

Fluidization Engineering
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Lecture – 19
Entrainment Characteristics (Part 1): Entrainment Characteristics

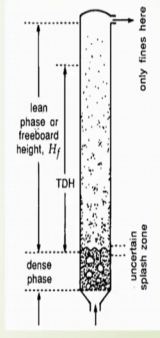
Welcome to massive open online course on fluidization engineering. Today's lecture will be on entrainment characteristics in a fluidized bed part 1 and previously, we have discussed about the bubble characteristics include as bed, in gas solid fluidization and gas liquid solid fluidization system. Their different characteristics of bubble and that is it is size what is the rise velocity of the bubble and for to be the solid movement surroundings as the bubble? And how the coalitions and breakup of the bubble is being happened in the fluidized bed?

All those characteristics has already been discussed and also the slugging characteristics are based on the flow of the bubble inside the bed. Now, whenever the bubbles will movement bubbles will move when the fluidized bed. How the solid particles will be entrained because we know that the bubbles that are formed, it will have some wake region and from that wake region solid particles will be entrained by the bubbles and now the characteristics of that entrainment in the fluidized bed will be learning something in this lecture. In this case of course, how this entrainment happens that is mechanism of the entrainment also why should why should we learn this entrainment characteristics?

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What is Entrainment?

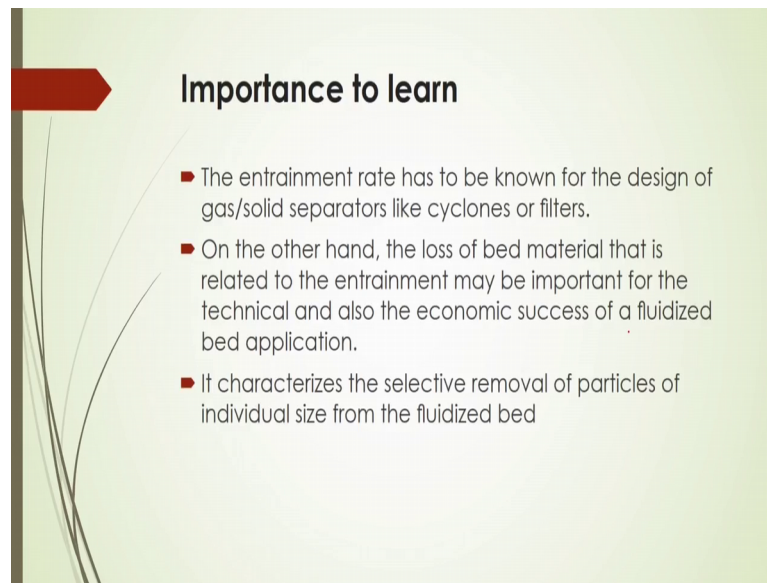
- **Definition:** Entrainment means the flux of solids carried out of the fluidized bed by the gas in kg per unit cross-sectional area and second
- **Denoted** by G_s ($\text{kg}/\text{m}^2\cdot\text{s}$)



Now, before going to that first of all we have to know; what is that entrainment. Now as a definition you can say that the entrainment means the flux of solids that is being carried out the fluidized bed by the gas in kg per unit cross sectional area and second; that means, per unit time how much solids per unit cross sectional area will be actually carried out by the gas that is called entrainment.

It is generally denoted by the symbol G_s for solid and G for that is flux of gas flux of solids that is being carried out by the gas. Now, see this picture here the fluidized bed in this case this fluidized bed of course, you know that that already, we have discussed there are 2 regions one is dense region and another is lean region they are. Above this dense region of course, the solid particles will be entraining along the height of this bed and then distribution of the solids also will be there the concentration of the solid particles along this axis, it will be varying because of the increment of characteristics of the solid particles by gas.

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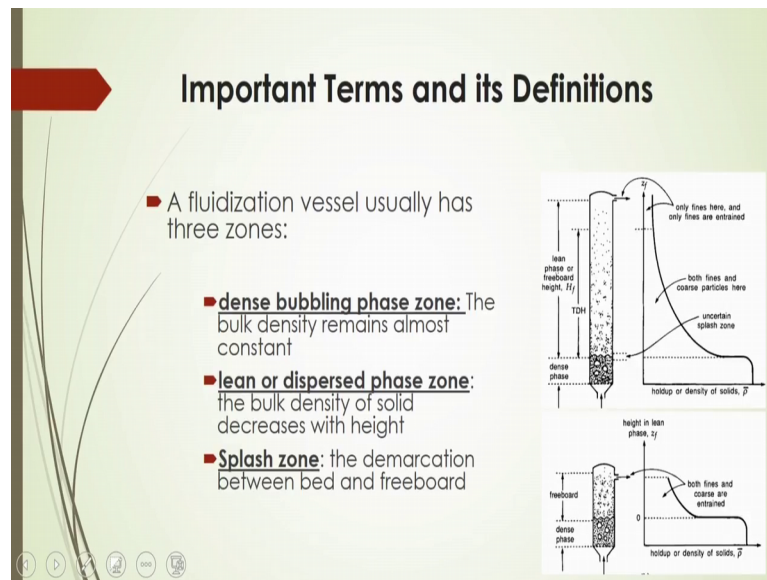
Now, the entrainment rates that has to be known for the design of gas solid separation like cyclones or filters. You have to know this what should be; that means, entrainment rate before going to design the gas liquid solid or gas solid fluidized bed.

On the other hand, you can say the loss of bed material that is related to the entrainment may be important for the technical and also the economic success of the fluidized bed application. Sometimes you will see some materials will be lost because of these entrainment characteristics. Some materials will be coming out from the top of the bed those who are very fine in size and also without losing these materials you have to optimize that loss by just recirculating the solid particles; that means, reusing that solid particles.

Now, what will be the amount of solid particles that is coming out from the top of the bed? That you have to know that is called entrainment rate based on which you will be able to calculate what will be the amount of solid that will be coming out from the top of the bed. On this entrainment characteristics also, it will govern the selective removal of particles of individual size from the fluidized bed.

We have to know that before going to design the fluidized bed what should be the entrainment characteristics inside the fluidized bed. Now, before going to that in details. Of course, you have to know some terms.

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And conditions also what will be the definitions of the different terms that is generally being used for analysis of these entrainment characteristics and fluidized.

Now, a fluidization vessel that usually has generally chi zones here you know that dense bubbling phase zone the bulk density remains almost constant here and the lean or dispersed phase zone the bulk density of the solid that decreases with height. Another impo another one important that is zone that is very narrow zone, it is called splash zone. This actually this is the demarcation between the bed and the freeboard; that means, this one here it is shown in the figure here this portion this is this is called the splash zone.

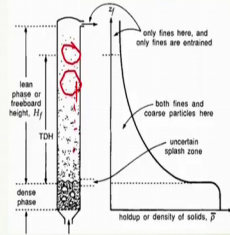
This is not a zone in this case the label will govern this the separation of this 2 zones of one is dense and another is lean. Separation of the separation label of this lean and the dense bubbling phase zone will be remarked as the splash zone this of course, it will be not in a linear, but it will be just may be in shaw shape.

In this case of course, we are we have to know these things that what is dense bubbling phase zone? What is lean dispersed phase zone? And the splash zone inside the bed.

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Important Terms and its Definitions

- **Freeboard:** The section of the vessel between the surface of the dense phase and the exit of the gas stream. The bulk density of solids decreases with height in the freeboard, increasing the freeboard decreases the entrainment from the bed.
- **Transport disengaging height (TDH):** a freeboard height above which entrainment does not change appreciably. When the gas stream exits above the transport disengaging height, or $H_f > TDH$, then both the size distribution and entrainment rate are close to constant.



Now, free board what is that free board the section of the vessel section of the vessel between the surface of the dense phase and the exit of the gaseous stream the bulk density of the solids decreases with height. In the free board where as it will increase the free board; that means, here decreases by the entrainment on the bed. This case you will see that very interesting point that if you increase the free board height, there will be a decrease of entrainment from the bed.

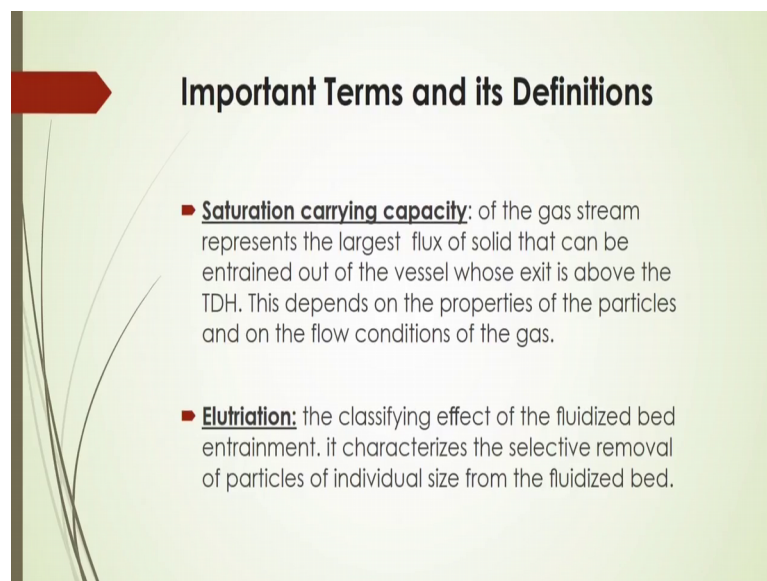
Very immediate zone of the dense you will see there will be a concentration or highly dense solid particles will be there by the entrainment; that means, here immediate zone of these dense there will be high concentration of solid particles though there will be there entrainment, but before after that; that means, along with the axis so; that means, here at a certain height there will be a decreasing trend of the solid particle concentration. There of course, this concentration sometimes it will be represented by the bulk density they are inside the bed.

