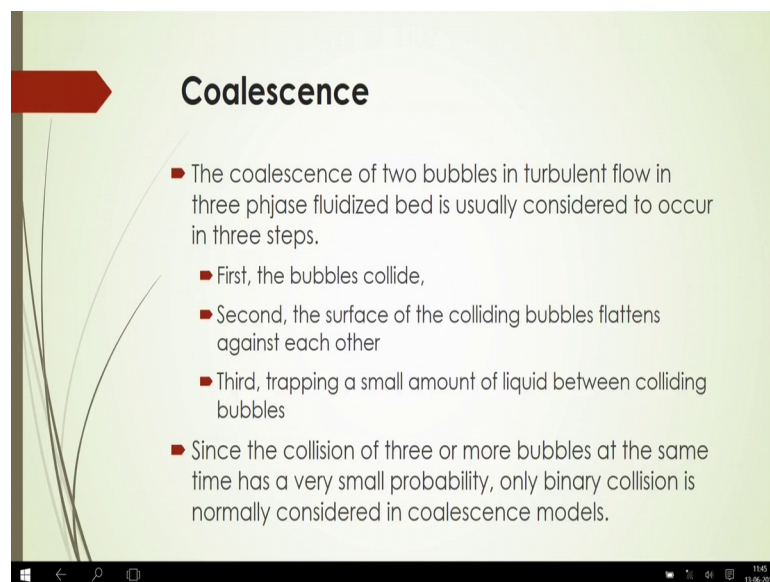


Fluidization Engineering
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Lecture – 15
Bubbling Fluidization Part 3
Bubble coalescence in 3-phase fluidization

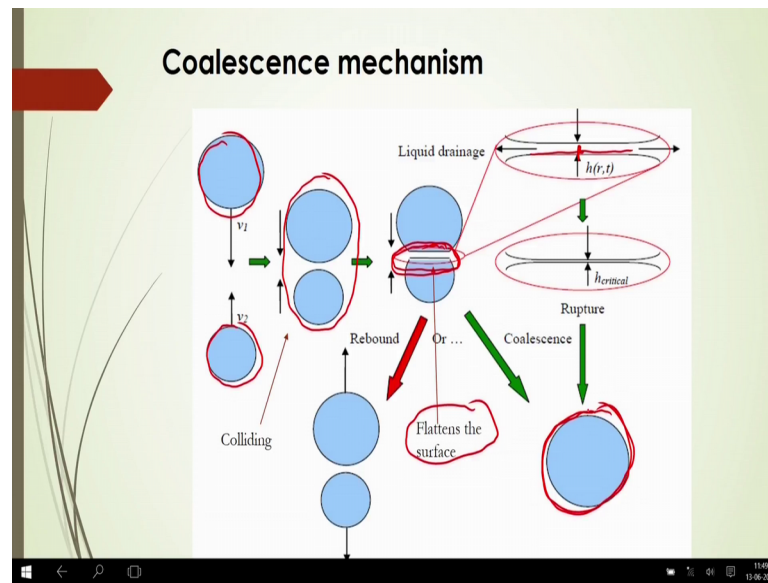
So, welcome to massive open online course on fluidization engineering. So, today's lecture will be on again bubbling fluidization part 3, here the bubble coalescence and breakup in 3 phase fluidization will be discussed and then what is coalescence.

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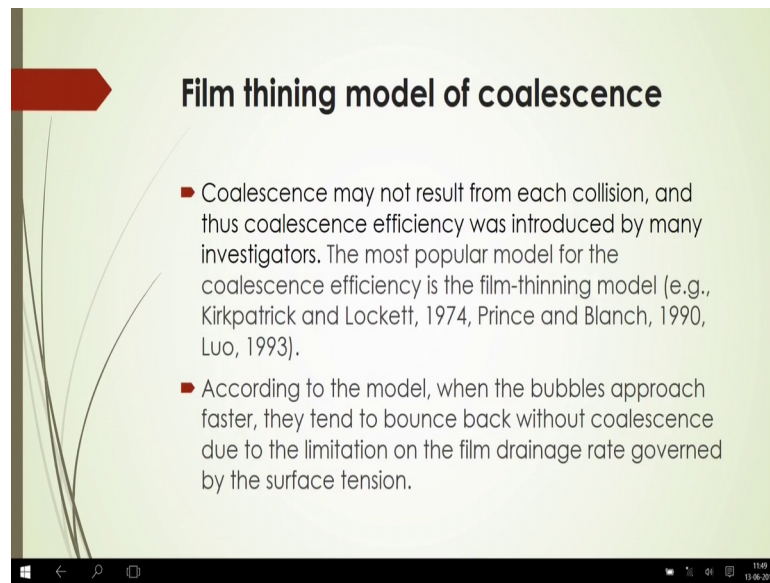
This coalescence actually is the phenomena in which the 2 or 3 bubbles will come to each other and make a bigger bubble. So, this phenomenon happens in bubbling fluidized bed as bubbling fluidized bed in 2 and 3 phase flow system. So, the coalescence of 2 bubbles in turbulent flow, generally in the gas liquid solid fluidized bed is usually considered to occur in 3 steps; first the bubble collide and second the surface of the colliding bubbles flattens against each other and third the trapping a small amount of liquid between bubbles, which are colliding to each other. And in this case since the collision of 3 or more bubbles at the same time has a very small probability only binary collision is normally considered in coalescence models. So, in this case we will discuss different fluidization

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Now, see this coalescence mechanism here in this case see suppose 2 bubbles are colliding to each other at a velocity approaching to each other at V_1 and V_2 respectively. And these 2 bubbles are colliding and colliding here in this case and they are coming to each other at a certain distance, after that they will be coming to touch and sometimes at a certain gap when they are coming to each other, if this gap will be a certain gap that will be critical gap in that case because, of this critical gap there will be certain rupture of the surface of this 2 bubbles and because of which there will be a coalescence happening. Now during this actually rupture there will be a flattens surface come to each other and then interchanging the liquid molecules in this case and joining this surface of this 2 bubbles and making it bigger bubbles here. So, here see in this case the surface of this 2 bubbles before colliding they are flattens this surface and then they are reaching to a certain distance of this 2 level and whenever again it will be becoming this small gap and there will be a liquid drainage in between these 2 surfaces and after a certain critical distance there will be a thin film liquid, which will be just sharing this liquid molecule to each other and then they will actually they will making one surface and becomes bigger bubbles, so in this way coalescence is happened.

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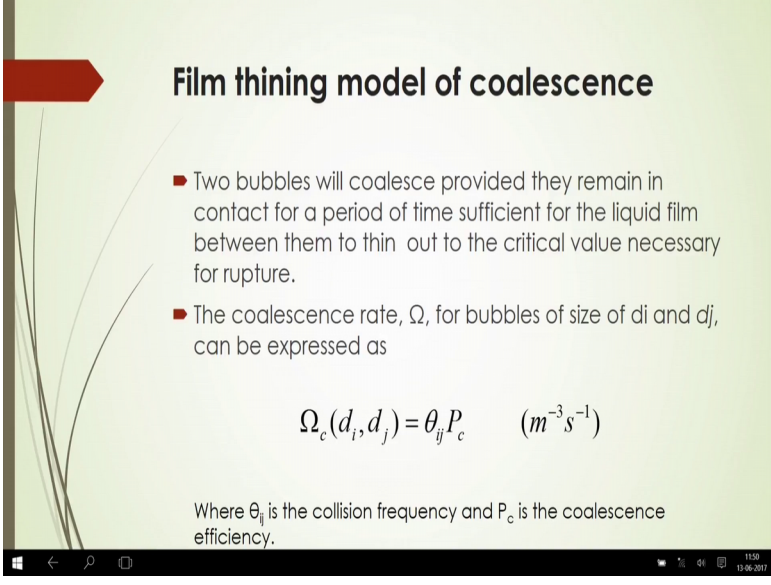
Film thinning model of coalescence

- Coalescence may not result from each collision, and thus coalescence efficiency was introduced by many investigators. The most popular model for the coalescence efficiency is the film-thinning model (e.g., Kirkpatrick and Lockett, 1974, Prince and Blanch, 1990, Luo, 1993).
- According to the model, when the bubbles approach faster, they tend to bounce back without coalescence due to the limitation on the film drainage rate governed by the surface tension.

