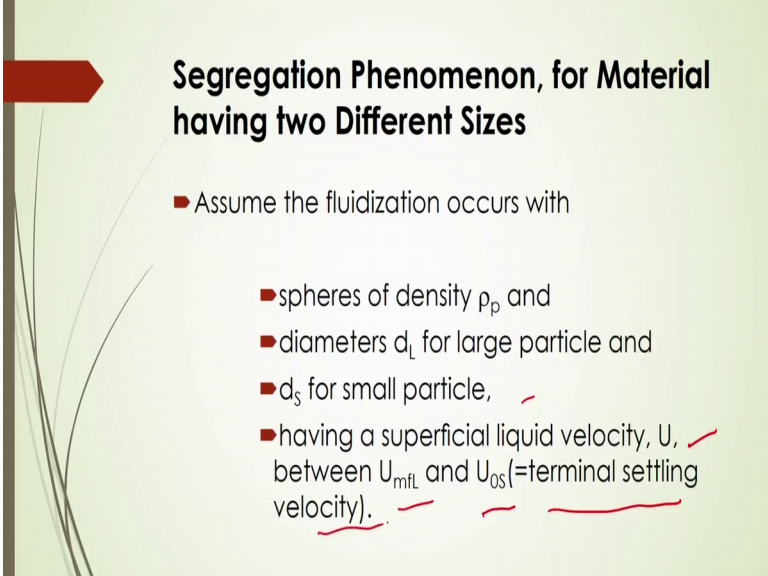
Phenomena Involved in Particle Mixing and Segregation, In Case of Binary Mixture of Particles (contd.)

- **Density difference**: If, however, the density difference is in the opposite direction, then depending on the magnitude of this difference, the segregation may be attenuated throughout the entire voidage range as the liquid velocity is increased
- It may be reversed over the entire voidage range, or it may be reversed at the lower voidages and attenuated at the higher.
- The last situation is the interesting bed (or layer) inversion phenomenon

And, phenomena that involved in particle mixing and segregation that also depends on that density difference of that fluid particle, and also the voidage inside the bed or the density differences in the opposite direction then depending on the magnitude of this difference, the segregation may be may be attenuated a throughout the entire voidage range, as the liquid velocity is increased.

Now, it may sometimes happened in reverse direction over the entire voidage range or it may be reversed at the, lower voidage and attenuated at the higher that depends on the, fluid density. The last situation is that the, interest that interesting bed inversion phenomena is there. So, sometimes that bed fluids will be of inverting the, they are phenomena of that the mechanism of that what is that fluid particle segregation and part solid particle segregation inside that. Totally, that depends on that diameter and also the flow rate and also the fluid properties of the, a system.

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Segregation Phenomenon, for Material having two Different Sizes

- Assume the fluidization occurs with
 - spheres of density ρ_p and
 - diameters d_L for large particle and
 - d_S for small particle,
 - having a superficial liquid velocity, U , between U_{mfl} and U_{os} (=terminal settling velocity).

Now, how this segregation of phenomena for material having two different sizes can be obtained and can be actually quantified then. Let us, consider the fluidization that occurs with that spheres of density ρ_p and the diameters d_L for large particles, and the diameter d_S for the small particles. And, and having a spherical liquid velocity U they are between U_{mfl} and U_{os} . U_{os} called terminal settling velocity for the particle of smaller size.

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Segregation Phenomenon (contd.)

- In the absence of the smaller particles, the fluidized bed suspension density
$$\rho_{BL} = \rho_p(1 - \epsilon_L) + \rho \epsilon_L \quad (5)$$
- In the absence of the larger particles, the suspension density is given by
$$\rho_{BS} = \rho_p(1 - \epsilon_S) + \rho \epsilon_S \quad (6)$$

ρ_p = Particle density, ρ = liquid density, ϵ_s = volume fraction of smaller particle, ϵ_L = volume fraction of larger particle

And, in the absence of smaller particles, so the fluidized bed suspension density to be actually calculated based on this equation 5 here.

Now, here in this case this the fluidized bed suspension density will be is equal to rho p; that means, density of the particles into 1 minus epsilon L 1 minus epsilon L is nothing, but the volume fraction of the, smaller particles there. And, then density of the liquid into epsilon L is the, what is that porosity of this only liquid volume fraction they are. And, in the absence of larger particles the suspension density will be given as rho p into 1 minus epsilon S into rho into epsilon S here.

So, epsilon L and epsilon S is the volume fraction of the larger particles, and smaller particles they are in the bed. And, if you known is volume fraction of this smaller and larger particles they are, then what should be the, suspension density based on that larger particles and smaller particles there. So, rho p is the particle density rho is the liquid density, epsilon S is the volume fraction of smaller particles, epsilon L is the volume fraction of larger particles. By which you will be able to calculate what will be the, fluidized bed suspension density by this equation number 5 and 6 here is shown in slides.

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Subtracting above two equations one gets

$$\rho_{BL} - \rho_{BS} = (\rho_p - \rho)(1 - \varepsilon_L) - (\rho_p - \rho)(1 - \varepsilon_S) \quad (7)$$

Defining the dimensionless density ratio as

$$\gamma = (\rho_{BL} - \rho_{BS}) / (\rho_p - \rho) \quad (8)$$

Then $\gamma = \varepsilon_S - \varepsilon_L \quad (9)$

where ε_S and ε_L are the voidages displayed by the respective monocomponent beds when they are each separately fluidized by the same liquid at the same superficial velocity, U , as the binary bed.

Now, if you subtract these two equations of this 5 and 6, then you can simplify it as by this equation number 7 and defining the dimensionless density ratio and defining by this equation number 8, then you can represent this equation here as gamma is equal to epsilon S minus epsilon S L. Where, epsilon S and epsilon L are the voidage displayed by the respective monocomponent beds when they are each separately fluidized by the same liquid at the same spherical velocity, U , as the binary particle bed mixer.