That is why the free board section is that that beyond the dense zone. The section of this zone actually represents the trend of the solid particles concentration along this height of the free board. Another important point is that that transport disengaging height that is very important design parameter here a free board height above who is that entertainment that does not change appreciably that is very important.

Because, here you seen at a certain height there will be no change of concentration of the solid particles. They are so; that means, here entrainment does not change appreciably. When the gas extreme exists above the transport disengaging height; that means, HF if it is greater than TDH. TDH means here transport disengaging height.

When both the size distribution entrainment rate are close to constant. This is the case that what is the transported disengaging height that is called to the a free board. Height above who is entrainment does not change appreciably when the gas stream exists above the transport disengaging height; that means, you have to represent it that HF. HF means height of free board that will be if it is greater than transport disengaging height then the both the highest go to the size distribution and the entrainment rate are close to constant they are; that means, uniform particles those are very fine that will be there in the keyboard height.

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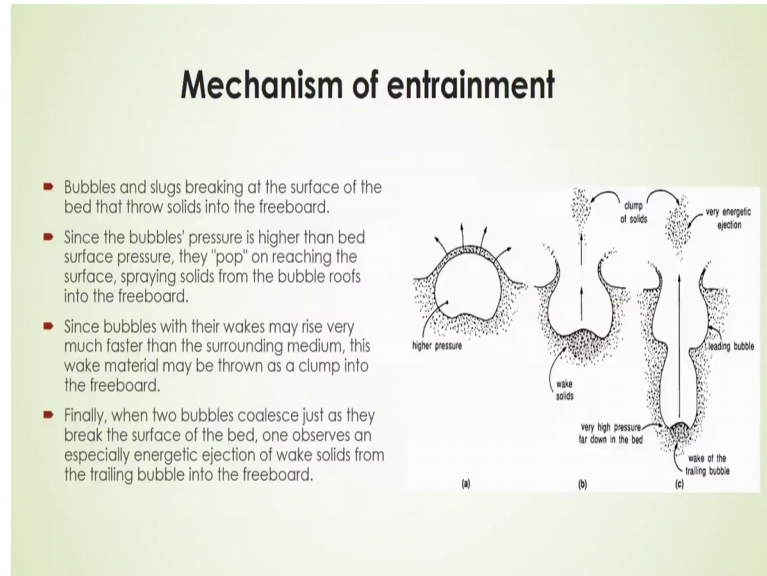
Important Terms and its Definitions

- **Saturation carrying capacity:** of the gas stream represents the largest flux of solid that can be entrained out of the vessel whose exit is above the TDH. This depends on the properties of the particles and on the flow conditions of the gas.
- **Elutriation:** the classifying effect of the fluidized bed entrainment. it characterizes the selective removal of particles of individual size from the fluidized bed.

Another important definition is that terms for he for it is definition you can say that saturation cap carrying capacity. The saturation carrying capacity is the gas stream represents the largest flux of solid that can be entrained out of the vessel whose exit is above the TDS this depends on the properties of the particles and on the flow conditions of the gas. Another important is point another important parameter is called elutriation the classifying effect of the fluidized bed entrainment. The it characterizes the selective removal of particles of individual size from the fluidized bed.

Saturation carrying capacity and the elutriation these are 2 important parameters which will actually affect the solid distribution inside the fluidized bed.

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Now, of course, that also because of the mechanism of the entrainment what is that mechanism of the entrainment how this mechanism of entrainment is happened. In this case you will see that bubbles and slugs. In break at the surface of the bed that will throw the solids into the free board and since the bubbles pressure is higher than bed surface pressure. Then you can say they pop up on reaching the surface and spraying solids from the bubble roofs into the free board.

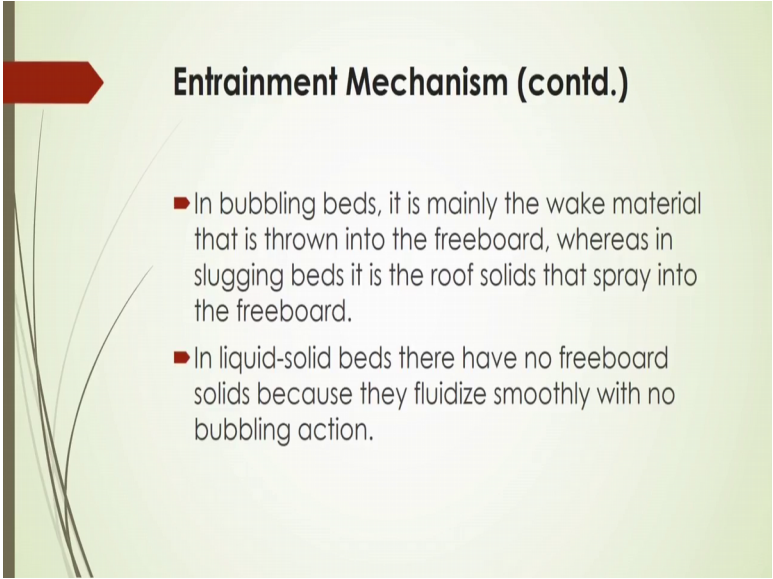
Here like this at the higher pressure here at the surface it will be breaking up and the solid will be throwing from this surface. Here this entrainment then here entrainment occur here and these solid fine solid particles it will be ejecting and then it will be going up that to the bed. Since, the bubbles which their wakes may rise very much faster than the surrounding medium this wake material may be thrown as a clump into the free board. Finally, you can say when 2 bubbles coalesce just as they break the surface of the bed then one observes an especially energetic ejection of the wake solids from the trailing bubble into the free board.

This is the mechanism by which that entrainment occurs. Basically, bubbles or slugs it is their. These bubbles or slugs it will break at the surface of the bed and then it is through the solid particles to the free board and again you will see that inside the bubble there

will be a high pressure then the bed pressure. Of course, in that case it happens. That there will be some pop. Whenever it will reach to the surface of the bed and then a spring the solids from the bubble roofs into the free world.

Also since the bubble with their wakes may rise very much faster than the surrounding medium, this wake material then may thrown as a as a clump you can say into the freeboard and finally, you can say that when 2 bubbles coalesce of course, they will be made as a bigger bubble and when they just reach, but to the surface it will break up and then you can see there will be some energetic that is rejection of the solid particles by breaking these bigger bubbles there and because of which their entrainment occurs. This is the common mechanism by which the solid particles can get entrained into the free board.

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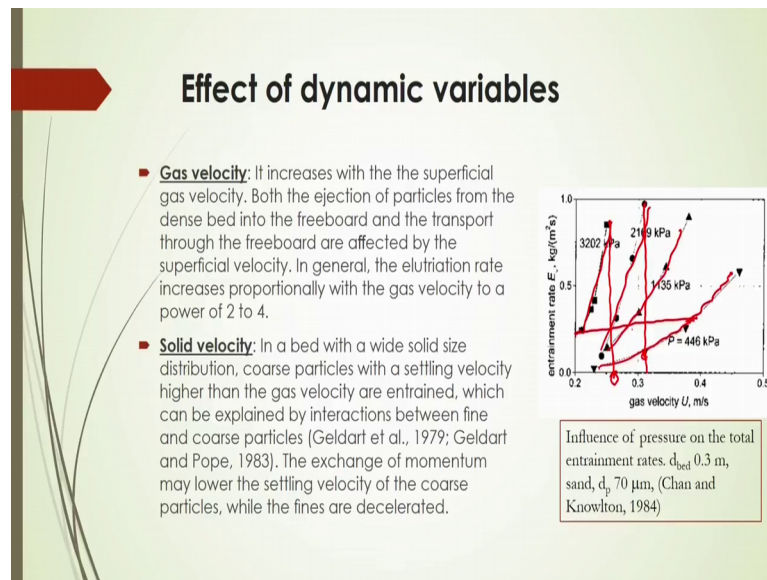
Entrainment Mechanism (contd.)

- In bubbling beds, it is mainly the wake material that is thrown into the freeboard, whereas in slugging beds it is the roof solids that spray into the freeboard.
- In liquid-solid beds there have no freeboard solids because they fluidize smoothly with no bubbling action.

In bubbling beds, you will see that it is mainly the wake material that is thrown into the free board whereas; in slugging beds it is the roof solids that sprayed to the free board. In solid beds sorry in in liquid solid beds you will see there have no free board of course, solids. Because they fluidize smoothly with no bubbling formation or action.

Of course there are some effect on this entrainment what are the effects or what are the different variables that affect on this entrainment. Now of course, the gas velocity is important to one.

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Because the that will give the that is kinetic energy supply by which that solid particles will be ejecting from the surface of the bed by breaking the bubble of course, that is why the gas velocity is important factor there when gas velocity increases, we will see that this entrainment characteristics will be increases; that means, here entrainment of solids will be higher. Both the ejection of the particles from the dense bed into the free board and the transport through the free board are affected by this gas velocity.

