

Fluidization Engineering
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Lecture – 22

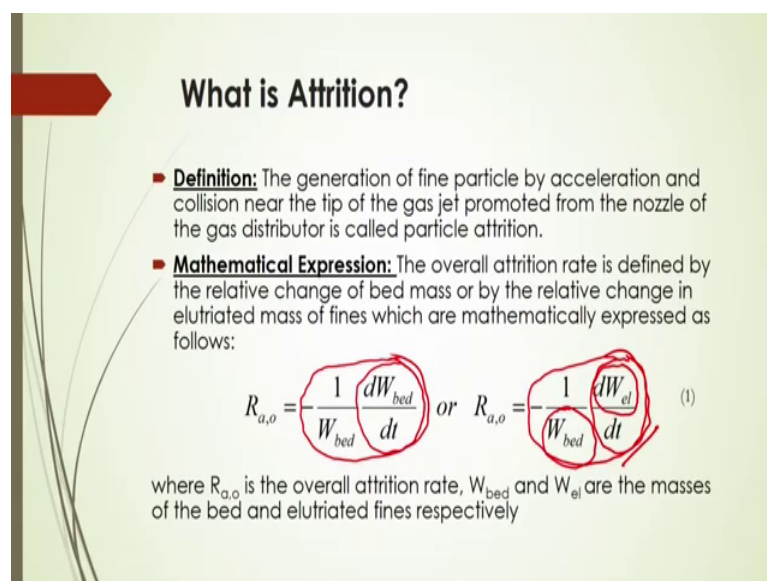
Entrainment Characteristics (Part 2): Attrition in Fluidized Bed (Part 1)

Welcome to a massive open online course on fluidization engineering. So, today's lecture will be on attrition in fluidized bed, I think, we have discussed about that particle elutriation and entrainment in a fluidized bed in the previous last two lectures and there we have also discussed the mechanism of entrainment and how these entrainment characteristics can be modeled and also elutriation, how it is affected by different particle size and also attrition mechanisms there.

So, in this lecture, we will be discussing about the attrition of the particles when those particles will be interacting during the operation of the fluidization. So, there may be due to this attrition, the particles will be broken up into a smaller one and then entrainment of this particles will be happening there and also this attrition may be depending on the particle characteristics even size distribution etcetera in the fluidized bed.

So, we will discuss about this here.

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What is Attrition?

- **Definition:** The generation of fine particle by acceleration and collision near the tip of the gas jet promoted from the nozzle of the gas distributor is called particle attrition.
- **Mathematical Expression:** The overall attrition rate is defined by the relative change of bed mass or by the relative change in elutriated mass of fines which are mathematically expressed as follows:

$$R_{a,o} = -\frac{1}{W_{bed}} \frac{dW_{bed}}{dt} \quad \text{or} \quad R_{a,o} = -\frac{1}{W_{bed}} \frac{dW_{el}}{dt} \quad (1)$$

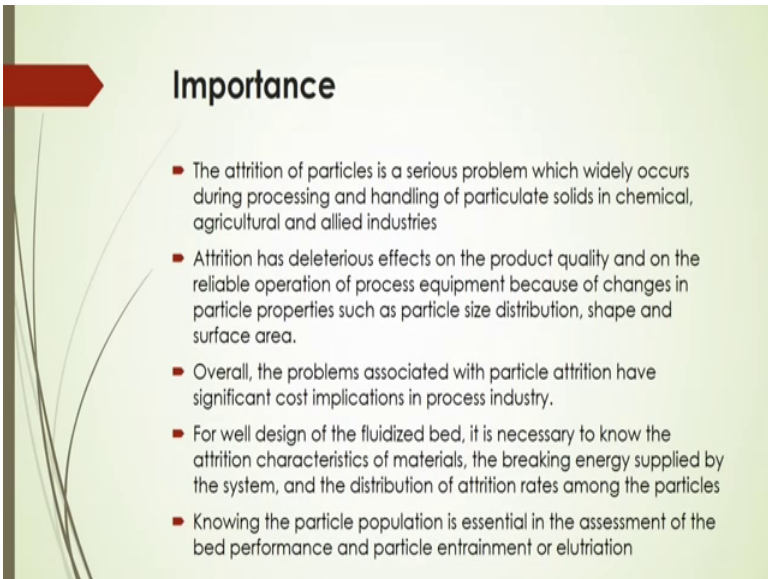
where $R_{a,o}$ is the overall attrition rate, W_{bed} and W_{el} are the masses of the bed and elutriated fines respectively

So, before going to details of this attrition, we should know, what is the definition of a attrition and also how this attrition can be expressed, mathematically, there you know that what is the attrition, basically, if you are going; if you are saying about that attrition, it will be just a generation of fine particle by acceleration and collision near the tip of the gas jet that is promoted from the nozzle of the gas distributor, this is called the particle attrition.

So, actually basically this is nothing, but the generation of fine particles by collision of the particles and this attrition rate that is the overall attrition rate is defined by the relative change of the bed mass or by the relative change in elutriated mass of fines which are mathematically expressed as follows here. Now here, this R_a is the overall attrition rate and W_{bed} and W_{el} or W_{el} , you can say that are the masses of a bed and elutriated fines respectively.

So, R_a is being represented by this here as $\frac{1}{W_{bed}} \frac{dW_{el}}{dt}$; that means, weight of fine beds, here in the fluidized bed per unit time or this can be represented by this elutriated fines rate of elutriated fines formation that is $\frac{dW_{el}}{dt}$, this is the weight of the bed of solids there and this dW_{el} means here, what will be the weight of elutriated fines? That is generated by the collision of a particles per unit time here. So, this will be your rate.

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Importance

- The attrition of particles is a serious problem which widely occurs during processing and handling of particulate solids in chemical, agricultural and allied industries
- Attrition has deleterious effects on the product quality and on the reliable operation of process equipment because of changes in particle properties such as particle size distribution, shape and surface area.
- Overall, the problems associated with particle attrition have significant cost implications in process industry.
- For well design of the fluidized bed, it is necessary to know the attrition characteristics of materials, the breaking energy supplied by the system, and the distribution of attrition rates among the particles
- Knowing the particle population is essential in the assessment of the bed performance and particle entrainment or elutriation

So, this in this way that the mathematical expression of the overall attrition rate can be given here and what is the importance to know the attrition mechanism and while attrition characteristics to be actually required to study for this the attrition of the particles, actually, it is a serious problem which widely occurs during the processing and handling of particles solids in a in chemical even agricultural or in allied industries.

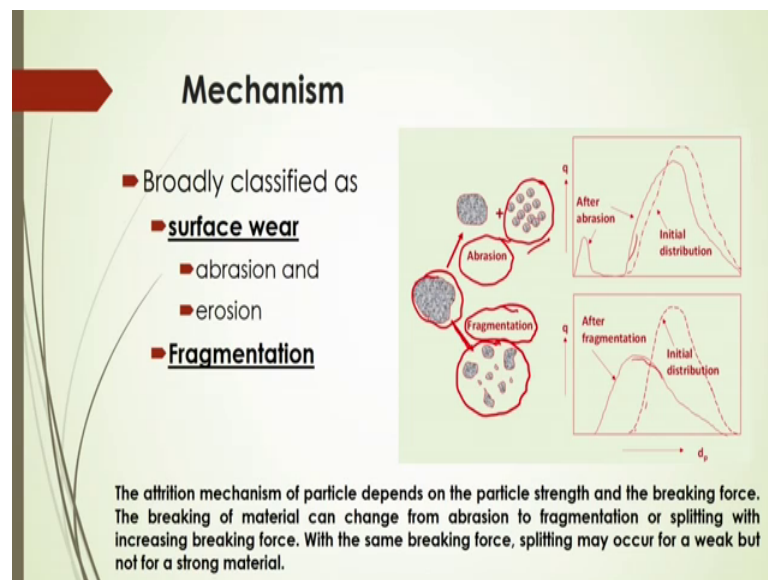
So, attrition has deleterious effect on the product quality and on the reliable operation of a a process equipment because of that some changes in particle or properties such as a like a particle size distribution shape and surface area which will govern the change of a mass transfer or heat transfer characteristics in the fluidized bed, even if is there any loss of the fines are during the operation of the fluid operation, then how much fines is formed by the attrition or by that elutriated fines, how they are coming out, what will be the mechanism for that of course, you should know because all those things to be taken care of whenever you are operating fluidized bed with some catalyst particles for the specific application.

Because some loss of particles may hinder the performance of the fluidized bed, even somewhere, you will see if some operations is well defined or well characterized by the fine particles, whereas, if you are having the coarser particles, then you have to know how much coarser particles will be there, how much fine particles will be there inside the bed, then based on which you we will have to actually judge that what will be the performance of the fluidized bed there because this is very important because what will be the amount of fine particles is coming out from the bed that is I think, we have discussed in the previous classes that what will be the exit entrainment rate there and this is coming because of that attrition not only attrition after attrition, there will be a some mechanism of that ejection of the fines by the bubbles whenever it will be dissolving the surface of the fluidized bed.

Ah now in this case of course, that is why you have to know some extent of the characteristics of the attrition inside the bed and overall the problems associated with the particle attrition have significant cost implication in implications in process industries, of course, and for well design of the fluidized bed, it is necessary to know the attrition characteristics of materials and the breaking energy supplied by the a system and the distribution of the attrition rates among the particles, also you have to know before going to design the fluidized bed.

And knowing the particle population also is essential in the judgment of the bed performance and the particle entrainment or elutriation inside the bed. So, we have to actually know something about that, attrition characteristics inside the bed because of this.