Now, there are different models to represent this coalescence phenomenon, here film thinning model of coalescence is very important in this regard. So, coalescence may not result from each collision here and thus coalescence efficiency was induced by many investigators; the most popular model for the coalescence efficiency is the film thinning model, like Kirkpatrick and Lockett, 1974 Prince and Blanch 1990 and also Luo 1993, they have nicely presented this film thinning model to interpret the coalescence of the bubbles. Accordingly based on this model when bubbles approach faster they tend to actually bounce back, without coalescence due to the limitation on the film drainage rate governed by the surface tension, so that will be discussed here.

Now, according to that phenomena of this film thinning, 2 bubbles will coalesce and provide and they actually remain in contact for a period of time.

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Film thinning model of coalescence

- Two bubbles will coalesce provided they remain in contact for a period of time sufficient for the liquid film between them to thin out to the critical value necessary for rupture.
- The coalescence rate, Ω , for bubbles of size of d_i and d_j , can be expressed as

$$\Omega_c(d_i, d_j) = \theta_{ij} P_c \quad (m^{-3} s^{-1})$$

Where θ_{ij} is the collision frequency and P_c is the coalescence efficiency.

That will be sufficient for the liquid film between them to thin out the critical value which is necessary for rupture; the coalescence rate which is denoted by Ω of size of d_i and d_j ; here d_i and d_j generally the class of 2 bubbles here which diameter within the range this is d_i and d_j . So, this coalescence rate Ω can be expressed as Ω_c , c for coalescence and which is a function of class i bubble and class j bubble; class i bubbles is means that bubble of diameter d_i and then the bubble of diameter d_j .

So, this coalescence rate that will be is equal to what should be the coalescence efficiency and the collision frequency. So, this coalescence rate depends on this collision frequency and the coalescence efficiency. So, this coalescence efficiency is represented by P_c and collision frequency is represented by θ_{ij} , collision between the particles of diameter d_i and d_j , so this will be coalescence rate.

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Collision Frequency

- Collision may occur due to a variety of mechanisms. One is the random motion of bubbles due to turbulence (see Figure). Saffman and Turner (1956) proposed a collision frequency model which only takes turbulent collision into account

$$\theta_{ij} = \frac{\pi}{4} (d_i + d_j)^2 n_i n_j (\epsilon d_i)^{1/3} \left[1 + (d_i / d_j)^{-2/3} \right]^{1/2} \quad (m^{-3} s^{-1})$$

- where θ_{ij} is the collision frequency between bubbles with diameter of d_i and d_j .
- n_i and n_j is bubble number density (m^{-3}),
- ϵ is local turbulent dissipation rate ($m^2 s^{-3}$)

And then what is that collision frequency then you have to calculate first; what should be the collision frequency this collision may occur due to a variety of mechanisms, 1 is the random motion of the bubbles due to turbulence and in that case saffman and turner 1956 proposed a collision frequency model, which only takes turbulent collision into account.

So, in that case that this collision frequency is a function of the bubble diameter of a different classes and the number of bubbles in that particular classes and also the what will be the energy actually distributed in that system that this collision efficiency also depends on this particular energy level and so based on this collision frequency theta ij is correlated this way here this equation is given this is phi by 4 into di plus dj whole square into ni and nj into epsilon di to the power 1 by 3 into 1 plus di by dj to the power minus 2 by 3 whole to the power 1 by 2.

So, in this case what happened this collision efficiency depends on only frequency depends on only that collision between 2 bubbles. So, that is a function of this bubble diameter number of bubbles in that particular classes and energy distribution inside the bed. So, here this di and dj are the bubble diameter of the class i and j and ni nj are the number of bubbles of class i and j here epsilon is the local actually turbulent dissipation rate; that means, what will be the energy dissipation per unit volume.

So, this will be represented by this epsilon and so from this equation you can calculate what should be the collision frequency for that particular collision efficiency and or what is the coalescence rate.

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- In addition to turbulence, bubbles of different sizes have different rise velocities that may lead to collision
- Also bubbles located in a region of relatively high liquid velocity may collide with bubbles in a slower section of the velocity field
- Prince and Blanch (1990) proposed a collision frequency model based on the summation of the **turbulent collision rate**, **buoyancy-driven collision rate** and **laminar shear collision rate** as follows:

$$\theta_{ij} = \theta_{ij}^T + \theta_{ij}^B + \theta_{ij}^{LS}$$

$$\theta_{ij}^T = 0.089 \pi n_i n_j (d_i + d_j)^2 \varepsilon^{1/3} \sqrt{(d_i^{2/3} + d_j^{2/3})}$$

$$\theta_{ij}^B = \frac{\pi}{16} n_i n_j (d_i + d_j)^2 (u_{ri} - u_{rj})$$

$$\theta_{ij}^{LS} = \frac{1}{6} n_i n_j (d_i + d_j)^3 \left(\frac{du_i}{dR} \right)$$

$$u_r = \sqrt{\frac{2.14 \sigma}{\rho_l d_b} + 0.505 g d_b}$$

(a) Buoyancy-driven and (b) laminar shear induced collision.

And in addition to turbulence the bubbles of different sizes have different rise velocities, that may lead to collision of course 2 different bubbles will have different velocity, because they have different buoyancy they are they are different in size, because of which they will have different velocity in that particular energy level and because of which those bubbles will reach to each other with different velocity also.

Bubbles located in a region of relatively high liquid velocity may collide with bubbles in a slower section of the velocity field. In this case what happen you will see that liquid velocity also is very important because, this liquid velocity what will be the kinetic energy supplied in that particular system that also governs this collision frequency and prince and blanch 1990 they have proposed a collision frequency model, based on the summation of the turbulent collision rate buoyancy driven collision rate and laminar shear collision rate.

So, what we have seen that there are 3 types of collision actually there, one is the turbulent collision another is buoyancy driven collision and another is the laminar shear

collision. So, summation of those 3 collision rate will give you the collision frequency, here in this case by this equation you can calculate what should be the collision frequency here. So, θ_{ij} that will be is equal to θ_{ij} this is turbulent and this is called θ_{ijb} means here buoyancy driven collision rate θ_{ij} means θ_{ijl} , here the collision rate due to the laminar shear. So, θ_{ij} the turbulent shear depends on that how many numbers of the bubbles are in the particular classes and also diameter and also energy level here.

So, in the turbulent collision rate is represented by this equation $0.089 \phi n_i n_j \left(\frac{d_i^2 + d_j^2}{d_i^3 + d_j^3} \right)^{1/3}$ here. So, this case you can get this turbulent collision rate and also the buoyancy driven collision rate, what should be that it depends on this of course, buoyancy driven means here 2 bubbles will be moving at a certain buoyancy depending on the turbulent velocity of the depending on the sorry terminal velocity of the bubbles and also what is the relative velocity of that bubbles, that relative velocity that depends on the physical properties of the system and diameter of the bubble etc. So, in that case if there are 2 classes bubbles then, what should be the relative velocity of that plus i and the relative velocity of plus j.

So, what should be the effective velocity of these 2 classes, that will be is equal to $u_{ri} - u_{rj}$ what is this u_r is the relative velocity of the sorry rise velocity of the bubble here this rise velocity is calculated in this case $\sqrt{2.14 \frac{\sigma}{\rho d_b}} + 0.505g$ into d_b . So, this 1 is the rise velocity here this depends on the sigma, means here surface tension of the liquid and the d_b bubble diameter and also liquid density.

So, from this equation you will be able to calculate what should be the rise velocity and now what should be the relative velocity of this 2 classes i and j, this is i what is the u_{ri} that you can calculate in this u_{ri} means here what will be the bubble diameter of i th classes, then here d_{bi} and d_{bi} here similarly u_{rj} you will be able to calculate u_{rj} will be is equal to here just substitution of d_{bj} and d_{bj} here other remain same.