The gas velocity may be considered as superficial gas velocity or actual gas velocity. This actual gas velocity will be equals to that is superficial gas velocity divided by the porosity of the bed. If you increase the gas velocity you will see that more entrainment of the solid particles va to be inside the freeboard and in general the elutriation rate increases proportionally with the gas velocity to a power of 2 to 4.

Here elutriation of course, there the solid particles which will which are going to be elutriated or entrained to the bed that depends on this gas velocity. How it will be there of course, this elutriation rate increases proportionality with this gas velocity to the power of 2 to 4 there. Now, inside the bed of course, the solids of course, will be moving. What should be the solid velocity that also va affect the entrainment characteristics inside the bed.

In a bed, you will see with a wide solid size distribution coarse particles with a settling velocity that is more higher than the gas velocity are entrained. Very important that here

you have to have v_a the particular that is solid size distribution. Then here you will see that if the particles are very coarse; that means, the higher size; that means, settling velocity will be higher of course, than the gas velocity.

In that case, that those particles will not be entrained at that particular gas velocity even if the gas velocity is greater than that settling velocity of that coarser particles then only that entrainment of the particles will be happened and in that case, you will see sometimes if the wider size distribution what will happen that coarser particles and finer particles will be mixed mixture of those coarser and finer particles will be there; that means, wide size distribution.

In that case, the higher sized particles will be coming downward whereas, the finer sized particles should be going upward. Because, of the settling velocity which are greater than the gas velocity will be coming down. Whereas, the lower settling velocity are relative to the gas velocity that will be coming up. There will be a entrainment phenomena of only those fine particles who has that the settling velocity lower than the that is gas velocity. In that case, though because of that movement of the solid particles downward and upward based on that settling velocity and the relative velocity of the gas to the solid particles.

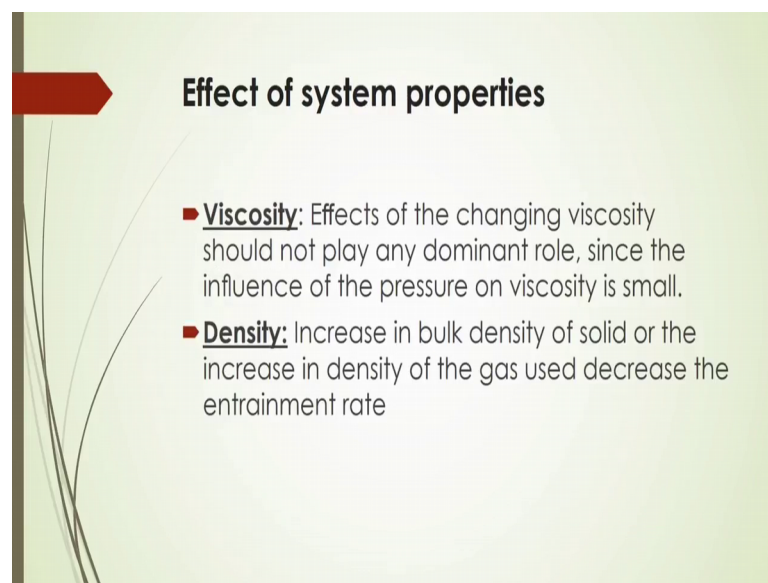
Then there will be an interactions between the fine particles and the coarser particles and then exchange of momentum between this coarser and the solid particles will happen and sometimes it may lower the settling velocity of the coarser particles and while the fines are decelerated. There is a phenomena in that case there will be an effective entrainment rate there. Because, of which that entrainment what entrainment will be there and what will be the interact interaction between the coarse particles and fine particles? And what will be the net rate of the entrainment? Depends on that interactions of the solid.

Very interesting is that if there wider size distribution you have to you will see there will be interaction. Otherwise, for uniform size distribution there will be no mass interaction which will hinder the entrainment rate. Only for wider size distribution there will be the decrease of entrainment rate there. Now, here in this picture you will see that that entrainment rate is increases with gas velocity it is seen that influence of pressure on the total entrainment rates also there.

It is seen that if you increase the pressure in the system what will happen that the entrainment rate will decrease. Because, you will see that for a particular gas velocity for a particular gas velocity you will see there. Higher the pressure inside that the entrainment rate will be higher for a particular gas velocity and here, of course, you will see their at a certain gas velocity here again this this higher pressure will give you the entrainment rate accordingly.

This is, but the gas velocity increases for a particular pressure then also the entrainment rate also will increase. Very interesting for a particular gas flow rate if you decrease the gas velocity there will be a decrease in entrainment rate. This results has been shown by this chan and knowlton 1984 for the particle size of 70 micrometer and the bed diameter of 3 centimeter or 30 centimeter.

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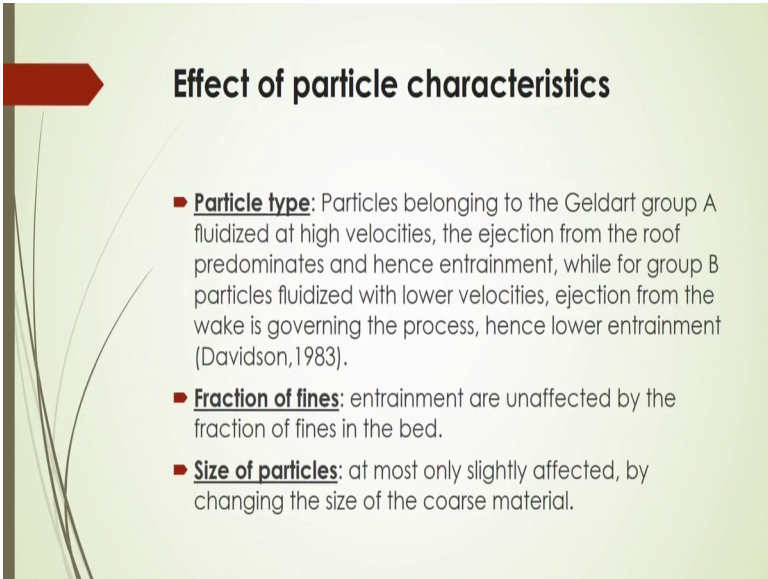


Now, effect of system properties of course, what is the system properties that viscosity and density you will see that viscosity should not play any dominant role for this gas solid fluidization system for it is entrainment.

Because this air is mostly used or some other gases whose viscosity is not that compared to that liquid. There will be a less effect there. Effect of changing viscosity should not play in that case any dominant role. Since, the influence of the pressure on the viscosity is a small and increase in bulk density of the solid or the increasing density of the gas used here in this case it will decrease the entrainment rate.

Density fact density factor is very dominant here because that density of the bulk solid or bulk density of the solid or the increasing density of the gas that will decrease the entrainment rate. Particle type, is there any effect on that particle type? That means, if you are using sand particles or glass particles or some other cool particles you can see that there will be a some change of that is entertainment rate. The particles belonging to the gilded group A.

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The slide features a light green background with a dark green vertical bar on the left. A red arrow points to the right, highlighting the title. The title is 'Effect of particle characteristics'. Below the title, there are three bullet points, each starting with a red square. The first bullet point is about 'Particle type', the second is about 'Fraction of fines', and the third is about 'Size of particles'.

Effect of particle characteristics

- **Particle type:** Particles belonging to the Geldart group A fluidized at high velocities, the ejection from the roof predominates and hence entrainment, while for group B particles fluidized with lower velocities, ejection from the wake is governing the process, hence lower entrainment (Davidson, 1983).
- **Fraction of fines:** entrainment are unaffected by the fraction of fines in the bed.
- **Size of particles:** at most only slightly affected, by changing the size of the coarse material.

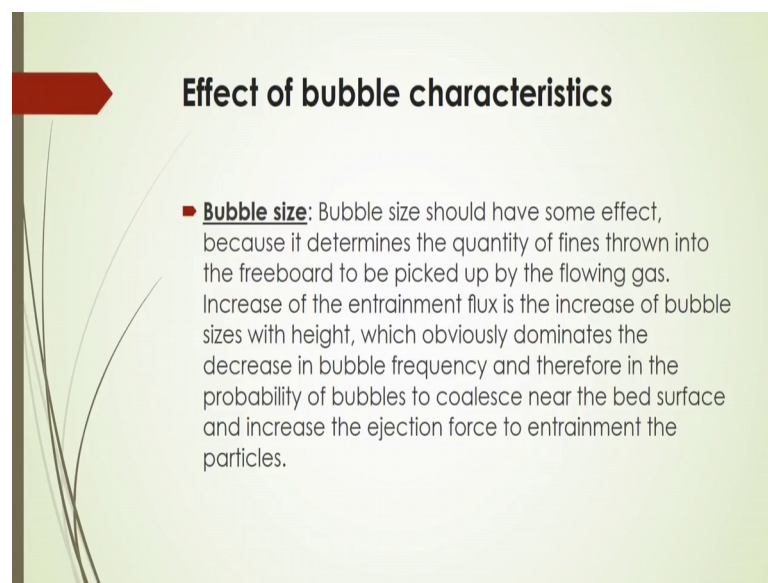
Of course, that aside, but group a fluidized at high velocity the ejection from the roof predominance and hence entrainment. While for group B particles fluidized lower velocities ejected ejection from the wake is governing the process. Hence, lower entrainment there. This a type or B type or C type that also based on that size of the particles and type sometimes it oak is there any adsorption characteristics of the solid particles of that. Then you will see there will be some hindering of the entrainment characteristics.