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Now, what should be the mechanism of this attrition, you will see this mechanism can broadly be classified as a one called surface wear another is called fragmentation.

Now, what is surface wear? Surfaces wear maybe two types abrasion and the erosion. So, attrition mechanism of particles that depends on the particle strength and the breaking force the breaking of the material can change from abrasion to fragmentation or you can say that the splitting with increasing breaking force and with the same breaking force, you will see that the splitting may occur for a weak, but not for a strong material there.

So, material property is very important here, whether this, the particles will be attriting attrition will be there or not because if the higher strength particles may not be very easy to broken up into a finer ones. So, there may be some when sometimes you know that in weight solids may not be easier to broken into finer particles there because of some abrasion or adhesive necessary nature of the particles inside the bed here know. So, in this case these two mechanisms are there; one is called abrasion, here see one particles just breaking into one and then another finer particles of most uniform in size whereas these particles may be broken into fragmented solids.

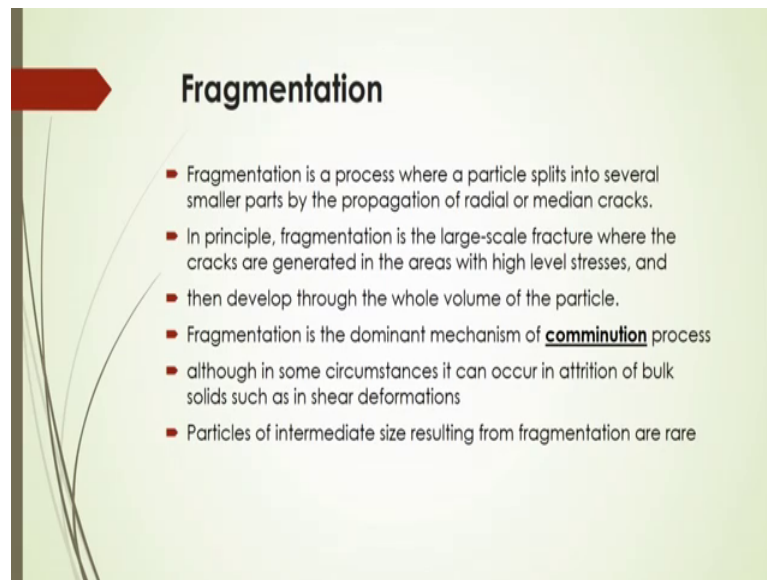
now the size of the solids may not be equal in size there. So, the distribution of the solids after attrition after attrition you will see two types of distribution will be having based on these two types of a mechanism of attrition there after abrasion, you will see initially or after abrasion this type of mechanism this type of distribution is coming here. So, initial distribution is like dotted line is represented by initial distribution of the solid particles inside the bed, whereas, after attrition, you will see the distribution, how it will be changing because of formation of fines there inside the bed.

Again for the fragmentation as per fragmentation here how sees this after attrition how it will be changing this you will see that the variation of this distribution will be changing because of the attrition; so, this is the distribution of the initial distribution means that you are putting the catalyst particles or some other particles may be inert particles that is not taking part in to the reactor of a reaction or any other physical operation that may be.

So, those mixture will be having some certain distribution. So, initially this distribution will be like this and after that you will see that after fragmentation it will be distributed with wide spreading like this type of whereas, in the case of abrasion there may not be that much of spreading of that particles after abrasion here. So, these mechanisms actually will be having that quantify the attrition extent of attrition based on this distribution function after attrition.

Now, this attrition depends on what basically whether this particle are in very strength or that is the depends on the properties of the particles whether particles are very strong or it may broken into fine particles a very easily with a very small force also, you can say and the breaking of the material can change from that abrasion to that fragmentation; that is two natures just like that because of this, there particle properties or it may split with the increasing breaking force there and with the same breaking force, you will see that splitting may occur for a week or not for a strong material there and what is that fragmentation.

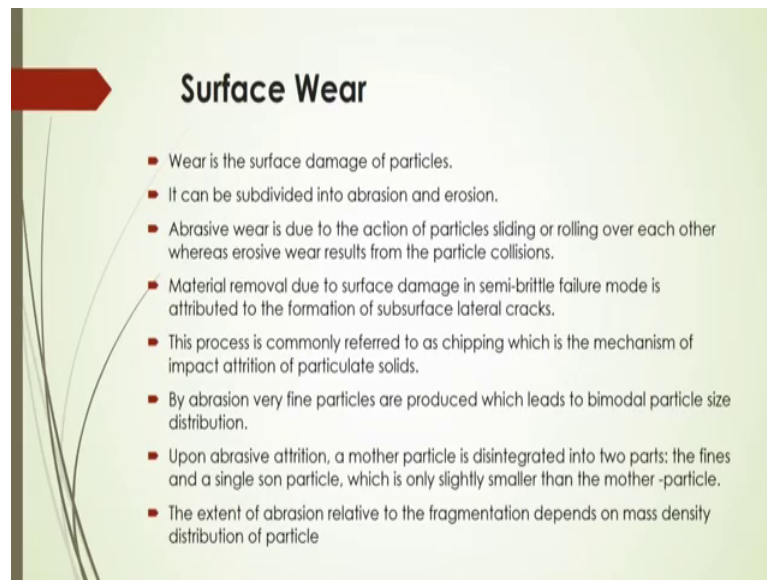
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Fragmentation is nothing, but a process where a particle splits into several smaller particles by the propagation of the a radial or a median cracks in principle. This plan a fragmentation is the large scale fracture whether the cracks are generated in the areas with high level of stresses and you can say the develop this will develop through the whole volume of the particle. Now fragmentation may be dominant mechanism of that contribution of the attrition process and in that case although in some circumstances, it can occur in attrition or bulk solids such as in shear deformations.

And particles of intermediate size may result from the fragmentation and that may be a very rare in case, but there may be possibility of that, particles of intermediate size resulting from that fragmentation. So, we can say that this fragmentation is nothing, but the splits into several smaller parts of that particle by the propagation of radial or median cracks and what is that and this is being developed to the whole volume of the particles here and this maybe the dominant in that nature compared to that abrasion mechanism.

(Refer Slide Time: 14:17)



Surface Wear

- Wear is the surface damage of particles.
- It can be subdivided into abrasion and erosion.
- Abrasive wear is due to the action of particles sliding or rolling over each other whereas erosive wear results from the particle collisions.
- Material removal due to surface damage in semi-brittle failure mode is attributed to the formation of subsurface lateral cracks.
- This process is commonly referred to as chipping which is the mechanism of impact attrition of particulate solids.
- By abrasion very fine particles are produced which leads to bimodal particle size distribution.
- Upon abrasive attrition, a mother particle is disintegrated into two parts; the fines and a single son particle, which is only slightly smaller than the mother -particle.
- The extent of abrasion relative to the fragmentation depends on mass density distribution of particle

And what is the surface wear? Surface wear is the surface damage of particles here a surface is being damaged; it can be subdivided into abrasion and erosion. Now abrasive wear is due to the action of a particles that is being slides or you can say roll over each other, whereas, whereas, whereas, erosive wear that may results from the particle collision. So, we can say that this abrasive wear is nothing, but the action of particles sliding or rolling over to each other and whereas, erosive wear this is two types of wear surface wear are there.

So, erosive wear results from the particle collisions and maybe, you will see that the material removal will be happens due to the surface damage in semi brittle failure mode and which is attributed to the formation of substance in lateral cracks and this process is commonly referred to as the chipping which is the actually mechanism of impact attrition of the particulate solids and by abrasion very fine particles that are produced needs to by model particle size distribution there and because of that abrasive attrition a a mother particle or you can say that these initial particle is disintegrated into two parts.

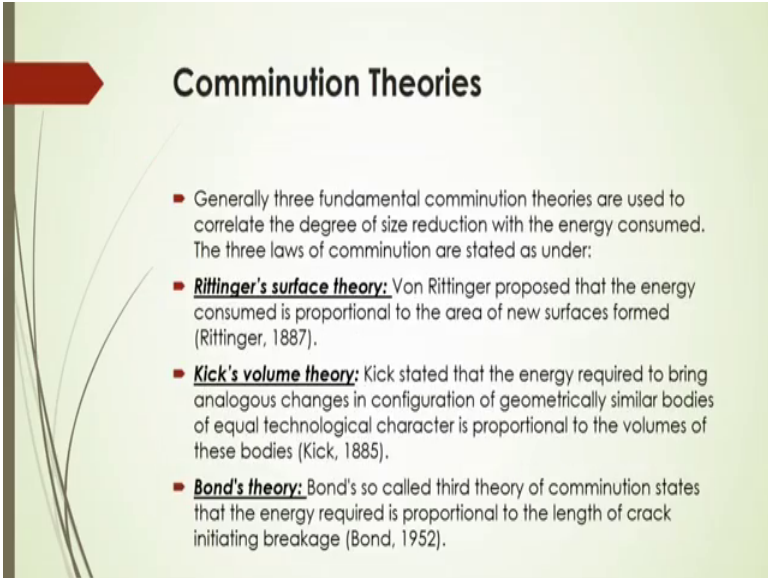
And one is the fines and another is single strong particles, there which is only slightly smaller than the mother particles and the extent of this abrasion that is related to the fragmentation depends on the density mass density, you can say mass density distribution of the particles. So, so you can you can observe or you can identify, what should be the

extent of abrasion related to the fragmentation based on the mass size distribution of the particles there?