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As per **Richardson-Zaki (1954)** theory applied for monocomponent beds

$$\varepsilon_S = \varepsilon_L \left(\frac{d_L}{d_S} \right)^{(3-m)/mn} \quad (10)$$

Therefore

$$\gamma = \varepsilon_L \left[\left(\frac{d_L}{d_S} \right)^{(3-m)/mn} - 1 \right] \quad (11)$$

The index n can be obtained from the relation

$$\frac{4.8 - n}{n - 2.4} = 0.043 Ar^{0.57} \quad (12)$$

The exponent m varies from 1 in the Stokes regime to 2 in the Newton regime

$(3-m)/mn$, varies from 0.43 to 0.22 as one moves from the Stokes to the Newton regime.

As if the inertial effects and turbulence increases, the degree of segregation decreases (**Epstein and Pruden, 1999**), even without complicating effects such as bulk circulation and hydrodynamic instability

As per Richardson-Zaki 1954 they have developed one correlations for this gamma, which is applied for the monocomponent beds here. So, they have given this here gamma is equal to $\epsilon L / d_p^3$ to the power three minus m/n . Where, this m and n are the parameters, so that depends on the other properties of the fluid.

So, the index n here given by this equation 10 or 11 here, this n can be calculated by this equation number 12 $4.8 - n$ by $n - 2.4$ that will be is equal to 0.043 into Archimedes number to the power, 0.57 . Now, this m and n so, n can be calculated from this equation 12 whereas, m also it depends on the different fluid actually a condition whereas, this high velocity fluid or low velocity fluid; that means, here the regime of this condition. Sometimes, whether for laminar regime then it will be Stokes regime and sometimes Newton regime may be more than Stokes regime they are.

So, that varies based on that fluid flow condition they are. Now, this exponent m generally varies from 1 in the Stokes regime and it will be 2 in the Newton regime. And, according to that, that $3 - m/n$ this coefficient here in this equation number 10 and 11 it is seen, as per they are experimental data and they have stated that, that this coefficient this $3 - m/n$ will be varying from 0.43 to 0.22 as one moves from Stokes regime to the Newton regime.

So, by this equation number 10 and 11 you will be able to calculate what should be the porosity of the fluidized bed for the smaller particles and larger particles by knowing this equation number 11 and 10 and, of course along with this index n for this by this equation number 12.

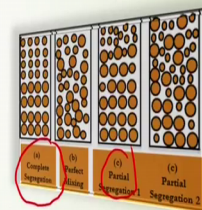
As if the inertial effects and the turbulence increases, you will see that the degree of segregation decreases that a given by this Epstein and Pruden, 1999 in their article. And, even without complicating effects such as bulk circulation and hydrodynamic instability, you will see that inertial effects and turbulence also will change and because of who is this segregation and it is degree will be affected by that phenomena.

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Qualitative Measure of the Degree Segregation

- The value of γ can function as a crude qualitative measure of the degree of particle segregation as follows (Epstein and Pruden, 1999):

Range of γ	Degree of segregation
$\gamma < 0.015 \pm 0.005$	little segregation, good perfect mixing (Fig. b).
$0.015 \pm 0.005 < \gamma < 0.045 \pm 0.015$	partial segregation with no interface between layers (Fig. d).
$0.045 \pm 0.015 < \gamma < 0.1$	segregation with fuzzy interface and some intermixing (Fig. c).
$\gamma > 0.1$	clean-cut segregation with sharp interface and little intermixing (Fig. a).



Now, some qualitative measurement of the by degree of segregation is given by Epstein and Pruden there, and the value of gamma they reported that may be a function as a crude qualitative measure of the degree of particle segregation as follows here. Now, they have given this some range of that gamma value, if gamma is less than 0.015 then used the it would be noted that, that there will be a little segregation and good perfect mixing as shown in figure b. Whereas, this gamma value if it is within the range of 0.015 to 0.045 and as per as per, as per figure here in they are d. Then, you will see that partial segregation with knowing no interface between layers will happen there.

Whereas, 0.05 and up to 0.1 it is seen that this gamma value, will be there then segregation with fuzzy interface and some inter mixing as shown in figure c they are in the slides. And, if suppose gamma value is greater than 0.1, it is seen that, that there will be a clear cut segregation with sharp interface and little inter mixing they are as shown in figure a in the slides here. So, based on this gamma value qualitatively at least you can say how much a segregation or what is the degree or intensity of the segregation in the bed that you can calculate, that you can of course, identify and interpret on that.

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Kennedy and Bretton (1966) Model

- Binary particle stratification is based on the more traditional concept of axial dispersion according to Fick's law of diffusion in competition with segregation
- Consider semibatch liquid fluidization that results in partial segregation of the type represented by Fig. c or d,
- then at any bed level within the partially mixed region, the volumetric particle mixing flux for species i must equal the particle segregation flux for that species, i. e.,

$$\frac{D_i dc_i}{dz} = c_i U_{pi} \quad (13)$$

where
 D_i is the axial dispersion coefficient of species i and
 U_{pi} the segregation (or classification) velocity of that species through a swarm of species j.

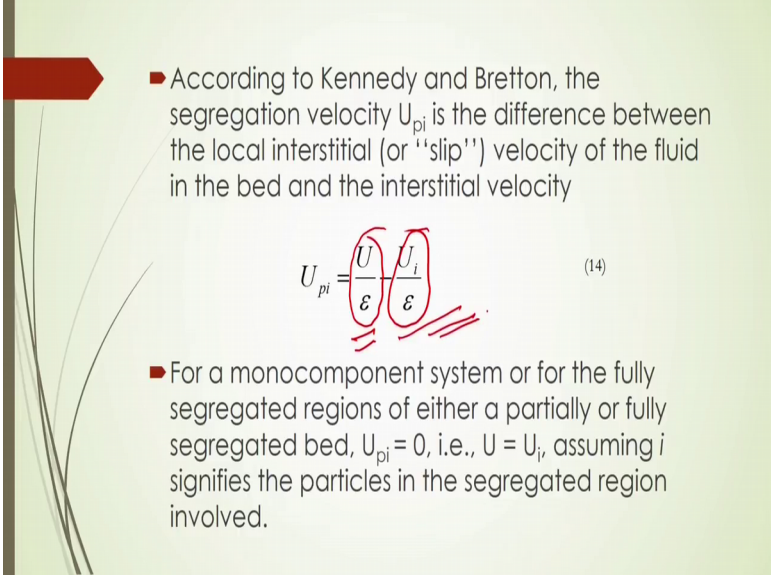
And, another important that, axial dispersion coefficient for this particle segregation will be there, then how this dispersion of the particles will be inside the bed Kennedy and Bretton 1966, they have actually interpret this phenomena that if binary particle stratification is in the fluidized bed, then based on this stratification axial dispersion according to the Fick's law of diffusion, in competition with the segregation can be explained.