So, in this case once you know this rise velocity of this class i and rise velocity of class j then what should be the effective velocity of this, at which this bubbles are moving then

this and also what should be the bubble diameter from this portion you have to from this equation, you can calculate what should be the buoyancy driven collision rate.

Another one is that laminar shear collision rate in that case 2 bubbles will be colliding parallelly at it is surfaces and that depends on the what should be the liquid velocity how it will be changing with respect to radius of the bubble. So, in that case that also depends on that diameter of the bubble of different classes and gradient of the liquid velocity.

So, here in this case this you will see the liquid velocity at a certain liquid velocity, how it will be moving this bubbles and then it will be coming to each other and then it will be colliding with this bubbles and then your joining this will be collision here and then because of which there will be a coalescence and here buoyancy driven to bubbles will be moving here smaller bubbles of course will be moving slower rate; whereas, bigger bubbles will be moving higher flow rate because of their buoyancy effect and then once upon a time they will be coming to each other and joining and then coalescence.

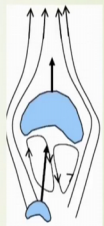
Here this relative velocity here 1 bubbles will be moving faster rate, another bubbles will be moving another faster rate that is rise velocity of this 2 bubbles and they once upon a time they will be colliding and then and then making a bigger bubbles or coalescence bubbles due this collision frequency.

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Lehr and Mewes (2001) model for collision frequency

- Lehr and Mewes (2001) suggested another expression for collision frequency following Prince and Blanch's (1990) model which take the turbulence and buoyancy driven induced collision into account:


$$\theta_{ij} = \frac{\pi}{4} (d_i + d_j)^2 n_i n_j \left(\max \left(\sqrt{2} (\varepsilon \sqrt{d_i d_j})^{1/3}, |u_{ri} - u_{rj}| \right) \right)$$



Now, Lehr and Mewes 2001 model for collision frequency. So, they have suggested another expression for collision frequency, just following the prince and Blanchs model which take the turbulence and turbulence and buoyancy driven induced collision into account. So, based on their model this collision frequency will be calculated based on this equation; here in this case this of course, you have to find out what is the relative velocity of the 2 classes of bubbles and what should be that this energy level here from this.

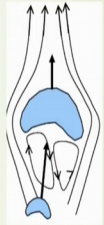
Now which 1 is the maximum from this 2 part based on that you have to calculate what will be the collision frequency, here also this frequency rate this collision frequency depends on the number of bubble in that particular classes of i and j; if you are dividing into i and j classes may be there are 2 classes your just you have to divide into 2 classes accordingly that based on the diameter of the bubble and see this Lehr and Mewes model exactly the same, but they have what blanch prince and Blanch they have actually proposed only thing is that there that relative velocity and also the energy level here based on which will be the maximum that will be considered here, because this those bubbles are colliding to each other they may have different energy level, because of their that is buoyancy effect and also the size of the bubble.

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Lehr and Mewes (2001) model for collision frequency

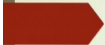
- If a bubble enters another bubble's wake (Figure), it will usually overtake the leading bubble in an inline collision that may result in coalescence.
- Although there are many observations of the wake effects, all of which showed that the wake effect is very important for coalescence, the wake-induced collision models are rare.



And if the bubble enters another bubbles wake according to the Lehr and Mewes, it will usually overtake leading bubble in an online in an inline collision that may result in

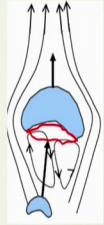
coalescence and although there are many observations of the wake effects, all of which showed that the wake effect is very important for coalescence the wake induced collision models are very rare.

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Collision frequency due to wake entrainment

- **Wu et al. (1998)** defined the effective wake volume, in which the bubbles that follow may collide with the leading one, as the projected bubble area multiplied by the effective length, L , which is 5-7 times the bubble diameter.
- They assumed that the wake structure of the leading bubble to be the same as that of a solid sphere



Now, collision frequency due to the wake entrainment this WU et al, actually defined the effective wake volume, in which the bubble that follow may collide with the collide with the leading one as the projected bubble area multiplied by the effective length l , which is generally 5 to 7 times the bubble diameter. In this case one important point is that you have to consider the projected bubble area and also what should be the effective length, effective length here actually based on the bubble diameter, it is generally 5 to 7 times of the bubble diameter is considered. So, they assume that this wake structure of the leading bubble to be the same as that of a solid sphere.

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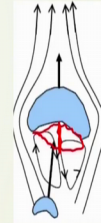
Collision frequency due to wake entrainment

- Wu et al. (1998) presented the following collision frequency due to wake entrainment (bubble of size d is the leading bubble):

$$\theta_{ij}^{WE} = C_{WE} d_i^2 \frac{u_r(d_i)}{L_w/d_i - 1/2} \left[\left(\frac{L_w}{d_i/2} \right)^{1/3} - 1 \right] n_i n_j$$

$$u_r = \left(\frac{d_i g (\rho_l - \rho_g)}{3 C_D \rho_l} \right)^{1/2}$$

$$C_D = 24 \frac{1 + 0.1 \text{Re}_D^{0.75}}{\text{Re}_D}, \quad \text{Re}_D = (1 - \alpha_g) \frac{\rho_l u_r d_i}{\mu_l}$$



Now, collision frequency due to this wake entrainment this WU et al. presented the following frequency, due to the wake entrainment here this θ_{ij} , this WE for wake entrainment, we represented the wake entrainment here and then this is a function of this here L_w bubble diameter and the number of bubbles in that particular classes. Now here this θ_{ij} , because of this wake entrainment if we have this proportionality constant will be is equal to C_{WE} and that depends on what will be the experimental condition that you are following.

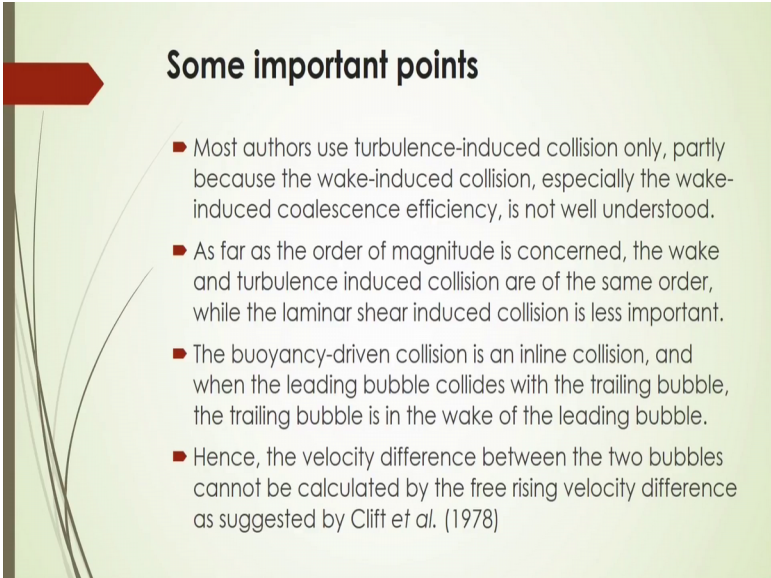
So, this is the constant and also here this L_w is the effective length, which is actually that we have already told that could be 5 to 7 times of the bubble diameter to be considered here and here this wake length this is wake length L_w , this wake length here wake length here very important this is the wake region and what will be the effective length here this is the wake length here.

So, this wake length is a main important factor how what will be the volume of the wake, that depends also that that also govern this wake length; here wake length if it is increases what will happen sometimes this collision frequency will increases and sometimes it may reduces, because of the bubble diameter and also here the rise velocity of this bubble this will be of course, this is a function of this $d_i g$ and the relative density of the liquid and also what is that drag coefficient there. So, C_D can be calculated C_D is the drag coefficient C_D can be calculated as $24 \text{ into } 1 \text{ plus } 0.1 \text{ Reynolds number}$, this is depends on the Reynolds number; by this equation you will be calculate that should be

the drag coefficient here and if you and Reynolds number will be defined as here this μ_r μ_l into $1 - \alpha_g$.