So, we are having that two types of mechanism here. One is fragmentation, another is surface wear, where a surface wear have two types, there is abrasive wear, another is called erosive wear, whereas, the abrasive wear is the resulting from the action of particles by sliding or rolling over and a erosive wear that is results from the particle collision even fragmentation also, it is being formed by the particle collision also, but they are the formation that is the size distribution will be different there and this process generally sometimes referred to as shipping of the material simple and in general cases, you observed that the by this surface wear mechanism whenever a smaller fine particles will be formed the distribution of that doctor particles will be bimodal in size.

So, we have to know that ok, if it is that size distribution like this for this surface wear and such distribution from this fragmentation what should be the difference from which you can say you can identify what extent of actually a attrition happens inside the fluidized bed.

(Refer Slide Time: 18:08)



Comminution Theories

- Generally three fundamental comminution theories are used to correlate the degree of size reduction with the energy consumed. The three laws of comminution are stated as under:
- **Rittinger's surface theory:** Von Rittinger proposed that the energy consumed is proportional to the area of new surfaces formed (Rittinger, 1887).
- **Kick's volume theory:** Kick stated that the energy required to bring analogous changes in configuration of geometrically similar bodies of equal technological character is proportional to the volumes of these bodies (Kick, 1885).
- **Bond's theory:** Bond's so called third theory of comminution states that the energy required is proportional to the length of crack initiating breakage (Bond, 1952).

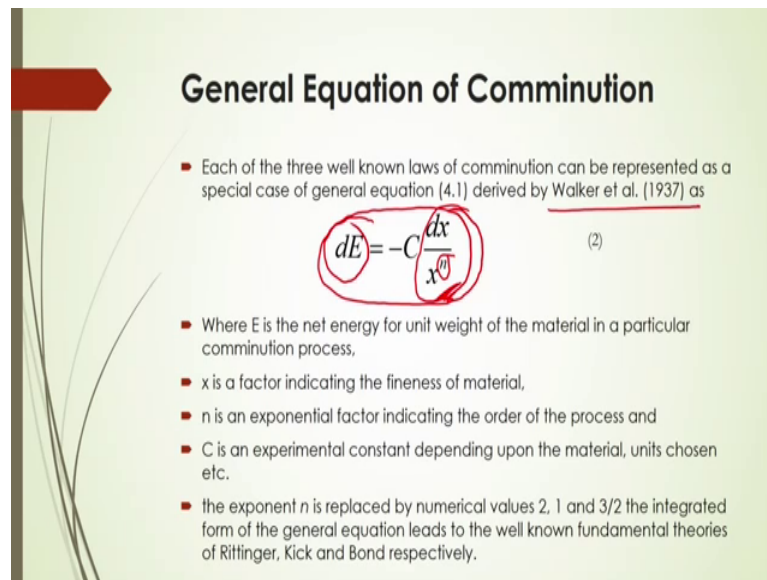
Now, before going to the actual mechanism, how this happens, you have to know some extent of that theories on that; what is the mechanism of; what is the theory behind it that particle is collision and then it will form the smaller particles.

Generally three fundamental you will see; that comminution comminution theories that are used to correlate the degree of size reduction with the energy consumed that. So, the formation of the fine particle that depends on the how much energy is consumed for this the splitting of the particles or breaking of the particles into a finer one. So, this degree of that size reduction, you can say that formation of fine particles that depends on the energy consumption, there are generally three general formula that is given by different investigator like Rittinger's and a Kick and then Bond there are three fundamentals theory by which you can explain how this actually these comminution of the particles is happened inside the bed.

So, Rittinger's surface theory as per that that the energy consumed is that is proportional to the area of that new surface formed during these attrition process and as per Kick's volume theory that the energy require to bring this analogous changes in the configuration of a geometrically similar bodies of equal you can say technological character is proportional to the volume of these bodies here and as per Bond's theory the that you can say that the energy required for this breaking up into finer ones is proportional to the length of the crack that is initiating for this breakage to form or to result the fine particles there.

So, that the, these three theories, all those theories actually basically based on the energy consumption now after breaking what will be the size, what will be the volume of the bodies that is formed and also, what should be the length of the crack that depends there based on which that this three separate or different theories are given or explained or proposed by these a investigators.

(Refer Slide Time: 20:47)



General Equation of Comminution

- Each of the three well known laws of comminution can be represented as a special case of general equation (4.1) derived by Walker et al. (1937) as

$$dE = -C \frac{dx}{x^n} \quad (2)$$

- Where E is the net energy for unit weight of the material in a particular comminution process,
- x is a factor indicating the fineness of material,
- n is an exponential factor indicating the order of the process and
- C is an experimental constant depending upon the material, units chosen etc.
- the exponent n is replaced by numerical values 2, 1 and 3/2 the integrated form of the general equation leads to the well known fundamental theories of Rittinger, Kick and Bond respectively.

Now, general equation of a comminution that that you can say each of the three well known theory of the comminution can be represented in the special case of general equation like derived by Walker et al 1937 here.

So, they have explained these theories the and the like this de; that means, here a net energy change that will be is equal to proportional to the what is that d x by x to the power n here. So, where E is called the net energy per unit weight of the material in a particular comminution process and x is a factor here that indicates the fitness, sorry, fineness of the material. Here fineness that is what extent of fine particles are being formed inside the bed and here exponent that is called n, n is the factor indicating the order of the process there and C is an constant that can be obtained by the experimental which depends on the material properties and also other physical operation operating conditions.

And the exponent n is a is or you can replace this exponent n by numerical values of two one and three by two the integrated form of the general equation that is leads to the well known fundamental theories of Rittinger's kick and born respectively. So, if you replace this n by two that will be actually as per the equation of Rittinger's or theory of Rittinger's and if you substitute this value of one then it will be simple kicks theory and if you substitute n is equal to 3 by 2. It may be expressed or it can be referred to that bond theory.

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Net energy to bring about size reduction

- For all practical purposes measurement of net energy to bring about size reduction of known feed size to desired product size is of importance. The corresponding net energies (E) per one ton (kWh/t) required for feed x_1 to product x_2 are

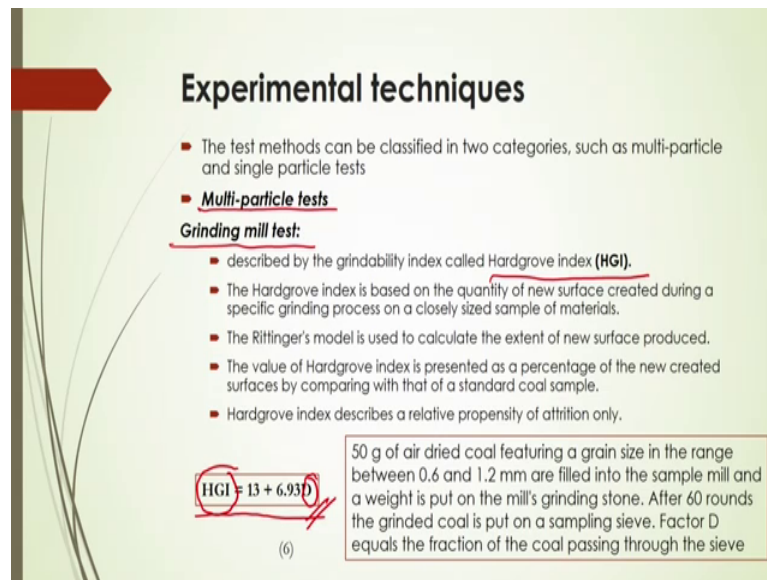
$E = C(1/x_2 - 1/x_1)$	(Rittinger's theory)	(3)
$E = -\ln(x_2/x_1)$	(Kick's theory)	(4)
$E = 2C(1/\sqrt{x_2} - 1/\sqrt{x_1})$	(Bond's theory)	(5)

Now, net energy to bring about the size reduction what should be the net energy that is to reduce the size of the particle inside the bed. So, for all practical purposes, you can say this measurement of the net energy to bring about this reduction of this sides of the particles to the desired product size that is very crucial to know, here the corresponding net energies per unit of or you can say per unit ton required for the feed x_1 to the product x_2 here.

So, if you are expressing that product size is x_1 here and a product size is the x_2 and the initial size is x_1 , then x_1 to x_2 to reduce this x_1 to x_2 , what should be the amount of energy required per 1 ton that can be expressed here as per Rittinger's theory, it will be is equal to E is equal to C into $1/x_2$ minus $1/x_1$ as per Kick's theory, you can express this energy consumption as E will be is equal to minus \ln that is logarithm of that is ratio of product final product to the initial size.

Now, here as per bonds theory you can say that this energy consumption will be is equal to two times of C into $1/\sqrt{x_2}$ minus $1/\sqrt{x_1}$; so, this three equation, you can use how much energy will be consumed if you know that initial and final size of the particles.

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Experimental techniques

- The test methods can be classified in two categories, such as multi-particle and single particle tests
- Multi-particle tests
- Grinding mill test:
 - described by the grindability index called Hardgrove index (HGI).
 - The Hardgrove index is based on the quantity of new surface created during a specific grinding process on a closely sized sample of materials.
 - The Rittinger's model is used to calculate the extent of new surface produced.
 - The value of Hardgrove index is presented as a percentage of the new created surfaces by comparing with that of a standard coal sample.
 - Hardgrove index describes a relative propensity of attrition only.

$HGI = 13 + 6.93D$

(6)

50 g of air dried coal featuring a grain size in the range between 0.6 and 1.2 mm are filled into the sample mill and a weight is put on the mill's grinding stone. After 60 rounds the grinded coal is put on a sampling sieve. Factor D equals the fraction of the coal passing through the sieve

Now, experimental how to obtain this extent of this attrition here; so, the test methods can be classified into categories such as multi particle and then single particle tests here.

