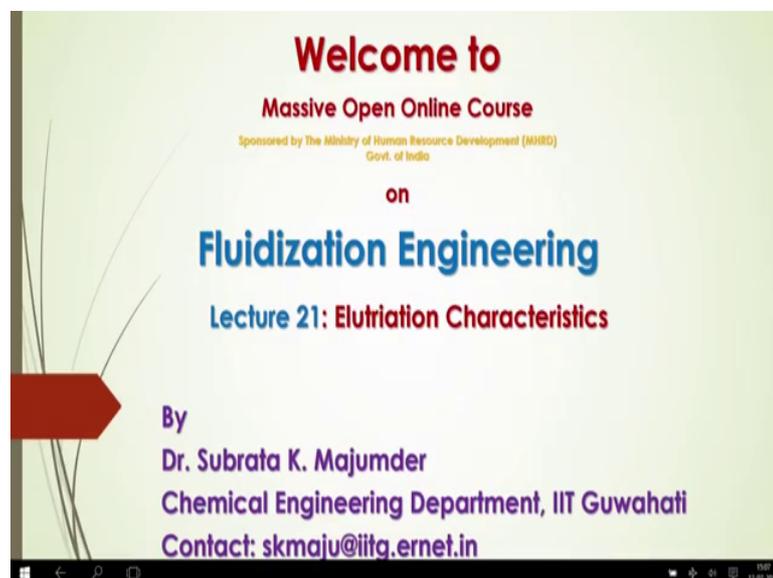


Fluidization Engineering
Dr. Subrata K. Majumder
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture – 21
Entrainment Characteristics (Part 2): Elutriation Characteristics

So welcome to massive open online course some fluidization engineering. This lecture will be on elutriation characteristics in the fluidized bed. We have discussed the entrainment characteristics in the bubbling as well as a first fluidization condition.

(Refer Slide Time: 00:40)



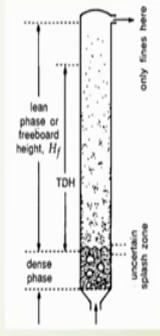
So, this elutriation characteristics also is the same way to discuss how to actually the particle is separating along the axis of the bed. Here also that mechanism is the same way that, the fine particles will be just segregating from the fluidized bed along the axis and the entrainment mechanism also will be some extent it will be useful to analyze this.

So, what is that actually elutriation? Elutriation that defined as the classifying effect of the fluidized bed entrainment it characterizes the selective removal of the particles of individual size from the fluidized bed. So, this is the definition and this elutriation way it is generally being denoted by notation W_s .

(Refer Slide Time: 01:47)

What is Elutriation?

- **Definition:** the classifying effect of the fluidized bed entrainment. it characterizes the selective removal of particles of individual size from the fluidized bed.
- **Denoted** by W_s ($\text{kg}/\text{m}^2\cdot\text{s}$)
- **Elutriation rates** from vessels with freeboard heights greater than the TDH



The diagram illustrates a fluidized bed reactor. It shows a vertical column with a dense phase at the bottom, a transition zone (TDH), and a lean phase or freeboard height (H_f) at the top. The top of the column is labeled 'only fines here'. The bottom of the column is labeled 'uncertain splash zone'.

Which is exactly the unit for that solid flux here; W_s unit is the; that means, kg per meter square second. And this elutriation rates from the vessel with freeboard height is greater than the TDH that will be should be remember.

(Refer Slide Time: 02:06)

Importance

- The entrainment rate has to be known for the design of gas/solid separators like cyclones or filters.
- On the other hand, the loss of bed material that is related to the entrainment may be important for the technical and also the economic success of a fluidized bed application.
- It characterizes the selective removal of particles of individual size from the fluidized bed

Now, what is the importance that for which you have to know that elutriation characteristics in the fluidized bed. Now you know that entrainment rate that has to be known for the design of gas solid separators like cyclones or filters, in the same way elutriation is also important because this give you the classification of the solid particles

which particles will go up, which particles will go down what should be the interaction of the larger particles or coarser particles with the smaller particles and because of that interactions, how the solid particles will be segregating by the entrainment mechanism there.

On the other hand, you can say the less amount of a solid particles will be coming down, relative to the finer particles if there is high interaction inside the bed and there will be a formation of the fine particles because of the interaction. And in that case, the very fine particles will be coming out easily relative to the coarser one. But if coarser one particles the strength of the particles actually in main factor if the strength of the particle is not that much enough. So, that the coarser particle sometimes by their individual interactions may be forming finer particles and then it will go up.

So, based on which that you can say, how the solid particles also will be segregating from these two classes of the particles there. So, the loss of bed material that is related to the entrainment may be important for the technical and also the economic success of a fluidized bed applications there. Sometimes you have to operate the fluidized bed in such a way that, there is no enough elutriation or there is no attrition of the solid particles because of which that, entrainment characteristics will go down. So, sometimes this fine particles the internal circulation of the fine particles is required for the better transport operation and in that way, sometimes the a economic operation of the fluidized bed can be done in such a way by designing without considering and those entrainment and elutriation characteristics of the particles inside the bed.

And it characterizes the selective removal of particles of the individual size from the fluidized bed. If you are going to segregate the particles from this bed, then there will be a certain selection or selective removal of the particles of the individual size from the fluidized bed by this elutriation mechanism.