This α_g is the gas hold up in the gas liquid solid 3 phase fluidized bed. So, you know this is very important that this wake induced collision rate depends on the gas hold up inside the bed and the number of bubbles in the particular classes and also what should be the rise velocity of the bubbles what will be the wake length and what will be the rise velocity of the bubble and also what will be the drag coefficient that you have to calculate and Reynolds number in this case if it is a turbulent flow then only C_D should be considered as 0.44, whereas other than this turbulent flow you have to calculate the drag coefficient from this equation here.

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Some important points

- Most authors use turbulence-induced collision only, partly because the wake-induced collision, especially the wake-induced coalescence efficiency, is not well understood.
- As far as the order of magnitude is concerned, the wake and turbulence induced collision are of the same order, while the laminar shear induced collision is less important.
- The buoyancy-driven collision is an inline collision, and when the leading bubble collides with the trailing bubble, the trailing bubble is in the wake of the leading bubble.
- Hence, the velocity difference between the two bubbles cannot be calculated by the free rising velocity difference as suggested by Clift *et al.* (1978)

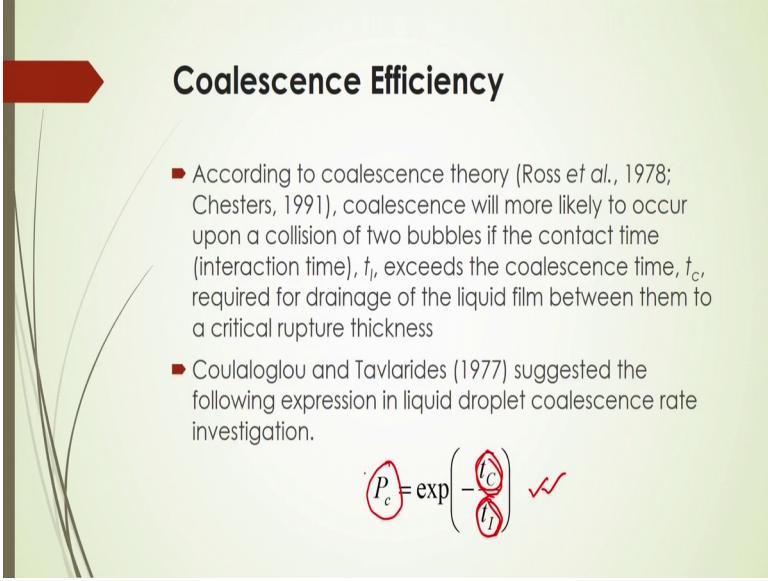
Now, some important which is to be noted also remember that in this case most authors used turbulence induced collision only partly because, the wake induced collision especially the wake induced collision efficiency is not well actually understood, because of that wake structure is not actually is not well defined and also it is not well predicted in particular operating system. So, what should be that collision frequency that is not actually well predicted for this coalescence rate; as far as the order of magnitude is concerned that the wake and turbulence induced collision are of the same order it is seen from that simulation and also from the experimental condition; while the laminar shear induced collision is sometimes is not important in the high turbulent region.

The buoyancy driven collision is in inline collision and when the leading bubble collides with the trailing bubble is in the wake of the leading bubble and hence the velocity difference between these 2 bubbles contact 2 bubbles that may not be suitable may not be suitable for colliding efficiency because, it cannot be calculated by the free rising velocity difference as suggested by Clift et al 1978.

Also in turbulent flow there are the only 2 important collision mechanisms that the wake entrainment and the turbulence. So, in summary we can say that there are 2 important collision mechanisms the wake entrainment and the turbulence; but turbulence and laminar shear induced collision sometimes used for in laminar condition and also turbulent conditions in between that wake entrainment, is sometimes taking into account for the refinement of the simulation one can either assume that the wake induced coalescence efficiency is the same as turbulence induced coalescence efficiency.

So, that the summation of these 2 collision frequency can then be used to calculate the coalescence efficiency, that is actually reported by Hibiki and Ishii 2000 or one can consider only the turbulent induced collision as the existing coalescence efficiency models are actually based on the turbulence induced coalescence that reported by Luo et al 1993 now up to this, so we have discussed that what should be the collision frequency.

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Coalescence Efficiency

- According to coalescence theory (Ross et al., 1978; Chesters, 1991), coalescence will more likely to occur upon a collision of two bubbles if the contact time (interaction time), t_i , exceeds the coalescence time, t_c , required for drainage of the liquid film between them to a critical rupture thickness
- Coulaloglou and Tavarides (1977) suggested the following expression in liquid droplet coalescence rate investigation.

$$P_c = \exp\left(-\frac{t_c}{t_i}\right)$$

✓

Now what should be the coalescence efficiency, then coalescence efficiency actually will be more likely to occur when the collision of 2 bubbles will be more intensive in that

particular operation, if the contact time and the interaction time they are actually these 2 times are mainly governing this coalescence efficiency.

So, according to coalescence theory that coalescence will more likely to occur upon a collision of 2 bubbles, if the contact time that is interaction time exceeds the coalescence time that is t_i and t_c and also the required to drainage of the liquid film it is required between them to a critical rupture thickness. Now of course, then what should be that t_i and t_c , it is very important now based on which this Coulaloglou and Tavlariides 1977 suggested the following expression in liquid droplet coalescence rate, for their investigation here this P_c is equal to exponent of that is minus t_c by t_i , this t_c is the coalescence time and t_i is the interaction time and P_c is the coalescence efficiency.

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Interaction and coalescence times

- the interaction time, t_i , is proportional to the characteristic lifetime of an eddy of size equal to the sum of the sizes of the approaching bubbles which can be estimated as

$$t_i \propto d^{2/3} \varepsilon^{-1/3}$$
- the coalescence time, t_c , can be estimated from the film drainage model

$$t_c \propto \rho_c^{1/2} d^{3/2} \sigma^{-1/2}$$

Then the coalescence efficiency can be estimated as

$$P_c \propto \exp\left(-\frac{t_c}{t_i}\right) = \exp\left(-\rho_c^{1/2} d^{5/6} \sigma^{-1/2} \varepsilon^{1/3}\right)$$

Now, what should be that interaction and coalescence time, this interaction time is actually proportional to the characteristics lifetime of an eddy of size equal to the equal to the sum of the sizes of approaching bubbles which can be estimated as this here. So, interaction time depends on the size of the eddy; that means size of the bubble or bulk of the liquid that drifts the bubbles and also the take per's to actually make a collision between 2 bubbles, that depends on which this interaction time. So, this interaction time is a function of then what is the eddy size that is and also what will be the energy there energy distribution or energy dissipation for this collision and then here this t_i can be calculated from this relationship.

So, it depends on d_i to the power 2 by 3 and epsilon to the power minus 1 by 3, the coalescence time t_c can be estimated from the film drainage model; this again is a function of this diameter of the bubble and continuous phase density and the surface tension of the continuous phase that is liquid here.

So, from this equation also you can calculate what should be the t_c and then the coalescence efficiency after substituting this t_c and t_i , value there then coalescence efficiency will be equals to that in terms of this all variables like density of the continuous phase and the diameter of the eddy or we can see bubble size here and the surface tension of the liquid and the energy dissipation for the happening of this collision to give you coalescence efficiency.