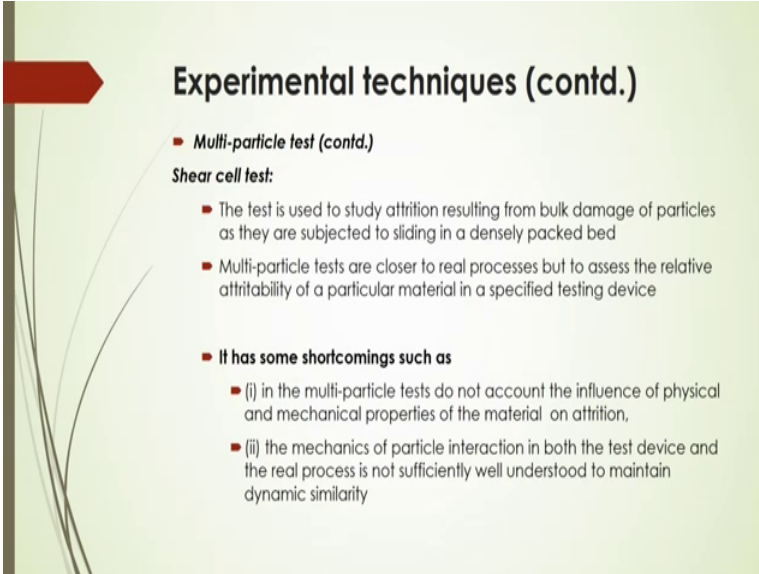
If you are using different types of particles may be coal and with sand mixture or may be similar type of particles, there collision will be there, may be two types of particles. So, that is why multi particle or the single particle will be there. So, if you are doing the test with multi particle, then of course, you will get different types of actually different characteristics there. So, in case of multi particle test; so, there we will see, it is called grinding mill test here. So, it is generally described by the grind ability index which sometimes it is called Hardgrove index, there HGI, this HGI is based on the a quantity of new surface. So, that is created during the during a specific grinding process that on a on a closely sized sample of materials there.

The Rittinger's model generally is used to calculate the extent of this new surface produced the value of this Hardgrove index is presented as a percentage of new that is generated surface by comparing with that of a standard coal sample there and Hardgrove index describes a relative propensity of the attrition only there. So, Hardgrove index can be represented by this here this is HGI that will be is equal to $13 + 6.93D$; that means, 13 plus 6.93 into D and this ah; that means, here it is the factor here 50 gram of air, if you dried coal featuring a grain size in the range between 0.6 and 1.2 millimeter or filled into the

sample mill and a weight is put on the mills grinding stone after 60 rounds, the grinded coal is put on a sampling sieve.

Factor D equals the fraction of the coal that is passing through the sieve. So, if you are able to after grinding, if you pass it to the sieve, then what should be the fraction of the coal that is passing through the sieve, if you represented by D, then Hardgrove index will be is equal to 13 plus 6.93 into this fraction of coal passing through the sieve here ah.

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Experimental techniques (contd.)

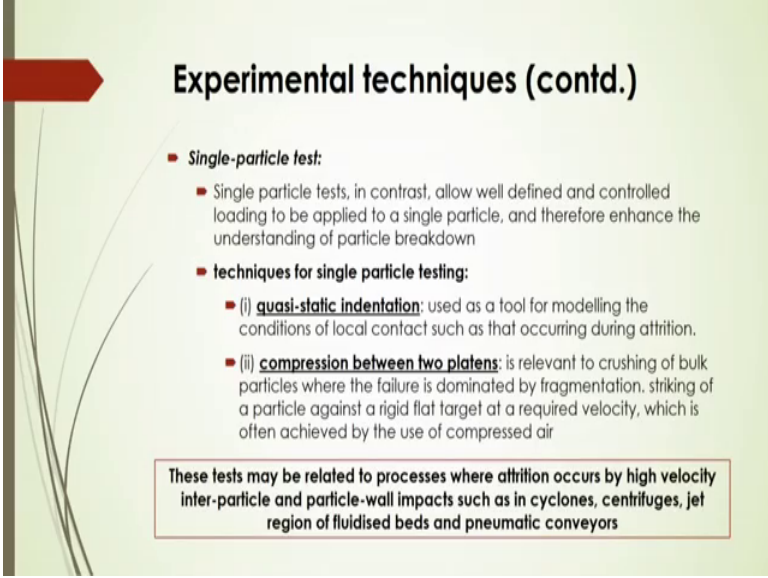
- **Multi-particle test (contd.)**
 - Shear cell test:**
 - The test is used to study attrition resulting from bulk damage of particles as they are subjected to sliding in a densely packed bed
 - Multi-particle tests are closer to real processes but to assess the relative attritability of a particular material in a specified testing device
 - **It has some shortcomings such as**
 - (i) in the multi-particle tests do not account the influence of physical and mechanical properties of the material on attrition,
 - (ii) the mechanics of particle interaction in both the test device and the real process is not sufficiently well understood to maintain dynamic similarity

As per shear cell test, this test is used to study the attrition resulting from bulk change of particles as they are subjected to sliding in a closely packed bed there and multi particle tests are closer to the real process, but to assess the assess the a relative relative you can say attri tability of a particular material in a specified testing device.

It has some disadvantage such as in the multi particle test that do not account the influence of a physical phenomena and the a mechanical properties of the material that are being attrition that are being used for attrition the mechanics of particle interaction in both the test, actually that devices and the real process is not sufficiently well understood to maintain the dynamic similarity inside the test devices.

So, generally, you can you can do that the shear cell test ah, but you may not be getting the exact results because of this some shortcomings of the a device that you are being used and experimental techniques for the single particle test you will see that.

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Experimental techniques (contd.)

- **Single-particle test:**
 - Single particle tests, in contrast, allow well defined and controlled loading to be applied to a single particle, and therefore enhance the understanding of particle breakdown
- **techniques for single particle testing:**
 - (i) **quasi-static indentation:** used as a tool for modelling the conditions of local contact such as that occurring during attrition.
 - (ii) **compression between two platens:** is relevant to crushing of bulk particles where the failure is dominated by fragmentation. striking of a particle against a rigid flat target at a required velocity, which is often achieved by the use of compressed air

These tests may be related to processes where attrition occurs by high velocity inter-particle and particle-wall impacts such as in cyclones, centrifuges, jet region of fluidised beds and pneumatic conveyors

Single particle test in contrast, you can say allow well defined and controlled loading to be applied to a single particle there and therefore, you can say that entrance ah or you can say that the understanding of the particle breakdown is very easier by this single particle test.

And a techniques for single particle testing here there are several techniques of course, mainly two techniques are being used for this testing. One is quasi static indentation techniques that is used as a tool for modeling the conditions of a local that is contact such as that occurring during the attrition and compression between 2 platens that is relevant to the crashing of bulk particles where the where the failure that is dominated by fragmentation and you can say striking of the particles that against a rigid flag target at a required velocity in that case and mean be often assumed by the use of compressed air.

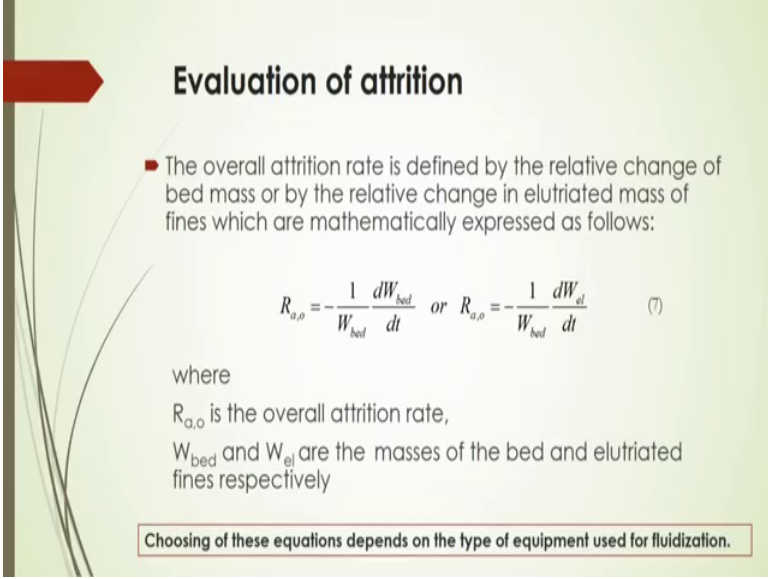
So, these two techniques are being used for single particle test here because of that here in this case, the a local contract such that during that attrition, we will be actually obtained by some mathematical simulation and also, you can say that sometimes bulk particles that are failure which will be dominated by the fragmentation and then you can obtain by that compression of the two platens by this compressing air.

So, this tests may be related to the process where attrition occurs by the velocity and inter particle and particle all impacts such as in a cyclone or centrifugal device and then the jet region of the fluidized bed and geometric converge, you will see that attrition base

basically it is predominant in the region of that distributor where the gas jet is coming from the distributor from the hole distributor hole they are you will see that the velocity of the jet will be very higher and if your hole size is very low and gas velocity so high, in that case, you will get the high a jet and because of that higher jet the particles particle are interaction and all the collision between the particles would be higher and because of which there will be the attrition.

So, this attrition actually depends on that energy related and also there will be some wall effect, of course, if there is a particles of that is smooth surface, then attrition will be less compared to that if surface of the particles would be that is not in smooth that is rough surface, then attrition will be more there because the attraction sorry because of that collision and also that inter particle that is the hitting of the surface will be more intensive than the regular surface.