Now, as per they are model that if you consider the semibatch liquid fluidization, that results in partial segregation of the type that is represented in figure c and d here in these slides. Then, you will see that at any bed level within the partially mixed region, the volumetric particle mixing flux for the species, I that must be equal the particle segregation flux for that species.

So, in that case equation 3 will be useful to actual interpret that axial dispersion they are inside the bed. Now they have actually concluded inside in this way, that the as per their as per Ficks law of diffusion, they have given this equation of 13 by that the density or you can say that concentration of that species with respect to z at a steady state condition will be actually depending on the what is that particles flow rate they are; that means, or you can say that segregation velocity of the species through, a swarm of species j in the mixer. So, in that case this proportionality constant may be will be expressed by this axial dispersion coefficient.

So, based on their model this equation 30 can be explained here. So, D_i into $d c_i$ by $d z$ that will be equal to c_i into U_{pi} there so, c_i is the proximity constant and D_i is called the axial dispersion coefficient of the species i . And U_{pi} here is the segregation, or you can say classification velocity of that species in the bed.

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- According to Kennedy and Bretton, the segregation velocity U_{pi} is the difference between the local interstitial (or "slip") velocity of the fluid in the bed and the interstitial velocity

$$U_{pi} = \frac{U}{\epsilon} - \frac{U_i}{\epsilon} \quad (14)$$

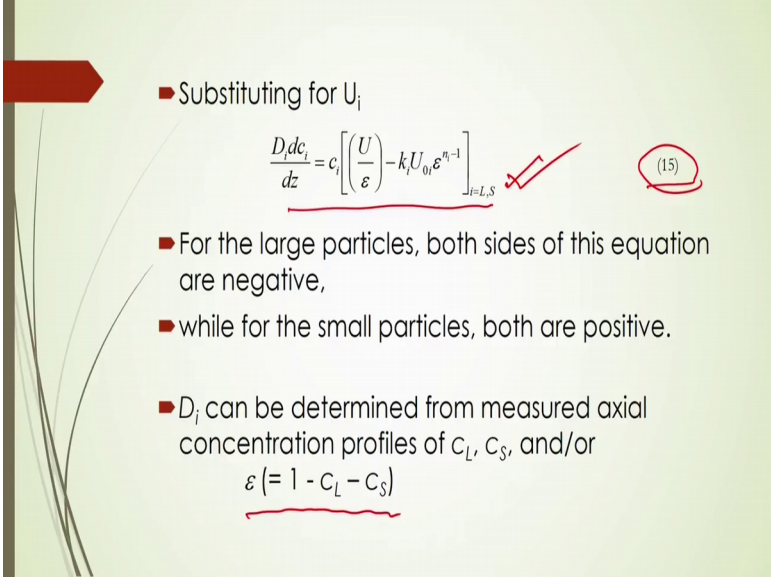
- For a monocomponent system or for the fully segregated regions of either a partially or fully segregated bed, $U_{pi} = 0$, i.e., $U = U_i$, assuming i signifies the particles in the segregated region involved.

Now according to their model that segregation of velocity U_{pi} can be expressed by the difference between the local interstitial velocity of the fluid in the bed and, the and the interstitial velocity can be expressed by that here, it will be is equal to U by ϵ by U_i by ϵ here. So, U is the gas velocity and ϵ is the porosity of the gas. So, U by ϵ if this is called, but actual velocity of the gas this U_i by ϵ ; that means, here U_i is called the species segregation velocity and ϵ is that risk, what is that voidage inside the bed.

So, based on these you can say that what should be the sleep velocity of that segregation velocity, or relative velocity of the segregation velocity, you can calculated in terms of this actual or interstitial velocity. So, for a mono component system or for the fully segregated region of either a partially, or fully segregated bed, then you can say that the U_{pi} value should be is equal to 0 and whereas, as this U will be equals to U_i ; that means, gas velocity will be is equal to that individual species velocity in that case you will see that particles in the segregated reasons that will be involved in the bed.

So, in this case will be actually expressing how that the sleep velocity will change that, that extent of segregation or degree of segregation, whether it will be that partially or fully segregated bed are there or not. So, find you have to first you have to calculate what should be that means, a local, or you can say that sleep velocity of that local velocity of that segregated particle they are.

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- Substituting for U_i

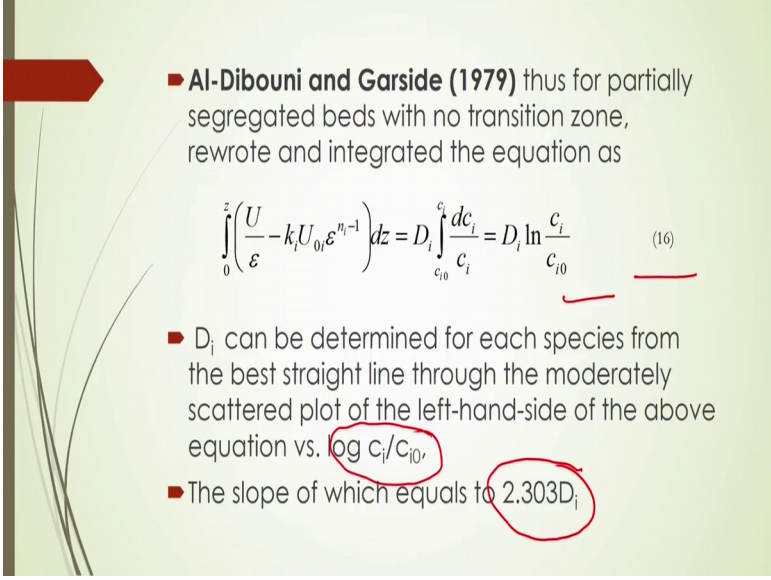
$$\frac{D_i dc_i}{dz} = c_i \left[\left(\frac{U}{\varepsilon} \right) - k_i U_0 \varepsilon^{n-1} \right]_{z=L,S}$$