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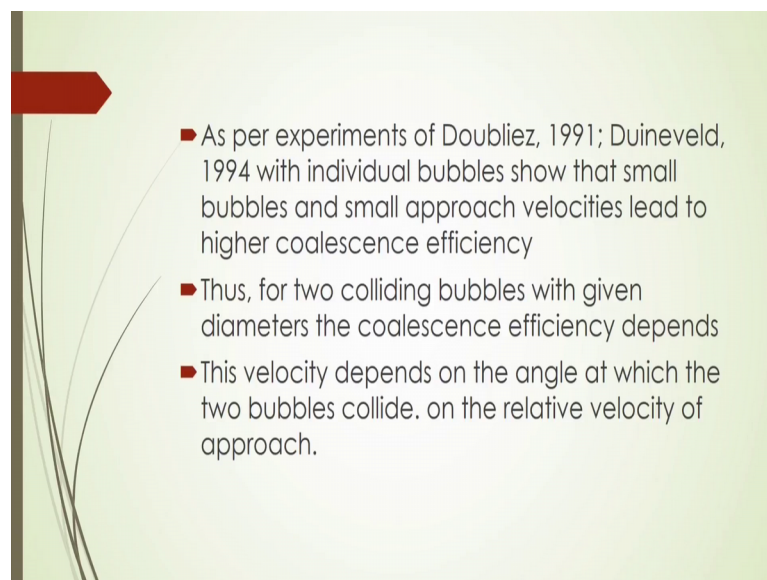
Different Models of bubble coalescence efficiency	
Lee et al., 1987a	$P_c(d_i, d_j) = \exp \left[-\frac{1}{4} \left(\frac{R}{d_i + d_j} \right)^{2/3} t_c \right]$ $t_c = \left(\frac{R}{4} \right) \left(\frac{\rho_c d}{2\sigma} \right)^{1/2} \ln \left(\frac{h_0}{h_f} \right) + 24\pi^2 M \sigma \mu h_f^2 A_0^{-2}$
Prince and Blanch, 1990	$P_c(r_i, r_j) = \exp \left[-\left(\frac{r_i^3 \rho_c}{16\sigma} \right)^{1/2} \epsilon^{1/3} r_i^{-2/3} \ln \frac{h_0}{h_f} \right]$, $r_e = \frac{1}{2} \left(\frac{1}{r_i} + \frac{1}{r_j} \right)^{-1}$
Chesters, 1991	$P_c = \exp \left[-c \left(\frac{We}{2} \right)^{1/2} \right]$, $We = \frac{\rho_c (\epsilon d)^{1/2} d}{2\sigma}$
Luo, 1993	$P_c(d_i, d_j) = \exp \left[-\frac{0.75 \left(1 + \frac{\epsilon^2}{\sigma^2} \right) \left(1 + \frac{\epsilon^2}{\sigma^2} \right)^{1/2}}{\left(\frac{\rho_d}{\rho_c + \rho} \right)^{1/2} \left(1 + \frac{\epsilon^2}{\sigma^2} \right)^{1/2}} We^{1/2} \right]$, $We = \frac{\rho_c d \bar{u}_e^2}{\sigma}$, $\frac{\epsilon}{\sigma} = d_i / d_j$, $\bar{u}_e = \left(\bar{u}_i^2 + \bar{u}_j^2 \right)^{1/2} = \bar{u}_i \left(1 + \frac{\epsilon^2}{\sigma^2} \right)^{1/2}$, $\bar{u}_i = \beta^{1/2} (\epsilon d_i)^{1/2}$, $\beta = 1.69$

Now, different other models of bubbles coalescence efficiency has given by different investigators here lee et al 1987, they have given this equation for calculating the coalescence efficiency; here one important parameter is c and this c to be calculated from the experimental data and also this t_c is the coalescence time this coalescence time can be calculated based on this equation. And prince and blanch they have given this equation this coalescence efficiency depends on the physical properties of the system and the what that is radius of this effective radius of this classes i and j bubble, which will be calculated from this equation that is r_{ij} ; this r_{ij} is equal to half of 1 by r_i plus 1 by r_j this Chester's 1991 he has given this coalescence.

In terms of that is way one number, this way one number is defined here ρc into ϵd to the power $2/3$ $\bar{d}^2 \pi$ is this way one number, this will be able to calculate the coalescence efficiency from this equation. Luo et al 1993 they have developed other models like this; this models actually one parameter that is c which will be calculated from this experimental condition.

And also this depends on the way one number and also other parameter like ξ that is ration of this diameter of bubbles of i and j classes and then what will be the relative velocity or relative or velocity of this what is that this 2 classes bubbles and what should be the rise velocity of the particular classes of i bubbles there. So, from this correlation also you can calculate the coalescence efficiency.

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Now, as per experiments of Doublez 1991 and also the Duineveld 1994 with individual bubbles show that the small bubbles and small approach velocities lead to higher coalescence efficiency. And 2 colliding bubbles with given diameters than coalescence efficiency, that depends on this particular velocity of this bubble classes, and this velocity depends on the angle at which they come to each other and collide to each other at a certain relative velocity, and from which will be able to calculate what should be the coalescence efficiency.

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■ Lehr and Mewes (2001) assume that the relative probability is equal for all steradians. Thus, the coalescence efficiency can be written as:

$$P_c(d_i, d_j) = \min\left(\frac{u_{crit}}{u'}, 1\right)$$

where $u_{crit} = \sqrt{\frac{We_{crit} \sigma}{\rho d_{eq}}}$, $u' = \max\left(\sqrt{2} \left(\varepsilon \sqrt{d_i d_j}\right)^{1/3}, |u_i - u_j|\right)$ and $d_{eq} = 2 \left(\frac{1}{d_i} + \frac{1}{d_j}\right)^{-1}$

$We_{crit} = 0.06$

Lehr and Mewes 2001 assumed that the relative probability is equal to all Steradians, thus the coalescence efficiency can be written as this here; from this equations as per Lehr and Mewes you will be able to calculate what will be the coalescence efficiency.

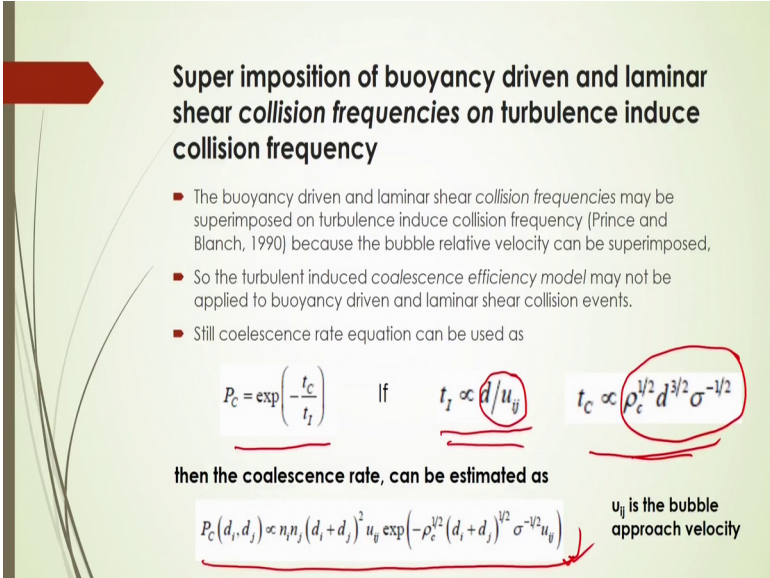
In this case very important point that some critical velocity is required to make a coalescence of this, 2 bubbles; this critical velocity depends on the critical way one number this which is defined as root over of WE critical sigma by rho 1 d equivalent here, in this critical velocity the equivalent diameter.

If bubble diameter is very small than critical velocity will be something else this, critical velocity actually that depends on what will be the critical way one number generally in 3 phase system in this critical number will be considered as 0.06 for this bubble coalescence phenomena. So, from which you have to calculate what should be the critical rise velocity there. This again depends on that equivalent bubble diameter this equivalent bubble diameter.

Since you are considering this 2 classes of bubbles, then what should be the equivalent bubble diameter that will be calculated as this equation here 2 into 1 by di plus 1 by dj to the power minus 1. So, from this equation you will be able to calculate what should be the equivalent bubble diameter and assume the equivalent bubble diameter and also what will be the bubble diameter of this classes i and j and they are respective velocity will be able to calculate what will be the coalescence efficiency from this equation. So, this may

be this coalescence efficiency will be either minimum of this 2, whether this critical by this actual velocity or 1. So, either minimum of this 2 you have to take as coalescence efficiency for your calculation.