(Refer Slide Time: 04:42)

Determination of elutriation rates

- Two different approaches to find elutriation rate
 - First one: Zenz et al. approach
 - Second: elutriation constant approach
- Both approaches rest on the assumption that the flux rate of any particular size of solid i is proportional to its weight fraction x_i in the bed, all other factors kept constant, or

$$G_i = x_i G_{si}^* \quad (1)$$

- where G_{si}^* is the flux rate from an imaginary bed of solids, all of size i . Thus it is the saturation carrying capacity of the gas for that particular size of solid.

Determination of the elutriation rates you have to know, how to actually estimate the elutriation rate. Two different approaches actually you will get to find the elutriation rate, one is called that is Zenz et al approach, another is elutriation constant approach.

Now, both the approaches actually based on the assumption that the flux rate of any particular size of solids i is proportional to its weight fraction x_i in the bed, and all other factors kept constant. So, this is very important that you have to know the elutriation rate. Generally in the literature it is given two types, one is Zenz et al model another is the elutriation constant model there; in that case, for both the cases you have to actually assume that the flux rate of any particular size of the solid that should be proportional to the weight fraction of that particular classes solid in the bed.

All other factors if you are keeping constant and G_{si} is equal to $x_i G_{si}^*$ in that case, G_{si}^* is the flux rate from an imaginary bed of solids all of size i thus it is saturated carrying capacity of the gas or that you can say that the exit concentration dilute exit concentration at a fixed rate there and for that the particular size of solids will be coming out from the bed.

So, in this case the rate of the elutriation you have to calculate by this denoting by G_{si} for this particular class of particles, that will be is equal to x_i into G_{si}^* . So, i means here class of particles, at a particular class of particles here i . So, G_{si} means here, the solid entrainment of that particular class; that means, if the particle size is suppose d_{pi} ;

that means, that class then it will be represented that finer or coarser of that particular classes that will be is equal to, what should be the weight fraction of that particular classes i particle? And also, what should be the carrying capacity or saturated carrying capacity of that particular gas for those solid particles of plus i?

So, from this equation we will be able to calculate the elutriation rate.

(Refer Slide Time: 07:38)

Procedure of Zenz et al.

- Step 1:** Divide the size distribution of particle into narrow intervals and find intervals
 - For elutriable solids $u_t < u$
 - For non-elutriable solids $u_t > u$
- Step 2:** Find G_{it}^* for each interval of elutriable solids from the curve shown. The Fig shows that one curve refers to fine particle (Geldart A) systems (more than 90% entrainable), the other to fines removed from larger particle (Geldart AB) systems.
- Step 3:** Under the assumption of Eq. (1), the total entrainment is

$$G_t = \sum_{\text{all elutriable intervals, } i} x_i G_{it}^* \quad (2)$$

$$G_t = \int_{\text{all particles}} G_{it}^* p_r(d_p) d(d_p) \quad \text{In terms of the continuous size distribution} \quad (3)$$

The graph plots $\frac{(u_p - u_t)^2}{g d_p}$ on the y-axis (log scale from 10⁻¹ to 10²) against G_{it}^* on the x-axis (log scale from 10⁻¹ to 10²). Two curves are shown: one for Geldart A systems (higher entrainment) and one for Geldart AB systems (lower entrainment). A horizontal line is drawn at $G_{it}^* = 10^0$, and a vertical line is drawn at $\frac{(u_p - u_t)^2}{g d_p} = 10^0$. The intersection of these lines is marked with a circle and labeled 'u'. The region to the left of this intersection is labeled 'For Geldart A systems in which most particles are entrained' and the region to the right is labeled 'For fines removed from larger particles'.

Now, Zenz et al model, as per that Zenz et al model you will get the different step to calculate that elutriation rate here. Now as a step 1, you can say that this distribution of the particle that will actually role enough there. So, you have to divide the size distribution of the particle into narrow intervals and then you have to find the intervals in such a way that, for elutriate for elutriable solids; that means, U_f is less than u ; that means, here or U_t here.

So, this U_t elutriable solid in that case, the terminal velocity should be is less than a fluid velocity, then only those solids will be elutriable. And for a non elutriable solids, the terminal velocity of the solids will be is greater than fluid velocity of course, the coarser particles those are all those a terminal velocity is higher than the fluid velocity those particles will not be entrained at all.

So, it cannot be actually elutriable or cannot be segregated from the fine particles there. And if it is less than suppose terminal velocity then only it will be elutriable and then

those particles will be segregated from the coarser particles whose terminal velocity will be higher than the fluid velocity there. And in the step 2, you find then G_{si}^* for each interval of the elutriable solids. So, first you have to find out which one is the elutriable, just from the terminal velocity of the solids and what will be the fluid velocity, whether it is greater or less than then you can find out, which one will be the elutriable.

Once you know the elutriable solids and then find the, what is the G_{si}^* for each individual of elutriable solids from the curve shown here in this case. And this is this curve actually this figure is representing the, this saturated carrying capacity versus this what is that some parameters here. So, then from which you will be able to calculate, what should be the? So, this figure you can get it from that Kunil and Levenspiel also there it is given.

So, once you know this G_{si}^* from this figure and then you have to calculate the G_s here from this equation like this here. So, for each class of particles you have to get the respective G_s star. So, once you know that you have to sum up all this parameter here in this case. So, G_s to be calculated by this equation 2. Here again, for this continuous size distribution you can calculate this elutriation rate by this equation, by this equation 3 here.

