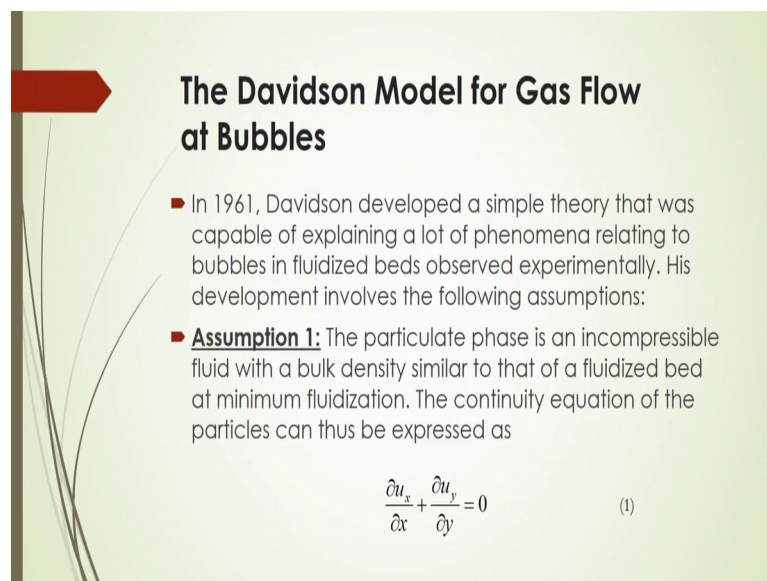


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**Lecture – 17**  
**Bubbling Fluidization Part 5: Gas and Solid Movements at Bubble**

Welcome to massive open online course on fluidization engineering, today's lecture will be on bubbling fluidization part 5, gas and solid movement at bubble, earlier we have discussed about the bubble coalesce and the breakup in a fluidized bed, in gas solid and gas liquid fluidized bed. In this lecture again, we will be discussing something about bubble characteristics, how the solid particles will be moving around these bubble in the bubbling fluidized bed.

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**The Davidson Model for Gas Flow at Bubbles**

- In 1961, Davidson developed a simple theory that was capable of explaining a lot of phenomena relating to bubbles in fluidized beds observed experimentally. His development involves the following assumptions:
- **Assumption 1:** The particulate phase is an incompressible fluid with a bulk density similar to that of a fluidized bed at minimum fluidization. The continuity equation of the particles can thus be expressed as

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \quad (1)$$

And here of course, we will see whenever solid particle will be moving surrounding the bubbles, there the solids will follow some pattern that is what should be the velocity or potential surrounding this bubble, and also what should be the pressure gradient surrounding the bubble, that depends on the bubble moving up or down based on the or according to the solid particle movement.

Now, sometimes means whenever the bubble will be moving up, there will be gradient of pressures surrounding the bubble, that may governs the solid particles moving upward or downward, even sometimes you will see there will be a some energy distribution

surrounding the bubble, because of how there will be some change of velocity pattern of the solid particles whenever the bubbles will be moving up.

Also, the relative velocity of the bubble if compared to the fluid velocity, sometimes this bubbles will be moving fast and sometimes this bubbles will be moving slow, and because of that movement of the bubbles related to the liquid velocity, it will change the flow pattern of the solid particles.

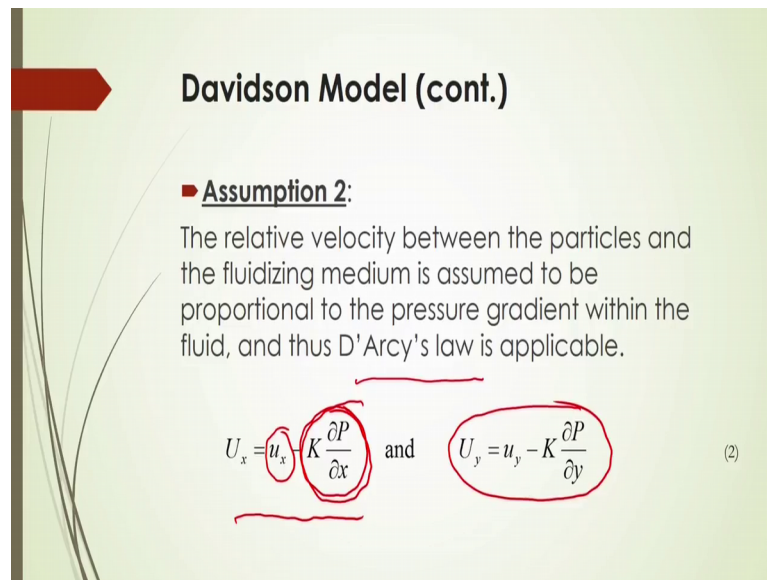
So, because this change of solid particle movement surrounding the bubbles, the enhancement or you can say the change or transport of the mass, from the bubble phase to the emersion phase or emersion phase to the bubble phase will be changed. So, we have to know what should be the characteristics of the solid movements surrounding this bubble.

Now, Davidson 1961 she developed a simple theory that was capable of explaining a lot of phenomenon of this solid movement relating to the bubble movement in the fluidized bed, and they have observed this phenomenon experimentally and they have suggested some models based on some assumptions, now the model actually basically based on this assumption there are several assumption. So, they the model consist the suggested by Davidson model before developing the Davidson model.

Now, what are those assumptions, let us see Davidson he assumed that, particulate phase surrounding the bubble is an incompressible fluid with a bulk density that is similar to that of the fluidized bed at a minimum fluidization. So, he assumes this one first. So, that is why assumption as a assumption one, that he actually considered that that particular phase surrounding this bubbles, will be acting as an incompressible fluid with a bulk density, that is similar to that of the fluidized bed, which is running at a minimum fluidization condition.

The continuity equation of the particle can then be expressed as this one, the equation number one here is given in the slides here, what is that  $\frac{d u_x}{d x} + \frac{d u_y}{d y}$  that will be is equal to 0, this is called continuity equation of the solid movement inside the bubbling fluidized bed, and in this x and y direction, what should be the velocity gradient that summation of this velocity gradient that will be equals to 0 so, this is called continuity equation. So, this assumption has been taken by this Davidson before developing this model.

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**Davidson Model (cont.)**

■ **Assumption 2:**  
The relative velocity between the particles and the fluidizing medium is assumed to be proportional to the pressure gradient within the fluid, and thus D'Arcy's law is applicable.

$$U_x = u_x - K \frac{\partial P}{\partial x} \quad \text{and} \quad U_y = u_y - K \frac{\partial P}{\partial y} \quad (2)$$

Now, assumption 2 they, that is Davidson he also assumed that, the relative velocity between the particles and the fluidizing medium, is assumed to be proportional to the pressure gradient within the fluid, and thus they have actually observed that this phenomenon is applicable as far as D'Arcy's law.

So, this what is that a D'Arcy's law? The D'Arcy's law is nothing but that what should be the relative velocity of the fluid particle, whenever it will be moving inside the bed surrounding this bubble, that is relative velocity will be expressed as what should be the velocity of the liquid, and what should be the pressure gradient which is so, this is here  $\frac{\partial P}{\partial x}$  that is sometimes  $K$  times this pressure gradient.

Now, this is your velocity relative velocity between the particles and fluidizing medium. So, based on this relative medium velocity of course, she developed the nice relationship between the fluid part solid particles, which is moving surrounding the bubble, and also in the  $y$  direction this D'Arcy's law will be is equal to  $U_y$  that will be equals to small  $U_y$  minus  $K$  of  $K$  of  $\frac{\partial P}{\partial y}$ . So, as per this D'Arcy's law, the relative velocity between the particles and the fluidizing medium can be calculated.

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### Davidson Model (cont.)

- **Assumption 3:**  
Fluidizing fluid is assumed to be incompressible, and thus the continuity equation with the assumption that the voidage is constant at  $\epsilon_{mf}$  can be written as

$$\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} = 0 \quad (3)$$

Now, as an assumption 3 the fluidizing fluid is assume to be incompressible here, and thus the continuity equation with the assumption that the voidage is constant at  $\epsilon_{mf}$ , this minimum porosity of the fluidizing condition that can be retained as here, the summation of this relative velocity gradient in x and y direction that will be is equal to 0.

So, this continuity equation in terms of relative velocity of the fluid particle compared to this liquid medium; then can be expressed by this continuity equation 3.

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### Davidson Model (cont.)

- **Assumption 3 (cont.):**
- Eliminating the velocities from Eqs. (1) through (3), we have

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0 \quad (4)$$

- Equation (4) is exactly similar to that for a fixed bed, and thus the pressure distribution in a fluidized bed is unaffected by the motions of the particles in a fluidized bed.