Because of that coalescences nature or suppose some solid particles they have some moisture adsorption is there. Because of which some the change of settling velocity and the interactions then there will be some change of entrainment rate inside the bed and fraction of fines entrainment are unaffected by the fraction of fines in the bed what will be the fraction of the solid particles is there.

It does not have any effect on the entrainment rate. Size of particles of course, at most only slightly affected by changing the size of the coarser material. That depends on the interaction how interaction of the solid particles. Whether, it is smaller or larger or finer then depends on that the entrainment characteristics. Very interesting point is that very and dominant factor is that bubbles size. Bubble size of course, you will see.

Because of these there will be entrainment rate change when because of that entrainment rate change.

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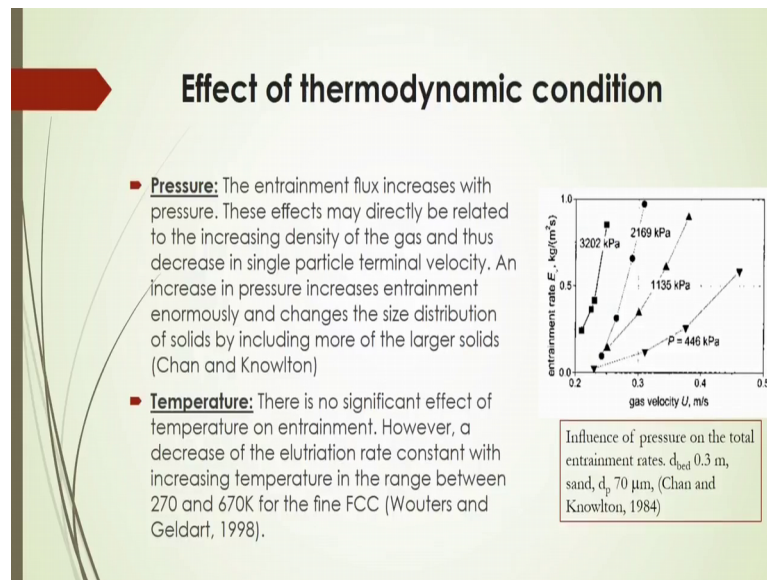


There will be heat transfer rate change also there will be a mass transfer rate change that huge effect of this of this bubble size there. Bubble size should have some effect here like because it determines the quantity of fines thrown into the freeboard to be picked up by the flowing gas.

Increase the entrainment flux is the increase of bubble size with height. We will see along with the height of the column there will be size bubble size will increase because of the coalesce and because of which there will be a entrainment rate increase. Which; obviously, dominates the decrease in bubble frequency of course, you will see you will get the less bubble there and therefore, in the probability of bubbles to coalesce near the bed surface and increase the ejection force there to entrain the solid particles there.

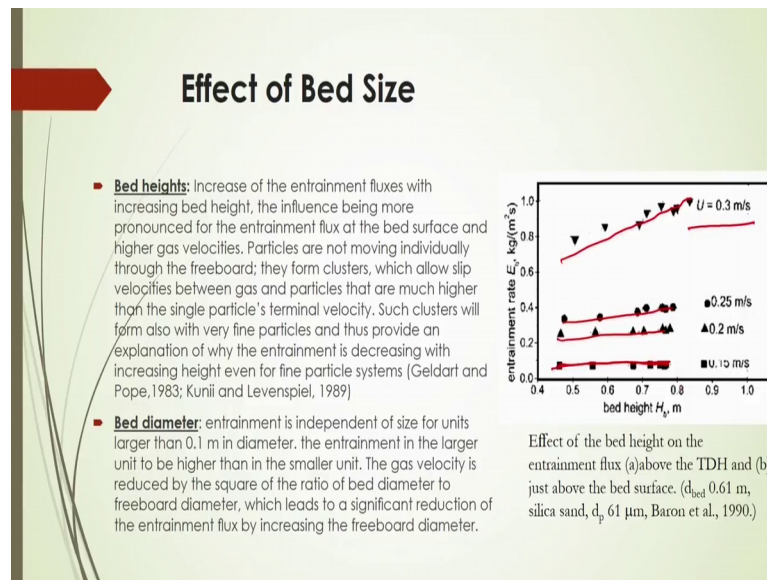
In this way, the bubble size will affect on the entrainment rate.

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Now, is there any effect of thermodynamic condition like is there any effect of pressure is there any effect by temperature or not the entrainment rate increases with pressure of course, you see these effects may directly be related to the increasing density of the gas or thus decrease in a single particle terminal velocity and increasing pressure increases the entrainment raised enormously and changes the size distribution of solids by including more of the larger solids according to chan and knowlton and there is no significant actual effect of the temperature on entrainment; however, you will see there will be a decrease of a entrainment rate constant or elutriation rate constant will be discussing later on that what is that elutriation rate. Elutriation rate constant to the increasing temperature in the range between 270 and 670 k for the fine fluidized catalytic cracking fluidized bed. In that case, wouters and geldart 1990 had 98, they have reported to the phenomena of the entrainment characteristics based upon the temperature.

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We have seen that this pressure will have effect on entrainment increasing the pressure entrainment will be more their effect of bed size here, you will see that increase of the entrainment fluxes with increasing bed height. The influence being more pronounced for the entrainment flux at the bed surface and higher gas velocities and you will see that particles will not move individually through the free board. They form some clusters they may allow slip velocities between gas and particles that are much higher than the single particles terminal velocity.

Such clusters will form will form also with fine very fine particles and thus provide and explore explanation of why the entrainment is decreasing with increasing height even for fine particle system there. Very interesting is that here you will see that along the axis if that gas velocity is changing and then you will see that there will be a kinetic energy distribution change and because of which there will be a change of entrainment rate and also if there is a cluster formation then there will be a slip between the between the gas and the solids and because of which there will be a some change of terminal velocity because of formation of clusters and then there will be increase in interaction of the solid particles with the clusters and the gas there and which will change the entrainment inside the bed. There may be some decrease in entrainment with increasing height there.

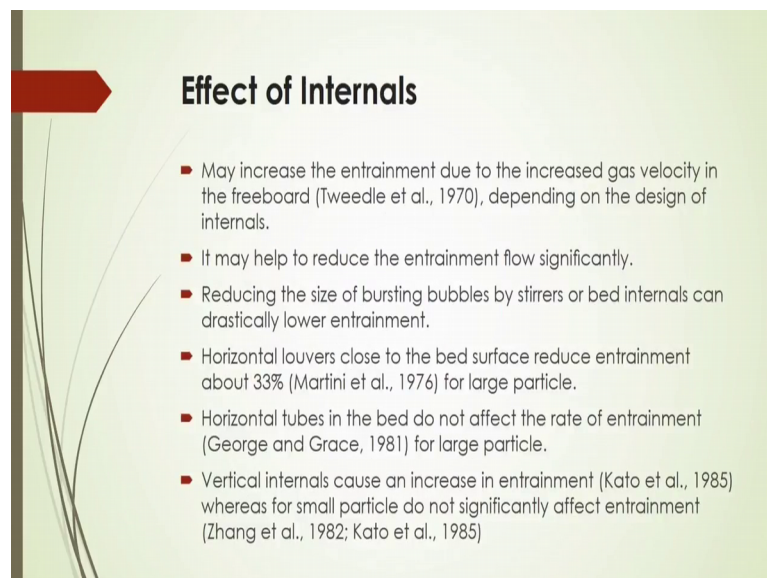
Because of that formation of the cluster and the interaction there bed diameter of course, you will see that independent of that bed diameter here the entrainment is independent of

size for units which is larger than 10 centimeter in diameter and the entrainment in the larger units of the bed will be there than the smaller unit because the larger diameter there will be a internal circulation of the solid particles which may govern the which may affect which may results in that is interaction of the solid. Is there any cluster then there will be a interaction and also that hindering the solid particles to move upward and downward and because of which there will be a change of entrainment inside the bed. The gas velocity is reduced by the square of the ratio of the diameter to the free board diameter which leads to a significant reduction of the entrainment flux by increasing the free board diameter. Here, in this figure, it is seen that how this enchantment rate is actually changing with bed height. It is seen that this bed height if you increase the bed height this entrainment rate will increase here at different gas velocity.

Here also you will see higher the gas velocity of course, the entrainment rate will be higher. Where as that lowest lower the higher gas velocity the entrainment will be lower, but also the increase of the entrainment with the bed height parallel will be there. This has been observed in that it is reported by baron et al 1990 by from their experimental data.

Above the TDH here you will see TDH and the just above the bed surface there they have done the experiment and they got these experimental results for the entrainment to bed height is there any effect of entrainment.