So, here G_s will be is equal to integration, for all particles it will be G_{si}^* into p_e into d_p into function of p_e as a function of particle diameter into d_p . So, in this case, you will be able to calculate, what should be the, that means, elutriation rate. This here $p_e d_p$ is nothing, but the particle size distribution function.

(Refer Slide Time: 11:07)

The Elutriation Constant Approach
[Called as Leva (1951) and Yagi and Kunii (1952) approach]

- With particles of wide size distribution in a bed with a freeboard ($>TDH$), the flux of particles of size i out of the bed, can be expressed as

$$-\frac{1}{A_t} \frac{dW_i}{dt} = k_i^* x_i = k_i^* \frac{W_i}{W} \quad (4)$$

where k^* [$\text{kg}/\text{m}^2 \text{ s}$] is called the elutriation rate constant.

$k_i^* = G_i^*$ as per equation (1), refers to the flux rate of solids i if they are alone in the bed.

$k_i^* = 0$ refers to flux for those solids that are not removed at all by entrainment

$k^* = \text{large}$ refers rapid removal of that size of solids from the bed

The elutriation constant approach here, it is called something is a Leva 1951 and Yagi and Kunli approach also. So, now, with particles of wide size distribution in a bed with a freeboard, which transport disengagement height is actually less than these a freeboard. The flux of particles of size i out of the bed can be expressed as by equation 4 here.

See here, this is the 1 minus 1 by A_t , A_t is the cross sectional area of the bed and here dW is the what would be the amount; that means, mass of a solids of size i , which is changing with respect to time, that the rate of that mass of this elutriation that will be is equal to k_i^* into x_i or is equal to k_i^* into W_i by W here. This x_i is the mass fraction. So, this is W_i of the W weight of the or mass of the i th size particles and W is the total mass of the bed here all for all classes.

Where k^* is called the elutriation rate constant, this k_i^* it will be is equal to G_i^* as per equation 1, which refers to the flux rate of solids i , if they are alone in the bed. And k_i^* will be is equal to 0 which refers to the flux for those solids that are that are not removed at all by entrainment. And k^* that will be very large, which refers to the rapid removal of that size of the solid from the bed.

(Refer Slide Time: 13:13)

Alternate way of elutriation rate

$$\left(\frac{\text{Rate of removal of solids } i}{\text{Weight of that size of solid in the bed}} \right) = k$$

Or

$$-\frac{dW_i}{dt} = k_i W_i ; k_i = [s^{-1}] \quad (5)$$

Comparing eqs. (4) and (5)

$$k_i^* = \frac{k_i W}{A_t} = k_i \rho_i (1 - \epsilon_m) L_m \quad (6)$$

This k is different from k*

Now is there any other alternative way or not that there is one alternate way of elutriation rate that can be calculated, now if we represent this rate of removal of the solid of class i which is proportional to the weight of that size i of the solids in the bed, then what should be the proportionality constant that will be is k. So, this k is different from here k star.

So, if we represent this elutriation rate by this simple equation and this rate which is directly proportional to the weight of that size not weight fraction there. So, by in that case, this proportionality constant will not be the same as whatever in the previous equation it is represented. So, or you can say that minus dW i by dt will be is equal to k i W i, k i the unit will be a second inverse here. So, comparing this 4 and 5 equation number, then k i star what will be is equal to k i W by A t, it means cross sectional of the bed and then it will be is equal to k i rho i into 1 minus epsilon m m into L m.

So, from this what should be the relation between k i star and k i you can get it. So, k i is simple that it is that elutriation directly proportional to the weight of the solids whereas, k i star it will be represented by that the rate equation will be actually proportional to the mole fraction or you can say that is are the weight fraction of the solids of class high there.

(Refer Slide Time: 14:43).

Relationship between k and G_s

- For a elutriable single size i of solids, the carryover from the vessel is

$$k_i^* = G_{s_i} = \left(\frac{\text{Saturation carrying capacity}}{\text{of gas for solids } i} \right) \quad (7)$$

- If the fluidized bed consists of coarse solids plus only one size of elutriable solids of mass fraction x_i , then the total carryover is

$$G_{s_i} = x_i k_i^* = x_i G_{s_i}^* \quad (8)$$

Now, significance of this k_i and k_i^* , that is the elutriation rate constant. Now k_i^* is equal to $G_{s_i}^*$ as per equation 1 which refers to the flux rate of solids, if they are along in the bed. And k_i^* is equal to 0 which refers to the flux for those solids that are not actually removed at all by the entrainment. And this elutriation rate will be very large if the rapid removal of that size of the solids from the bed is happen and k_i varies inversely with a bed height. In batch or unsteady state in experiments it is seen that, k_i should change during a run as the bed weight W is changed.

On the contrary, you can say that k_i^* is unaffected by these changes. It is the true rate constant and is the quantity to use when reporting data and presenting correlations there and relationship between k and G_s . For a elutriable single size i of solids, you can say the carryover from the vessel is represented as this k_i^* , that will be is equal to $G_{s_i}^*$ are is equal to saturation carrying capacity of gas for solids i . If the fluidized bed that consists of a coarse solids plus only one size of elutriable solids of mass fraction x_i , then the total carry over will be is equal to x_i into k_i^* .

So, that will be is equal to x_i into that means, $G_{s_i}^*$.

