Process Design Decisions and Project Economics

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Module - 08
Chemical Project Economics
Lecture - 42
Time Value of Money

Welcome in the previous lecture, we saw a simple cost model where you can estimate the total capital investment for a particular process, only on the basis of the onsite cost. Then we saw the concept of depreciation, various methods of depreciation, the straight line method, double declining balance, M A C R S method and finally, we developed a simple expression for the net cash accruals or the net cash flow to the company from the process. Now in this lecture we take ahead this thing and see the concepts of interest and time value of money.

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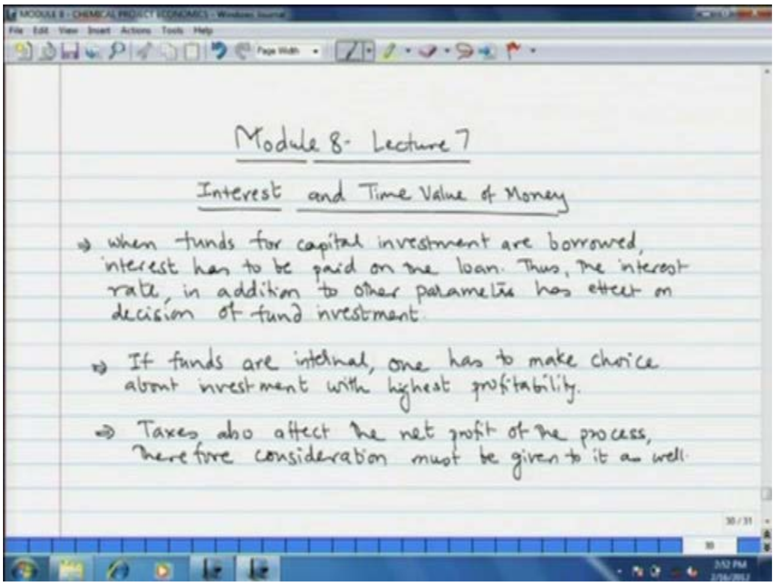

In most of the cases the funds for capital investment are borrowed and therefore, we have to pay interest and that loan, it could be form of a equity where dividend displayed or the loan can be borrowed from different financing institutions, like H D F C's housing development, I D B I industrial development bank of India.

And then we have to pay interest on that loan or dividend on the equity therefore, the interest in addition to other parameters has effect on the decision of fund investment in some cases, the funds are completely internal especially for a big companies and in that case one has to make decision, only about investment with highest profitability, then there are taxes like in the previous lecture, we saw income tax, but in addition to income tax, there are several other charges and which also affect the net profit of the process.

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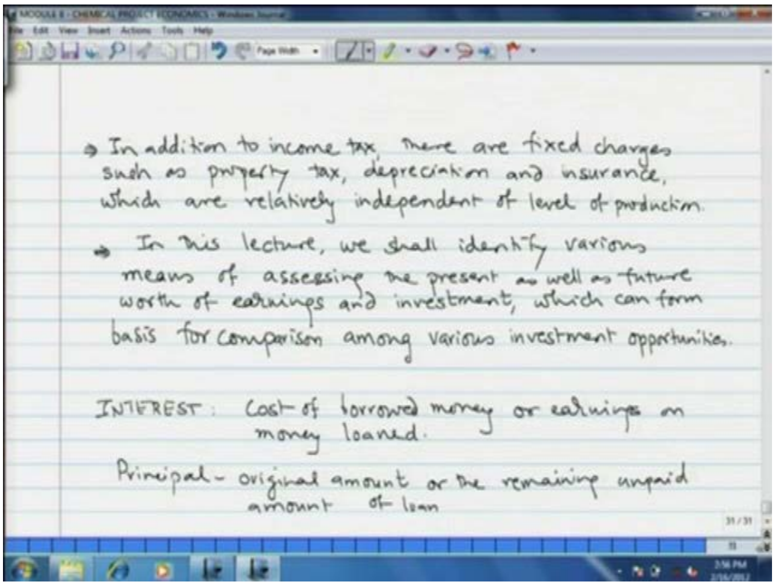

Now these are the in addition to income tax, these are like fixed charges like property tax depreciation and insurance, which are relatively independent of the level of production. So, what we will do in this lecture is that, we shall identify various means of assessing the present as well as future worth of earning and investment, which can form basis for comparing among various investment opportunities.

We are going to make an investment at time zero for a particular process and then we are going to earn certain money or net cash flow, that we develop that, we saw in the previous lecture over the operation, a period of the plant which would be 10 years, 15 years, 20 years. However while making investment at time zero, we have to make an estimate of the future worth of the investment and the present worth of the earnings, that is what we are going to do in this lecture, let us first see the concept of interest.

It is the cost of borrowed money earnings money or earnings on money loan, cost of borrowed money or earnings if you are financing any project, then you are going to earn interest, earnings on money loan. There are two other things, here the principal which is the original amount that is borrowed or the remaining unpayed amount of the loan.