- For the large particles, both sides of this equation are negative,
- while for the small particles, both are positive.
- D_i can be determined from measured axial concentration profiles of c_L , c_S , and/or $\varepsilon (= 1 - c_L - c_S)$

And substitution of this U_i value here, you just substitute it in equation number 13 there, then you will be having this equation which is expressed by equation 15, they are and in this case for the large particles both sides of this equation are negative whereas, for the small particles both may be positive.

So, D_i can be determined from the measured axial concentration profiles of this C_L and C_S and or by this epsilon is equal to 1 minus C_L minus C_S they are. So, based on who is a so, equation 15 can be expressed and by which you will be able to calculate, this axial dispersion coefficient this D_i just by plotting this gradient of this concentration with this U or epsilon there inside the bed.

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- **Al-Dibouni and Garside (1979)** thus for partially segregated beds with no transition zone, rewrote and integrated the equation as

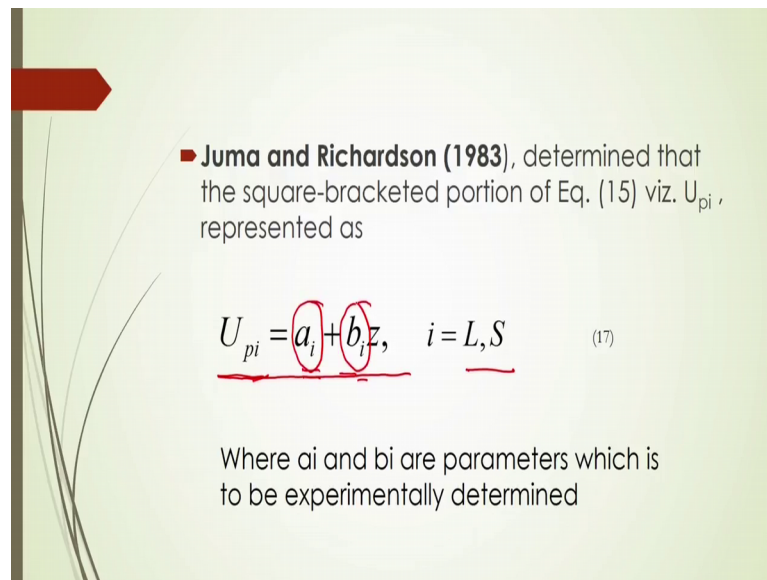
$$\int_0^z \left(\frac{U}{\varepsilon} - k_i U_{0i} \varepsilon^{n_i-1} \right) dz = D_i \int_{c_{i0}}^{c_i} \frac{dc_i}{c_i} = D_i \ln \frac{c_i}{c_{i0}} \quad (16)$$

- D_i can be determined for each species from the best straight line through the moderately scattered plot of the left-hand-side of the above equation vs. $\log c_i/c_{i0}$.
- The slope of which equals to $2.303D_i$.

Al-Dibouni and Garside 1979 for partially segregated beds with no transition zone, they have developed another equation by integration of equation like this here, it is expressed in equation number 16 and based on this distribution 16, you can also be able to calculate what should be the axial dispersion coefficient in the bed for that particular species. And from the best straight line through this moderately scattered plot of this left hand side of the above equation versus this log of C_i by C_{i0} , if you plot then you will be able to calculate this D_i value.

And this slope of who is equals to that 2.303 into D_i for this equation number 16 and, then from which you will be calculate what will be the D_i .

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■ **Juma and Richardson (1983)**, determined that the square-bracketed portion of Eq. (15) viz. U_{pi} , represented as

$$U_{pi} = a_i + b_i z, \quad i = L, S \quad (17)$$

Where a_i and b_i are parameters which is to be experimentally determined

Now Juma and Richardson 1983, they have also given another equations to calculate this that means, they are segregation velocity that is that along the axis of the fluidized bed and, they have given this equation of this here, U_{pi} is equal to a_i plus b_i into z this a_i and b_i are the parameters which is to be experimentally determined, here i means L and S ; that means, large and smaller particles. .

So, here for the larger particles so, U_{pi} that would be equal to a_L plus b_L into z and also, for smaller particles U_{pi} is equal to a_S plus b_S into z . Now if you are able to find out experimentally what to how this U_{pi} for large particles and, then not for smaller particles these velocities are changing along with the z , then you can mathematically express that segregation velocity for this large and smaller particles by this equation number 17. And from the experimental data you will be able to find out what should be the a_i parameter of this a_i and b_i for this equation 17. Once you know this parameters a_i and b_i , then automatically you can say that what should be the predicted value of this U_{pi} for any operating conditions there.

Once you know this predicted value of U_{pi} ; that means, segregation velocity for these large and smaller particles by equation 17 and if you substitute there of course, you will be able to find out what should be the axial dispersion of the this larger and smaller particles inside the bed for this binary mixture. And equation 16 can then be integrated

by knowing this parameter a_L or a_S as then and substitute here you will get this type of equation.