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Effect of Internals

- May increase the entrainment due to the increased gas velocity in the freeboard (Tweedle et al., 1970), depending on the design of internals.
- It may help to reduce the entrainment flow significantly.
- Reducing the size of bursting bubbles by stirrers or bed internals can drastically lower entrainment.
- Horizontal louvers close to the bed surface reduce entrainment about 33% (Martini et al., 1976) for large particle.
- Horizontal tubes in the bed do not affect the rate of entrainment (George and Grace, 1981) for large particle.
- Vertical internals cause an increase in entrainment (Kato et al., 1985) whereas for small particle do not significantly affect entrainment (Zhang et al., 1982; Kato et al., 1985)

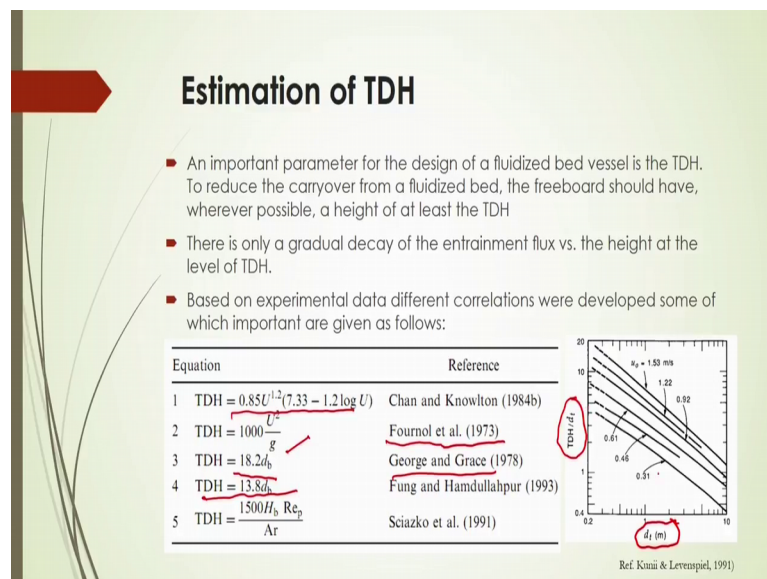
If there is any mechanical device is inserted into the fluidized bed or not if the suppose any internals is internal is the attached or added or insert or that is attached inside the bed. Then, what should be the rate of change of entrainment into the in the fluidized bed.

it may increase the entrainment due to the increased gas velocity. If you increase suppose some tube inside the bed or some baffles inside the bed then actual velocity of the gas will change and because of which the entrainment will change. There. That entrainment will increase with the gas velocity because of that increase of gas velocity by just internals depending on the design of that internals of course.

It may help the reduce the entrainment flow significantly and it reduces the size of busting bubbles by starrers or bed internals that can drastically lower the entrainment rate. Sometimes this horizontal lowers close to the bed surface reduce entrainment about 33 percent that is reported by martini et al 1976 for larger particle and horizontal cubes in the bed do not have any effect on the rate of entrainment that is reported by George and grace 1981.

That is also for large particles particle internals that cause an increase in entrainment that is wroted by kato et al 1985 where as for small particles do not have any significant effect on entrainment rate that as per that kato et al 1985.

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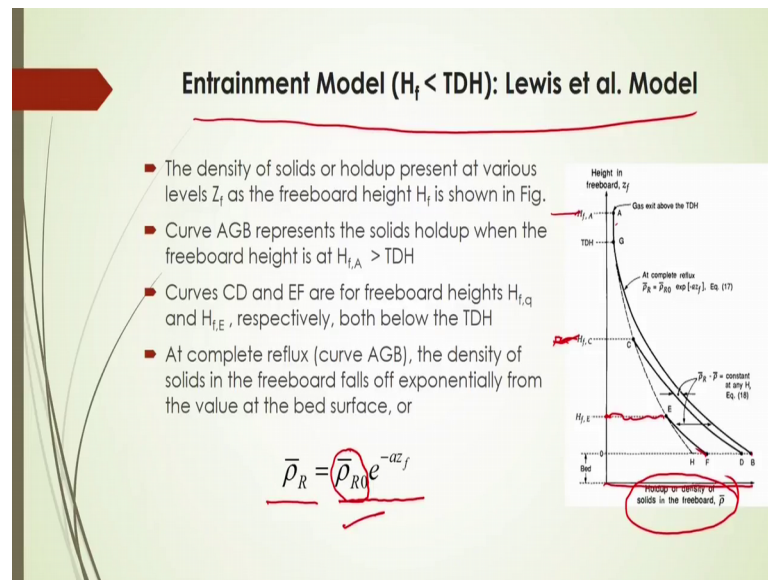


Now, how to estimate the TDH in the fluidized bed an important parameter it is called that TDH for the design of fluidized bed vessel and to reduce the carryover from the fluidized bed free board should have wherever possible a height of at least the TDH there.

TDH of course, there is only a gradual decay of the entrainment flux versus the height at the level of TDH based on experimental data different correlations that are proposed by several investigators, who serve very important some important correlations are given here for estimating the transport disengaging height inside the of the fluidized bed. Chan and knowlton 1984, they have given these correlations to find out this transport disengagement height that depends on the velocity of the fluid and also another important that is called $4nol$ et al TDH should be is equal to $1000 u$ square by g and George and grace 1978 they told that only TDH should be as a function of only what does that bubble diameter. It is only 18.2 times of bubble diameter. Whereas, fung and hemdalay ahpur 1993 they have given this another correlations that also they have explained that this TDH will be a function of bubble diameter and sciazko et al 1991 they actually proposed the correlation for TDH in terms of Reynolds number of the particle and Archimedes numbers.

They have stated that that TDH will be proportional to the Reynolds number of the particle and also the height of the bed and also the inversely proportional to the Archimedes number there. In this figure as per kunii and levenspiel that how this TDH is changing with tube diameter or you can say that fluidized bed diameter it is seen that this TDH ratio of TDH to bed diameter it is decreases with a bed diameter.

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And there are different models to express these what is that entrainment characteristics.

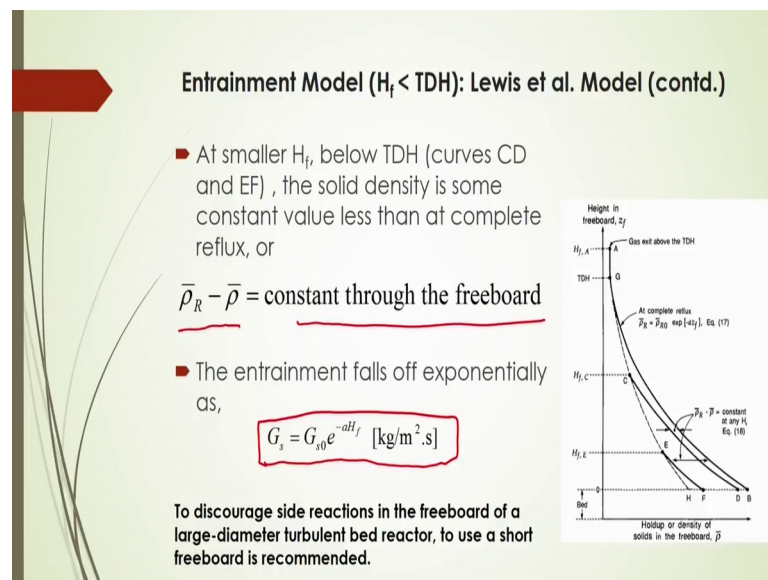
That means here to actually absorb or analyze the rate of entrainment in the fluidized bed now the here in this case entrainment model one entrainment model that is given by Lewis et al. Lewis et al they have given this entrainment characteristics if they consider that the height of the freeboard if it is less than TDH, then this density of the solids or hold up presents at various levels Z_f denoted by Z_f as the free board height H_f that is shown in the figure here if this y axis is represented by the height of the bed and x axis is represented by the holdup or density of the solids in the free board.

They have actually observed and from their experimental results and they have plotted it and they got this type of profile of this holdup along with this height of the bed. Now this in this case curve a g b in the figure curve a g and b in this figure you will see that curve a g b represents this solids hold up when the free board height is at is greater than TDH; that means, here height of the bed up to this here and curve c d in this curve c d here; that means, a solid particles c solid particles will coming out from this location and that is a TDH up to this curve c d and e f here also e f this e f r for free board heights of s f q and a s f e respectively.

Both below the TDH at complete reflux that is curve a g b complete reflux, the density of solids in the freeboard falls off exponentially from the value at the bed surface here. You can represent this exponential fall of this bulk density or you can say that hold up that

will be represented by this ρ_r that will be equal to ρ_{R0} into e to the power minus into Z_f . This is the exponential decay of this here a is the constant and Z_f is the height of the free board and ρ_{R0} also is the constants and that will be represented by the initial density of the solids inside the bed.

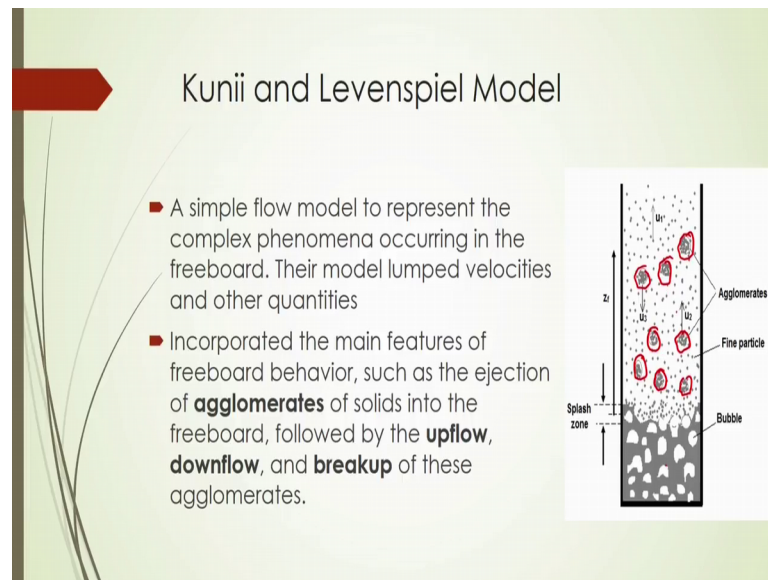
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Now, here at a smaller height of the free board that is below TDH this curve c d and e f the solid density is some constant value less than at complete reflux or you can represent it as ρ_R minus ρ that will be the constant through the free board the entrainment falls off exponentially which can be represented by this also this here G_s that will be equal to G_{s0} into the power minus $a H_f$.