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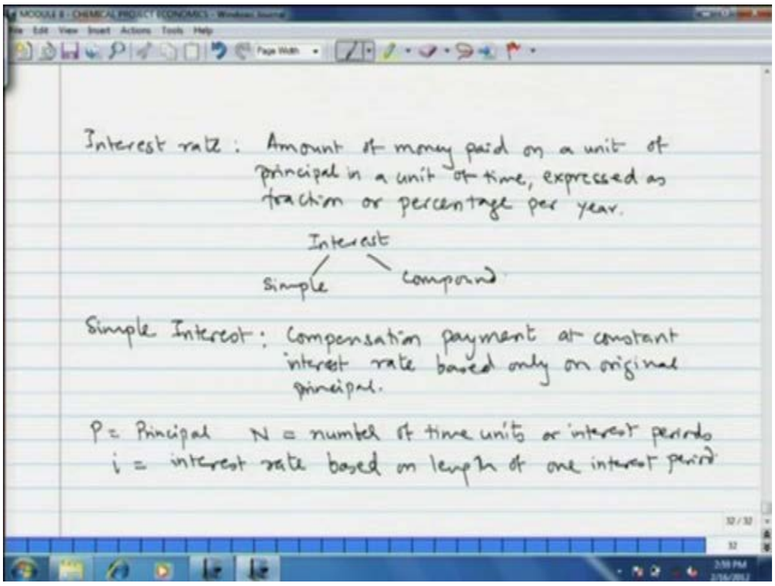

And next is the interest rate and this could be defined as amount of money, paid on unit principal or a unit of principal in a unit of time expressed as fraction or percentage per year. Now interest could be calculated in two ways, first is the simple interest and second is compound interest.

Simple interest it could be defined as compensation payment at constant interest rate, based only on the original principal. So, we can easily developed mathematical expression, if P is the principal that we borrowed, N is the number of time units or interest periods and if, i is the interest rate based on length of one interest period.

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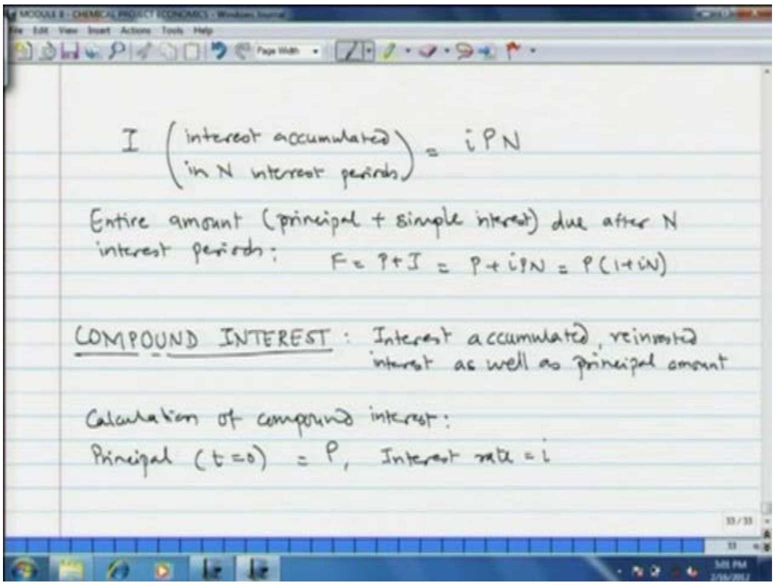

Then the interest that is accumulated that we denote by i, interest accumulated in N interest period is i into P into N finally, we have to pay back the principal as well the entire amount, that is principal plus simple interest due after N interest periods is F is P plus i where, P plus i into P into N, which is equal to P into 1 plus i N in compound interest, which is universally used the interest earned or accumulated, the reinvested interest as well as the principal amount. So, in this case the interest is always added to the principal. The calculation of compound interest is rather straight forward, let us say at principal at time equal to 0 is capital P the interest rate is i.

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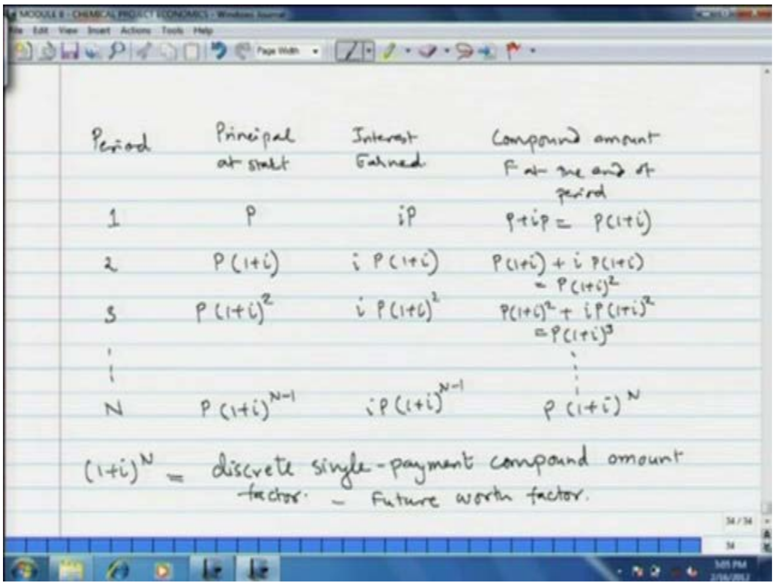

We can make a table for calculation period principal at start interest earned and the compound amount F, at the end of the period. So, for the period 1 from 0 to 1, the principal is P interest earned is i P and now this is added to the principal. So, the compound amount F at the end of period is P plus i P, which is equal to P plus then between period of 1 and 2 F becomes the initial principal, at start is P into 1 plus i interest earned over, it is i into P into 1 plus i and then at the end of the period you have P into 1 plus i, i into P, into 1 plus i that is equal to p into 1 plus i square, then between period 2 and 3, the total amount F becomes the initial principal.