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Eq. (16) can then be integrated by knowing the parameters a_L , b_L , and a_S , b_S , as

$$D_L \int_{1-\epsilon_L}^{c_L} \frac{dc_L}{c_L} = D_L \ln \frac{c_L(z)}{1-\epsilon_L} = \int_0^z (a_L + b_L z) dz$$

$$= a_L z + \frac{b_L z^2}{2} \quad (18)$$

$$D_S \int_{\epsilon_S}^{1-\epsilon_S} \frac{dc_S}{c_S} = D_S \ln \frac{1-\epsilon_S}{c_S(z)} = \int_z^H (a_S + b_S z) dz$$

$$= a_S (H - z) + \frac{b_S (H^2 - z^2)}{2} \quad (19)$$

where the voidage at $z = 0$ (the bottom of the transition zone) and below is ϵ_L and the voidage at $z = H$ (the top of the transition zone) and above is ϵ_S .

D_L and D_S best fitted to the experimental measurements according to Eqs. (18) and (19) will then be obtained and adjusted to best-match the profile of $c_L + c_S (= 1 - \epsilon)$ vs. z .

And from who is individually a for individual component of this larger and smaller particles are the dispersion coefficient you will be able to calculate. Here it is important that where, the voidage at z is equal to 0 the bottom part of the transition zone and below is the epsilon L and the voidage at that is at the top of the transition zone and above is epsilon S.

Now D_L and D_S best fitted to the experimental measurements according to equations 18 and 19, then you can obtain and obtain the value and adjusted to best mass the profile of that a C_L and C_S that will be equal to one minus epsilon versus a length of the z the length of the fluidized bed there.

So, from this equation 18 and 19, you will be able to calculate what should be the dispersion phenomena dispersion it is extent of dispersion of this large and smaller particles and, along that z how it will be changing also.

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■ The U_{pi} varies linearly with c_i from zero in a monocomponent bed of i particles in which $\varepsilon = \varepsilon_i$, $c_i = 1 - \varepsilon_i$ to U_{pi0} in a monocomponent bed of j particles in which $\varepsilon = \varepsilon_j$, $c_i = 0$, so that

$$U_{pi} = U_{pi0} \left(1 - \frac{c_i}{1 - \varepsilon_i} \right) \quad (20)$$

Van der Meer et al. (1984); Di Felice et al. (1987) and Asif and Petersen (1993)

Now the U_{pi} varies linearly with C_i from 0 in a mono component made up for that i particles in which that epsilon will be is equal to epsilon i and C_i will be equal to 1 minus epsilon i to U_{pi0} in a mono component bed of z particles in which, you can say that epsilon is equal to epsilon j and C_i will be equals to 0. So, that that U_{pi} should be is equal to U_{pi0} into 1 minus c_i by 1 minus epsilon i .

So, this relation has been given by actually Van Der Meer et al 1984 and the Di Felice et al 1987 also Asif and Petersen they have also are this 1993 they have also studied, this is the how that is segregated velocity for the large and smaller particles will change with it is the concentration. And they have also suggested this is the same type of relationship with the concentration and, which is expressed in equation number 20.

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Substituting this equation (20) into Eq. (13), taking $i = L$ and the upward direction as positive,

$$D_L \frac{dc_L}{dz} = c_L U_{pL0} \left(1 - \frac{c_L}{1 - \epsilon_L} \right) \quad (21)$$

that is,

$$\frac{dx_L}{x_L(1 - x_L)} = \frac{U_{pL0}}{D_L} dz \quad (22)$$

Integrating Eq. (22) with $x_L = 0.5$ [i.e., $c_L = 0.5(1 - \epsilon_L)$], $z - \bar{z}_L$,

One gets

$$\ln \frac{x_L}{1 - x_L} = \frac{U_{pL0}(z - \bar{z}_L)}{D_L} \quad (23) \quad x_L = c_L / (1 - \epsilon_L)$$

A plot of $z - \bar{z}_L$ vs. $\ln [x_L / (1 - x_L)]$ yields a slope equal to D_L / U_{pL0} , where U_{pL0} is taken as the measured or predicted velocity of a single large particle moving through a bed of the smaller particles fluidized by the same superficial velocity U as the binary under investigation.

Now, substituting this equation number 20 into again in equation 13 just by taking i is equal to L for larger particles and the upward direction is positive, then you will get this equation number 21 and after simplification of this 21, then you can express this here by equation number 20. And if you integrate this equation number 20, then you can get this x_L ; that means, your mass fraction of these large particles they are in terms of that concentration, then at z is equal to z_L , then you can express this equation to predict this what will be the mass concentration of these particles they are along with the axis of the bed.

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From Eq. (23) it follows that

$$x_L = \frac{1}{1 + e^{-Pe_L(z - \bar{z}_L)}} \quad (24)$$

$$Pe_L = L U_{pL0} / D_L \quad z - \bar{z}_L = (z - \bar{z}_L) / L$$

Similarly it can be shown that

$$x_S = \frac{1}{1 + e^{-Pe_S(z - \bar{z}_S)}} \quad (25)$$

$$Pe_S = L U_{pS0} / D_S \quad z - \bar{z}_S = (z - \bar{z}_S) / L$$

Now, from equation number 23 follows also that x_L should be equal to this here, we are this it will be related to this Peclet number here, this Peclet number is denoted by that axial dispersion coefficient of these large particles they are. So, if you know that axial dispersion coefficient of the large particles then you will be able to calculate what should be the mass fraction of the larger particles there in the bed. .

Along also, if you know that the z that is here by equation that z minus z_L that z_L by L . Similarly it can be shown for smaller particles also these x_S will be equal to these by knowing that the dispersion coefficient for the smaller particles which is expressed by this Peclet number here, in this equation number 25 whereas, this expected Peclet number of the small particles will be defined by this here, L here this length of the bed into U a $U_p S_0$ means here, the separation velocity for this a smaller particles divided by this dispersion coefficient of the smaller particles there.