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Evaluation of attrition

- The overall attrition rate is defined by the relative change of bed mass or by the relative change in elutriated mass of fines which are mathematically expressed as follows:

$$R_{a,o} = -\frac{1}{W_{bed}} \frac{dW_{bed}}{dt} \quad \text{or} \quad R_{a,o} = -\frac{1}{W_{bed}} \frac{dW_{el}}{dt} \quad (7)$$

where
 $R_{a,o}$ is the overall attrition rate,
 W_{bed} and W_{el} are the masses of the bed and elutriated fines respectively

Choosing of these equations depends on the type of equipment used for fluidization.

Now, you have to evaluate the attrition after this testing and you can say that attrition after a attrition process, you have to evaluate; what would be the actual overall attrition rate in the process. So, the overall attrition rate is defined by the relative change of the mass or by the relative change in elutriated mass of fines which are mathematically expressed as follows here. So, this already we have given earlier also this $R_{a,o}$ which the rate overall attrition rate and that will be is equal to $-\frac{1}{W_{bed}} \frac{dW_{bed}}{dt}$.

So, or you by this equation you can this. So, R_{a0} is the overall attrition rate and W_{bed} and W_{el} are the masses of the bed and the elutriated fines that is being found after attrition. Now choosing of this equation, that depends on the type of equipment also that are being used for the fluidization.

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Evaluation of attrition

- As per **Gwyn (1969)** the degradation of catalysts in a small scale test apparatus as the elutriated particles can be considered as the attrition product.
- The increase of the elutriated mass, W_{el} with time by an equation based on the initial solid bed mass, $W_{bed,0}$ which is known as "Gwyn equation" (**Gwyn, 1969**)

$$\frac{W_{el}}{W_{bed,0}} = K_a t^b \quad (8)$$

- the exponent b is constant whereas,
- the attrition constant K_a decreases with mean particle size.
- The variable t is called the shearing time of the particle

So, as per Gwyn 1969 that the degradation of the catalyst in a small scale tests apparatus, they have done as they are actually observation the elutriated particles can be considered as the attrition product and the increase of the elutriated mass that is W_{el} with time by an equation based on the initial solid based mass that is $W_{bed,0}$ which is known as Gwyn equation here.

So, in equation this as per that as per his actually suggestion or proposal that the attrition rate can be expressed in that way, this W_{el} by $W_{bed,0}$ that will be is equal to k into t to the power b here now the variable t here is called the shearing time of the particles how long this particles will take part for the shearing with other particles there and the attrition constant. It is K_a actually is the constant that will be obtained by experimentally and this decreases with the mean particle size also.

If the mean particle sizes are smaller then you will see that this k value also will be decreased. So, in that case the exponent b is constant here in this case. So, actually basically this K value and this b will be obtained from the experimental a results.

(Refer Slide Time: 35:54)

Ouwerkerk (1991) and Kenter (1992) Model

- They suggested that the attrition rate varied with the applied normal stress and the shear strain in a way that could be expressed by the following formula

$$W_{el} = K_o \left[\Gamma \left(\frac{\sigma}{\sigma_{ref}} \right)^{\alpha} \right] \quad (9)$$

- where K_o and α are empirical fitting constants;
- σ_{ref} is a reference normal stress at which the amount of attrition product is the same for all types of material to be tested
- The effective shear strain is defined as

$$\Gamma = \frac{\theta}{360} \frac{2\pi \bar{R}}{h} f \quad (10)$$

where

θ = the rotational angle (in degree),

h is the effective sample layer thickness,

R is the mean radius of the shear cell and

f is the slip factor which is equal to 0.86

And Ouwerkerk 1991 and Kenter 1992 model; as per their model, they suggested that the attrition rate; that will be vary with the applied normal stress and the shear strain in a way that could be expressed by the following formula here.

Here W_{el} is nothing, but the elutriated mass of the fines that is formed by attrition and K is the constants α is also a constants here, this Γ is a one factor that is called a shear strain this is the effective shear strain. So, this will be as defined by this θ by 360 into $2\pi \bar{R}$ by h into f where θ is the rotational angle in that device of that a test in which this attrition of in which this attrition test is being done and h is the effective sample layer thickness also and R is the mean radius of the shear cell in which this test is being done and f is the slip factor which is equal to 0.86.

So, after substitution of this value here during the experiment, you will get this shear strain, once you know that the shear strain and then what is the shear this normal stress the σ , a σ here reference is called a reference normal stress at which the amount of attrition product is the same for all types of material that is being to be tested and here α and K a to be experimentally obtained after substituting those parameters here.

(Refer Slide Time: 37:52)

Ghadiri et al. (2000) Model

- Neil and Bridgwater (1994) have recently proposed that the rate of attrition should be related to σ/γ^ϕ , where ϕ is an exponent, which only depends on the material type.
- Based on this concept, Ghadiri et al. (2000) developed a correlation by fitting his experimental data as

$$W_{el} = 3.858 \left[\sigma/\gamma^{0.5} \right]^{1.053} \quad (11)$$

In this equation, Ghadiri et al 2002, they have actually proposed a another model actually based on this proposal of Neil and Bridgwater 1994.

So, Neil and Bridgwater 1994, they have proposed that the rate of attrition should be related to the sigma into gamma to the power 5 where 5 is an exponent which only depends on the material type. So, this one and based on this concept. So, Ghadiri 2; Gghadiri et al 2000s, they developed a correlation by fitting their experimental data like this here W_{el} that will be the 3.858 into sigma gamma to the power 0.5 to the power 1.053.

So, this exponents this come constraints they got it from their experimental results. So, here this five is nothing, but that here 0.5 here, whereas, this other exponents that this proportionality constants will be is equal to 0.3; 0.858. So, basically from this correlation you also be able to calculate what should be the elutriable mass of fines that are formed after a attrition process.

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Zenz and Kelleher (1980) Model

- In fluidized bed experiment, different investigators assessed the attrition rate in different ways based on the examination of a group of particles. For FCC catalyst and glass beads, **Zenz and Kelleher (1980)** developed a correlation for attrition rate without shroud for upwardly directed jet which is given by

$$\phi' = K_a \left(u_{or} \sqrt{\rho_{g,or}} \right)^{2.5} \quad (12)$$

- Where ϕ' is the flux of attrition per unit hole (flux is based on cross sectional area of the nozzle), (kg/min.hole per unit cross section of the nozzle),
- K_a is the attrition rate constant depends on particle diameter

d_p (μm)	K_a
0-2	1.11×10^{-6}
0-23	9.03×10^{-6}
0-50	22.29×10^{-6}

for FCC catalyst and glass beads

Once you know this normal stress and the shear strain during the process, Zenz and Kelleher, 1980, they also proposed some correlation to interpret the elutriation sorry attrition rate inside the fluidized bed. So, based on their fluidized bed experiment different investigators also assessed the attrition rate in different ways based on the examination of a group of particles for FCC catalyst and glass beads Zenz and Kelleher developed a correlation for the attrition rate without using in his shroud for upwardly directed jet which is coming out from the distributor hole and based on their experimental data or their concept, they have proposed this correlation that is 5 dashed that is called flux of attrition per unit hole through which this gas is coming out and K_a into u or 2 to the power root over g or 2.5 .

Here K_a is the attrition rate constant that depends on the particle diameter u or is nothing, but the velocity of the gas which is coming out from the orifice and this ρ_g or means here density of the gas or fluid at that orifice location because due to that higher a jet. And there will be a some change of temperature because of this collision or because of the a friction of these two particles and there will be change of temperature instantly and then there will be change of density of the gas.

So, at that orifice hole, what should be the temperature and based on which what should be the density of the gas there. So, that depends on this attrition rate that it attrition rate depends on that; that means, density of the gas here. So, here and this attrition rate per

unit hole that is flux is based on the cross sectional area of the nozzle, you can also define that way. So, hole that is kg per minute per a whole per unit cross section of the nozzle.

So, in that way you can calculate. So, this K a that is the attrition rate constant that depends on the particle diameter you will see that 0 to 2 micrometer particle diameter these attrition rate constant will be is equal to 1.11 into 10 to the power minus 6 as per their experimental data and 0 to 23 micrometer diameter their attrition test gives their 9.03 into 10 to the power minus 6 and for 0 to 50 diameter, this attrition rate will be 2 into 2.29 into 10 to the power minus 6.

So, in 10 to the power minus 6 range, here the rate constants will be obtained. So, this is the typical values they have obtained from their experimental data and so, after the substitute in this K value and if you know that or if this velocity and the density of the gas then you can easily say what should be the flux of the attrition rate flux of the attrition per unit hole that can be obtained from this equation 12.

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Extent of attrition

- The extent of attrition is defined as the time integral of the attrition rate which is expressed as

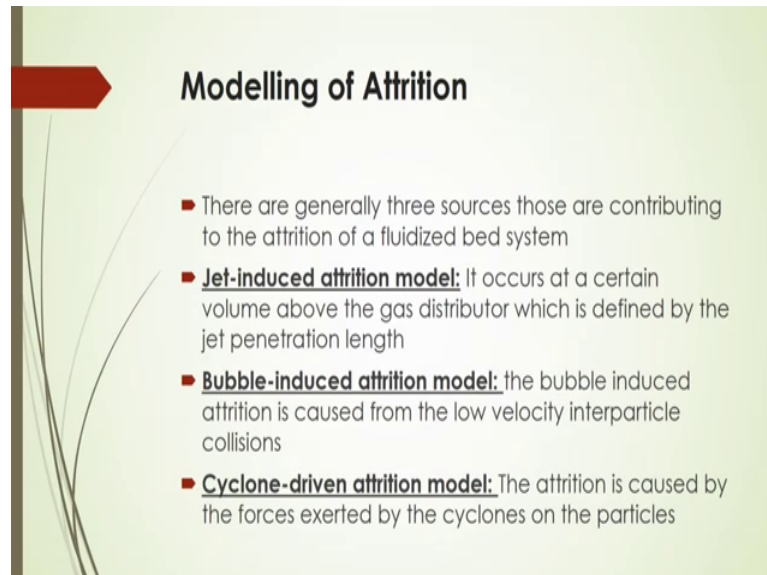
$$R_{a,ex}(t) = \int_0^t R_{a,o}(t) dt \quad (13)$$

- The extent of the attrition depends on the initial breakage and initial fines content in the bed

Now, what is the extent of attrition extent of attrition means the as the time integral of the attrition rate which is expressed as this R a ex for extent which is the that is depending on time. So, as a function of time it will be 0 to 1 into R at dt.