Now, to discourage side reaction in the free board of a large diameter turbulent bed reactor to use a short free board is recommended. It is to be noted that when I should use when I should use the smaller; that means, freeboard height; that means, less than TDH. Generally, in the free board of a large diameter turbulent bed you can use this smaller free board of fluidized bed.

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Kunii and Levenspiel model they have also represented the characteristics of the entrainment and they proposed their model the how this entrainment rate will change with respect to height of the fluidized bed. They have suggest a simple model flow model to represent this complex phenomenon of the entrainment inside the bed here.

There modern actually their model lambda velocities and other quantities they have incorporated in their model. They consider the actually some agglomerates or what will be the up flow of the solids? What should be the break up phenomena of the agglomerates? Or how the ejection of the solid agglomerates will take place? All these things they have actually incorporated there.

They incorporated the main features of free board behavior such as the ejection of agglomerates of solids and into the free board and that followed by the up flow or down low and the breakup of these agglomerates there. These are the some agglomerates inside the bed they have considered here and whenever the solid particles would be moving upward by that entrainment mechanism. Here, these are the agglomerates and these this white portions are called bubble.

Bubbles are coming from the bottom and this is the black or escolar is called as the solid particles these solid particles, due to this breakup of the gaseous particles by ejection of this bubble break up at the surface. The solid particles will be thrown to the top and it

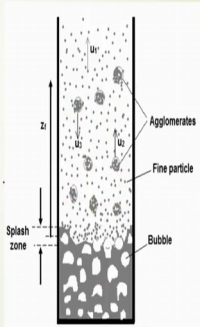
will be entrained. Now during that entrainment some solid particles will make agglomerates and these agglomerates will move here.

Now, in this case then agglomerates and the solid particles some solid particles will move downward so some solid particles will move upward, some agglomerates will go downward because of their higher settling velocity. Based on these, they have actually developed the model.

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Generalized version of Kunii and Levenspiel Model: Freeboard-Entrainment Model

- Consider a freeboard above bubbling or turbulent fluidized bed with the following assumptions:
- **Assumptions 1:** Three distinct phases are present in the freeboard.
- **Phase 1:** Gas stream with completely dispersed solids. The fines are carried upward and out of the bed at velocity u_1 while the coarse material rains back into the bed.
- **Phase 2:** Agglomerates, coming from the bed, and moving upward at velocity u_2 .
- **Phase 3:** Agglomerates and thin wall layers of particles moving downward at velocity u_3 .



Now, let us consider here, now, whenever they are just proposing this model or never derive in this model they consider a free board above the bubbling or turbulent fluidized bed with some assumptions.

What are those assumptions they have given some assumptions like assumption 1, they assume that 3 distinct phases are present in the free board that we have already talked about that some agglomerates like phase 1, phase 2, phase 3, what is that phase 1? That phase 1 is gaseous stream with completely dispersed solids there and the fines are carried upward and out of the bed. At velocity U_1 while the coarse material rains back into the bed.

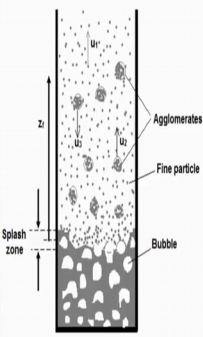
Here, very interesting that some fines that will be going upward at a certain velocity U_1 . Here, it is shown in the figure also the second phase there they are assumption that there will be some agglomerates and that will be coming from the bed and moving upward at a

velocity U_2 and phase 3 that agglomerates and thin wall layers of that particles moving downward at a velocity U_3 here. This U_1 U_2 and U_3 for this phase 1 phase 2 and phase 3 respectively.

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Freeboard-Entrainment Model (contd.)

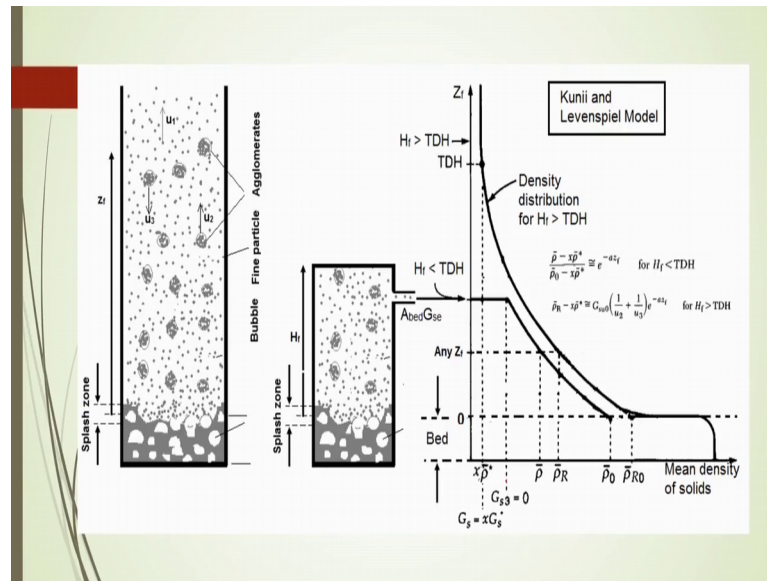
- **Assumption 2:** At any level in the freeboard the rate of removal of fines from the agglomerates to form dispersed solid of phase 1 is proportional to the volume fraction (or solid density) of agglomerates at that level.
- **Assumption 3:** Upward-moving agglomerates will eventually reverse direction and move downward, the frequency of change from phase 2 to phase 3 being proportional to the volume fraction of phase 2 at that level.



The diagram illustrates a vertical column representing a freeboard. It shows upward and downward arrows indicating flow directions. Agglomerates are depicted as larger, irregular shapes, while fine particles are smaller dots. A 'Splash zone' is indicated at the bottom of the column. Labels include 'Agglomerates', 'Fine particle', and 'Bubble'.

Assumption 2 that here at any level in the free board the rate of removal of fines rate of removal of fines from the agglomerates to form dispersed solids of phase 1 which is proportional to the volume fraction of the agglomerates at that level, then very interesting that some solid particles are also moving from the agglomerates to the dispersed solids of phase 1. What should be that criteria how much solids will be that will be proportional to the volume fraction of the agglomerates at that level there assumption 3 that upward moving agglomerates will eventually reverse direction because of if it if it is certain velocity will be going higher than the frequency of the change from this phase 2 to phase 3 being proportional to the volume fraction of the phase 2 at that particular level.

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Now, we have seen this figure. Very interesting that here we see how this profile is coming that whenever there will be a simultaneous movement of the phase 1 phase 2 and phase 3 even if TDH is greater than s_f is greater than TDH and s_f less than less than TDH. In that case, the what should be the profile this is the same exponentially decay profile of this bulk density. Because, of this dense entrainment. From this you can get the nature of this entrainment rate there from this calculation.

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Now consider at any level Z_i in the freeboard Mass flux $[kg/m^2.s]$

- G_{s1} = the mass flux of phase 1
- G_{s2} = the mass flux of phase 2
- G_{s3} = the mass flux of phase 3
- ρ_1 = mass of phase 1 per unit volume of freeboard $[kg/m^3]$
- ρ_2 = mass of phase 2 per unit volume of freeboard $[kg/m^3]$
- ρ_3 = mass of phase 3 per unit volume of freeboard $[kg/m^3]$

Then at steady state conditions the net upward flux of solids at any level in the freeboard is given by

$$\bar{G}_s = \underbrace{G_{s1} + G_{s2}}_{G_{su}} - \underbrace{G_{s3}}_{G_{sd}} = \underbrace{(\rho_1 u_1 + \rho_2 u_2)}_{G_{su}} - \underbrace{\rho_3 u_3}_{G_{sd}} \quad (1)$$

Now, consider at any level suppose z in the freeboard mass flux is represented by a k g meter k g per meter square second and if we consider that $G_s 1$ is equal to the mass flux of the phase 1 and $G_s 2$ the mass flux of the phase 2 and $G_s 3$, the mass flux of phase 3 and also, if we consider that ρ_{o1} is the mass of phase one per unit volume of the freeboard and ρ_{o2} is the mass of mass of phase 2 per unit volume of free board and ρ_{o3} is the mass of phase 3 per unit volume of free board, then at steady state conditions the net upward flux of the solids at any level in the fluid freeboard of this fluidized bed. That can be expressed as that G_s that will be equal to net upward flux of the solids at the level z that will be is equal to what will be the summation of that what is that mass flux of phase 1 and phase 2 those who are moving upward and what will be the mass flux of phase 3? That is moving downward what will be the net rate? That means, upward mass flux and the this is downward mass flux what will be the net that is this minus this.

From which you will be able to calculate what will be the net upward flux of the solids that is being carried out by the entrainment in the fluidize free board of these fluidized bed. Here this $G_s 1$ will be defined as ρ_{o1} into U_1 and $G_s 2$ will be defined as ρ_{o2} into u_2 . So, this will be your upward mass flux of the solid and $\rho_{o3} U_3$ that is $G_s 3$ igit it is a downward portion of the solid. That is downward mass flux of the solids in the free board.