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in accord with Di Felice et al. (1987), c_L and c_S are linearly related, so that

$$\frac{c_S}{1-\varepsilon_S} = 1 - \frac{c_L}{1-\varepsilon_L} \quad (26)$$

Thus

$$x_S = 1 - x_L = \frac{e^{-Pe_L(z-\bar{z}_L)}}{1 + e^{-Pe_L(z-\bar{z}_L)}} = \frac{1}{1 + e^{+Pe_L(z-\bar{z}_L)}} \quad (27)$$

Comparing Eqs. (25) and (27)

$$Pe_L = -Pe_S \quad \bar{z}_L = \bar{z}_S$$

D_i values vary from 0.24 to 515 cm²/s

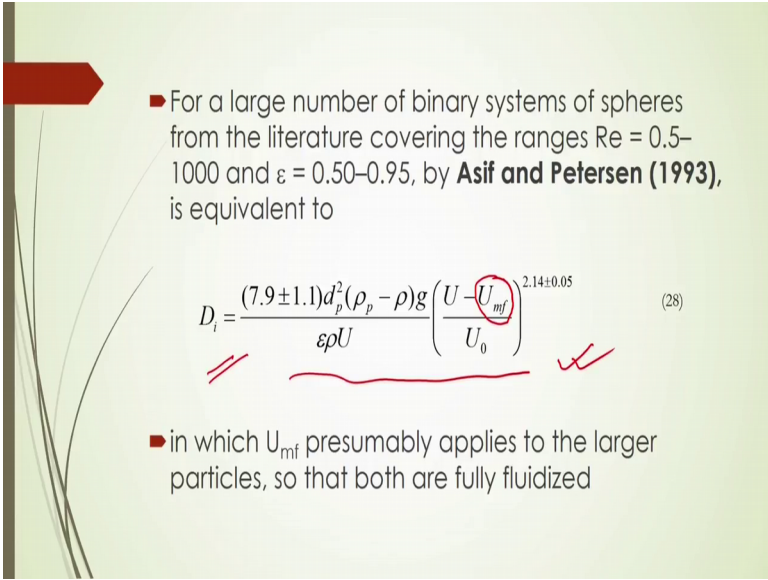
Now, Di Felice et al 1987, they have also are correlated with this C_S and C_L by they are what is that a porosity of the smaller and larger particles by equation number 26. And if you substitute this c_S from this equation number 26, then x_S can be calculated in terms of x_L they are by this equation number 27. So, this is equation 27 actually is coming after simplification by substituting this concentration profile for the smaller particles in terms of larger particles.

Now comparing this equation number 25 and 27, you can simply say that Peclet number of the large particles will be equal to Peclet number of smaller particles in magnitude, but opposite in direction because, the flow of these smaller and larger particles will be in the opposite direction they are. And of course, that what will be the length of occupation of the larger particles inside the bed and what will be the occupation of the smaller particles inside the bed and, then that should be equal to almost the same.

So, at the middle of this fluidized bed, it will be seen that there will be demarcation of these larger particles and smaller particles. If the Peclet number of these smaller particles and larger particles are same, or you can say that dispersion coefficient of the larger particles and smaller particles are same in the bed. And if in this condition this is happening only then, only when there will be the, that is concentration of the smaller particles would be based on this correlation with the larger particle concentration.

Now, Di Felice et al 1987, they have reported that this axial dispersion coefficient for these larger or smaller particles generally varies from 0.24 to 515 centimeter square per second, from the literature data, and they are also experimental data.

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■ For a large number of binary systems of spheres from the literature covering the ranges $Re = 0.5-1000$ and $\varepsilon = 0.50-0.95$, by **Asif and Petersen (1993)**, is equivalent to

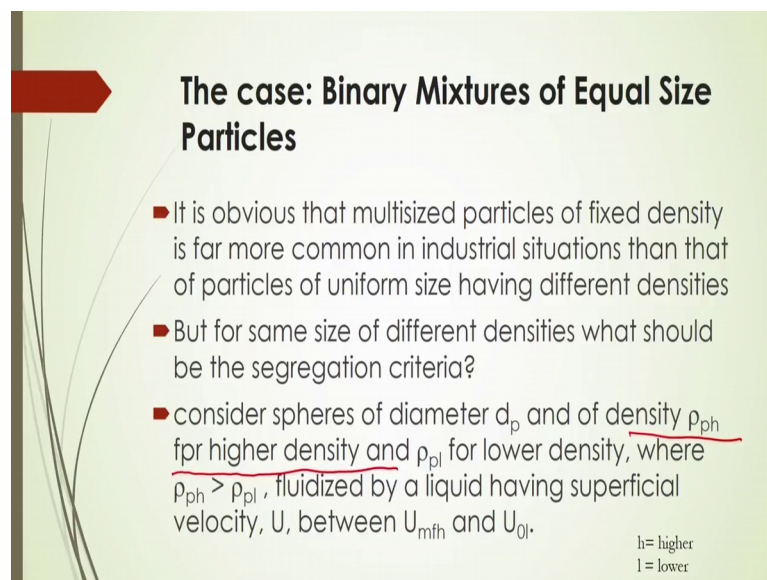
$$D_i = \frac{(7.9 \pm 1.1) d_p^2 (\rho_p - \rho) g \left(\frac{U - U_{mf}}{U_0} \right)^{2.14 \pm 0.05}}{\varepsilon \rho U} \quad (28)$$

■ in which U_{mf} presumably applies to the larger particles, so that both are fully fluidized

For a large number of binary systems of spheres from the literature covering, the range of Reynolds number that is 0.5 to 1000 and porosity of 0.50 to 0.95 Asif and Petersen 1993, they have suggested these correlations for this, dispersion coefficient of these larger or smaller particles inside the bed for their segregation phenomena.

So, this will be expressed by this equation and, which is a function of this gas, velocity a density of the liquid porosity of the fluid and, also what is that here the particle diameter and of course, the minimum velocity for the fluidization condition. In which you will see that U_{mf} is presumably applies to the larger particles. So, that the both are fully fluidized they are in that case. So, here important is that this U_{mf} should be considered on based on the large circulate larger particles there. So, once you substitute this U_{mf} and use 0 there and you will be able to calculate what should be the diameter sorry, what should be the dispersion coefficient of the particular large particles or smaller particles inside the bed.