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Super imposition of buoyancy driven and laminar shear collision frequencies on turbulence induce collision frequency

- The buoyancy driven and laminar shear collision frequencies may be superimposed on turbulence induce collision frequency (Prince and Blanch, 1990) because the bubble relative velocity can be superimposed,
- So the turbulent induced coalescence efficiency model may not be applied to buoyancy driven and laminar shear collision events.
- Still coalescence rate equation can be used as

$$P_c = \exp\left(-\frac{t_c}{t_i}\right) \quad \text{If} \quad t_i \propto \frac{d}{u_{ij}} \quad t_c \propto \rho_c^{1/2} d^{3/2} \sigma^{-1/2}$$

then the coalescence rate, can be estimated as

$$P_c(d_i, d_j) \propto n_i n_j (d_i + d_j)^2 u_{ij} \exp\left(-\rho_c^{1/2} (d_i + d_j)^{3/2} \sigma^{-1/2} u_{ij}\right)$$

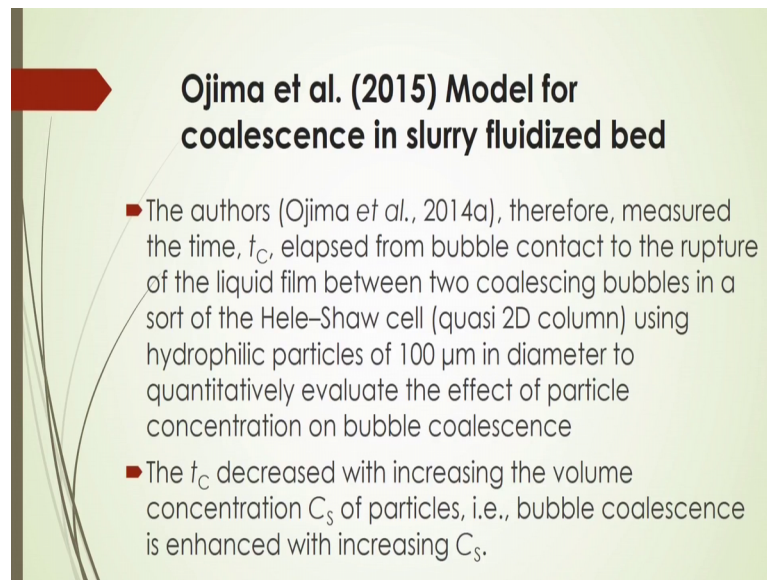
u_{ij} is the bubble approach velocity

Now, another important that super imposition of buoyancy driven and laminar shear collision frequencies on turbulence induce collision frequency, the buoyancy driven and laminar shear collision frequencies may be superimposed on turbulence induce collision frequency as per prince and blanch 1990 and because the bubble relative velocity can be there superimposed. So, the turbulent induced coalescence efficiency model may not be applied to buoyancy driven and laminar shear collision models.

So, that is to be noted down still coalescence rate equation can be used as here this P_c as is equal to exponent of minus t_c by t_i . If t_i is proportional to this ratio of this d by u_{ij} and t_c as this equation here given, so then the coalescence rate can be estimated as from this equation here after substitution of this t_i and t_c and finally, you can get this coalescence efficiency. So, which is to be noted down that here the buoyancy driven and laminar shear collision frequencies, sometimes may be superimposed they are for your simulation and for further calculation.

So, the turbulent induce collision efficiency model may not be applied to the buoyancy driven and laminar shear collision events because of their superimposition.

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Ojima et al. (2015) Model for coalescence in slurry fluidized bed

- The authors (Ojima et al., 2014a), therefore, measured the time, t_c , elapsed from bubble contact to the rupture of the liquid film between two coalescing bubbles in a sort of the Hele-Shaw cell (quasi 2D column) using hydrophilic particles of 100 μm in diameter to quantitatively evaluate the effect of particle concentration on bubble coalescence
- The t_c decreased with increasing the volume concentration C_s of particles, i.e., bubble coalescence is enhanced with increasing C_s .

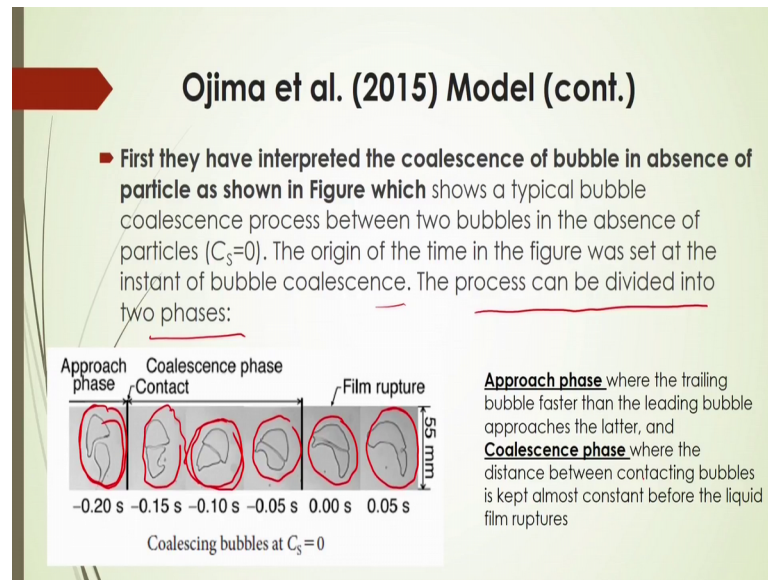
Ojima et al 2015 they have also presented 1 coalescence model in the slurry fluidized bed. So, in that case according to that ojima et al that they actually measured the time of coalescence and also based on which they actually derived the model in the quasi 2d columns by their experiment and they have interpreted that coalescence frequency according to their experimental results.

So, from their experiment they observed that the coalescence time that decrease with the increasing the volume concentration of solid particles and that is why this bubbles coalescence is enhanced with the increasing of solid concentrations in the fluidized bed. So, here in this case it is important to note that that whenever you are using certain consideration of the solids, that for this coalescence efficiency for their experiment you cannot do with that d type or b type particles, you have to use the particle size at least within the range also 0 to 100 micrometer.

So, this will be better to get this coalescence efficiency in this slurry fluidized bed and for this other type like d type and b type this will see that there will be chant turbulent region and also high velocity there will be very difficult to calculate that critical diameter of the bubble to get it coalescence and make a the bigger bubbles to observing the particular experimental condition. So, with the smaller particles it will be easier to actually observe or interpret the coalescence efficiency the fluidized bed and also this because of this there will be certain concentration to be also maintained; according to

this ojima et al they also represented they also stated that this coalescence time depends on the concentration of the particle and that is why you can say that bubble coalescence will be changed with this particle concentration.

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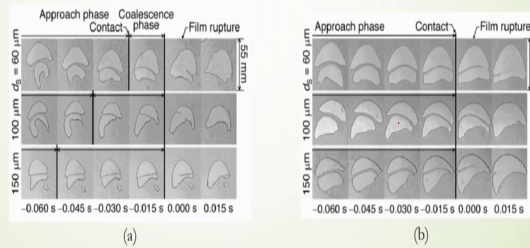
Now, first they have interpreted the coalescence of bubbles in absence of particle as shown in figure, here which shows a typical bubble coalescence process between 2 bubbles in absence of particles, initially they have not taken any particles and they got some phenomena of this coalescence like this 2 bubbles approaching at a certain time and after that they are contacting and then they are just after with respect to time they are becoming different shape and conjugating and then after that film rupturing and then after that they become a bigger bubble 1 with respect to time.

So, it will take very smaller time to take to get this coalescence after getting final stage of film rupture here. So, this happens only this there is no that means particle concentration is 0 there. So, the origin of the time in the figures that was set at the instant of bubble coalescence the process can be divided into 2 phases here, one is called approach phase where the trailing bubble faster than the leading bubble approaches the latter one, whereas this coalescence phase in this case the distance between this contacting bubbles is kept almost constant before the liquid film rupture. So, in this way these 2 approaches they have explained the coalescence efficiency in the fluidized bed.