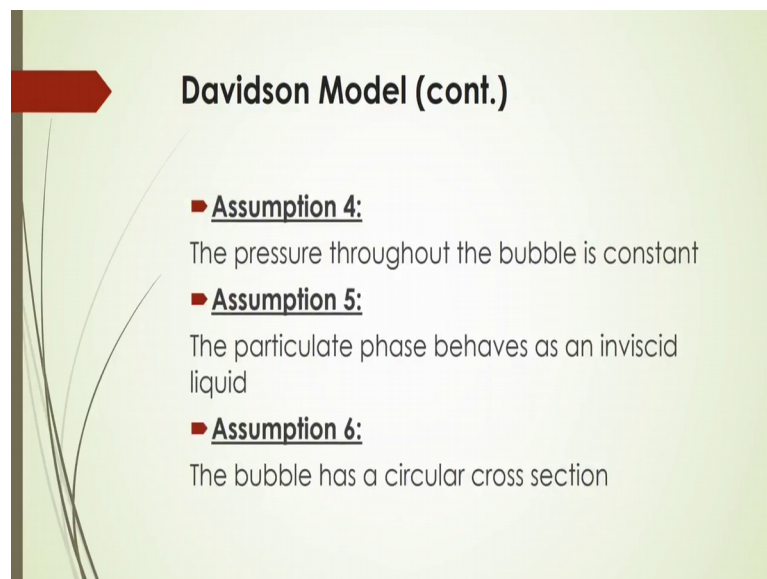


Again, this based on these the eliminating the velocity from the equation one through equation 3, we can get these, this in terms of this pressure what should be the continuity equation, and we can then express here this  $\rho u^2$  or  $\rho u^2$  by  $\rho x^2$  plus,  $\rho u^2$  by  $\rho y^2$  is equal to 0.

So, eliminating the velocity in terms of pressure, this equation can be expressed, now this equation is exactly similar to that for a fixed bed, actually in fixed bed condition this relationships satisfy very well, the pressure distribution in a fluidized bed that will be unaffected by the motion of the particle which is of course, the same as per fixed fluidized bed.

So, this is very important that we are considering here or as per David Davidson model, the assumption is see that that of course, the pressure distribution in a fluidized bed as per this equation will be unaffected by the motion of the particles in the bed.

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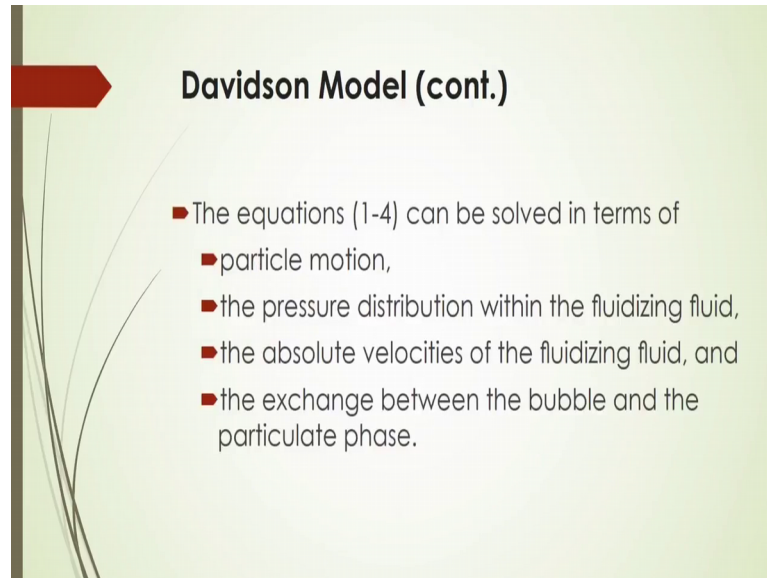
**Davidson Model (cont.)**

- **Assumption 4:**  
The pressure throughout the bubble is constant
- **Assumption 5:**  
The particulate phase behaves as an inviscid liquid
- **Assumption 6:**  
The bubble has a circular cross section

Now, other assumptions like here assumption 4, the pressure throughout the bubble is constant; that means, what will be the pressure surrounding the bubble, the through the bubble or through the bubble surface that will be constant, and assumption 5 the particulate phase that will behave as an inviscid liquid, there will be no viscous effect of the particulate phase that he has considered as in assumption 5.

Now, assumption 6 that the bubble has a circular cross section of course, it is obvious we have to consider that the bubble cross sectional area, what should be that should be circular cross section.

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**Davidson Model (cont.)**

- The equations (1-4) can be solved in terms of
  - particle motion,
  - the pressure distribution within the fluidizing fluid,
  - the absolute velocities of the fluidizing fluid, and
  - the exchange between the bubble and the particulate phase.

And here then from this equations one to 4, can be solved in terms of that like particle motion, what will be particle motions surrounding this bubble, what should be pressure distribution within then fluidizing fluid and the absolute velocity of the fluidizing fluid, and also the what should be there exchange of the gas between the bubble and the particulate phase, and sometimes this exchange will not be only gas may be small amount of particle also may be exchanged from the bubble to the emersion phase or the emersion phase to the bubble.

So, based on this or in term of this like a particulate motion pressure distribution, absolute velocity and the exchange rate. So, equation one to 4 can be solved to get the bubble characteristics, or you can say that solid movement behavior or gas movement behavior or relating to the bubble motion, what should be that can be actually accessed, now as per this Davidson model, this Davidson model that it is it can be derived that the fluid distribution function what should be the fluid distribution function here.

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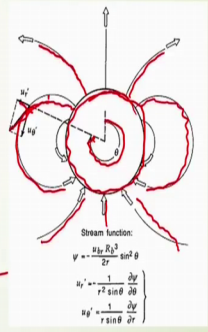
### Davidson Model (cont.)

■ Davidson derived the fluid stream function as

$$\psi_f = \left( U_B - \frac{U_{mf}}{\epsilon_{mf}} \right) \left[ 1 - \left( \frac{R}{r} \right)^3 \right] \frac{r^2 \sin^2 \theta}{2} \quad (5)$$

$$\left( \frac{R}{R_B} \right)^3 = \frac{U_B + 2(U_{mf} / \epsilon_{mf})}{U_B - 2(U_{mf} / \epsilon_{mf})} \quad (6)$$

Source: Yang, 2003



Stream function:  
 $\psi = \frac{U_B R_B^3}{2r} \sin^2 \theta$   
 $u_r' = -\frac{1}{r^2} \sin \theta \frac{\partial \psi}{\partial \theta}$   
 $u_\theta' = \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r}$

Source:  
Kunii and Lenenspiel, 1991

This fluid distribution function see the picture here this is one bubble, this bubble and surrounding this bubbles there will be a pressure gradient, and also the velocity of the gas, and also fluid you can say there will be some gradient kinetic energy distribution, because of this which the solid particle will here from this location will move aside and from the bottom solid particles will come to the bubble surface and.

Here at this location exactly heat this solid particle at the surface, now here in this case the movement of the solid particles will be like this here this. So, this is the stream line or stream path you can say of the solid particles surrounding this bubbles, how it will be moving the by this equation, equation 5 and 6 you can observe or you can predict what should be stream fluid, stream function which is behaving inside outside this bubble..

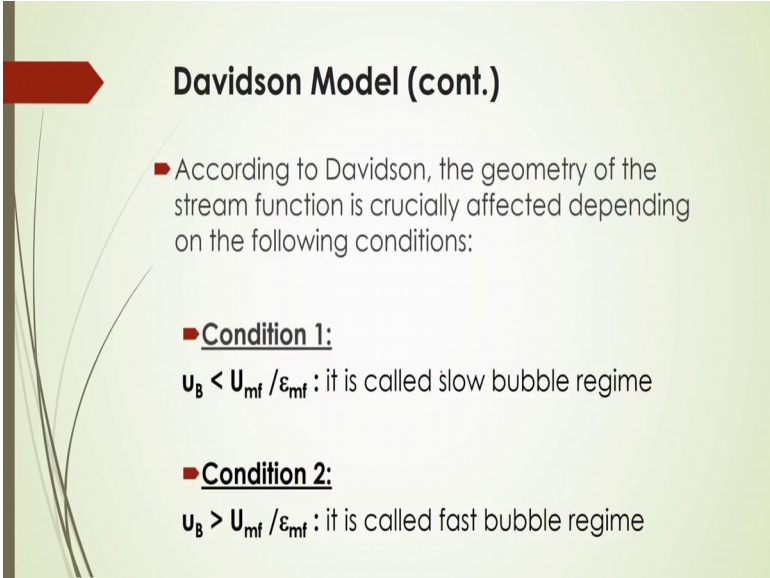
And based on that fluid stream solid particles will be moving surrounding this bubble in this particular fashion, now in this case what should be that stream function this is stream function depends on this stream line, or this velocity pattern of the solid depends on the fluid velocity,  $U_B$  fluid velocity that that is  $U_f$  and then bubble rise velocity here, this bubble rise velocity is very important here whether this bubble rise will be fast or the bubble rise will be slow relative to the fluid velocity, that is also important and  $U_m$  by epsilon m a this is nothing but the  $U_f$  that is fluid velocity actual fluid velocity at this minimum fluidization condition.

So, this is your relative velocity and into what should be that here at particular location, what should be that fluid stream function here, that will be is equal to 1 minus R by small r, here this small r capital R is the bubble radius, and the small r is that certain radial location and here theta is nothing but here as per this equation given this is your theta.

So, from this theta angle at a particular r location, and if we know this stream function and what should be the velocity at this direction  $U_\theta$  and  $U_r$  from which you will be able to calculate what should be the stream function and based on this pre-stream functions also you will be able to calculate, what should be the relative velocity of the particles that is moving aside the bubble.

And then this how to calculate this then radius at a particular R, what should be the value this R by R B to the power cube that will be is equal to here, this will be is equal to  $U_b$  by  $U_b^3$  plus  $2 U_m a$  by  $U_\epsilon m f$  divided by  $U_b$  minus  $2 m U_f$  by  $\epsilon$ . So, this will be your relationship for this radius, whenever it will be changing here at a particular R radial location.