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The case: Binary Mixtures of Equal Size Particles

- It is obvious that multisized particles of fixed density is far more common in industrial situations than that of particles of uniform size having different densities
- But for same size of different densities what should be the segregation criteria?
- consider spheres of diameter d_p and of density ρ_{ph} for higher density and ρ_{pl} for lower density, where $\rho_{ph} > \rho_{pl}$, fluidized by a liquid having superficial velocity, U , between U_{mfh} and U_{0l} .

h = higher
l = lower

Now, some case let us see here for binary mixture of equal sized particles there, it is obvious that multi sized particles are fixed density is far more common, in industrial situation then that a particle of uniform size having different are densities of course, in different applications will not be having the same size of particles they are. For mixture of particles of course, there in that way that whenever you are making grinding and, then you not be having the same size particles there.

And, but for the same size of different densities what should be the segregation criteria also that should be known there, consider the spheres of diameter d_p and, density the ρ_{ph} for higher density and, ρ_{pl} for lower density where this ρ_{ph} is greater than ρ_{pl}

particle fluidized by liquid having the spherical particle, superficial velocity of the liquid U between U_{mf} and the U_o here.

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Then the degree of segregation by particle density

$$\gamma_h = \frac{\rho_{Bh} - \rho_{Bl}}{\rho_{ph} - \rho} = 1 - \varepsilon_h - \frac{\rho_{pl} - \rho}{\rho_{ph} - \rho} (1 - \varepsilon_l) \quad (29)$$

where $\rho_{Bh} = \rho_{ph}(1 - \varepsilon_h) + \rho \varepsilon_h$ $\rho_{Bl} = \rho_{pl}(1 - \varepsilon_l) + \rho \varepsilon_l$

$$\varepsilon_h = \varepsilon_l \left(\frac{\rho_{pl} - \rho}{\rho_{ph} - \rho} \right)^{1/mn} \quad (30)$$

$$\gamma_h = 1 - \frac{\rho_{pl} - \rho}{\rho_{ph} - \rho} - \varepsilon_l \left[\left(\frac{\rho_{pl} - \rho}{\rho_{ph} - \rho} \right)^{1/mn} - \frac{\rho_{pl} - \rho}{\rho_{ph} - \rho} \right] \quad (31)$$

mn equals 4.8 both in the Stokes and the Newton regimes

Then the degree of segregation by that particle density can be expressed by this equation number 29 given here. In this case this ρ_{Bh} for the larger particles here to be expressed by this equation here and, then ρ_{Bl} to be expressed by this equation there and, equation 30 is denoted for that porosity for that higher density particles they are in in terms of that lower density particles, they are inside the bed. And this n of course, that we have already shown that how n should be calculated this here and m and n should be what will be the value mn equals to the 4.8 both in the Stokes and Newton regime.

So, by this equation 29 to 31 will be able to calculate the degree of segregation by particle density they are.

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Non-dimensionalized Bulk Density Differences of the Two Particle Species in Two Types of Binaries

	$\rho_p = \text{constant}$ $d_1 / d_2 = \sqrt{2}$			$d_p = \text{constant}$ $(\rho_{ph} - \rho) / (\rho_{pl} - \rho) = \sqrt{2}$		
	ϵ_l	ϵ_i	γ	ϵ_l	ϵ_h	γ_h
	Eq. (10)	Eq. (9)		Eq. (30)	Eq. (29)	
Stokes regime: $M=1, n=4.8$	1.0	0.866	0.134	1.0	0.930	0.070
	0.5	0.433	0.067	0.5	0.465	0.181
Newton regime: $M=1, n=4.8$	1.0	0.930	0.070	1.0	0.930	0.070
	0.5	0.465	0.035	0.5	0.465	0.181

Yang (2003) ✓ ✓

Here non dimensionalized bulk density difference of the two particle species in two types of binaries in this table are shown, how this epsilon S and epsilon l and also the gamma are changing a for different density and for different particle size there. So, from this table you will be able to calculate, or you can get the different values for this a porosity and what is that gamma value.

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Binary Mixtures of Particles Differing in Size and Density: Bed Inversion

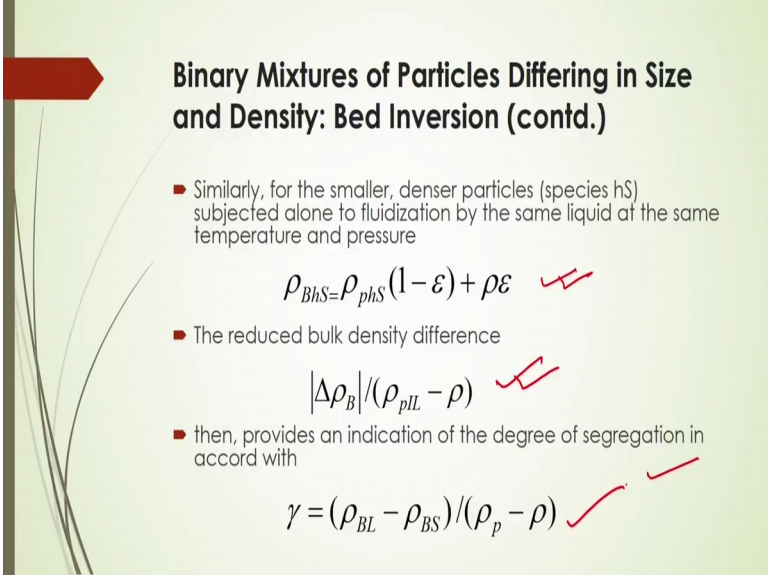
- Consider a binary mixture of spheres of diameters d_{iL} and d_{hS} ($< d_{iL}$) and densities ϵ_{pIL} and ϵ_{pHS} ($> \epsilon_{pIL}$), respectively
- For the larger, less dense particles (species IL) subjected alone to liquid fluidization,

$$\rho_{BIL} = \rho_{pIL}(1 - \epsilon) + \rho\epsilon$$

And also another important that, if you are using that binary mixture of particles that differing in size and density and, there will be bed inversion there.