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Ojima et al. (2015) Model (cont.)

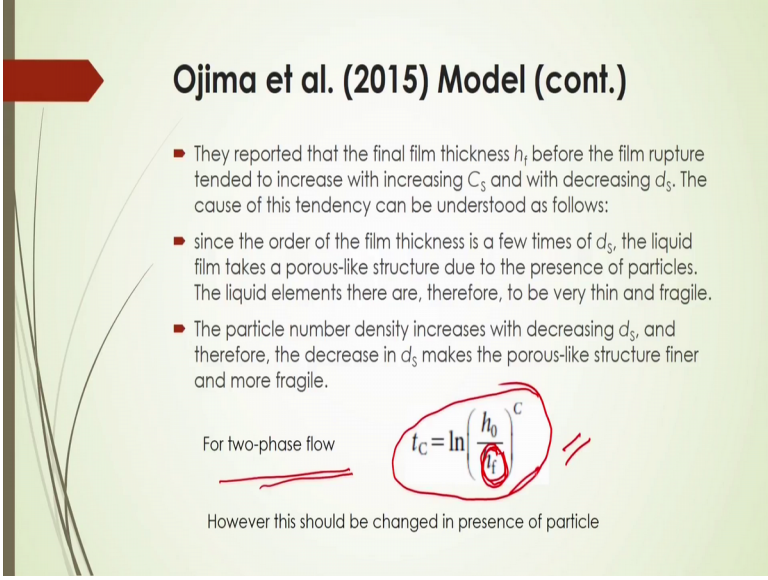
- They also done the experiment with different concentration of particles and explained as:
- The stated that coalescing bubbles at $C_s = 0.20$ as shown in **Figure (a)**, compared with the bubbles at $C_s = 0$, the coalescence phase is shorter at all d_s . The decrease in d_s also decreases the time duration of the coalescence phase, i.e., bubble coalescence is enhanced.
- The effect of d_s , however, disappears at the higher concentration, $C_s = 0.40$, as shown in **Figure (b)**. In this case, the bubbles coalesce immediately after the contact, so that d_s has little influence on coalescence.



And they also done the experiment with the different concentration of the particles and explained that that coalescence bubbles at concentration of the particles of 20 percent as shown in figure a; compared with the bubbles at no concentration of the particles the coalescence phase is in that case they observe that the shorter at all size of particles and the decrease in this solid diameter also decrease the time duration of the coalescence phase, that is the bubble coalescence is enhanced there.

So, it is important to note that if you decrease the particle diameter you can have the enhancement of the bubble coalescence, the effect of this particle diameter; however, disappears at the higher concentration at the particle as shown in figure here b, the in this case the bubble coalescence immediately after the contact, so that the size of the solid has a little influence on the coalescence.

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Ojima et al. (2015) Model (cont.)

- They reported that the final film thickness h_f before the film rupture tended to increase with increasing C_s and with decreasing d_s . The cause of this tendency can be understood as follows:
- since the order of the film thickness is a few times of d_s , the liquid film takes a porous-like structure due to the presence of particles. The liquid elements there are, therefore, to be very thin and fragile.
- The particle number density increases with decreasing d_s , and therefore, the decrease in d_s makes the porous-like structure finer and more fragile.

For two-phase flow

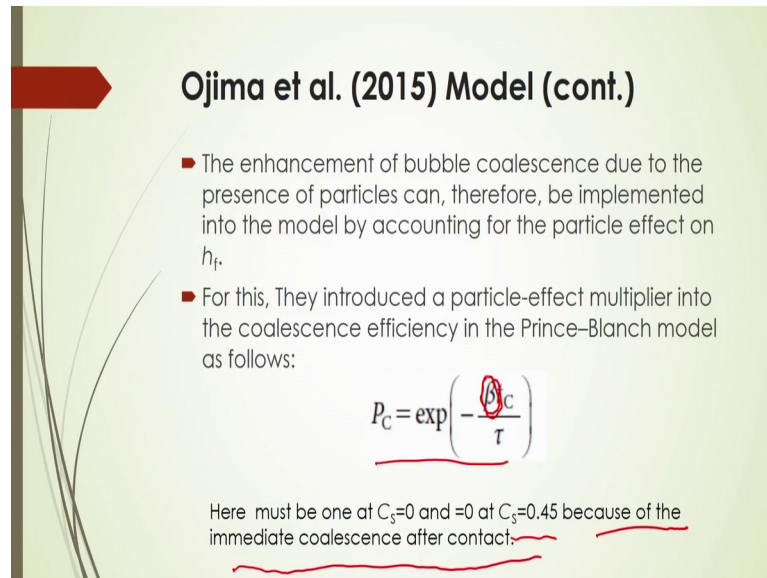
$$t_c = \ln \left(\frac{h_0}{h_f} \right)^C$$

However this should be changed in presence of particle

And they reported that that final film thickness, that is represented by h_f final film thickness before the film rupture, that tended to increase with the increase of solid concentration and it will decrease with the diameter of the solid. The cause of this tendency can be understood as like this here, since the order of the film thickness is a few times of that particle diameter, the liquid film takes a porous like structure due to the presence of particles and the liquid elements there are therefore to be very thin and fragile.

The particle number density increases within with decreasing particle diameter and therefore the decrease in particle diameter makes the porous like structure finer and more fragile. So, for 2 phases flow you can apply this is coalescence time based on the film thickness and these 2 phase flow for this coalescence time you can apply for this 3 phase flow because, here this h_f depends on that final film thickness depends on the concentration of the particle, so you can apply it in the 3 phase system also; however, this should be changed in presence of particles because of this film thickness.

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Ojima et al. (2015) Model (cont.)

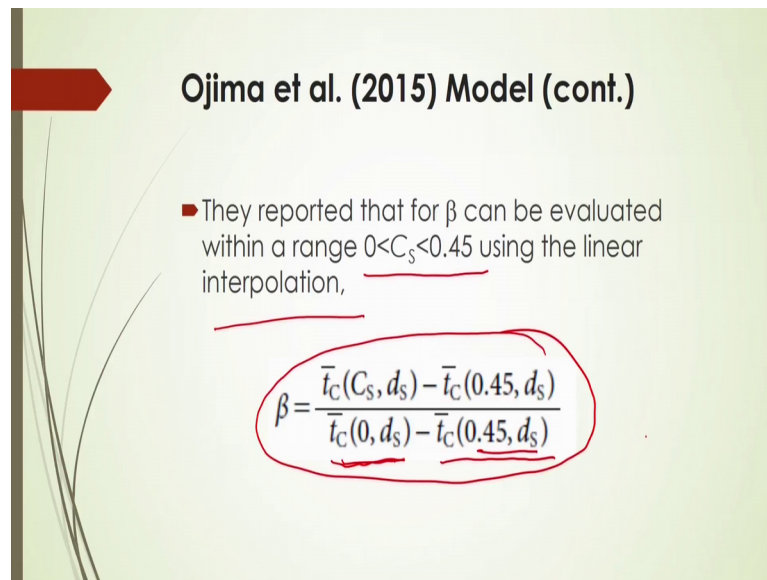
- The enhancement of bubble coalescence due to the presence of particles can, therefore, be implemented into the model by accounting for the particle effect on h_f .
- For this, They introduced a particle-effect multiplier into the coalescence efficiency in the Prince-Blanch model as follows:

$$P_c = \exp\left(-\frac{\beta_c}{\tau}\right)$$

Here β_c must be one at $C_s=0$ and $=0$ at $C_s=0.45$ because of the immediate coalescence after contact.