(Refer Slide Time: 16:39)

Relationship between k and G_s (contd.)

- If the bed contains coarse solids plus sizes 1, 2, ... , n of elutriable solids, then the total entrainment is

$$G_s = \sum_{i=1}^n x_i k_i^*; \sum x_i < 1 \quad (9)$$

If all the bed solids are elutriable

$$\sum x_i = 1$$

- Alternatively, if one considers a continuous size distribution in the bed, then the total flux of elutriated solids is

$$G_s = \int_{\text{all particles}} k^* p_b(d_p) d(d_p) \quad (10)$$

These expressions for fluxes are applicable only for bed height is greater than Transport disengagement height

Now, here if the bed contains coarse solids plus sizes like 1, 2, 3 dot dot dot n; that means, n different size of a of solids which are elutriable and then, the total entrainment will be is equal to G s that will be is equal to their summation of i is equal to i to n, x i into k i star; that means, you have for all solid are classified a particles are there you have to sum up all those things.

Now, summation of xi should be less than 1 there, and if all the bed solids are elutriable then only you can say that summation of this mass fraction of all classified will be is equal to 1. Now alternatively, if one considers a continuous size distribution in the bed, then the total flux of elutriated solids will be calculated as these G s will be is equal to here k star into p b d p into d of d p here. Now these expressions for fluxes are applicable only for the bed height that is greater than the transport disengagement height. So, it is to be noted down.

(Refer Slide Time: 18:15)

How to estimate k and k* experimentally?

- Solids Flow Experiment
 - steady state recirculating system;
 - steady state once-through system;
- Batch Experiment

Now how to estimate k and k star experimentally? Now, solids flow experiments here you will see that one is steady state operation and steady state re circulating system and there will be a steady state once through system and another important experiment that as experiment. You can estimate this, this elutriation rate by this two types of a experiment there.

(Refer Slide Time: 18:43)

Solids Flow Experiment

- In this experiment you have to measure the carryover rate and weight fraction of size *i* in the bed. Then from Eq. (4), or in terms of a continuous size distribution of bed solids, you have to determine k_i^* as

$$k_i^* = \frac{\text{Flux of } i \text{ from the bed, kg/m}^2 \cdot \text{s}}{\text{Weight fraction of } i \text{ in the bed}} = \frac{G_i p_i(d_p)}{p_b(d_p)} \quad (11)$$

So, solid flow experiment as per solid flow experiment, you will see that you have to measure, the carry over rate and the weight fraction of the size in on the bed and then

from equation 4 or in terms of a continuous size distribution, if it is given to you of the bed solids, then you have to determine k_i^* as here this k_i^* that will be is equal to flux of i from the bed in kg per meter square second upon weight fraction of a weight solid particles in the bed.

And that mathematically you can say that G_s into that means, p into d_p ; that means, a solid particle size distribution and divided by p_b into d_p here. This again this void fraction of the bed; that means, represented by this distribution.

(Refer Slide Time: 19:46)

Batch Experiment

- In this experiment, you have to first find out the initial bed composition and the composition with respect to time.
- If the total mass of bed does not change significantly (< 20%) in a time,
- Then integration of Eq. (4) to get

$$\frac{W_i}{W_{i0}} = \exp\left(-\frac{k_i^* A_{bed} t}{W}\right) = \exp(-k_i t) \quad (12)$$

Now as per batch experiment, this experiment you have to first find out the initial bed composition and the composition with respect to time. Now, if the total mass of bed does not change significantly; that means, less than it is 20 percent in a time then integration of the equation 4 will give you that W_i by W_{i0} that will be is equal to exponent of; that means, here minus k_i^* into A_{bed} into t by W and; that means, your exponent of minus $k_i t$.

So, from this equation 12, you can get what should be the actually a elutriation amount of solids of class i from this equation.

(Refer Slide Time: 20:43)

Batch Experiment (contd.)

- If the bed weight changes significantly during the experimental run, i.e. if most of the bed solids are elutriable
- Then measure all the size fractions before and after the run and naming the elutriating size fractions 1, 2, . . . , n and the nonelutriating fraction "l"
- After that integrate Eq. (4)
- After rearrangement, you calculate

$$\frac{W_1}{W_{10}} = \left(\frac{W_2}{W_{20}} \right)^{k_1^*/k_2^*} = \left(\frac{W_3}{W_{30}} \right)^{k_1^*/k_3^*} = \dots \quad (13)$$

$$k_1^* A_{total} = W_1 \ln \left(\frac{W_{10}}{W_1} \right) + \sum_{i=1}^n W_{i,0} \frac{k_i^*}{k_1^*} \left[1 - \left(\frac{W_{10}}{W_1} \right)^{k_1^*/k_i^*} \right] \quad (14)$$

Measuring the bed composition before and after a run and inserting into Eq. (13) gives the ratio of k^* values. Then Eq. (14) gives k_1^* , from which all other k^* values can be found.

Now, if the bed weight changes significantly during the experimental run that is if most of the bed solids are elutriable. Then measure all the size fractions before and after the run. And naming the elutriating size fractions 1, 2, to n and the nonelutriating fraction of i.

After that you have to integrate the equation 4, and then rearrangement you can calculate W_1 by W_{10} ; W_2 by W_{20} , that will be is equal to W_3 by W_{30} and to the power k_1^* by k_2^* is equal to this as per equation 13. And from which you can calculate that k_1^* star, or that will be is equal to this as per equation 14, you can calculate this k_i^* star.