So, this is 1 plus P into 1 plus i square and the interest earned over it is i times P into 1 plus i square. And then the final amount F is P into 1 plus i cube and so on. So, after n periods we can easily write the principal at start it does not P into 1 plus i raise to N minus 1 interest turn will be in the last period will be P i into P into 1 plus i to the power N minus 1 and the total amount F will be P into 1 plus i to the power N. Now here the term 1 plus i to the power N is the discrete single payment compound amount factor or it is also known as discrete single payment future worth factor. Now because of the compounding the effective interest rate over certain period is different, than the nominal interest rate.

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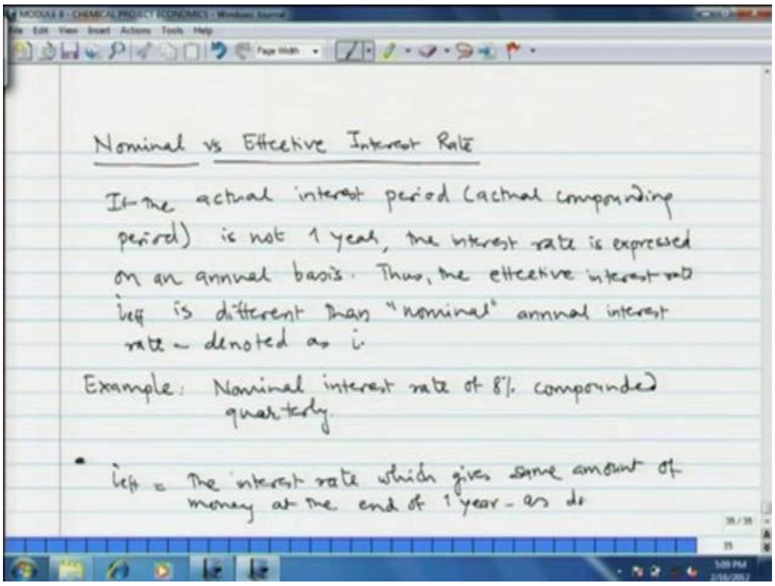

So, that relation we shall see now nominal versus effective interest rate like, for example, Indian bank compound the principal deposited with them on quarterly basis. So, the compounding period is not one year however, the interest is expressed on the basis of one year and because of compounding in period lesser than one year effective interest rate, that we earn over the deposits is higher than the nominal interest.

So that point we note here the actual interest period or actual compounding period is not one year, interest rate is expressed on an annual basis, thus the effective interest rate let me denote as, I E F F subscript is different than nominal annual interest rate and this we denote the nominal annual interest rate we denote as i. Now, let us try to derive a relation between i and i effective for example, we say that nominal interest rate of 8 percent per year compounded quarterly, the what we do now is that we define i effective adds the interest rate, which gives same amount of money at the end of year, if the compounding was only once in year ((Refer Time: 16:14)).

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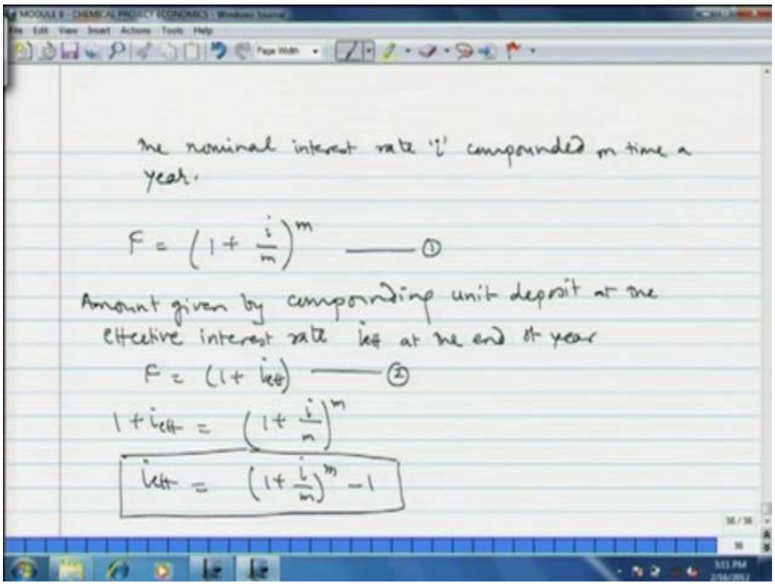

Nominal interest rate I is not r nominal interest rate I compounded m times a year, if compounding is m times a year, then the final amount F for unit deposit will be 1 plus i by m raise to m as per the previous formula. Now the amount given by compounding at effective interest rate at the end of one year just single compounding will be the same amount given by compounding unit deposit, we assume deposit to be 1 at the effective interest I E F F at the end of year will be F is equal to 1 plus I E F F and now we have to equate the two equations number 1 number 2.

So, 1 plus I E F F is equal to 1 plus i divided by m raise to m. So, i effective is 1 plus i by m to the power 1 plus i divided by m to the power m minus 1. So, this is the relation between i effective and i, now let us go back to the problem. That the example, we had seen that nominal interest rate of 8 percent compounded quarterly. So, what is the effective interest rate.