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Also, the average (or bulk) density of solids at any level in the bed is

$$\bar{\rho} = \frac{W_s}{V_{bed}} = \frac{W_{s1} + W_{s2} + W_{s3}}{V_{bed}} = \frac{W_{s1}}{V_{bed}} + \frac{W_{s2}}{V_{bed}} + \frac{W_{s3}}{V_{bed}} = \rho_1 + \rho_2 + \rho_3 \quad (2)$$

Let x be the fraction of fines in bed for which $u_i < u$.

Due to vigorous turbulence or mixing, the particles may inter-transfer the phase from each other like

Rate coefficients for transfer:

- xK_1 for the transfer of fines from phases 2 and 3 to 1, and
- K_2 for the transfer from phase 2 to phase 3.

Now, what should the average or you can say bulk density of the solids at any level in the bed. That will be represented by $\bar{\rho}$ here what is that $\bar{\rho}$? $\bar{\rho}$ is thus defined as what will be the mass of the solids and what will be the volume of the bed. That is divided mass of the solids by volume of the bed that is denoted by the average bulk density of the solids in the bed at any level.

This will be is nothing but what is the mass of the solids that will be what is the mass of solids for phase 1 mass of solids for phase 2 what will be the mass of solids for phase 3 if you sum of these and if you divide that summation by this volume of the bed, then you will get the average bulk density now individually if you divide it by this bed volume then you can get it as ρ_1 ρ_2 and ρ_3 .

Now, let x be the fraction of fines in the bed for which U_t is less than U ; that means, here some solid particles whose setting velocity will be less than U and those particles would be going upward now that fraction is represented by x here. If there is a fraction of the fines which is denoted by x and it will be going upward if they are setting velocity will be less than fluid velocity.

Due to vigorous turbulence or mixing the particles may inter transfer the phase from each other like here. If suppose this is in this case the here this is phase 1 phase 2 phase 3 there will be inter transfer from phase 2 to phase 1 there will be a transfer of solid particles similarly from phase 2 to phase 3 there will be a solid transfer and phase 3 to phase 2 there will be a solid transfer.

Rate coefficients for this transfer can be expressed as that xK_1 that is for fine solids for the transfer of fines from phase 2 to phase 3 and to phase 1; that means, here xK_1 for the transfer of fines that is here it will be represented for this rate coefficients that is that is for only transfer of fines from phase 2 to phase 3 and phase 1. For this this is your transfer coefficient this is your transfer coefficient here again for phase 3 to phase 1 what should be the transfer coefficient that will be represented by K_2 .

This K_2 define for the transfer from phase 2 to phase 3 here.

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Mass Balances at Level Z_f

For phase 1: $\left(\begin{array}{c} \text{Increase of solids} \\ \text{in phase 1} \end{array} \right) = \left(\begin{array}{c} \text{Transfer of solids from} \\ \text{phase 2 and 3 to 1} \end{array} \right)$

$$u_1 \frac{d\rho_1}{dz_f} = xK_1(\rho_2) + xK_1(\rho_3) \quad (3)$$

For phase 2: $\left(\begin{array}{c} \text{Decrease of solids} \\ \text{in phase 2} \end{array} \right) = \left(\begin{array}{c} \text{Transfer of solids from} \\ \text{phase 2 to 1 and 3} \end{array} \right)$

$$-u_2 \frac{d\rho_2}{dz_f} = xK_1(\rho_2) + K_2(\rho_2) \quad (4)$$

Now, if we do the mass balance at level Z_f then you will see for phase 1 this will be your; that means, here increase of solids in phase 1 that will be transfer of solids from phase 2 to 2 and 3 to 1. By then you can represent this equation that increase of solids in solids by this and then transfer will be represented by this.

Similarly, for phase 2 you can represent the transfer equation by this equation 4 and for phase 3.

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for phase 3: $\left(\begin{array}{c} \text{Decrease of solids} \\ \text{in phase 3} \end{array} \right) = \left(\begin{array}{c} \text{Transfer of solids} \\ \text{from phase 2 to 3} \end{array} \right) - \left(\begin{array}{c} \text{Transfer of solids} \\ \text{from phase 3 to 1} \end{array} \right)$

$$-u_3 \frac{d\rho_3}{dz_f} = K_2(\rho_2) - xK_1(\rho_3) \quad (5)$$

Since all solids that reach the freeboard height H_f leave the vessel, there is no downflow there, so

$$\text{At } z_f = H_f, \quad \rho_3 = 0 \quad (6)$$

Only upflow agglomerates of solids projected into the freeboard from the dense bed, so

$$\text{At } z_f = 0, \quad \rho_1 = 0 \text{ and } G_{s0} = \rho_2 u_2 \quad (7)$$

Solving equations (3 to 5), one can get expression for ρ_1, ρ_2, ρ_3 with these boundary conditions

Again, the decrease of solids in phase 3 that will be transfer of solids from phase 2 to 3 minus the transfer of solids from phase 3 to 1 which can be represented by this equation 5 now since all solids that reach the free board height at Z_f leave the vessels there is no down flow therefore, we can say that at Z_f is equal to H_f there will be no ρ_3 that is density of the phase 3.

Only up flow agglomerates of the solids projected into the free board from the dense bed. You can write here is Z_f is equal to 0 ρ_1 should be 0 and you can say initial mass flux of the solids that is upward that will be is equal to only U_2 into ρ_2 now solving this then equation 3 to 5 1 can get expression for row 1 row 2 and row 3 with these ever boundary conditions.

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With Eq. (1), the net flux of solids at any freeboard level and at the vessel outlet can then be expressed as

$$\frac{G_s - xG_s^*}{G_{s0} - xG_s^*} = e^{-aH_f} \quad (8)$$

where G_s^* is the flux of carryover from a very tall vessel fluidizing only entrainable solids when $H_f > TDH$.

$$\frac{xG_s^*}{G_{s0}} = \frac{x(K_1/K_2)(1+u_2/u_3)}{1+x(K_1/K_2)(1+u_2/u_3)} \quad (9)$$

$$a = \frac{K_2}{u_2} \left[1 + \frac{xK_1}{K_2} \left(1 + \frac{u_2}{u_3} \right) \right], \quad [\text{unit is m}^{-1}] \quad (10)$$

Now, with equation one the net flux of the solids at any free board level and at the vessel outlet that can be expressed by substituting this ρ_1 and ρ_2 and expressing by mass flux which can be expressed as here by equation H. Here, in this case, then you can say what will be the mass flux of the solids that is being entrained by that bubble inside the bed that will be represented by this.

This is a function of a h_f that is as that is along the height of the bed how these mass flux will be changing here. In this case, where G_s^* is one important parameter which is represented as the flux of carryover from a very tall vessel fluidized that will only entry level solids here when s_g is greater than TDH .

xG_s star by G_{s0} that will be is equal to this in this case and a what is that that is called some constants in this case. This decay constants will be represented in terms of this transfer coefficient by this equation 10.

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Special case

- For a vigorously bubbling bed or a turbulent bed with few fines

$$xG_s^* \ll G_{s0} \quad (11)$$

- So, from equation (9) and (10) one can get

$$\frac{xK_1}{K_2} \left(1 + \frac{u_2}{u_3} \right) \ll 1 \text{ or } \frac{xK_1}{K_2} \ll 1 \text{ implies } a = \frac{K_2}{u_2} \quad (12)$$

- Therefore the equation (8) becomes

$$G_s - xG_s^* \cong G_{s0} e^{-\left(\frac{K_2}{u_2} \right) H_f} \quad (13)$$

These expressions provide a physical interpretation of the parameters reported by Lewis et al.

And special cases for a vigorously bubbling bed or a turbulent bed with few fines you can say that xG_s star that will be very less than the initial mass flux of the solids that is coming upward.

From equation 9 and 10 with this equation 11 you can say that this truth of this equation here xK_1 by K_2 into $1 + u_2$ by u_3 that would be very less than equals to 1 or you can say xK_1 by K_2 any less than less than equals to 1 which implies that a ; that means, decay constant this a will be is equal to K_2 by U_2 what is that a here in this equation 8 that a is some constant here that will be that will be governing the entrainment characteristics there inside the free board.

Which you can say. This a is a function of actually the velocity of the phase there 2 therefore, the equation 8 we can write that G_s minus xG_s star that may be very small here that will be the G_{s0} in G_{s0} ; that means, into e to the power minus K_2 by U_2 into in to $s \cdot m$.

Now, this a is represented by K_2 by U_2 . These expressions provide a physical interpretations of a parameters that is reported by Lewis et al. From this relation will be

able to calculate you will be able to know what should be the actually entrainment rate as a function of height of the fluidized bed.