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**Davidson Model (cont.)**

- According to Davidson, the geometry of the stream function is crucially affected depending on the following conditions:
  - Condition 1:**  
 $U_b < U_{mf} / \epsilon_{mf}$  : it is called slow bubble regime
  - Condition 2:**  
 $U_b > U_{mf} / \epsilon_{mf}$  : it is called fast bubble regime

And then according to Davidson the geometry of the stream function is crucially effected depending on the following conditions, now there are several conditions, you will see now let us consider one as  $U_b$  less than equals to  $U_{mf} / \epsilon_{mf}$  what is this? This is called slow bubble regime, what does it say? Bubble velocity is less than the fluid

velocity, then it will be called as a slow bubble regime, and if this bubble rise velocity is greater than fluid velocity it will be called as fast bubble regime.

So, under this 2 flow regimes, what should be the bubble movement or solid movement, and also gaseous stream function or fluid stream function that depends on.

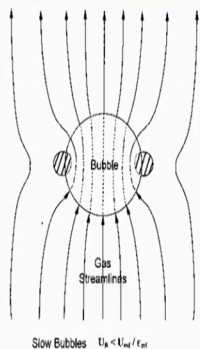
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### Slow Bubbles Regime

- In this case, the fluidizing fluid moves upward relative to the bubble motion.
- This case is usual for beds of large particles and small bubbles.
- The majority of fluidizing fluid enters the void at the base and leaves from the roof.
- The fluidizing fluid, in essence, uses the bubble void as the shortcut.
- The fluidizing fluid penetrates the particulate phase freely from the bubble except for a small fraction of fluid in the shaded area, as shown in Fig. in a circle of radius  $a'$ , expressed by

$$a' = \frac{|R|}{2^{1/3}} = 0.8|R| \quad (7)$$

Davidson's bubble model  
—slow bubbles



Slow Bubbles  $u_b < u_{mf} / \epsilon_{mf}$

Now, you will see slow bubble regimes, in this case the fluidizing fluid moves upward relative to the bubble motion, and this case is usual for beds of large particles and small bubbles, and the majority of fluidizing bed, that enters the void at the base and leaves the roof here.

Very important point that the whenever bubble is there will be some void. So, at the base of the this void or you can say bubble, the fluid enters to this void, and fluid enters to this void from the bottom of this bubble and leaves from it is roof.

So, the fluidizing fluid whenever moving surrounding this bubble, always try to follow the shortcut roots. So, that is why the fluidizing fluid in a sense uses the bubble void as it is as the shortcut way, the fluidizing fluid that penetrates the particulate phase, this particulate phase means what surrounding this bubble what will be the gas end particle will be mixture. So, that phase here this is called particulate phase this surrounding this bubble that will be called particulate phase.

So, the fluidizing fluid that penetrates the particulate phase, freely from the bubble, except for a small fraction of fluid in the shaded area, as shown in figure this in it is circle of radius  $a$  dash here expressed like this, suppose this circle of radius  $a$  dash here, this as per figure this circle of radius  $a$  dash here gas stream line is there. So, this gas stream line reaches to this bubbles from this bottom side, from it is bottom and it will be coming out from it is roof, that is called from this upper surface of this bubble.

So, that sometimes it will be always it should follow the shortcut way to fill up, but some solid particles it will try to move this far away from this bubble, but there will be a some inclination of the pattern or velocity fluctuation, in such way that there will be a it is tendency to go to touch of this bubble here.

So, in this case some particles will cross over the bubble surface, and some particles will not be able to cross over this bubble surface, now there will be certain velocity of that particles, those who are cross over this bubble surface, that will be given later on. So now, in this case what should be the bubble radius or bubble radius in which that very small fraction of the fluid, in the shaded area that crossed by this solid particles.

So, that is shown in that circle of radius  $a$  dash here, in this case it has to be is equal to  $R$  divided by 2 to the power 1 by 3, that will be equals to  $0.8$  into  $R$ , now in this case very important point is that now that the bubble rise velocity of course, should be less than the liquid velocity; that means, bubble will be slower than the liquid velocity. So, in that case only that will be observing this type of phenomenon.



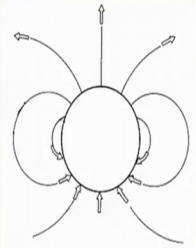
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### Slow Bubbles Regime (cont.)

- In both these regimes, i.e.,  $u_B > u_{mf}/\epsilon_{mf}$  and  $u_B < u_{mf}/\epsilon_{mf}$ , the gas exchange between bubble and particulate phase reported by Davidson through an analogy to a fixed bed:

$$q = 3\pi R_B^2 U_{mf} \quad \text{for a 3-D bed} \quad (8)$$

The gas exchange rate  $q$  is the total volumetric flow rate of gas passing through the bubble void. It can either pass through the bubble void or be recirculated back to the bottom of the bubble, depending on the relative magnitude of the bubble velocity and the minimum fluidizing velocity



And also, in both these regimes, you will see if bubble rise velocity is greater than liquid velocity or fluid velocity, or that the bubble rise velocity is slower than the fluid velocity, there will be a gas change there will be a certain amount of gas, that will be change exchanged between bubble and particulate phase, that is reported by Davidson through a through an analogy to a fixed bed.

In that case, what should be that gas exchange rate that of course, you have to consider, what does it mean; that means, whenever gas will be moving upward at a certain velocity, you will see that some solid particles will be will be moving over this surface of these bubble. So, and also there will be a circulation of the solid particles over this surface of this bubble.

Now, those only those particles which rate that those particles will be actually passing through this bubble surface, from for exchange of this bubble to the emersion phase or particulate phase, now this that rate is called  $q$   $q$  can be calculated as this  $3.5 R_B^2 U_{mf}$ , what is this  $R_B$ , is nothing but the radius of the bubble and  $U_{mf}$  is the minimum fluidization velocity if you are considering the 3-dimensional fluidized bed.

Now, the gas exchange rate  $q$  is the total volumetric flow rate of gas that passes through the bubble void, it can either pass through the bubble void, or it can recirculate back to the to it is bottom that depends on the relative magnitude of the bubble velocity, and the minimum fluidization velocity.

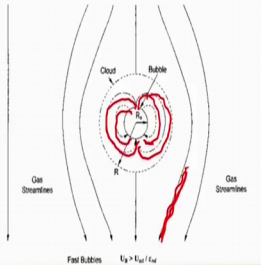
So, what should be the relative velocity that of course, gives this circulation velocity of the solid surrounding this bubble, whenever if is there any solid circulate solid particulates now in case of fast bubble you will see the fluidizing fluid.

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### Fast Bubbles Regime

- The fluidizing fluid in this case moves downward relative to the bubble motion. The fluid flows past the fictitious sphere of radius  $R$  with velocity  $-(u_b - u_{mf}/\epsilon_{mf})$  at  $r$ .
- Inside the sphere of penetration of radius  $R$ , the fluid leaves the roof of the bubble and recirculates back to the base of the bubble as shown in Fig.

Davidson's Model  
(fast bubble regime)



the particulate phase streams past a sphere of radius  $R_b$ . The relative velocity of the particulate flow to the void is  $u_b$  at  $r = \infty$ .

In this case moves downward related to the bubble motion, and the fluid flows past the fictitious sphere of radius  $R$ , with velocity that will be  $U_b$  minus  $U_{mf}$  by  $\epsilon_{mf}$  at  $r$ , here negative is because of this downward movement of the solid particles.

So, here this fluidizing fluid moves downward, and the bubbles will rise fast. So, that there will be a certain relative velocity at which the fluid flows past the sphere of this radius of  $R$  of the bubble, and with the certain velocity of these relative velocity  $U_b$  minus  $U_{mf}$  by  $\epsilon_{mf}$ .

Now, inside this sphere, this inside the sphere of penetration of radius  $R$ , the fluid leaves the roof of the bubble, and the circulates back to the base of the bubble as shown in the figure. So, here see here at this surface, the solid particles move aside, and recirculate and it will come back through these bottom of this bubble here.

This same way here this is this circulate like this is also recirculate like this, now this recirculation velocity that depends on the bubble rise velocity, and if this bubble rise velocity or bubble rise velocity, is for that is for greater than this fluid velocity then this

type of phenomenon you will observe whereas, this gas stream will be moving downward at this relative velocity.

And because of which this of course, due to this gas stream there will be a flow pattern of this solid particles follow the same fashion of this gas, stream line here in this case, the particulate phase streams pass a sphere of radius  $R_B$ , the relative velocity of the particulate flow to the void is  $U_b$  at  $r$  is equal to infinity; that means, at radius at  $R$  is equal into infinity at the radial direction  $R$  at infinity, you will see that relative velocity of the particulate flow to the void is only the bubble rise velocity.

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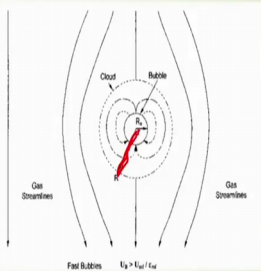
### Fast Bubbles Regime (cont.)