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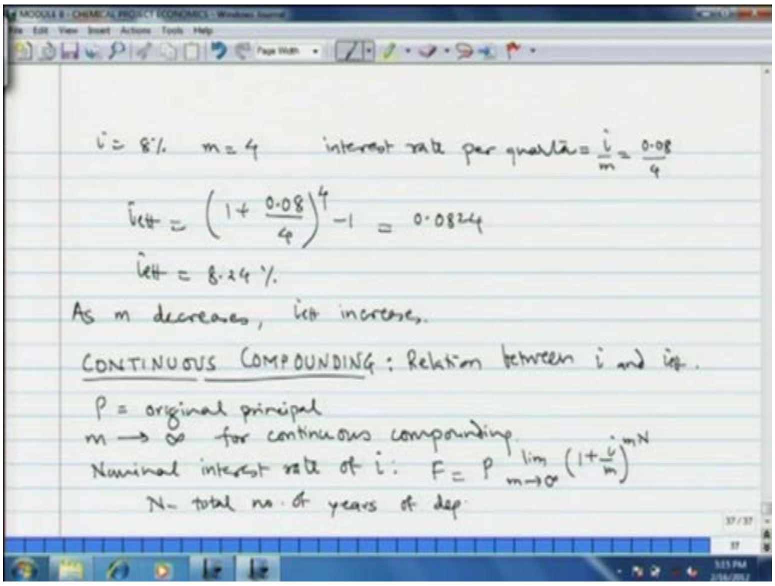

So, here we have i equal to 8 percent m is 4 because it is compounded quarterly. So, 4 quarters a year. So, then the effect the interest rate per quarter is i by m, which is 0.08 by 4 and then i effective, if you substitute that in the formula, we just derived 1 plus 0.08 by 4 raise to 4 minus 1, this becomes 0.824.

So, the effective interest rate corresponding 8 percent nominal, annual interest rate is 8.4 cent 8.24 percent. So, that is the relation between i and i effective in some cases, there is continuous compounding as m decreases i effective increases. So, compounding could be per quarter or per month or per week or per day or in the worst in the limiting case, it is per second. Then in that case, what is the relation between i effective and i.

So, that thing we note some banks like American banks, they do continuous compounding. So, for that case we have to find relation between i and i effective, let us say P is the original principal, now when compounding is continuous, the compounding period m tends to infinity and then for nominal interest rate of i, the total amount F that is accumulated is P limit m tends to infinity, 1 plus i by m to the power m into N, where m is the compounding period and N is the total number of years of deposit.

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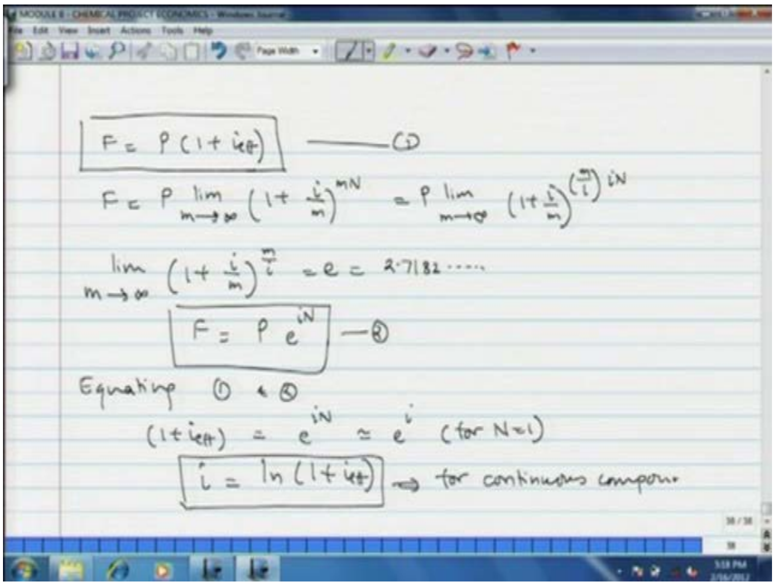

For effective interest rate the definition remains, the same F is equal to P into 1 plus i effective and now we have to equate the 2 quantities, but before that we try to find the limit of that summation F is equal to P in case of continuous compounding P into limit m tends to infinity 1 plus i by m raise to m into N, this we write as P limit m tends to infinity 1 plus i by m plus 2 m by i into i into N and then we known, that m limit m tends to infinity 1 plus i by m raise to m by i is e exponential term 2.7182 and so on.

So, forth therefore, in case of continuous compounding the total F, that accumulates over principal P is F is equal to P into i into N with this, we equate with the regular formula equating again 1 and 2 we get 1 plus i effective is e to power i into N or when N is 1, we get this and then i is equal to L N 1 plus i effective, this is the relation between i and i effective for continuous compounding.

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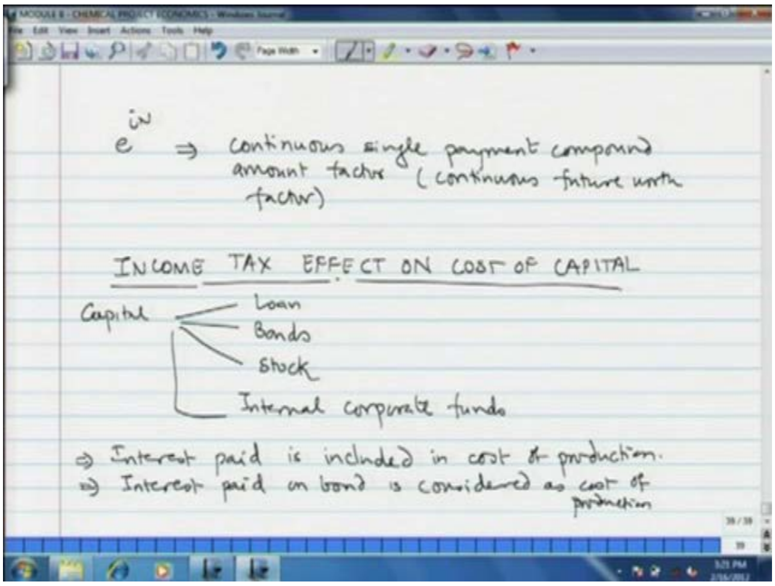

The factor e to power N i N is called as continuous single payment, compound amount factor or it is also known as continuous future worth factor having done this, let us try to see the income tax effect on cost of capital for business venture can come through various sources, first source as I had said at the being of lecture is loan could be in the form of bonds, it could be in the form of stock or it could be internal cooperate funds.