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Special case

upward flux can be obtained by substituting the densities ρ_1, ρ_2 in equation

$$G_{su} = G_{s1} + G_{s2} = \rho_1 u_1 + \rho_2 u_2$$

$$\frac{G_{su} - xG_s^*}{G_{su0} - xG_s^*} = e^{-aH_f} + e^{-[aH_f - (a-b)z_f]} \quad (14)$$

$$b = \frac{xK_1 + K_2}{u_2}, \text{ and } a-b = \frac{xK_1}{u_3}$$

Since for $xG_s^* \ll G_{su0};$
 $xK_1 \ll K_2; \quad a-b \ll a;$

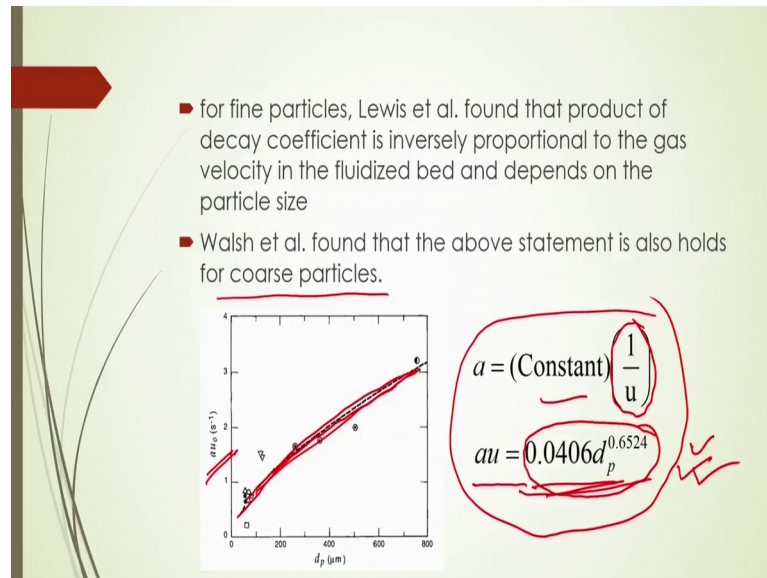
Since always $z_f \leq H_f$

$$\frac{G_s - xG_s^*}{G_{su0}} = e^{-az_f} \quad (15)$$

Another important that what should be the upward flux that can be obtained from this this concentration or bulk density if you know by solving those equations that already we have discussed earlier.

That the upward flux that can be obtained by substituting the densities ρ_1 ρ_2 in equation like this, here if you substitute here then you will get this one and then finally, you can represent these mass flux by this equation here and in this case some constants b and also a there which will be defined by these equations and this Z_f always should be less than equals to H_f . In that case $a - b$ into to Z_f that less than less than equals to Z into a $h f$ from the mass balance equation and since for this equals to xG_s^* less than less than G_{su} and xK_1 less than is K_2 $a - b$ should be very less than a . In this regard, you can express the mass flux profile by this equation 15 as $G_s - xG_s^*$ by G_{su0} is equal to the power minus aZ_f . From this equation this equation will give you the profile of that entrainment rate or entrainment flux as a function of height of the free board.

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Now, for fine particles Lewis et al found that the product of decay coefficient that is a is inversely proportional to the gas velocity in the fluidized bed and that depends on the particle size. Walsh et al found that above statement is also holds for coarse particles. It is it can be then said that that a will be equals to constant of $1/u$; that means, here is proportional to the inverse of the velocity of the fluid and then a you will be equals to here that constant you can say au is equal to constant that some time depends on the particle diameter that you can represent in this here by this equation.

You see this figure that how au changing with particle diameter here if au will increase with the particle diameter. From this you can say or this profile you can represent by this equation au will be equals to 0.0406 into d_p to the power 0.6524 here.

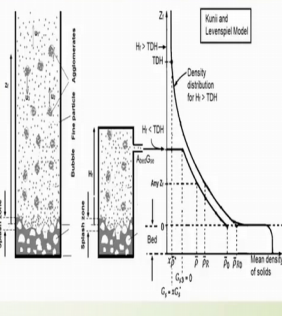
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Entrainment profile In terms of Density

- The entrainment profile can be expressed as density once one solved for the densities (ρ_1, ρ_2, ρ_3) from the entrainment model for different phases as

$$\bar{\rho}_R - x\bar{\rho}^* = G_{s0} \left(\frac{1}{u_2} + \frac{1}{u_3} \right) e^{-\alpha z_f} \text{ for } H_f > \text{TDH}$$

$$\frac{\bar{\rho}_R - x\bar{\rho}^*}{\bar{\rho}_0 - x\bar{\rho}^*} \cong e^{-\alpha z_f} \text{ for } H_f < \text{TDH}$$

$$\bar{\rho}_0 - x\bar{\rho}^* \cong G_{s0} \left\{ \left(\frac{1}{u_2} + \frac{1}{u_3} \right) - \frac{1}{u_3} e^{-\alpha z_f} \right\} \text{ for } H_f > \text{TDH}$$


Now, you can represent this entrainment profile in terms of density also the entertainment profile can be expressed as density once all the densities rho 1 rho 2 and rho 3 from the entrainment model for different phases as these. From this equation you will be able to calculate what will be the entrainment or bulk density profile.

This also from those from these equations you will be able to calculate the entertainment profile in terms of density here.

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Example

- Estimate the entrainment flux from a short free-board of a fluidized bed. Assume that the bed is operated at a certain velocity of about 167.66 times of minimum fluidization velocity for all particle to be entrainable.

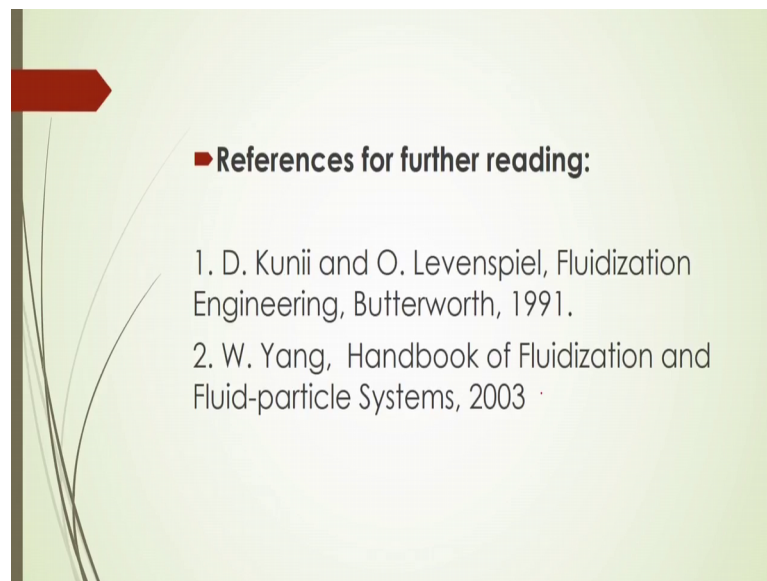
$$\bar{d}_p = 60 \mu\text{m}, \rho_p = 1.3 \text{ kg/m}^3, u_{mf} = 0.003 \text{ m/s}$$

$$\rho_s = 1500 \text{ kg/m}^3$$

Now, let us do an example. Estimated the entrainment flux from a short freeboard of a fluidized bed in this case assume that the bed the bed is operated at a certain velocity of about 167.66 times of minimum fluidization velocity for all particle to be entrainable.

D_p is given to you and then density of the gas is given to you minimum velocity of the fluidization is given to and the solid particle density is 15000 kg per meter cube.

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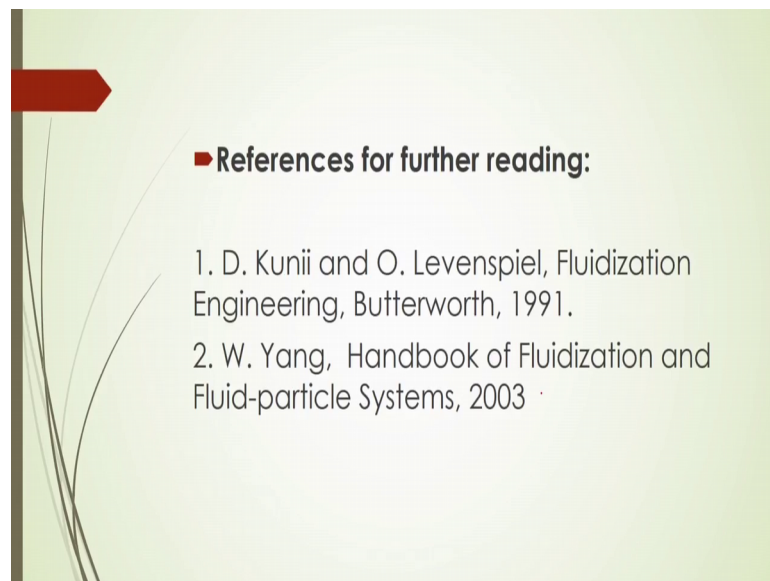


You just use what you have to find calculate what will be the U here and then calculate this u square by $G_d p \rho_s$ square and then find out this parameter G_a star into ρG_u that will be is equal to one from this figure one you can calculate and G_a star that you can calculate and $G_s U$. You can calculate from the figure 2 then what should be the a value as per a is equal to this equation and then $G_s u$ is equal to here as per this equation.

What we have learnt from this lecture that what should be the entrainment characteristics according to that model of that entrainment if we consider that phase phenomena inside the bed and the interaction of the phase if there is any agglomeration and fines and also the gaseous particles is there if there any interaction then based on their velocity what will be the net flow of the fines that is entrained by the movement of the gas bubbles inside the freeboard and how to express the profile of that entrainment characteristics there?

It is seen that that entrainment characteristics is decreasing exponentially that and also this entrainment characteristics depends on different other variables like pressure, temperature, even is there any internals or not even gas velocity, particle size distribution. We have learned a lot of things from this lecture about this entrainment characteristics. Next, we will discuss that what should the entrainment characteristics if the fluidization occurs very fast; that means, first fluidization case what should be the entrainment characteristics that will be discussed.

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In the next lecture and more about this information you can get from this 2 references of this text book then you will get more fundamentals of this entrainment characteristics.

Thank you.