- The bubbles in the fluidized bed are accompanied by a "cloud" while rising through the bed. The cloud is usually very thin.
- The **radius of the penetration** for a 3-D bed and **stream function** for the particles are:

$$\bar{R} = \left[ \frac{U_B / (U_{mf} / \varepsilon_{mf}) + 2}{U_B / (U_{mf} / \varepsilon_{mf}) - 1} \right]^{1/3} R_B$$

$$\psi_f = \left( U_B - \frac{U_{mf}}{\varepsilon_{mf}} \right) \left[ 1 - \left( \frac{R}{r} \right)^3 \right] \frac{r^2 \sin^2 \theta}{2}$$

Davidson's Model  
(fast bubble regime)



the particulate phase streams past a sphere of radius  $R_B$ . The relative velocity of the particulate flow to the void is  $U_b$  at  $r = \infty$ .

Now, the bubbles in the fluidized bed that are accompanied by a cloud that is of course, we have earlier also described, that whenever bubbles will be moving up it will having a one region of solid particles accumulating surrounding this surface, and that accumulation region of the solid particles will be called as cloud. So, that the radius of this region will be very thin, that in that case the cloud that it will be called the cloud thickness that thickness may be thin or thicker depending on this bubble velocity.

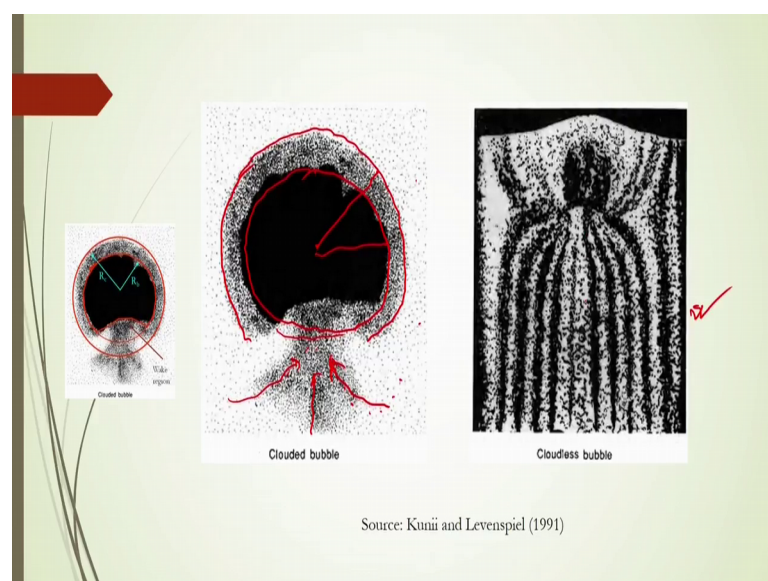
So, the bubbles in the fluidized bed are accompanied by a cloud whenever it will rise through the bed, the cloud is usually very thin whenever it will be going fast, the radius of the penetration of a 3 dimensional bed and the stream function for the particles are there then expressed as this  $R$  here this radius of the penetration for a 3 dimensional bed here this rate radius this  $R$ , here in this figure here it is seen that this is your cloud radius

and this is the bubble and surrounding this bubbles there will be a cloud, and then what should be the cloud radius that is  $R$  that will be is equal to  $U_b$  by  $U_m$  a  $f$  salon  $f$  plus 2 by  $U_b$  by  $\epsilon m f U_m$  by  $\epsilon$  by  $f$  minus 1.

So, from this equation you can calculate what should be the  $R$  of this cloud, that is the cloud here and of course, it depends on this radius of the bubble as well as the bubble rise velocity and fluid velocity, and this follows this the solid particles what should be the stream function of the solid particles whenever bubbles will be rising at a faster rate, it will be calculated as  $U_a$  b minus  $\epsilon$  by  $U_b$  minus  $U_m$  f by  $\epsilon$ , that is here the relative velocity of the bubble to the fluid velocity into here like this. So, from this relationship you will be able to calculate what will be the stream function of solid, that is exactly as that that as gas stream function here of fluid stream function surrounding this bubbles, now the particulate phase stream function fast sphere of radius  $R_B$  here the relative velocity of the particulate flow, to the void here is  $U_b$  at  $r$  is equal to infinity.

So, this is actually applicable for this Davidson model of fast bubble regime. So, in this case very interesting that, this there will be a formation of cloud region of thin cloud region, and then what should be the thickness of the cloud region, and also what should be the radius of the bubble that will be obtain, now what should be that cloud radius that is given by other models Kunii and Levenspiel also they have expressed in different way.

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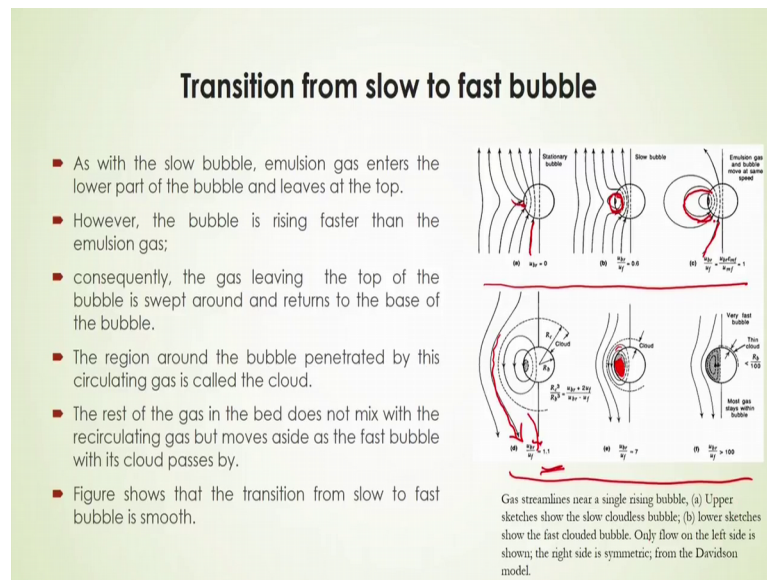
Let us see what is that actually clouded bubble, now as per Kunni and Levenspiel, you will see this is the bubble this dark region, you will see this is the void and here this is as a bubble and see here this you can say this is bubble, and surrounding this bubbles there will be a cloud of fine solid particles. Now this will be if you are emerging this surface that will be as a cloud ring, and then here this portion is called this wake region this wake regions solid particles will be coming from these, and then it will be just coming back at this region like this here.

So, this is the movement of the solid and this is your ark, now this is cloud radius from the centre cloud radius, and this is your this is called your bubble like here this will be your bubble radius now also this bubbles. So, very interesting that there will be a certain velocity by which you can get this type of observation of the bubble, which will carry this solid particles surrounding it as a cloud particle cloud and also as a wake region.

You can see there will be a accumulation of the solid particles from this wake region, and the stream level of the solid particles how this solid particles will be moving that will be as per the velocity potential, that is given by the equation previously in previous slides, then you can have some behavior of the solid particle in which location for the stimulation, it is very important to know that how this solid particles will be moving aside from this bubble, and also coming back from coming back into the back of this bubble here.

And at a certain velocity you will see some bubbles will be forming that that will not form any cloud there. So, this is called cloudless bubble as per this figure, this type of phenomenon also will be observed at a certain condition; that means, here the high velocity will be there, but the solid particles that is cloud, whatever it will be just as a ring it will just falling down from this wake region of this bubble here and there will be some a transition phenomenon, whenever bubble moving from slow to fast velocity.

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So, as with slow bubble emulsion gas enters this lower part of the bubble the, and leaves at the top; however, this bubble is rising faster than the emulsion gas. So, consequently the gas leaving the top of the bubble is swept around and returns to the base of the bubble, the region around the bubble that is penetrated by this circulating gas that is called cloud, the rest of the gas in the bed then does not mix with the recirculating gas, but moves aside as the fast bubble with it is cloud passed by.

Now, figure in this case shows that the transition from slow to fast bubble is very smooth, in this case if bubble rise velocity is that is 0 what does it mean that bubble will be stationary. So, in this case the solid particles how it will be moving up, you will see the bubble solid particles will cross this bubble surface, in this case at a certain velocity pattern whereas, at the surface some solid particles will come and then go aside just by touching one of the point.

Whereas the some other solid particles, some other particles they will not actually move aside just by touching one point, they will cross over the surface of the bubbles, in this way here the dust line is shown here that the solid particles fall in such way that they will move, just crossing this surface of the bubble, and where as in this case the solid particles will touch and then it will be moved out again.

So, there will be no cross over the surface of this bubble at this location or whereas, if bubble will be moving at a certain velocity; that means, some fraction of the fluid



velocity like may be the 60 percent of the fluid velocity, at that velocity bubble rise velocity bubble rise then you will see there will be a phenomenon of this here this some portion of this path here in this location, there will be a circulation internal circulation there will be a circulation of the solid particles at this surface of the bubble.

Whereas some other solid particles will be cross over through the bubble. So, in this case even if you increase the bubble velocity; that means, here bubble rise velocity is almost equals to the fluid velocity, in that case you will see the velocity pattern of the solid particles like this here, only few solid particles will be cross over this one.

Whereas other solid particles will be crossing over in such way that, there will be no straight forward to the cup passing this bubble, and they will not move out straight just by in the surface of the bubble they will immediately aside from this surface, and again with the circular path they will come back to this surface here.

So, from this point they will go aside or leave aside and then they will come back to this surface, again in this case here some solid particles will not move in downward like this, they will go up and they will just cross over this surface. So, here this phenomenon is very interesting that at the fluid velocity there will be a circulation of the solid particles at it is surface, in such way that solid particles when it those solid particles will be leaving from the surface, they will come back again to the bubble surface. So, in this fashion the solid particles just change the flow pattern from the slow bubble to the higher bubble rise velocity even. In fact, if the bubble rise velocity is more then the fluid velocity; that means, here the ratio of the bubble rise velocity to the fluid velocity if it is greater than one, that is at see here at this picture at 1.1.