So, consider a binary mixture of spheres of diameter d_{IL} and d_{hS} and densities ρ_{pIL} and ρ_{phS} respectively and, then for the largest or less than particles along to the liquid fluidization it can be represented, that this ρ_{BIL} will be equal to that ρ_{pIL} into $1 - \epsilon$ plus ρ_{phS} times ϵ they are.

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Binary Mixtures of Particles Differing in Size and Density: Bed Inversion (contd.)

- Similarly, for the smaller, denser particles (species hS) subjected alone to fluidization by the same liquid at the same temperature and pressure

$$\rho_{BhS} = \rho_{phS}(1 - \epsilon) + \rho\epsilon$$
- The reduced bulk density difference

$$|\Delta\rho_B|/(\rho_{pIL} - \rho)$$
- then, provides an indication of the degree of segregation in accord with

$$\gamma = (\rho_{BL} - \rho_{BS})/(\rho_p - \rho)$$

So, similarly for the smaller denser particles also you can express by this equation and, the reduced bulk density to be explained by this equation then, but it will provide the indication of the degree of segregation in accordance to this equation here shown in the slides.

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■ According to **Jean and Fan (1986)** the segregation velocity of species hS with respect to species IL is given by

$$U_{phS} = \left(\frac{U}{\varepsilon} \right) - k_{hs} U_{0hS} \varepsilon^{nhS-1}$$

$$\frac{1}{1-\varepsilon} = \frac{v_{IL}}{1-\varepsilon_{IL}} + \frac{1-v_{IL}}{1-\varepsilon_{hI}}$$

■ The inversion condition is then taken as

$$\frac{d(U_{phS})}{dU} = 0$$

At the inversion velocity, the entire bed attains a uniform composition equal to that of the overall solids composition, $x_{IL} = 0.4$, and a bulk density equal to ρ_{BB} .

Now, according to Jean and Fan 1986 this is the segregation velocity of the species h S with respect to the species I l that will be given by this equation here. Now the inversion coordination you will get, if you are just in the derivative and considering it will be equal to 0, then $d U_{phS} / d U$ is equal to 0. So, at this condition there will be inversion. So, at the inversion velocity what will happen the entire bed attains a uniform composition that will be equal to that of the overall solids composition; that means, x_{IL} that should be equal to 0.4 and the bulk density equals to ρ_{BB} there.

(Refer Slide Time: 55:49)

References

- Al-Dibouni MR, Garside J. Particle mixing and classification in liquid fluidised beds. Trans IChemE 57:94-103, 1979.
- Asif M, Petersen JN. Particle dispersion in a binary solid-liquid fluidized bed. AIChE J 39:1465-1471, 1993.
- Di Felice R, Gibilaro LG, Waldram SP, Foscolo SP. Mixing and segregation in binary-solid liquid fluidised bed. Chem Eng Sci 42:639-652, 1987.
- Epstein N, LeClair BP, Pruden BB. Liquid fluidization of binary particle mixtures—I. Overall bed expansion. Chem Eng Sci 36:1803-1809, 1981.
- Epstein N, Pruden BB. Liquid fluidisation of binary particle mixtures—III. Stratification by size and related topics. Chem Eng Sci 54:401-415, 1999.
- Jean R-H, Fan L-S. On the criteria of solids layer inversion in a liquid-solid fluidized bed containing a binary mixture of particles. Chem Eng Sci 41:2811-2821, 1986.
- Juma AKA, Richardson JF. Segregation and mixing in liquid fluidized beds. Chem. Eng. Sci. 38:955-967, 1983.
- Kennedy SC, Bretton RH. Axial dispersion of spheres fluidized with liquids. AIChE J 12:24-30, 1966.
- Kunii D. and Levenspiel O., Fluidization Engineering. Butterworth, 1991.
- Richardson JF, Zaki WN. Sedimentation and fluidization. Part I. Trans Instn Chem Engrs 32:35, 1954.
- Van der Meer AP, Blanchard CMRJP, Wesselingh JA. Mixing of particles in liquid fluidized beds. Chem Eng Res Des 62:214-222, 1984.
- Yang W., Handbook of Fluidization and Fluid-particle Systems, 2003

So, more informations you can get from other references that is here given and, so by this lecture what we conclude that we are able to know, then in liquid medium how actually the segregation and extent of segregation mixing are happening and how, the dispersion coefficient of that larger particle and smaller particles are based on the segregation velocity, how this dispersion coefficient is changing.

And also how different sizes and densities of that particles, in this liquid medium will effect that segregation velocity and, consequently the dispersion coefficient inside the bed. And also what should be the inversion case will have happen, if you know the inversion segregation velocity for that individual particles with different densities, then how it will be this inversion and what is that inversion condition, it is very important that when that larger particles and smaller particles will be mixed in such a way that that at a certain velocity, you will see that inter bed will attain any new for uniform composition that would be equal to that overall solid composition.

And so, based on who is you will be able to analyze that fluidized bed and, the fluid mixing inside the bed based on this segregation phenomena. So, I think the lecture may be very useful for that analyzing, the axial dispersion of the solid particles inside the bed in liquid solid system. Also same concept also you can used for if there is the gas is the applied in that gas liquid solid fluidized bed there. So, only thing is that you have to consider the density of the fluid, only for that they are a the mixture of gas and liquid as one fluid there.

So, and other properties will be almost same for the solid particles there and, then how this axial dispersion coefficient of that solids are in the mixture of gas liquid there will be happening and, then you will be able to calculate based on this concept. So, that will be caused or that that will be called that dispersion phenomena three phase fluidized bed system. So, and I think this lecture will be helpful for analyzing also for gas liquid solid system. And more reference are given here, to get more information about this, and in next lecture we will be considering about that gas dispersion interchange, they are in the fluidized bed there so.

Thank you for your attention.