The extent of attrition depends on the initial breakage and initial fines contained in the bed modelling of attrition.

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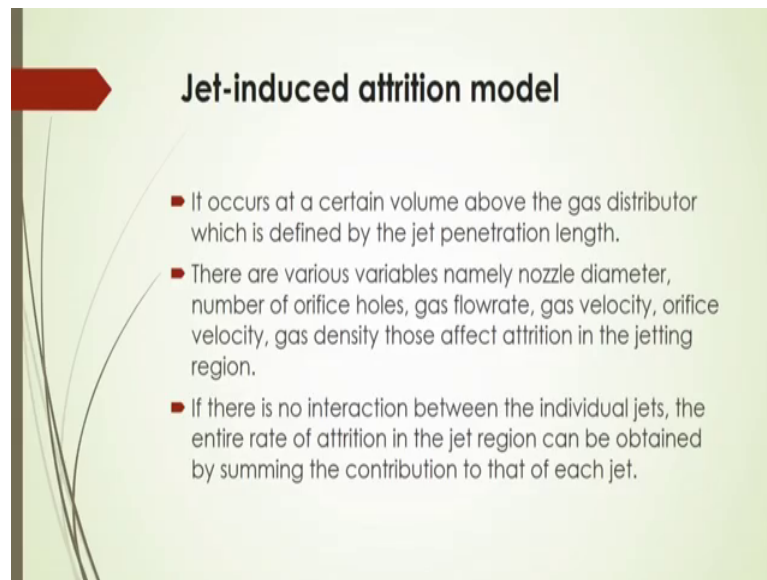
Modelling of Attrition

- There are generally three sources those are contributing to the attrition of a fluidized bed system
- **Jet-induced attrition model:** It occurs at a certain volume above the gas distributor which is defined by the jet penetration length
- **Bubble-induced attrition model:** the bubble induced attrition is caused from the low velocity interparticle collisions
- **Cyclone-driven attrition model:** The attrition is caused by the forces exerted by the cyclones on the particles

We have to do some modeling, how you have to develop some model in such way that you can predict what predict the attrition rate inside the bed there are generally 3 sources, those are contributing to the attrition of the fluidized bed system, one is jet induced attrition model another is bubble induced attrition model and then cyclone driven attrition model jet induced jet induced attrition model occurs generally at a certain volume above the gas distributor which is defined by the jet penetration length.

And bubble induced attrition model generally caused from the low velocity inter particle collisions and a cyclone driven attrition model is generally expressed by the forces that is exerted by the cyclone on the particles and jet induced attrition model occurs.

(Refer Slide Time: 44:47)



Jet-induced attrition model

- It occurs at a certain volume above the gas distributor which is defined by the jet penetration length.
- There are various variables namely nozzle diameter, number of orifice holes, gas flowrate, gas velocity, orifice velocity, gas density those affect attrition in the jetting region.
- If there is no interaction between the individual jets, the entire rate of attrition in the jet region can be obtained by summing the contribution to that of each jet.

That a certain volume above the gas distributor which is defined by the jet penetration link and there are various variables of course, namely nozzle diameter number of orifice holes, even gas flow rate and then gas velocity orifice velocity gas density and all other parameters, sometimes, you will see that even size of the particles also the affect the attrition in the jetting region.

If there is no interaction between the individual jets that is coming out from the whole of the distributor of the entire rate of attrition in the jet region can be obtained by summing the contribution to that of each jet, if suppose there are 3 jets that is used for the fluidization to distribute the gas through the jet and through this hole as a jet. So, if individual jet is giving their individual contribution for the attrition rate then you have to summing up if there is no actual interaction among the a jet.

If there is a there is a there is a you can say that the interaction between the individual jets, then there will be attrition rate maybe will be changed because of that the collision rate and also the efficiency of the breakup of the particles, there in the distribution region and also sometimes you will see if the interaction of the jet is there the energy which is coming out from that hole as a gas jet they are ah. So, it may be sometimes change that energy distribution in the fluidized region that is distributor region and because of which the attrition rate will be changing.

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Jet-induced attrition model (contd.)

- The overall attrition rate (kg/s) of the region above distributor can then be defined by

$$R_{a,distr} = N_{or} R_{a,j} \quad (14)$$
$$R_{a,j} \propto \rho_{g,or} d_{or}^2 u_{or}^n \quad (15)$$

- The proportionality factor depends on the properties of the solid material
- The attrition rate per jet is proportional to the square of the orifice diameter
- Jet velocity is one of the decisive factors for jet-induced attrition. The extent of the attrition depends on the jet gas velocity raised to power of n , where n varies between 1 and 5.1.

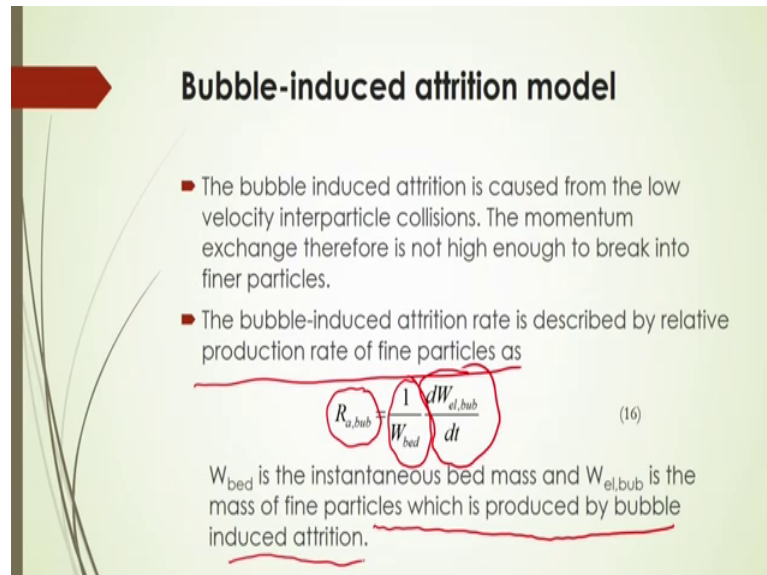
Now, the overall attrition rate you cannot of course, obtain what that is there any interaction or not it is very difficult to measure that whether interaction of the jet or not and it is not possible to measure also of that particular dynamic condition. So, overall attrition rate; we obtain after just getting what should be the fine formation is there. So, the overall attrition rate of the region above the distribute distributor can be defined by this $R_{a,distr}$ that would be equal to N_{or} into $R_{a,j}$ here N_{or} is the number of orifice whole in the distributor and $R_{a,j}$ is the rate of attrition for this jet.

This $R_{a,j}$ is proportional to this gas density and the square of the what is there or you can say that cross sectional area of the orifice hole and also what should be the flowrate of the gas through the orifice hole and this proportionality factor depends on the properties of the solid material the attrition rate per jet, this $R_{a,j}$ is proportional to the square of the orifice diameter and a jet velocity is one of the decisive factor, you can say for the jet induced attrition the extent of the attrition that depends on the jet gas velocity that is raised to power of n where n varies between 1 and a 5.1.

So, in this way, you can calculate or predict or interpret, what should be the overall attrition rate of the region above the distributor there only thing that you have to know, what will be the number of holes is there and also $R_{a,j}$, this $R_{a,j}$, you can get this what would be the fine formation and then at the different operating condition and after fitting this data with the models here, then you will get the proportionality constant, once you

know that proportionality constant, you can say, what should be the overall attrition rate of the region above the distributor.

(Refer Slide Time: 49:31)



Bubble-induced attrition model

- The bubble induced attrition is caused from the low velocity interparticle collisions. The momentum exchange therefore is not high enough to break into finer particles.
- The bubble-induced attrition rate is described by relative production rate of fine particles as

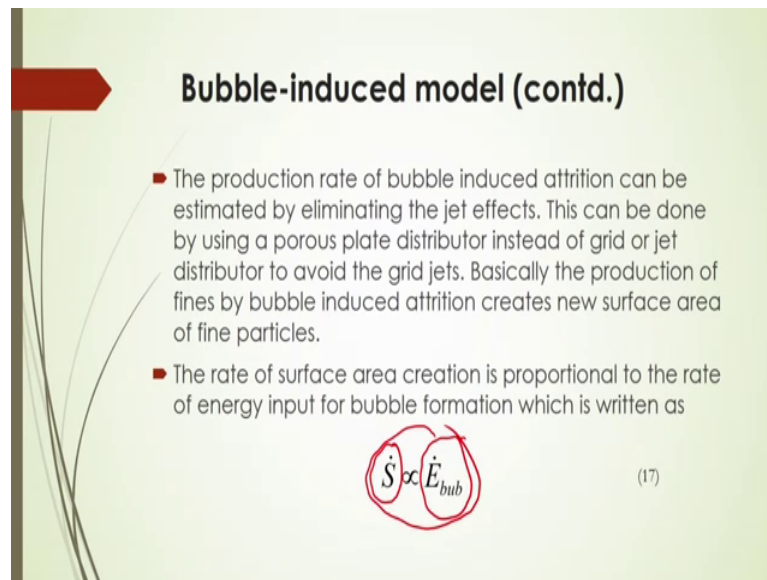
$$R_{a,bub} = \frac{1}{W_{bed}} \frac{dW_{el,bub}}{dt} \quad (16)$$

W_{bed} is the instantaneous bed mass and $W_{el,bub}$ is the mass of fine particles which is produced by bubble induced attrition.