So, in this case very interesting is that, some solid particles will be totally get the continuous loop of the circulation surrounding this bubble, where as some particles will not be getting the continuous loop whereas, those particles will cross over, but they will go downward here.

So, that is why in this case, since the bubble rise velocity is high, the solid particles maximum solid particles will try to get the recirculated inside the continuous way, but some other solid particles will disturb by this downward movement, and they will get again some particles will some come downward. So, in this fashion will see the velocity pattern of the solid particles will be changing because of the bubble rise velocity.

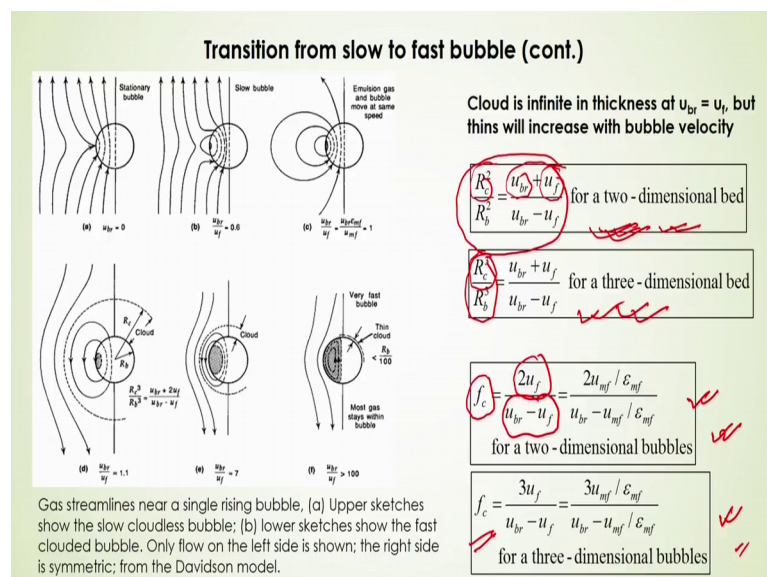
Here again if it is suppose that is more higher the fluid velocity there of course, inside the bubble there will be a some solid particles there will be depositing, and there will be a continuous circulation of the solid particles here, and in this case as per picture as the path of the solid particles will be moving like this, and some other solid particles which is far away from the bubble that will be going downward.

And if suppose 100 times of the fluid velocity is bubbled, rise velocity here no bubble rise no particles will be actually those who have try to cross over this bubble surface, no bubble will be coming downward they will be just moving this surface at the surface in the in the circulation way like this here in the picture.

So, here there will be a thin cloud formation, and where as other cases there will be a thicker cloud. So, very first bubble cases this type of phenomenon will have been. So, most gas here stays within the bubble in this phenomenon. Now gas stream line near a single rising bubbles in this case a case here upper stresses uppers cases shows here slow, cloudless bubble in this case slow bubble where as in the higher bubble velocity; that means, first bubbles in case of first clouded bubbles.

Here you will see there will be a circulation of the solid particles how it will behave surrounding this bubble. So, as per this Davidson model this flow pattern or you can say stream line of the solid particles, how it will be moving that you can calculate from the stream line function of the solid movement or the gaseous movement there.

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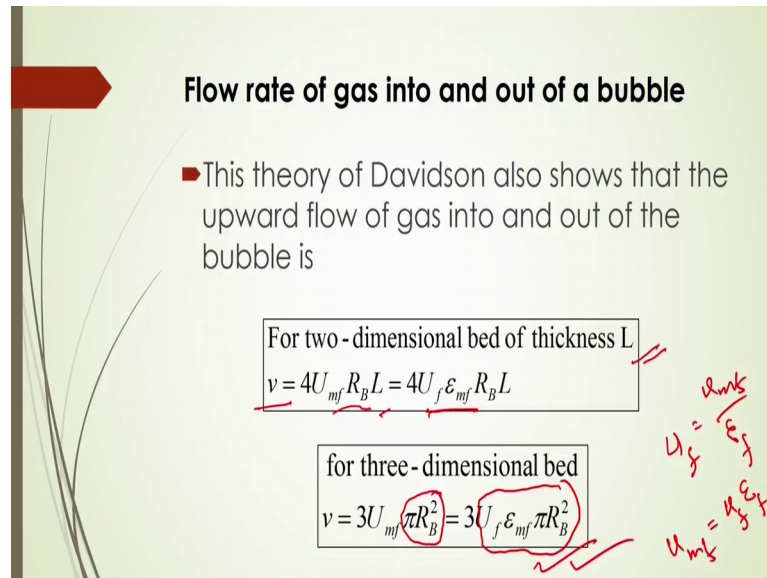
In this case again, now whenever the bubbles will move very fast of course, the formation of the cloud region and the internal circulation of the solid particles at the surface very interesting. So, in that case you will see, what should be cloud radius? And what should be the bubble size? This cloud radius or cloud thickness that depends on the bubble size where in if bubble size is very that is bubble size bigger, bubble size will give you the bigger cloud radius, whether it is 2 dimensional bed or 3 dimensional bed of course, the nature of this cloud formation cloud formation will be depending on the asymmetry of the bed, for the 2 dimensional bed you calculate the cloud radius this  $R_c$  square by  $R_B$  square that will be is equal to  $u_{br}$  plus  $u_f$  by  $u_{br}$  minus  $u_f$ , what is this, this is this is  $u_{br}$  means bubble rise velocity  $u_f$  means fluid velocity, and  $R_c$  is called the radius of the cloud.

Now, this you will be able to calculate only at the condition, that that  $u_{br}$  is equal to  $u_f$ ; that means, bubble rise velocity whenever it will be rising at the fluid velocity itself, then at that condition this cloud radius that will be given by this equation, for 2 dimensional bed where as in case of 3 dimensional bed, from the mass balance equation, you will be able to calculate, what should be the cloud radius? This cloud radius will be calculated from this equation here, and that also depends on the bubble rise velocity and fluid velocity.

What should be the fraction volume fraction of the; that means, cloud compared to that bubble volume here, that will be calculated here this  $f_c$  that will be is equal to here, this  $2 u_f$  by relative velocity of the bubble to the fluid velocity. So, from this relation we will be able to calculate for the 2-dimensional bubble, and 3 dimensional bubbles this what is that fraction of the bubbles clouds, that can be calculated from this equation.

Now, of course, here the only flow on the left side is shown in this case here the flow pattern. So, there may be a question that why that right side of the bubble it is not happened of course, the same pattern same fashion as per this davidson model this right side will be symmetric to the left side.

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**Flow rate of gas into and out of a bubble**

■ This theory of Davidson also shows that the upward flow of gas into and out of the bubble is

For two-dimensional bed of thickness  $L$

$$v = 4U_{mf} R_B L = 4U_f \epsilon_{mf} R_B L$$

for three-dimensional bed

$$v = 3U_{mf} \pi R_B^2 = 3U_f \epsilon_{mf} \pi R_B^2$$

Handwritten notes on the right side of the slide:

- $U_{mf} = U_f \epsilon_{mf}$
- $U_{mf} = U_f \epsilon_{mf}$

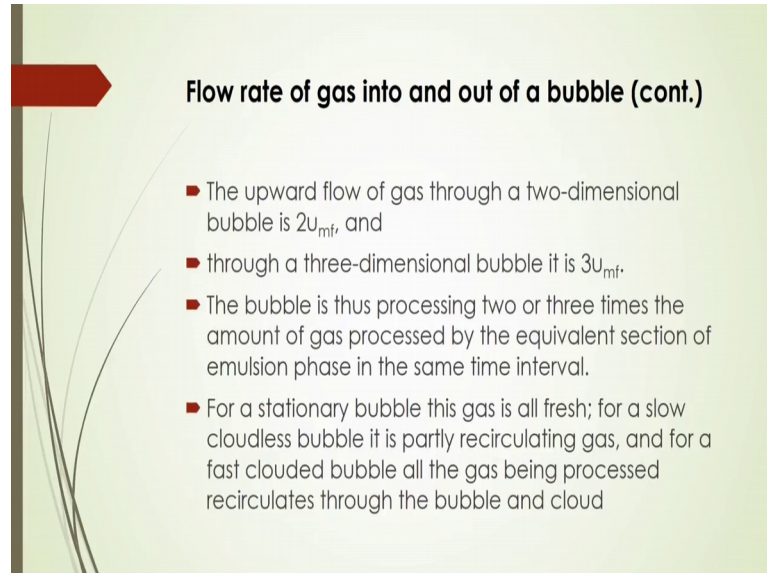
Now, what should be the flow rate of gas into and out of a bubble of course, whenever the bubbles will be moving as per that solid particles, since it is moving aside to the bubble and also into the bubble, and also cross over this bubble similar fashion of course, you will see the gas will also be moving, now that gas will be moving the upward motion, some gas will be coming into the that is bubble and also out of the bubble.