Now, measuring the bed composition before and after a run, and inserting into equation 13 that will give you the ratio of k^* values. Then equation 14 gives k_1^* star here and from which all other k^* values can be found here.

(Refer Slide Time: 22:12)

Some important correlations for k_i^*

$$\frac{k_i^* g_{pi}^2}{\mu(u_o - u_{ti})^2} = 0.0015 Re_1^{0.5} + 0.01 Re_1^{1.2}$$

$$\frac{k_i^*}{\rho_g(u_o - u_{ti})} = 1.52 \times 10^{-5} \frac{u_o - u_{ti}}{(g d_{pi})^{0.5}} \times Re_1^{0.725} \left(\frac{\rho_s - \rho_g}{\rho_g} \right)^{1.15}$$

$$\frac{k_i^*}{\rho_g(u_o - u_{ti})} = 0.046 \frac{(u_o - u_{ti})}{(g d_{pi})^{0.5}} \times Re_1^{0.3} \left(\frac{\rho_s - \rho_g}{\rho_g} \right)^{0.15}$$

$$\frac{k_i^*}{\rho_g u_o} = 0.0001 + 130 \exp \left[-10.4 \left(\frac{u_{ti}}{u_o} \right)^{0.5} \left(\frac{u_{mf}}{u_o - u_{mf}} \right)^{0.25} \right]$$

$$\frac{k_i^*}{\rho_g u_o} = 23.7 \exp \left(-5.4 \frac{u_{ti}}{u_o} \right)$$

$$k_i^* = 0.011 \rho_s \left(1 - \frac{u_{ti}}{u_o} \right)^2, \quad \rho_s \text{ (kg/m}^3\text{)}$$

$$\frac{k_i^*}{\rho_g(u_o - u_{ti})} = 2.07 \times 10^{-4} Fr^{\alpha} Re_1^{1.6} \left(\frac{\rho_s - \rho_g}{\rho_g} \right)^{0.61}$$

$\alpha = Re_1^{-0.6}, Fr = (u_o - u_{ti})^2 g d_{pi}$
 for Geldart group A particles
 $Re_1 = d_{pi} \rho_g u_{ti} / \mu$

[Source: Kunii and Levenspiel (1991)]

Now, some important correlations for k_i^* as per different investigators from the experimental results, it is given in Kunii and Levenspiel textbook and you can get. So, these are the different correlations from which you can get this k_i^* . So, these correlations will be helpful to know this k_i^* for designing any fluidized bed, and all these things they have all this k_i^* given by different investigators. They have correlated this with the experimental condition like here terminal velocity of the fluid particle diameter and velocity of the fluid particle size, density of the gas, and density of the solid and minimum fluidization velocity, and also the terminal velocity for that particular classes of solids.

And also what will be the significant group, as per that is given this terminal Reynolds number based on terminal velocity and a Froude number based on that particle diameter, and also relative velocity of the fluid, relative to that terminal velocity of the particle and also based on different type of solids. They have also got the different correlations you can get more information from the Kunii and Levenspiel book for more correlations also. So, these correlations very important to calculate how to calculate the k_i^* and from which, what should be the elutriation elutriable amount of solids in the fluidized bed for that particular operating condition you will be able to calculate.

(Refer Slide Time: 24:01).

		U , m/s	D , m	d_p , mm	Reference	Comments
1	$\frac{K_{tm}^* \cdot g \cdot d_p^2}{\rho_f(U - u_0)^2} = 0.0015 \cdot Re_d^{0.8} + 0.01 \cdot Re_d^{1.2}$	0.3-1.0	0.07-1.0	0.1-1.6	Yagi and Aochi (1955) as cited by Wen and Chen (1982)	Cited by different authors in different ways, original publication inaccessible
2	$\frac{K_{tm}^*}{\rho_f \cdot U} = \begin{cases} 1.26 \cdot 10^{-7} \cdot \left(\frac{U^2}{g \cdot d_p \rho_f}\right)^{1.88} & \text{for } \frac{U^2}{g \cdot d_p \rho_f} < 3.10 \\ 1.31 \cdot 10^{-8} \cdot \left(\frac{U^2}{g \cdot d_p \rho_f}\right)^{1.18} & \text{for } \frac{U^2}{g \cdot d_p \rho_f} > 3.10 \end{cases}$	0.3-0.7	0.05 x 0.53	0.04-0.2	Zenz and Weil (1958)	Correlation aiming at FCC fluidized beds
3	$\frac{K_{tm}^*}{\rho_f(U - U_0)} = 1.52 \cdot 10^{-11} \left(\frac{U - U_0}{g \cdot d_p}\right)^{0.5} \cdot Re_d^{0.728}$	0.6-1.0	0.102	0.7	Wen and Haslinger (1960)	
4	$\frac{K_{tm}^*}{\rho_f(U - U_0)} = 4.6 \cdot 10^{-12} \left(\frac{U - U_0}{g \cdot d_p}\right)^{0.5} \cdot Re_d^{0.7} \cdot \left(\frac{\rho_s - \rho_f}{\rho_f}\right)^{0.15}$	0.9-2.8	0.031-0.067	0.7-1.9	Tanaka et al. (1972)	
5	$\frac{K_{tm}^*}{\rho_f \cdot U} = A + 130 \cdot \exp\left[-10.4 \left(\frac{U}{U_0}\right)^{0.5} \left(\frac{U - U_0}{U - U_{mf}}\right)^{0.27}\right]$ with $A = 10^{-3} \dots 10^{-4}$	0.6-2.4	0.91 x 0.91	0.06-1.0	Merrick and Higley (1974)	Correlation derived for bubbling fluidized bed combustors
6	$\frac{K_{tm}^*}{\rho_f \cdot U} = 23.7 \cdot \exp(-5.4 \frac{U_0}{U})$	0.6-3.0	0.076	0.06-0.35	Geldart et al. (1979)	
7	$\frac{K_{tm}^*}{\rho_f \cdot U} = 9.43 \cdot 10^{-14} \left(\frac{U^2}{g \cdot d_p}\right)^{1.88}$	0.1-0.3	0.61 x 0.61	0-0.125	Liu et al. (1980)	