Now, as per bubble induced attrition model, you can say, this bubble induced our attrition model basically happened because of the low velocity inter particle collisions. So, the momentum exchanges, therefore, is not that much high to break into the finer particles and this bubble induced attrition rate is described by the relative production rate of the fine particles here. So, this $R_{a,bub}$ means here bubble induced attrition that will one by W_{bed} into a rate of change of this elutriation due to this bubble ejection of the solid particles in the bed.

Then W_{bed} is the instantaneous bed mass and $W_{el,bub}$ is the mass of fine particles which is produced by the bubble induced attrition.

(Refer Slide Time: 50:23)



Bubble-induced model (contd.)

- The production rate of bubble induced attrition can be estimated by eliminating the jet effects. This can be done by using a porous plate distributor instead of grid or jet distributor to avoid the grid jets. Basically the production of fines by bubble induced attrition creates new surface area of fine particles.
- The rate of surface area creation is proportional to the rate of energy input for bubble formation which is written as

$\dot{S} \propto \dot{E}_{bub}$ (17)

And the production rate of the bubble induced attrition can be actually obtained by eliminating the jet effects of course, and this can be obtained by using the porous play distributor for a distributor of a instead of grid or jet distributor to avoid the grid, grid jets, basically, the a production of the fines by bubble induced attrition that creates new surface area of fine particles.

So, in this case, you can say, what should be the amount of new surface area are formed when fines are formed there during this attrition process and then from this a surface area of the rate of surface area that is generated which would be proportional to the rate of energy input for the bubble formation which is written as this S dot, this here S dot is nothing, but the rate of surface generation which would be directly related to the energy consumed during this bubble induced attrition.

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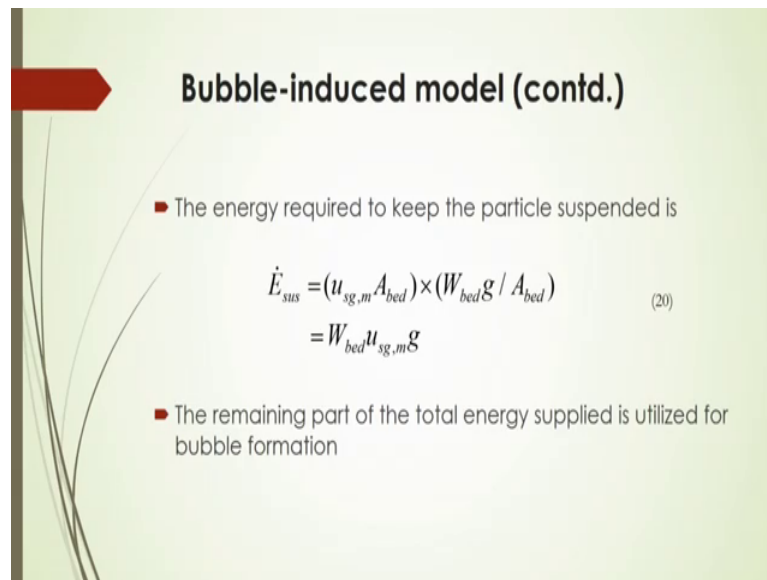
Bubble-induced model (contd.)

- The rate of production of new surface area can be represented by the mass rate of production of the fines as
$$\dot{S} = R_{a,bub} \cdot W_{bed} \quad (18)$$
- The total rate of energy input to the fluidized bed can be expressed as
$$\begin{aligned} \dot{E}_{total} &= \text{Volumetric flowrate} \times \text{Pressure drop} \\ &= (u_{sg} A_{bed}) \times (W_{bed} g / A_{bed}) \\ &= W_{bed} u_{sg} g \end{aligned} \quad (19)$$

The rate of production of this new surface area can be represented by this a rate of course, that mass rate of the production of the fines as this S dot is equal to R a bub into W bed as per equation 18 here given. So, the total rate of energy input to the fluidized bed for this attrition process then can be expressed as this E total will be is equal to volumetric flow rate into pressure drop, you know that pressure drop, these are already being actually discussed earlier in the lecture, how to calculate the pressure drop there and what should be the volumetric flow rate inside the bed, if the volumetric flow rate, if you know the velocity of this gas or fluid and then if you multiply by cross sectional area, then it will be volumetric flow rate and then pressure drop you can represent it by this.

And if you substitute this here and finally, after simplification, you can get the W bed into u sg into g this is your the total energy that is being consumed for the attrition process there.

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Bubble-induced model (contd.)

- The energy required to keep the particle suspended is

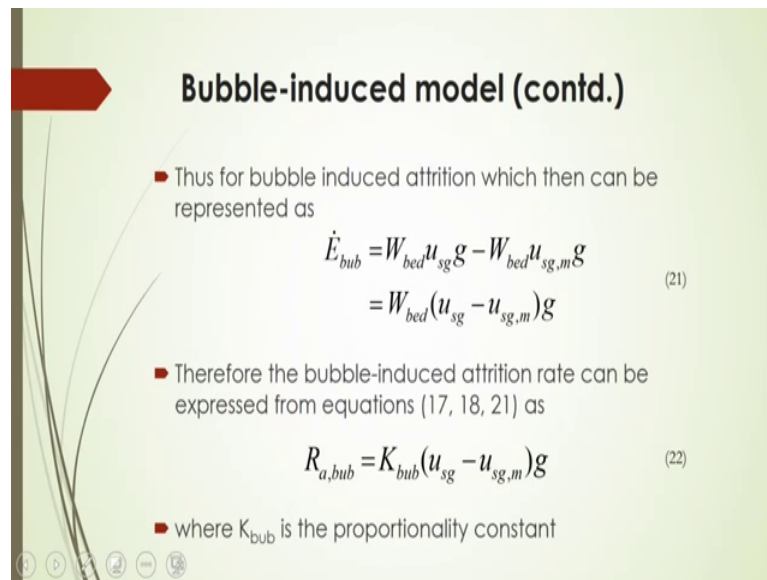
$$\begin{aligned}\dot{E}_{sus} &= (u_{sg,m} A_{bed}) \times (W_{bed} g / A_{bed}) \\ &= W_{bed} u_{sg,m} g\end{aligned}\quad (20)$$

- The remaining part of the total energy supplied is utilized for bubble formation

And the energy required to keep this particle suspended of course, is that is that is E suspension that will be is equal to $u_{sg,m}$ into a bed and then here this m power your minimum fluidization a condition and then this W_{bed} into g by a bed that will be is equal to $W_{bed} u_{sg,m} g$. So, this is the energy required to keep the particle just suspended.

So, what will be the minimum velocity of the fluid for getting the minimum fluidization and what should be the energy supplied for this, this will be called $\dot{E}_{suspension}$ and the remaining part of the total energy that will be supplied is which will be utilized for the bubble formation there in the bed.

(Refer Slide Time: 53:33)



Bubble-induced model (contd.)

- Thus for bubble induced attrition which then can be represented as
$$\dot{E}_{bub} = W_{bed} u_{sg} g - W_{bed} u_{sg,m} g$$
$$= W_{bed} (u_{sg} - u_{sg,m}) g$$
(21)
- Therefore the bubble-induced attrition rate can be expressed from equations (17, 18, 21) as
$$R_{a,bub} = K_{bub} (u_{sg} - u_{sg,m}) g$$
(22)
- where K_{bub} is the proportionality constant

Now, thus for bubble induced attrition which then can be represented as like E_{bub} is equal to what would be this here total energy here, this is called $W_{bed} u_{sg}$ into g , this is called total energy supplied here minus this is what would be the just energy consumed supplied for just getting the bed in fluidized condition.

Or you can say that suspend get the suspension. So, after subtracting you can get this equation for energy supplied for the attrition by these bubble induced mechanism therefore, the bubble induced attrition rate can be expressed from the equation 17, 18 and 21 has given that as per here after simplification, you can say this will be is equal to K_{bub} into u_{sg} minus $u_{sg,m}$ into g , here K_{bub} is the proportionality constants which will be obtained from the experimental data here ah.

(Refer Slide Time: 54:39)

Bubble-induced model (contd.)