So, the what should be the flow rate of the gas, that is coming into the bubble and out of the bubble. So, in that case the Davidson theory gives you the a flow rate of the gas, which is moving upward into and out of the bubble, for 2 dimensional bed if it is thickness is  $L$ , then you can calculate the flow rate of the gas as 4 times of  $U_{mf}$  into  $R_B$  into  $L$ , here  $U_{mf}$  is the minimum fluidization velocity,  $R_B$  is the bubble radius and the  $L$  is the thickness of the 2 dimensional bed, and then  $U_{mf}$  from this you can calculate, what will be the  $4 U_{mf}$  into  $U_f \epsilon_{mf}$  because, here  $U_{mf}$  that will be is equal to  $2 U_{mf}$  by  $\epsilon_{mf}$ , which will give you the  $U_{mf}$  will be is equal to  $U_{mf}$  that will be is equal to  $U_f \epsilon_{mf}$ . So, that is why here this instead of  $U_{mf}$ , you can directly calculate  $U_f \epsilon_{mf}$  into  $R_B$  into  $L$ .

For this 3-dimensional bed this will be is equal to 3 into  $U_{mf}$  into cross section that is here,  $\pi R_B^2$  a cross sectional area of the bubble here, into 3 times of this  $U_{mf}$  into  $U_f \epsilon_{mf}$  into  $3 U_{mf}$  into  $U_f \epsilon_{mf}$  into sorry from this equation you

can calculate, what is should be the gas flow rate that is moving into and out of the bubble.

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**Flow rate of gas into and out of a bubble (cont.)**

- The upward flow of gas through a two-dimensional bubble is  $2u_{mf}$ , and
- through a three-dimensional bubble it is  $3u_{mf}$ .
- The bubble is thus processing two or three times the amount of gas processed by the equivalent section of emulsion phase in the same time interval.
- For a stationary bubble this gas is all fresh; for a slow cloudless bubble it is partly recirculating gas, and for a fast clouded bubble all the gas being processed recirculates through the bubble and cloud

Flow rate of gas into and out of the bubble that of course, depends on that whether it is the 2-dimensional bed, or 3-dimensional bed and for 2-dimensional bed you will see that upward flow of gas through a 2-dimensional bed of course, it will be 2 times of minimum fluidization velocity.

Whereas for 3-dimensional bubble it is 3 times of  $U_{mf}$ , the bubble is thus processing 2 or 3 times the amount of gas, processed by the equivalent section of the emulsion phase in the same time interval, for a stationary bubble this gas is all fresh, here for a slow cloudless bubble it is partially recirculating gas, and for a fast-clouded bubble all the gas being processed recirculates through the bubble and cloud.

What will be the pressure distribution around a bubble, according to Davidson model the gas flows in the emulsion phase as an incompressible viscous fluid, hence the relative velocity between gas and solid must satisfy the D'Arcy's law, whatever it is given earlier also or you can represent this D'Arcy's law in this way like  $U_g - U_s$ .

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### The pressure distribution around a bubble

- According to Davidson model, the gas flows in the emulsion phase as an incompressible viscous fluid
- hence, the relative velocity between gas and solid must satisfy Darcy's law. Thus for any direction  $x$ ,

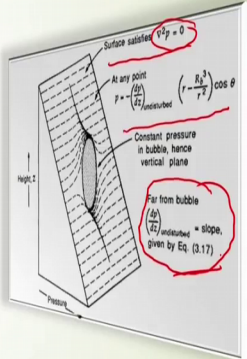
$$(u_g - u_s)_x = -K \frac{\partial p}{\partial x}$$

Now, that is what is the relative velocity of the solid particles relative to the fluid. Now, here  $u_g - u_s$  means here gas velocity is the  $u_g$  and  $u_s$  is the solid velocity, and that will be actually represented by this what will be the pressure gradient here.

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### Pressure distribution around a bubble (cont.)

- The pressure in the lower part of the bubble is lower than that in the surrounding bed,
- whereas in the upper part it is higher.
- Thus, gas flows into the bubble from below and leaves at the top.
- The resulting flow pattern is solely dependent on the relative velocity of bubble with emulsion gas.
- Figure shows a distinct difference in the gas flow pattern, depending on whether the bubble rises faster or slower than the emulsion gas.



The diagram illustrates a bubble in a fluid bed. It shows a vertical cross-section of the bubble with a central vertical plane. The pressure distribution is indicated by arrows pointing towards the bubble from the surrounding bed. The pressure is lower at the bottom of the bubble and higher at the top. The flow pattern is shown as gas entering from the bottom and leaving from the top. The diagram also shows the relative velocity of the bubble with the emulsion gas, which is indicated by a vector labeled  $u_g - u_s$ . The pressure distribution is given by the equation  $p = p_0 - \frac{\rho_b g}{2} \left( r - \frac{R_b^2}{r^2} \right) \cos \theta$ . The pressure is constant in the bubble, hence vertical plane. The pressure is also given by the equation  $p = p_0 - \frac{\rho_b g}{2} \left( r - \frac{R_b^2}{r^2} \right) \cos \theta$ . The pressure is also given by the equation  $p = p_0 - \frac{\rho_b g}{2} \left( r - \frac{R_b^2}{r^2} \right) \cos \theta$ .

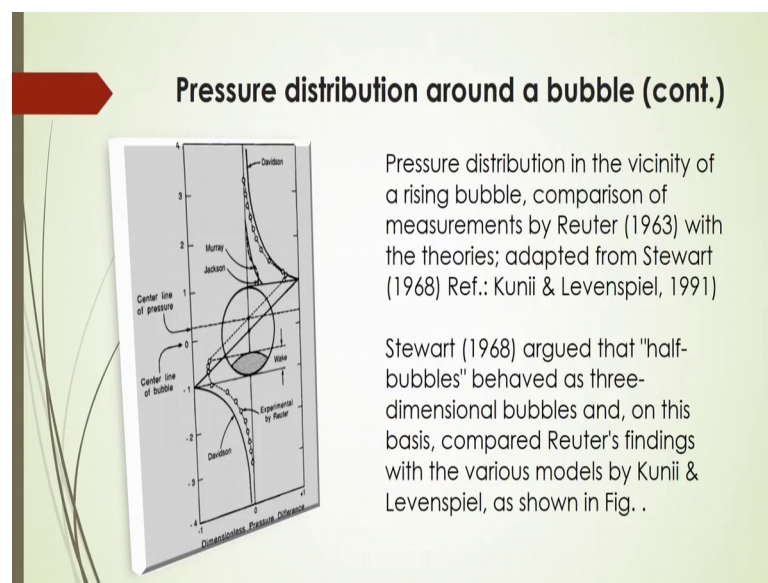
And then what should be that actual pressure distribution around a bubble, the pressure in the lower part of the bubble is lower than that in the surrounding bed here as per here figure it is shown.



Where as in the upper part it is higher, thus gas thus in this case gas flows into the bubble from the below and the top it leaves, the result in flow pattern is solely dependent on the relative velocity of bubble with emulsion gas, in this case a distinct difference in the gas flow pattern which is shown in the figure, that depends on the whether the bubble rise faster or slower than the emulsion gas.

Here see the plan of this bubble that is moving here at this direction and at particular location at a point, what should be pressure that can calculated that depends on the radial direction, and also what will be bubble radius there and here it is in this case the surface, satisfy this here this continuity equation that earlier shown here and perform the bubble the pressure difference will be here will be some constant here, as per slope that is given in equation 3.17 in your Kunii and Levenspiel text book.

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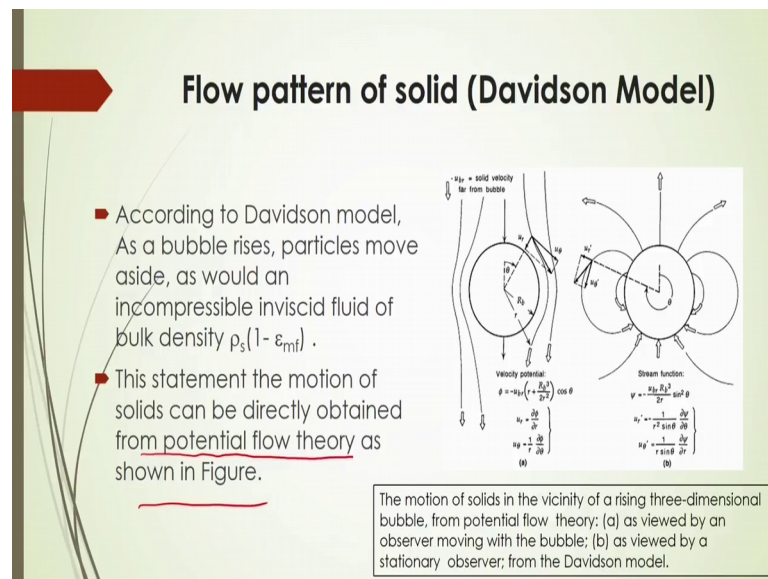


Here pressure distribution in the vicinity of a rising bubble, you will see in the figure comparison of the measurements by Reuter with the theories adopted from the Stewart, they have shown this phenomenon of the pressure distribution profile of the distribution surrounding the bubble.

Here see here this location is nothing but the centre of the bubble, now here and this location you will give you will get the here dimensional pressure is difference; there will be no pressure difference here.

So, at this case the dimensional pressure difference will be 0, at this location here at that is far away from the bubble, whereas at the bubble surface you will see it will be the maximum one this pressure difference. So, Stewart 1968 argued that this half bubbles, behaved as 3 dimensional bubbles, and on this basis compared Reuter's fittings with the various models by Kunii and Levenspiel as shown in this figure.