The interest that is paid on portion of an investment that comes from loan can be considered as the cost, that is the cost of production, then interest rate on the bonds can also be considered as cost of production, now when we do preliminary analysis.

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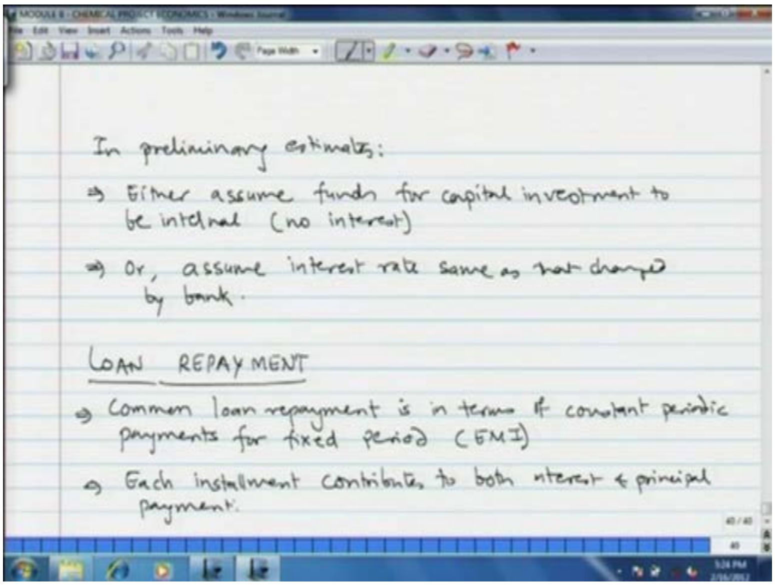

Of the economic feasibility of the project, we have to assume that either the funds are totally internal. So, that no interest is included or we have to assume that the interest is charged on total capital investment or a fraction of total capital investment at interest rate equivalent to the rate charged for bank loans. So, that point we note for preliminary estimates either assume funds for capital investment to be internal or so that. So, no interest on it or assume interest rate same as that charged by bank.

The interest rate itself could be a measure of profitability, that we shall see when we shall do the discounted cash flow rate of return analysis for profitable of the process, let us see the concept of loan repayment the loan that is borrowed from bank is paid back in different instalment, common loan repayment is in terms of constant periodic payments for fixed period, these are also known in many cases as E M I equated monthly instalment, when the period of payment is one year the total payment is constant.

However, the principal balance decreases with instalment the interest portion of each payment is smaller than the previous while the principal portion of each payment is larger than the previous. So, that point you note each instalment contributes to both interest and principal payment.

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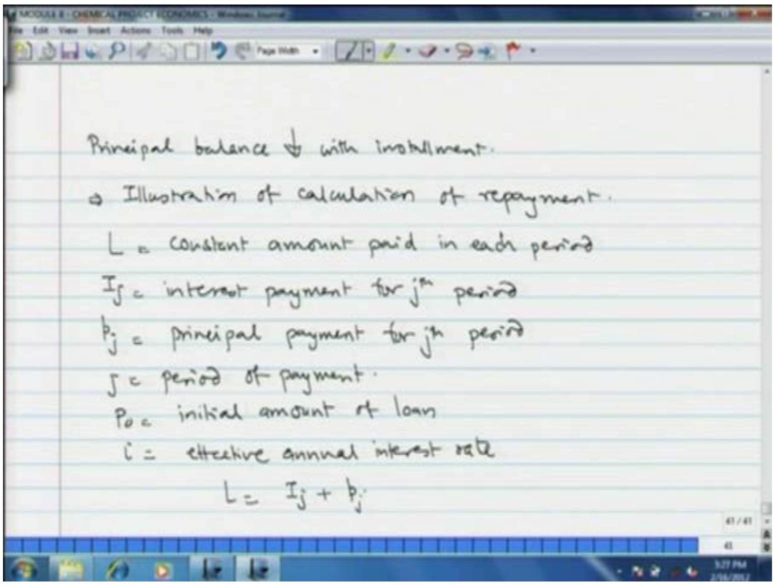

Principal balance decreases with instalment therefore, for each payment the interest portion of each payment is smaller than the previous while the principal portion of each payment is larger than previous, let us illustrate the calculation of repayment, let L be the constant amount paid in each period now as i said L contributes to both interest payment and principal payment.

So, let i j denote the interest payment for j th period and let P j denote the principal payment for j th period, j is the period of payment P not is the initial amount of loan and i is the effective annual interest rate, remember i is not nominal interest rate, but effective interest annually. So, then L is equal to i j plus P j.