for $58 \leq \left(\frac{U^2}{g \cdot d_p}\right) \leq 1000$; $0.1m/s \leq U \leq 0.3 m/s$

Some other correlations also given here, see in this table, there are different correlations also given for predicting the k_i star in the fluidized bed from their different experiment and within a certain range of their particular experimental condition. As per different investigators, they have suggested different correlations and they also obtain that experimental results of this k_i star, based on their by using different bubbling fluidized bed operating condition and solid particles there.

(Refer Slide Time: 24:42).

8	$K_{tm}^* \left[\frac{kg}{m^2 s}\right] = 0.011 \cdot v_0 \left(1 - \frac{u_0}{U}\right)^2$	0.9-3.7	0.92 x 0.92	0.3-1.0	Colakyan et al. (1981), Colakyan and Levenspiel (1984)	Focus on Geldart Group B and D particles
9	$K_{tm}^* \left[\frac{kg}{m^2 s}\right] = 2.8 \cdot 10^{-12} \left(\frac{U - u_0}{U}\right)^{1.8} \left(\frac{\rho_s - \rho_f}{\rho_f}\right)^{0.8425} \cdot D_p$				Kato et al. (1985)	
10	$\frac{K_{tm}^*}{\rho_f \cdot U} = 1.6 \left(\frac{U}{u_0}\right) \left(1 - \frac{u_0}{U}\right)$				Sciurco et al. (1991)	
11	$K_{tm}^* \left[\frac{kg}{m^2 s}\right] = 5.4 \cdot 10^{-6} v_0 \left(\frac{U}{u_0}\right)^{3.4} \left(1 - \frac{u_0}{U}\right)^3$ for $d_p \leq \frac{10325}{Re^{0.57}}$	0.2-0.7		0.03-0.78	Burgers et al. (1992)	Correlation takes cohesive forces into account and is focused on superlines in group A and C systems
12	$K_{tm}^* \left[\frac{kg}{m^2 s}\right] = 0.35 v_0 U (1 - \phi_{10})$ with $(1 - \phi_{10}) = 7.41 \cdot 10^{-3} g^{0.47} d_p^{0.11} \rho_f^{0.44}$ and $R = \sum_{i=1}^n v_i \left(\frac{U - U_{0i}}{U}\right)$ for $u_0 < U$	0.1-0.6	0.071	0.03-0.2	Nakagawa et al. (1994)	
13	$K_{tm}^* \left[\frac{kg}{m^2 s}\right] = \begin{cases} 23.7 \cdot \rho_f \cdot U^{1.2} \exp(-5.4 \frac{U_0}{U}) & \text{for } Re < 3000 \\ 14.5 \cdot \rho_f \cdot U^{1.2} \exp(-3.4 \frac{U_0}{U}) & \text{for } Re > 3000 \end{cases}$ with $Re = \frac{\rho_f \cdot U}{\mu}$	0.2-0.8	0.076, 0.132	0.017-0.077	Taxina and Geldart (1996c)	
14	$\frac{K_{tm}^* d_p}{\rho_f} = A v_0^{1.5} \exp(6.92 - 2.11 \frac{U_0^{0.10}}{U_0^{0.02}})$ with $F_g = g \cdot d_p (\rho_s - \rho_f)$ (gravity force per projection area)	0.3-7.0	0.06-1.0	0.05-1.0	Choi et al. (1999)	Correlation based on a wide range of different units, materials, and operating conditions, e.g., temperature and pressure

So, these are some other correlations by which you can calculate the k_i star elutriation rate constant.

(Refer Slide Time: 24:52)

Solution

$$\frac{u_0^2}{g d_p^2 \rho_s^2} = \frac{(0.61)^2}{(9.8)(1.3 \times 10^{-4})(1200)^2} = 0.203 \times 10^{-3} \text{ m}^6/\text{kg}^2 \quad \text{(Ans. (a))}$$

From Figure $G_s^*/(\rho_s u_0) = 1.2$ $\frac{G_s^*}{u_0} = (1.2)(5.51) = 6.61 \text{ kg/m}^3$
 $G_s^* = (6.61)(0.61) = 4.03 \text{ kg/m}^2 \cdot \text{s}$

From eq. (1) $G_s^* = (0.2)(4.03) = 0.806 \text{ kg/m}^2 \cdot \text{s}$ **Ans. (b)**