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Now, what should be flow turn flow pattern of the solids, according to Davidson model as a bubble rise particles move aside as would an incompressible invisible fluid of bulk density, we have discussed this statement the motion of the solids can be directly obtained from the potential flow theory as shown in figure, here that already this is given in the Kunii and Levenspiel text book as per Davidson model, that we have discussed here this velocity potential of the solid particles exactly that earlier what we have discussed.

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### Wake Region of bubble

- Typical bubbles are not spherical but have a flattish, or even concave, base, as shown in Fig.
- The region just below the bubble is the wake region and it most likely forms because the pressure in the lower part of the bubble is less than in the nearby emulsion.
- Thus, gas is drawn into the bubble, causing instability, partial collapse of the bubble, and turbulent mixing.
- For fast clouded bubbles, this is the reason for a leakage of circulating bubble gas into the wake
- This turbulence by fast clouded bubbles also results in solids being drawn up behind the bubble and forming the wake region.

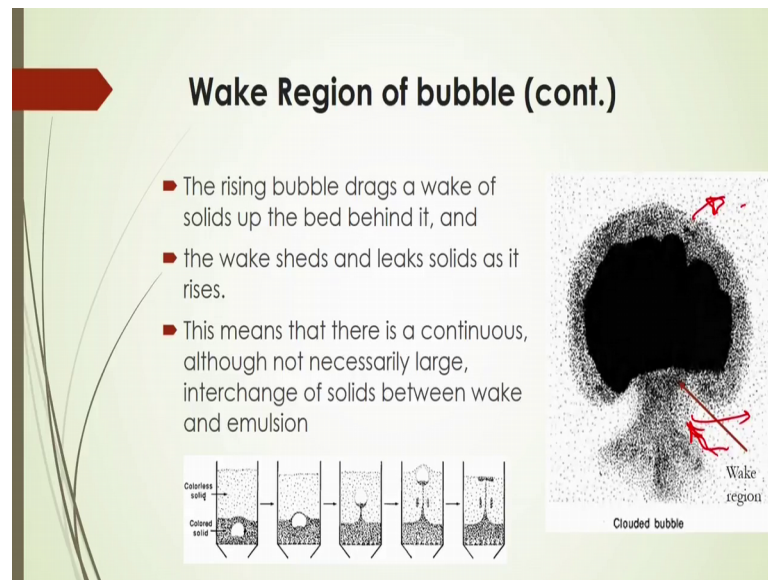


Here this wake region of the bubble, what that we have seen that bubble will whenever it will go up, it will carry some solid particles at it is wake region, and then what will be that wake region typical bubbles are not actually spherical at it is particular operation in the practical operation in this case, but have a flattish or even concave base as shown in the figure.

The region just below the bubble, is the wake region and it is mostly like forms because, the pressure in the lower part of the bubble is less than in the nearby emulsion, and gas is drawn into the bubble causing instability, and also you will see partial collapse of the bubble will be seen, and due to the turbulent mixing inside the bed.

For fast clouded bubbles this is the reason for a leakage of circulating bubble that will be into the wake, this turbulence by fast clouded bubbles also results in solids being, drawn up behind the bubble and forming the wake region.

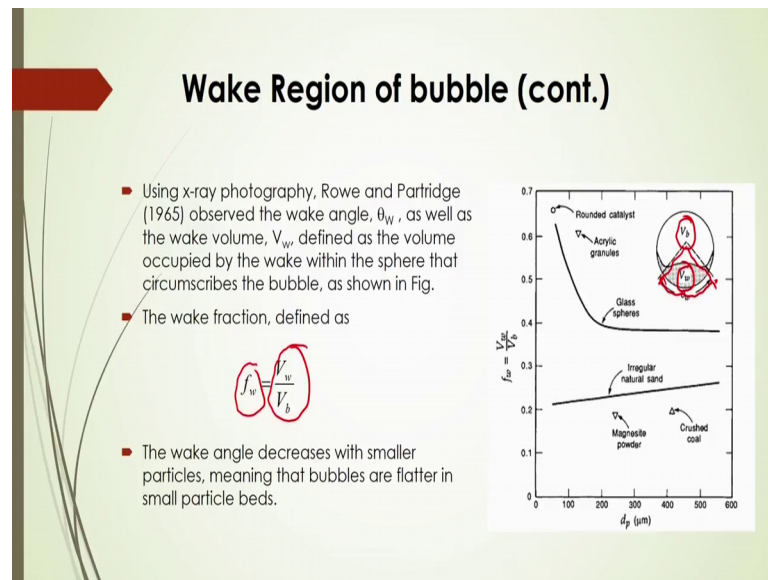
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Now, wake region of course, you will see that depends on the size of the bubble, and also what will be rising velocity, the rising bubble drags a wake of solids up the bed behind it and the wake sheds and leaks a solid as it rises, this means that there is a continuous that is although not necessary in the large cases the interchanges of the solids between wake and emulsion. So, there will be a some interchange of the solid particles between wake and emulsion.

From this wake region there will be a solid particles will be moving and also some particles will be moving in, moving out and moving in, from this location there will be a solid that is coming to the emulsion phase from this location, there will be a some solid particles of course, will be joining to this emulsion region at a highly clouded portion, that is at the wake region and that depends on the pressure gradient surrounding this bubble and also the velocity pattern of the gas or fluid here.

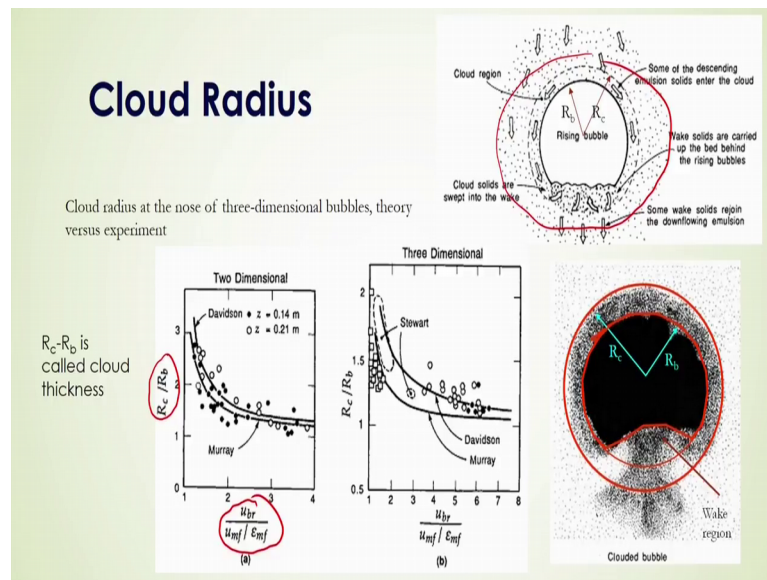
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Now, you will see using a x ray photography, Rowe and Partridge 1965 observed the wake angle there, here in this figure shown that wake angle, what is that wake angle, this, this to this part is wake angle. So, this is your wake angle. So, this wake angle if it is represented by  $\theta_w$ , as well as the wake volume as  $V_w$ , then the wake within the sphere that circumscribes the bubble, then what should be the wake fraction that will be defined by this  $f_w$ , that is nothing but the ratio of this wake volume to the bubble volume.

Now, this wake angle decreases with the smaller particles, what does it mean, this means that bubbles are flatter in small particle bed.

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Cloud radius this cloud radius at the nose of 3-dimensional bubbles theory versus experiment it is shown here, this  $R_c - R_b$ ; that means, what will be the cloud radius what will be the bubble radius what will be the difference this  $R_c - R_b$  is called cloud thickness.

No, this cloud thickness can be calculated from these here figure, in this case you will see that relative; that means, ratio of this cloud radius to the bubble radius is given from which you will be able to calculate what should be the cloud thickness, once you know this cloud radius and bubble radius of course, you will be able to.

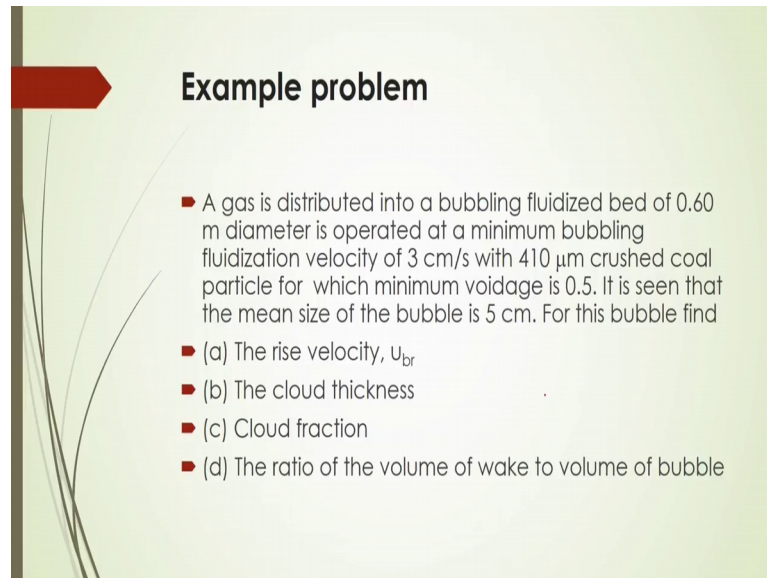
Now, this from this experimental radius from result from 2 dimensional and 3 dimensional bed, this is ratio of this  $R_c$  to the  $R_b$  versus this  $U_{br}$  by fluid velocity it is given, it is seen that if it is ratio of bubble rise velocity to the fluid velocity increases, this cloud thickness decreases of course, it is have it we have already discussed earlier, that this thickness of the radius of the bubble of course, will be decreasing whenever bubble rise velocity will be higher and in 3 dimensional cases also same pattern it is observed, that the cloud radius will be decreasing whenever bubble rise velocity increases.