- As the bubble velocity increases with the bed height, the bubble induced collision between particles increases with bed height. For this reason, Ulerich et al. (1980) proposed another model for bubble induced attrition rate as follows:

$$R_{a,bub} = K'_{bub}(u_{sg} - u_{sg,m})H_{bed} \quad (23)$$

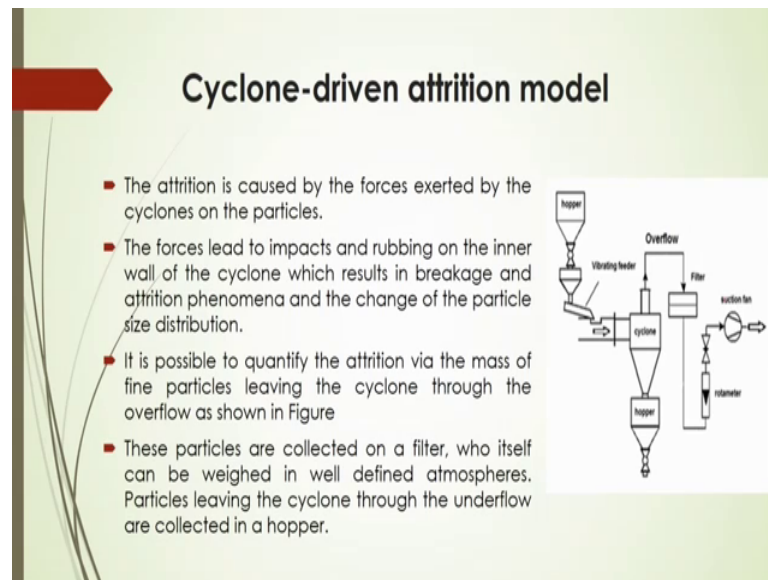
- Kono (1981) found that the attrition rate is exponentially increases with bed height with the exponent 0.78

$$R_{a,bub} \propto H_{bed}^{0.78} \quad (24)$$

The bubble velocity increases, of course, the bubble induced collision between the particles, of course, will be increases with a bed height and for this region Ulerich at al 1980 proposed another model for bubble induced attrition rate as per here given in equation 23 or above that will be is equal to K dash bub into u sg minus u sg m into H bed here.

Earlier one you have here u sg minus u sg m into g here whereas, here it will be h bed as per this height. So, this K bub dashed is also attrition rate constant as per their Ulerich at al in 1980 model, Kono 1981, founded that the attrition rate is exponentially increases with bed height with the exponent of 0.78 here which is expressed in expressed by this equation given in equation 24 here.

(Refer Slide Time: 55:39)



And cyclone driven attrition in this case, it is the actually happened because of the forces that exerted by the cyclones on the particles and the forces lead to impacts and rubbing on the inner wall of the cyclones which results in breakage and attrition phenomena and the change of the particle size distribution.

It is possible to quantify the attrition higher the mass of fine particles that is that is the being leave let the cyclone through the overflow as shown in the figure here. So, if you know the mass of fine particles that is leaving the cyclone through the overflow, you can quantify the attrition and these particles are collected on a filter who itself can be weighted in well defined atmosphere and particle form the cyclones through the underflow are collected in a hopper and from these, you will be able to calculate what should be the quantity of the attrition where this mass of the fine that is collected.

(Refer Slide Time: 56:57)

Cyclone-driven model (contd.)

- For an isolated cyclone, the attrition rate is measured as the mass flux of fines loss per unit mass flux of fed to the cyclone which is defined by
$$R_{a,c} = \frac{m_{c,loss}}{m_{c,in}} \quad (25)$$
- In the spiral or tangential cyclone, the particles are accelerated to a velocity which is related to the inlet gas velocity of cyclone. The kinetic energy of solids which is transported per unit time into the cyclone is calculated by
$$\dot{E}_{c,k} = \frac{1}{2} m_{c,in} u_{c,in}^2 \quad (26)$$

For separate cyclones or you can say isolated cyclones the attrition rate is measured as mass flux of the fines loss per unit mass flux of the feed to the cyclone which is defined by this equation 25 and in the spiral or you can say tangential cyclones. The particles are accelerated to a vertically to a vertically which is related to the inlet gas velocity of the cyclone and the kinetic energy of the solids which is transported per unit time into the cyclone is calculated by this equation 26 here half of m uc square in.

(Refer Slide Time: 57:48)

Cyclone-driven model (contd.)

- The production of free surface energy via abrasion is contributed by a fraction (η) of this kinetic energy supplied to the cyclone (Reppenhagen and Werther, 2000). The fraction (η) is function of the solid loading in the cyclone and it follows exponential function,
$$\eta = K_{c,l} M^n \quad (27)$$
- Where, $K_{c,l}$ is called loading constant. The solid loading (M) is defined by
$$M = \frac{m_{c,in}}{u_{c,in} A_{c,in} \rho_g} \quad (28)$$
- Reppenhagen and Werther (2000) found the exponent $n = -0.5$ from their experimental data.

And the production of the free surface energy via this cyclone remain or you can say abrasion which is contributed by the fraction eta of this kinetic energy supplied to the cyclone here and the fraction eta is a function of the solid loading in the cyclone and it follows the exponential function here as per equation 27 where here in this case K_{cl} is called the loading constant and this loading is defined by this m here then m will be is equal to $m_{c,in}$ by $u_{c,in}$ in $A_{c,in}$ here. So, see for cyclone here.

So, Reppenhagen and Werther 2000s found that this exponent of n will be a negative of 0.5 from their experimental data.

(Refer Slide Time: 58:46)

Cyclone-driven model (contd.)

- The production rate of attrited fines that is mass flux loss from the cyclone is then expressed as

$$m_{c,loss} = \frac{\eta m_{c,in}}{2\gamma S_m} u_{c,in}^2 = K'_c m_{c,in} u_{c,in}^2 M^n \quad (31)$$

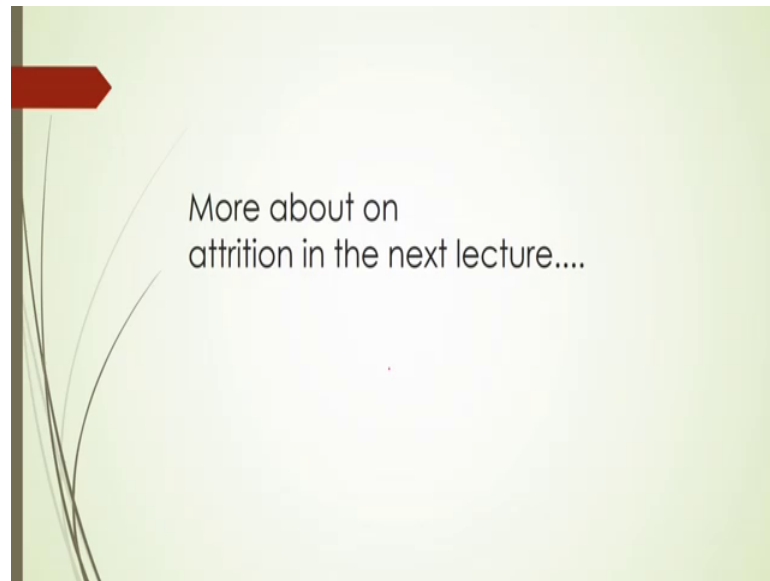
$$K'_c = K_c K_{c,l} \quad (32)$$

- Substituting the equation for M , the equation (31) becomes

$$m_{c,loss} = K'_c \frac{m_{c,in}^{1+n} u_{c,in}^{2-n}}{\rho_g^n A_{c,in}^n} \quad (33)$$

And also cyclone driven model as per that what should be the loss of fines from the cyclones that can be calculated from this equation here given as 31 eta $m_{c,in}$ by 2γ into $S_m u_{c,in}$ square. So, this will be basically depending on the cyclone velocity and the mass of the fines that is collected here and substituting the equation for m ; the equation 31, can be simplified and can be expressed by this equation 33 here.

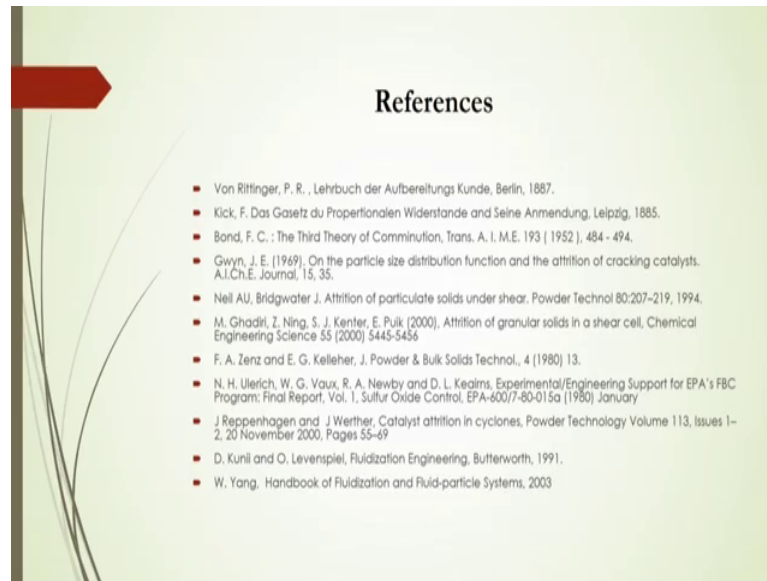
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So, this attrition, I think we have learned something about; what is the mechanism of that attrition process in the fluidized bed and how this attrition can be mathematically expressed and what the; what are the different tests of doing this attrition and attrition depends on what attrition depends on the particle size and also particle size distribution are also what will the energy consumption during this attrition and the theory given by this Rittinger's bond and then kick based on the energy consumption for a production of the a fine particles during this attrition process.

And also different models that are defined here you can use this model for a interpretation of this attrition process in the fluidized bed also you can predict the attrition process attrition extent of the attrition inside the bed and you can generate also your own a model by having a the different parameters based on their operating condition during experiment in the fluidized bed and also collecting the fines after attrition and measuring it and then fitting with the experiment fitting with some with your experimental data with some a suitable model or making the model in your from the basic theory you can developed also.

(Refer Slide Time: 61:16)



So, more about this attrition will be discussed in the next lecture, for this further reading, you can go through this reference we are given here so.

Thank you.