This enhancement of the bubble coalescence due to the presence of particle can therefore be implemented into the model by accounting for the particle effect on final film thickness; for this actually this ojima introduced a particle effect multiplier into the coalescence efficiency in the prince Blanch model as follows here. So, they have also followed the prince blanch model for this bubble coalescence just incorporating 1 multiplier here beta. So, this beta in this case depends on the particle concentration and here must be 1 at C_s is equal to 0 and it will be beta is equal to 0 whereas, this C_s will be is equal to 0.45 because, of the immediate coalescence after contact. So, this beta depends on the particle concentration and they reported that.

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Ojima et al. (2015) Model (cont.)

- They reported that for β can be evaluated within a range $0 < C_s < 0.45$ using the linear interpolation,

$$\beta = \frac{\bar{t}_c(C_s, d_s) - \bar{t}_c(0.45, d_s)}{\bar{t}_c(0, d_s) - \bar{t}_c(0.45, d_s)}$$

For beta can be evaluated within a range of particle concentration of this 0 to 0.45 using the linear interpolation like this here. So, this beta will be calculated from this equation here beta will be is equal to t_c that is a function of C_s and d_s minus t_c bar that is what is that at particle concentration of 0.45 at the same particle diameter and also this t_c 0 and d_s .

That means there will be no particle here at that condition what will be the t_c and also what should be the t_c that is coalescence time, when the particle concentration is at 45 percent and the diameter of the particle will be the same in size and also they have suggested.

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Ojima et al. (2015) Model (cont.)

■ They have suggested the following interpolation function based on their interpolation data as

$$\beta(C_s, d_s^*) = \begin{cases} f(C_s)g(C_s, d_s^*) & \text{for } C_s < 0.45 \\ 0 & \text{for } C_s \geq 0.45 \end{cases}$$

$$f(C_s) = \frac{1}{2} \left[\cos \left(\frac{20\pi}{9} C_s \right) + 1 \right]$$

$$g(C_s, d_s^*) = (0.187 d_s^{*-3.84} - 11.5) C_s^3 + (-3.63 d_s^{*-1.39} + 12.1) C_s^2 + 1$$

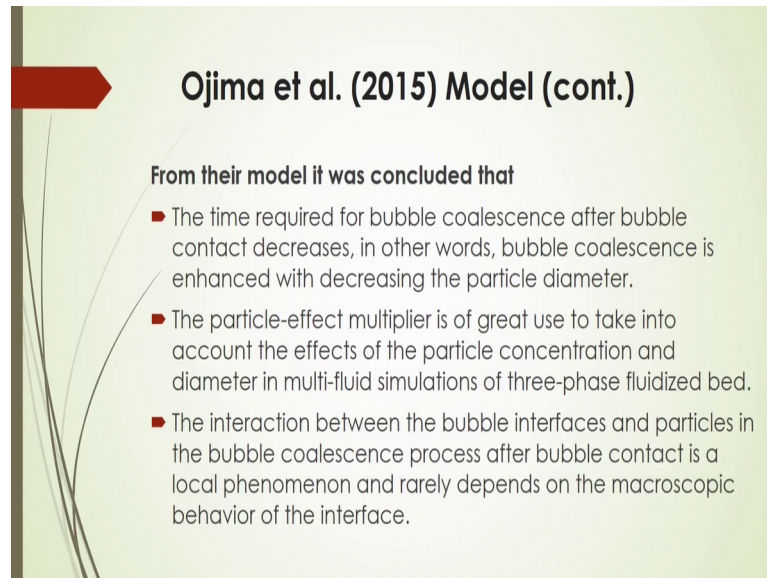
$$d_s^* = \frac{d_s}{150 \times 10^{-6}}$$

The following interpolation function based on their interpolation data as this beta C, beta is a function of that particle concentration and diameter of the particle, this diameter of this particle d_s^* is defined as this d_s by 150 into 10 to the power minus 6. So, this is again is a function of this concentration of the particle. So, it will be is equal to f as a function when the concentration is less than 0.45 and it will be 0 when the concentration is greater than equals to 0.45.

So, f C s will be is equal to this half of this cos into 25 phi by 9 into C_s plus 1. This function is depending on the concentration of the particle, and then again another function is called g C s; this g C s d_s^* is a again is a function of this critical diameter of this particle, here this is a 0.187 into d_s^* to the power minus 3.84 minus 11.5 into C_s cube and plus this.

So, from this correlation you will be able to calculate what should be the function of g and what should be the function of f; and then this beta will be is equal to product of this function f and g if the concentration you are taking as less than 45 percent. Whereas, this beta this parameter will be 0 if you are considering the concentration is greater than of 45 percent. And then form their model it can be concluded that the time required for bubble coalescence after bubble contact decreases.

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Ojima et al. (2015) Model (cont.)

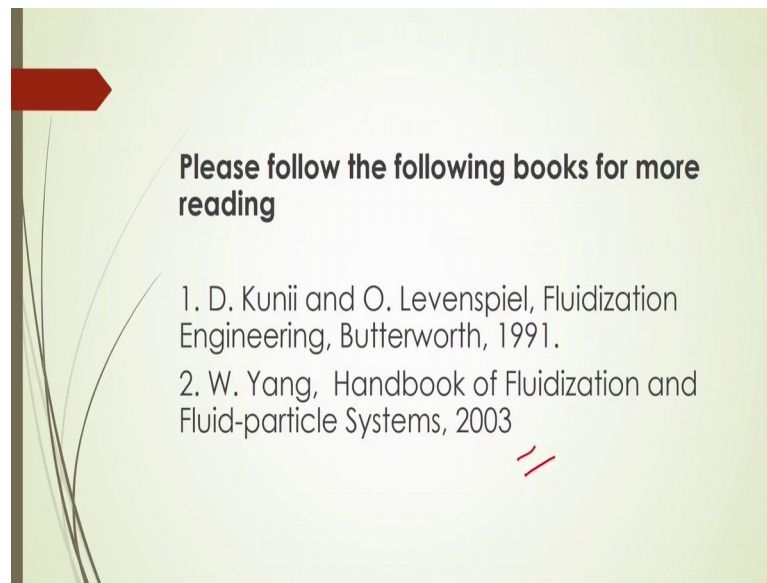
From their model it was concluded that

- The time required for bubble coalescence after bubble contact decreases, in other words, bubble coalescence is enhanced with decreasing the particle diameter.
- The particle-effect multiplier is of great use to take into account the effects of the particle concentration and diameter in multi-fluid simulations of three-phase fluidized bed.
- The interaction between the bubble interfaces and particles in the bubble coalescence process after bubble contact is a local phenomenon and rarely depends on the macroscopic behavior of the interface.

In other words bubbles coalescence is enhanced with decreasing the particle diameter, the particle effect multiplier is of great use in that case to take into account the effects of particle concentration and the diameter in multi fluid simulation of the 3 phase fluidized bed; the interaction between the bubble that depends on this concentration and also the particle size and governing equation of this bubble coalescence efficiency.

So, that bubble coalescence process after this contact of this bubbles, just modifying the equation of the 2 phase theory by taking the one parameter of beta, which depends on this concentration of the particles and it is seen that this presence of this particles that depends that effect the rupture of the film and which will enhance the bubble coalescence there.

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So, further discussion and further also information you can get from this reference this kunii and Levenspiel and yang also and also latest articles also can be followed to get the bubble coalescence more details in fluidized bed system. So, we have discussed what should be the bubble coalescence bubble collision rate in 2 phase 3 system and also what should be the phenomena in 3 phase system.

How to calculate this collision efficiency, how to calculate the bubble coalescence efficiency and collision frequency and what would be the mechanism that is approaching mechanism, there are 3 stages that what that wake induced collision laminar shear induced collision and also turbulent induced collision that will govern the coalescence efficiency and in presence of particles there increase of enhancement of the bubble coalescence that governs.

So, in the 3 phase system how to actually simulate the coalescence phenomena, you can use different software and also fluid software by taking into account this models, then you will get the different phenomena coalescence in and you can validate with your experimental observation, then you will interpret well in defined and for design of the 3 phase fluidized bed. So, next lecture onward we will be discussing more about this bed, so that is for all today.

Thank you.