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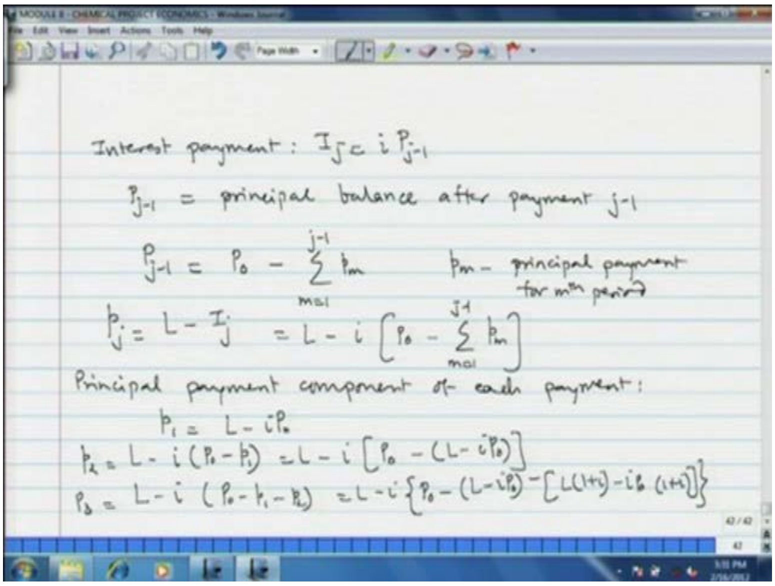

Now, principal for the j th period is P into j minus 1. So, the interest payment will be i j equal to i into P into j minus 1, where P is the principal for period j minus 1 to j or P is j minus 1 is the principal balance, after payment j minus 1 therefore, P j minus 1 is equal to P not the initial principal minus the sum of all payments for the principal till period j minus 1.

So, minus summation N equal to 1 to j minus 1 with P m, where P m is the principal payment for N th period, now P j is equal to L minus i j and now for i j you substitute expression i into P j minus 1 P j minus 1 is P not minus m equal to summation m equal to 1 to j minus 1 P m, the principal payment component of each instalment is then let us say for the first period p 1 is equal to L minus i into P not for the second period P 2 is L minus i into P not minus P 1, which is equal to L minus i into P not minus L minus i P not and so on.

So, P 3 is equal to L minus i into P not minus P 1 minus P 2 and then you substitute for that L minus i into P, not minus L minus i P not minus L to 1 plus i minus i P not to 1 plus i, this thing you just keep on do keep on substituting for P 1 P 2, so on.

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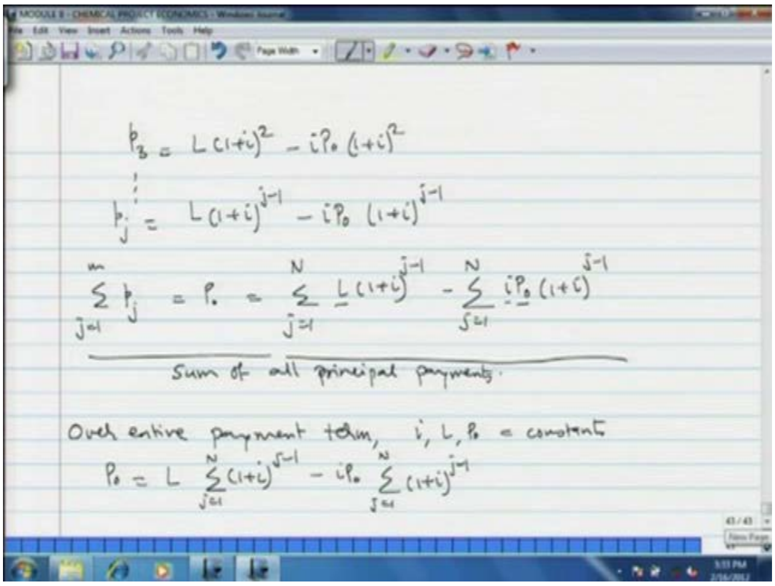

And then... For the j th period the term for P 3. We now we can, simplify as, if you expand this particular bracket and simplify. You get L P 3 equal to L into 1 plus i square minus i P not into 1 plus i square and in this way the j th principal payment is L into 1 plus i to the power j minus 1 minus i P not into 1 plus i to the power j minus 1. The sum of principal payment compo P not equal to summation j equal to 1 to N L into 1 plus i to the power j minus 1 minus summation j runs from 1 to N i into P not to 1 plus i to the power j minus 1, this is the total sum of all principal payments.

Now over entire payment term i is constant L is constant and P not is constant therefore, we can simplify for L, here we can take out L i and P not out of the summation sign therefore, P not equal to L into summation j runs from 1 to N, 1 plus i to the power j minus 1 minus i into P not summation j runs from 1 to N 1 plus i to the power j minus 1 and then we substitute for L.