From equation (3) $\frac{G_s^*}{\rho_s u_0} = 130 \left(\frac{G_s^*}{\rho_s u_0} \right)_{\text{max}} \left(\frac{d_p}{d_p} \right) d(G_s^*) = 5.88$
 $i_s = \frac{G_s^*}{u_0} = (5.88)(5.51) = 32.4 \text{ kg/m}^3$ (solid loading) **Ans. (c)**
 $A G_{s, \text{total}} = \left(\frac{\pi}{4} d^2 \right) (32.4)(0.61) = 559 \text{ kg/s}$ (total entrainment)

The graph plots $\frac{G_s^*}{\rho_s u_0}$ on the y-axis (log scale from 0.1 to 10) against $\frac{(u_0 - u_f)^2}{g d_p^2 \rho_s^2}$ on the x-axis (log scale from 10⁻⁴ to 10⁻¹). Two curves are shown: an upper curve for Geldart A fluids and a lower curve for coarse particles. A point is marked on the upper curve at approximately (0.2, 6.61).

Now, let us see one example to calculate this elutriation rate there. Now a fluidized bed of 6 meter diameter is operating with catalyst particle under the following operating conditions. Here the gas density is given here, density of the solids and the fluid velocity is given, particle size is given, and the freeboard height is a considered is greater than transport disengagement height. So, in this case you have to calculate, what should be the saturation carrying capacity? Calculate the saturation carrying capacity for a bed in which, the fines constitute just a 20 percent of the bed solid. Calculate the total entrainment and the solid loading at the exit port of a bed of fine catalyst here.

And it is given that; this particle size distribution is that for different particle size here in the table, this function is obtained from this table for this P_{dp} here. Now, just calculate the u_0 this group u_0^2 by $g d_p \rho_s^2$ and once you know this value. So, from this figure, you just calculate what should be the group G_s^* by $\rho_s u_0$ for that particular value of this u_0^2 by $g d_p \rho_s^2$ here and from which, you will be able to calculate, what should be the G_s^* ?; that means, saturated carrying capacity.

And then from equation 1, then G_s^* will be is equal to this once and from equation 3, then G_s by $\rho_s u_0$ will be calculated as per this here. After substitution of this, what is that size distribution, and integrating with a size range obtained to 130 micrometer here.

And what should be the density of the rho s here, it is burg density of the solids here, it is given here, solid loading you can calculate from which. Then after that total entrainment amount you can calculate from this equation.

(Refer Slide Time: 27:06)

Example

- Calculate the elutriation constant k^* for 40- to 120 μm particles from a experiments at 1040 kPa and $u = 0.381 \text{ m/s}$
- Solution:

From Fig. (a) $G_s = 0.9 \text{ kg/m}^2 \cdot \text{s}$.

Find the size distribution functions for bed particles, $p(d_p)$ and entrained particles $p_e(d_p)$ from Fig. (b) and Finally, the k^* values for the various particle sizes are found from Eq. (11) and are tabulated as follows

d_{p1} (μm)	40	60	80	100	120
$100p_b(d_{p1})$ (μm^{-1})	0.45	1.00	1.25	1.00	0.60
$100p_e(d_{p1})$ (μm^{-1})	1.20	2.00	1.25	0.45	0.10
k^* ($\text{kg/m}^2 \cdot \text{s}$)	2.4	1.8	0.90	0.41	0.15

Fig. (a)

Fig. (b)

Now, what should the elutriation rate constant k^* for 40 to 120 micrometer particles from a experiment at 1040 kilo Pascal and the fluid velocity is 0.381 meter per second. Now in this case, what is the G_s ? That it is very simple to calculate from this figure a, what should be the G_s you can calculate with respect to this velocity of fluid here. And after that, find the size distribution function for bed particles, here p_d and entrained particles p_e from figure b here. And Finally, then k^* values for the various particle size are found from equation number 11 and are tabulated as follows here.

(Refer Slide Time: 27:55).

Example

■ Calculate k^* values from various correlations in the literature for the fine elutriable fractions present in a bed of coarse and fine particles.

$\rho_g = 1.217 \text{ kg/m}^3$, $\mu = 1.8 \times 10^{-5} \text{ kg/m}\cdot\text{s}$, $u_{mf} = 0.11 \text{ m/s}$
 $\rho_s = 2000 \text{ kg/m}^3$, $u_0 = 1.0 \text{ m/s}$

d_p (μm)	30	40	50	60	80	100	>120
u_0 (m/s)	0.066	0.115	0.175	0.240	0.385	0.555	>1.0

$k^* = \frac{\rho_s d_p^2}{\mu(u_0 - u_{mf})} [0.0015 \text{Re}_p^{0.5} + 0.01 \text{Re}_p^{1.2}]$
 $\frac{k^*}{\rho_s(u_0 - u_{mf})} = 1.52 \times 10^{-5} \frac{u_0 - u_{mf}}{(\rho_s d_p)^{0.5}} \times \text{Re}_p^{0.725} \left(\frac{\rho_s - \rho_g}{\rho_g}\right)^{1.15}$
 $\frac{k^*}{\rho_s(u_0 - u_{mf})} = 0.046 \frac{(u_0 - u_{mf})}{(\rho_s d_p)^{0.5}} \times \text{Re}_p^{0.5} \left(\frac{\rho_s - \rho_g}{\rho_g}\right)^{0.15}$
 $\frac{k^*}{\rho_s(u_0 - u_{mf})} = 0.0001 + 130 \exp\left[-10.4 \left(\frac{u_0}{u_{mf}}\right)^{0.5} \left(\frac{u_{mf}}{u_0 - u_{mf}}\right)^{0.25}\right]$