So, this is the phenomenon how the solid particles moving down and up based on the bubble velocity, and depending on the bubble velocity what should be the cloud, what



should be the actual radius of the cloud, and also what should be the thickness of the cloud that we can calculate from this observation.

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**Example problem**

- A gas is distributed into a bubbling fluidized bed of 0.60 m diameter is operated at a minimum bubbling fluidization velocity of 3 cm/s with 410  $\mu\text{m}$  crushed coal particle for which minimum voidage is 0.5. It is seen that the mean size of the bubble is 5 cm. For this bubble find
- (a) The rise velocity,  $u_{br}$
- (b) The cloud thickness
- (c) Cloud fraction
- (d) The ratio of the volume of wake to volume of bubble

Now, let us do one how to calculate those this the bubble rise velocity, cloud thickness, cloud fraction the ratio of volume effect, the volume bubble that has already been discussed in this previous slides. So, let us see a gas is distributed into a bubbling fluidized bed of 0.60 diameter that is operated at a minimum bubbling fluidization velocity of 3 centimeter per second, with 410 micro meter cast coal particles, for which minimum void is seen as 0.5 and also it is seen that the minimum size of the bubble is 5 centimeter. So, for this bubble what should be the bubble rise velocity, what should be the cloud thickness, what should be the cloud fraction, and what should be the ratio of the volume of void, to the volume of bubble?

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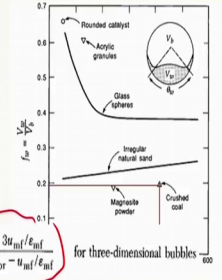
Since  $d_b/d_{bed} = 5/60 < 0.125$ ,  
the rise velocity of the bubble can be calculated as

$$u_{br} = 0.711 \sqrt{g d_{be}} = 49.8 \text{ cm/s}$$

Noting that  $R_b = 2.5 \text{ cm}$  and  
 $u_f = u_{mf}/\epsilon_{mf} = 3/0.5 = 6 \text{ cm/s}$ , Therefore

$$\frac{R_c}{R_b} = \left( \frac{u_{br} + 2u_f}{u_{br} - u_f} \right)^{1/3} = 1.12$$

So,  $R_c = 1.12(2.5) = 2.80 \text{ cm}$ ;  
 $R_c - R_b = 2.80 - 2.5 = 0.30 \text{ cm}$



$$f_c = \frac{3u_f}{u_{br} - u_f} = \frac{3u_{mf}/\epsilon_{mf}}{u_{br} - u_{mf}/\epsilon_{mf}}$$

$$f_c = \frac{V_w}{V_b} = 0.19$$

$$f_c = \frac{3u_f}{u_{br} - u_f} = 0.410$$

Let us see let us consider in this case the calculation like this, here since we know that this bubble diameter to the bed diameter is 5 by 60 here as per problem, 0.125 then you can calculate the bubble rise velocity from this equation, here that we have already discussed earlier in the previous lectures that our earlier lectures that how to calculate the bubble rise velocity.

So, bubble rise velocity can be calculated from this equation that is 0.711 to 2 to the power of g into equivalent bubble diameter. So, it is given here 5 centimeter and g you know that, then what should be the bubble rise velocity, it is seen that 49.8 centimeter per second.

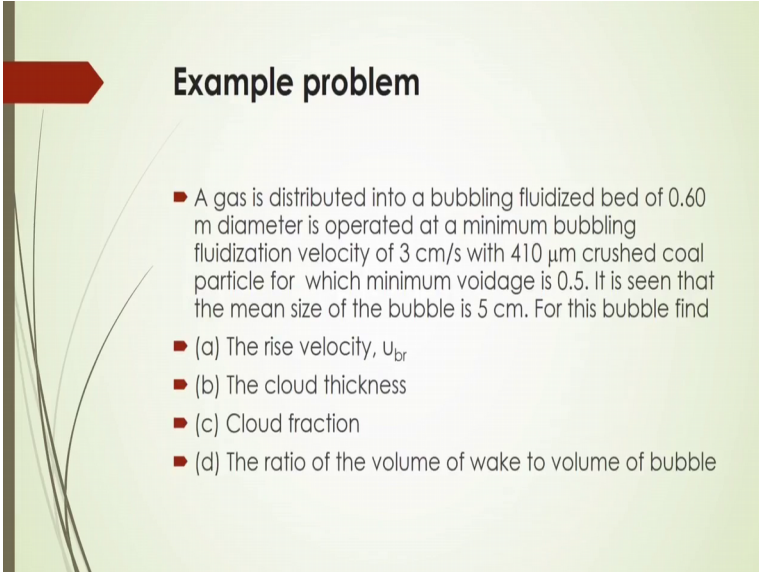
Now, in this case the bubble radius is 2.5 centimeter. So, what should be the fluid velocity, fluid velocity is the minimum fluidization velocity by minimum porosity. So, it is given here minimum fluidization velocity is 3, and the minimum porosity is 0.5. So, actual fluid velocity will be is equal to 6 centimeter per second as per calculation here.

Now, from the equation given in the slides that what should be the ratio of this bubble sorry cloud radius to the bubble radius is given here, in terms of bubble rise velocity and fluid velocity. So, from this relation you will be able to calculate what should be the ratio of this cloud to bubble radius, here is given 1.12. So, from this portion you will be able to calculate what should be the cloud radius.

So, if you multiply by this bubble radius by this 1.12 then you will be able to calculate what should be the  $R_c$  that will be is equal to 2.80 centimeter whereas, this then we know this  $R_b$  is equal to 2.5 and  $R_c$  is equal to 2.80. So, what should be the cloud thickness that will be is equal to just subtracting it you will get that 0.30 centimeter and now, what should be the wake volume to the bubble volume that will be given by this  $f_w$  this  $f_w$  will be equals to here  $V_w$  by  $V_b$ . Now this  $f_w$  that will be is equal to what and what should be the  $V_w/V_b$  wake region and that will be is equal to 0.19.

From this figure you can calculate from this figure as per this equation since this for coal particle here, then this,  $f_w$  is equal to 0.19 from this figure, and then  $f_c$  will be is equal to as per this equation  $f_c$  will be is equal to  $3, u_b$  by  $u_{br}$  minus  $u_b$  that will be is equal to 0.410.

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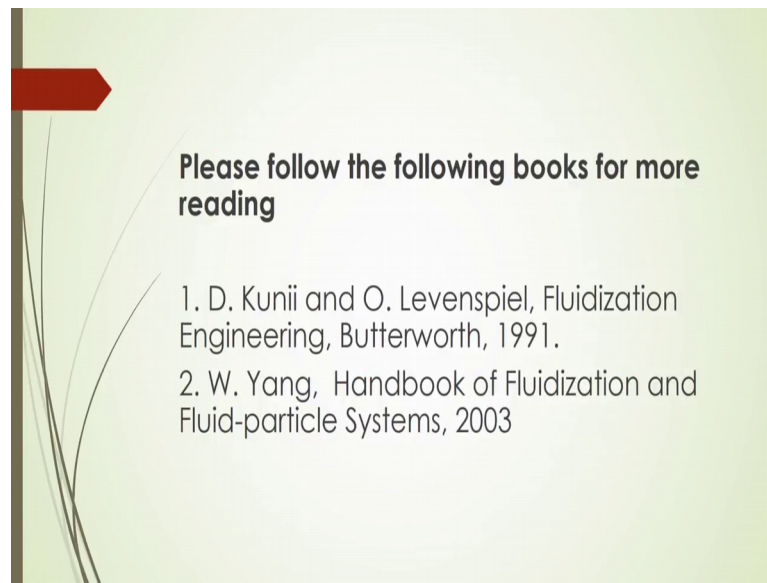


**Example problem**

- A gas is distributed into a bubbling fluidized bed of 0.60 m diameter is operated at a minimum bubbling fluidization velocity of 3 cm/s with 410  $\mu\text{m}$  crushed coal particle for which minimum voidage is 0.5. It is seen that the mean size of the bubble is 5 cm. For this bubble find
  - (a) The rise velocity,  $u_{br}$
  - (b) The cloud thickness
  - (c) Cloud fraction
  - (d) The ratio of the volume of wake to volume of bubble

So, in this way we can calculate what should be the value of cloud fraction, what should be the this is cloud fraction is  $f_c$ , and then what should be the ratio of volume of volume of to the volume of bubble, that is  $f_w$  and what should be the cloud thickness here.

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So, more information you can get from this text book of Kunii and Levenspiel and Wyang, for further reading we are it is suggested and also there will be a more information of other references you can get. So, in the next lecture will be discussing about the bubbling fluidization in slugging bed condition. So, so today's lecture, we have already then today's lecture, we came to know that what should be the solid and gaseous stream line at which the solid particles will be, moving surrounding this bubble, and what will be pressure distribution what will be the wake fraction what will be the cloud thickness, and also what should be the solid particles moving based on the bubble rise velocity, and next class of course, our next lecture will be again discussing about the some bubble characteristics in the slugging condition so.

Thank you.