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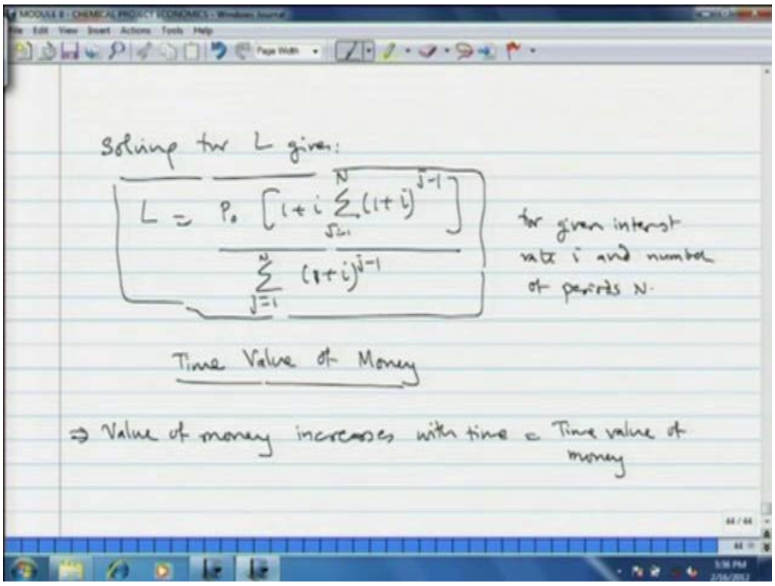

You get or Simplify for L solving for L gives L equal to just rearrangement of the previous expression P not into 1 plus i summation j running from 1 to N, 1 plus i to the power j minus 1 divided by summation j running from 1 to N, 1 plus i raise to j minus 1.

So, this is how the periodic payment L is calculated for a given interest rate i and number of periods N, let us see how we can evaluate the time value of money can, we predict the future value of present money or reverse the present value of future money, the amount of the value of money increases with time and that is the time value of money.

The investment could be in various forms, it could be savings account fixed deposit recurring deposit bonds stocks. So, wherever we invest money, we have to see that the money grows with time, we have to ensure that the business is profitable venture is profitable and then we have to calculate the future value of money.

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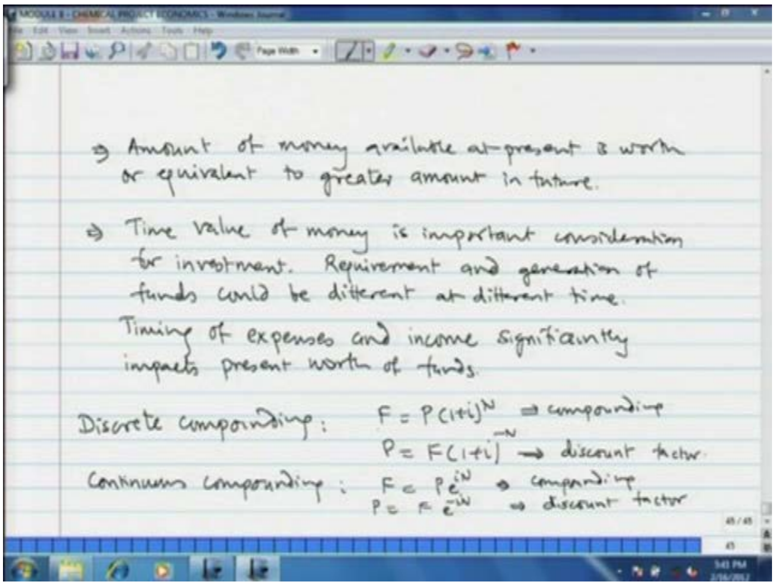

Amount of money available now or available at present is worth or equivalent to greater amount in future. Now, we have to consider the time value of money, when we invest in a particular business, we have to compare investment that require or generate different amounts of funds at different time. The timing of expenses and income may significantly impact the present worth of such funds investment may require that point to note is that the requirement or generation of funds could be different at different time.

Therefore, timing of expenses will become significantly impacts the present worth of such funds, now let us see how we can be relate the future value of present money and present value of future money. We have already seen the compounding of with simple interest and compound interest therefore, the future amount if the compounding is discrete such as, quarterly compounding then we have already derived the formula P is F is equal to P into 1 plus i to the power N. Future value of present money, we can reverse this and then we can write P is equal to F into 1 plus i raise to minus N.

So, 1 plus i raise to minus N, this factor is called as the discount factor, similarly for continuous compounding, we have derived a formula F is equal to P e to the power i N, we can reverse this and write P is equal to F e to the power minus i N.

So, minus i N is the discount factor or continuous compounding 1 plus i to the power N is compounding factor and 1 plus i to the power minus N is discount factor for discrete compounding and same holds for the continuous compounding e to the power i N is the compounding factor and e to the power minus i N is discount factor. Now, in many cases we have to make investment as single installments or a series of installment over regular time intervals.

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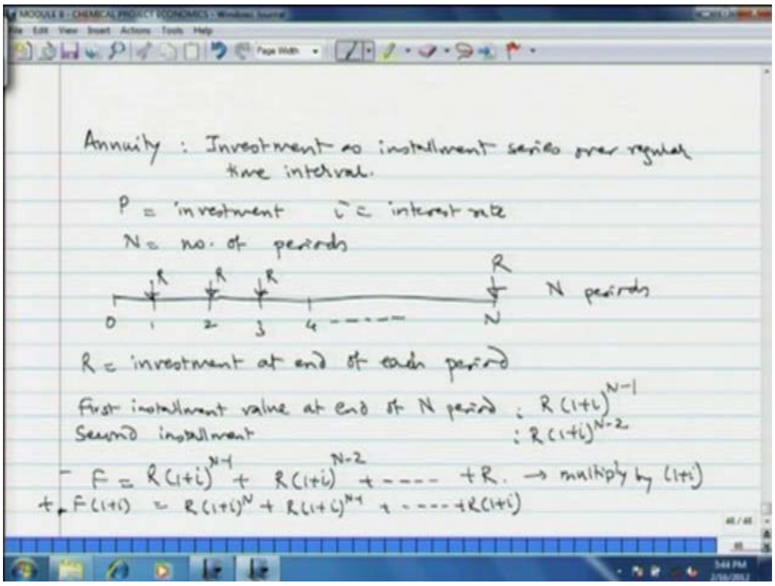

That is known as annuity or also recurring deposit investment as installments series, over regular time interval per month or per quarter or per year. Investment P is made at interest rate i and number of period is N. So, we have a time line like you open an account at time zero and then you make exact instalment at various period. So, we have here N periods and let us denote R as the investment at the end of each period.