$\frac{k^*}{\rho_s u_0} = 23.7 \exp\left(-5.4 \frac{u_0}{u_{mf}}\right)$
 $k^* = 0.011 \rho_s \left(1 - \frac{u_{mf}}{u_0}\right)^2 \rho_g \text{ (kg/m}^2\text{)} \cdot \rho_g \text{ (kg/m}^3\text{)}$
 $\frac{k^*}{\rho_s(u_0 - u_{mf})} = 2.07 \times 10^{-4} \text{Fr}_m \text{Re}_p^{1.4} \left(\frac{\rho_s - \rho_g}{\rho_g}\right)^{0.61}$
 $\alpha = \text{Re}_p^{0.6}$, $\text{Fr} = (u_0 - u_{mf})^2 g d_p / \mu$
 for Geldart group A particles
 $\text{Re}_p = d_p \rho_g u / \mu$

So, in this way, you can calculate the elutriation rate. Another example for calculate this k^* values from various correlations in the literature for the fine elutriable fractions present in a bed of coarse and fine particles there.

(Refer Slide Time: 28:44)

Solution

(a) With Yagi and Aochi's correlation

$$\text{Re}_p = \frac{(1.27)(0.240)(60 \times 10^{-6})}{1.8 \times 10^{-5}} = 0.9736$$

$$k^* = \frac{(1.8 \times 10^{-5})(1 - 0.024)^2}{9.8(60 \times 10^{-6})} [0.0015(0.9736)^{0.5} + 0.01(0.9736)^{1.2}] = 3.23 \text{ kg/m}^2 \cdot \text{s}$$

Wen and Hashinger's correlation

$$k^* = \frac{(1.52 \times 10^{-5})(1.217)(1 - 0.024)^2}{(0.9)(60 \times 10^{-6})^{0.5}} [0.9736]^{0.725} \left[\frac{2000 - 1.217}{1.217}\right]^{1.15} = 2.16 \text{ kg/m}^2 \cdot \text{s}$$

With Merrick and Hightley's correlation

$$k^* = (1.217)(1) [0.001 + 130 \exp\{-10.4 \left(\frac{0.240}{1}\right)^{0.5} \left(\frac{0.11}{1 - 0.11}\right)^{0.25}\}] = 0.834 \text{ kg/m}^2 \cdot \text{s}$$

With Geldart's correlation

$$k^* = (23.7)(1.217) \exp(-5.4 \times 0.240) = 7.89 \text{ kg/m}^2 \cdot \text{s}$$

With Zenz and Weil's procedure

$$\frac{u_0^2}{g d_p^3} = \frac{1^2}{(9.8)(60 \times 10^{-6})^3} = 4.3 \times 10^{-4}$$

$$G_0 / \rho_g u_0 = 5.0$$

$$k^* = G_0 = (1.217)(1)(5) = 6.09 \text{ kg/m}^2 \cdot \text{s}$$

From Guggioni and Zenz's procedure

$$\frac{u_0 - u_{mf}}{(\rho_s d_p)^{0.5}} = \frac{1 - 0.240}{(9.8)(60 \times 10^{-6})^{0.5}} = 31.3$$

$$G_0 / \rho_g u_0 = 1.6$$

$$k^* = G_0 = (1.217)(1)(1.6) = 1.95 \text{ kg/m}^2 \cdot \text{s}$$

d_p (μm)	30	40	50	60	80	100
Yagi and Aochi	—	—	3.21	3.23	2.87	1.87
Wen and Hashinger	—	—	1.94	2.16	2.12	1.52
Merrick and Hightley	10.1	4.19	1.80	0.83	0.21	0.05
Geldart et al.	—	—	—	7.89	3.85	1.44
Zenz and Weil	14.6	10.3	7.3	6.1	3.7	2.2
Guggioni and Zenz	—	7.3	3.7	1.9	0.49	0.1

So, in this case, again you can calculate here this correlation value in this case and then different correlations you just use, these correlations you can use from this table and based on which, just you can get that with Yagi and Aochi's correlations this you can get this k^* as this. And here Wen and Hashinger's correlations you can get k^* is equal

to 2.16 whereas, with Merrick and Highley correlations 0.834 and other correlations will give you the different value like this here.

So, as a summary, you can get that if you increase the particle diameter, you will see Yagi and Aochi, it will decrease the constant for this elutriation and for Wen and Hashinger, it is seen that also here it will increase and then decrease. And Merrick and Highley, as per that, it will decrease decrease and decrease very it is substantially decreasing that with respect to particle size. Geldart et al it is seen that, also the particle size will decrease the elutriation rate. Zenz and Weil it is seen that, that as per the particle size again this elutriation rate will decrease, but Gugnioni and Zenz et al as per their calculation purpose the correlation to seen that also this d_p will decrease the that is rate constant there.

So, very interesting that, we can then calculate what will be the elutriation rate from this lecture, and what is the elutriation rate? What are the different methods to calculate or estimate the elutriation rate and what is the elutriation rate constant and based on the different experimental condition how, how this elutriation rate can be estimated by different correlations that we can get from this. So, this will be helpful for the design further information and further understanding of the fluidization phenomena inside the bed ok. So, next class on what will be our next lecture will be will be something else on that fluidized bed we will be discussing later on.

Thank you.