So, we make investment at the end of each period not in the beginning we open an account at time zero, but make investment at the end of the period. Now in such cases, the total amount that is accrued in the bank can be calculated with simple summation like the first instalment, that we make at time 1 is under investment for N periods sorry, N minus 1 periods. So, for the first installment value at end of N period here is R into 1 plus i to the power N minus 1 because the deposit R is under investment for N minus 1 period, the second installment plus i to the power N minus 2. So, forth and the final deposit at the end of N period is R.

So, F the total amount that is accrued in the bank, now what you can do is that multiply this series by 1 plus i and then subtract from this. So, F into 1 plus i is equal to i into 1 plus i to the power N plus R into 1 plus i to the power N minus 1 plus R to the power 1 plus i and then subtract this, the second from first.

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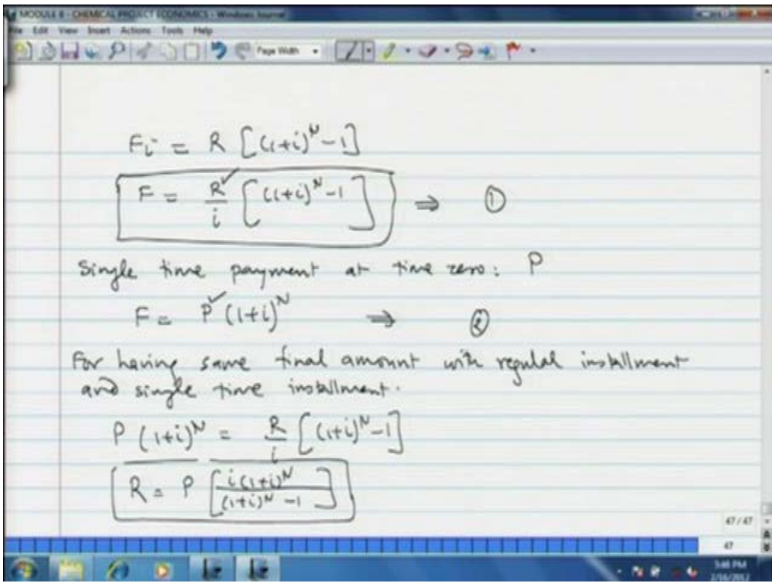

And then the summation gives into i is equal to R into 1 plus i to the power N minus 1 or F is equal to R by i into 1 plus i N to the powers. So, this is the final value of the installments regular instalments. If you compare this the final amount that we obtained with regular interval installment with a single instalment, let us say that we make a single time payment at time zero, which is equal to P, we invest amount P at time equal to zero single payment amount and this P remain centre investment for N periods, then the final amount F that is accrued is P into 1 plus i to the power N as we have already derived.

Now, suppose that we want to compare in this way that we want final amount F to be same with regular installment and single time instalment, then we have to equate the two expressions for having same final amount with regular installment and single time instalment, we have to equate the two.

So, P is 1 plus i to the power N is equal to R by i 1 plus i to the power N minus 1. So, we get a relation between R and P as R equal to P into i into 1 plus i to the power N divided by 1 plus i to the power N minus 1 and the term in the square bracket is called as capital recovery factor.

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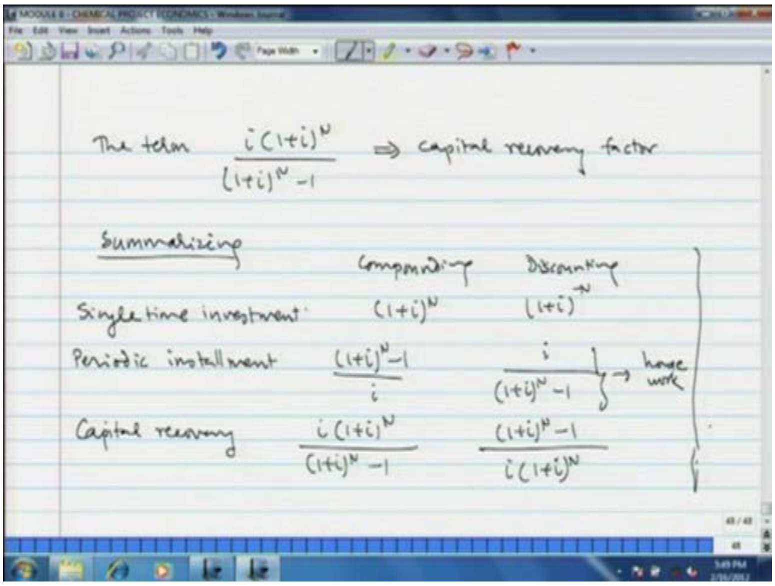

The term i into 1 plus i to the power N divided by 1 plus i to the power N minus 1 is called as capital recovery factor, we shall see the significance of this term in evaluation of the feasibility or profitability of a chemical project in the subsequent lecture. So, we can summarize the lecture today we have driven the time value of money the interest the compounding discounting loan repayment.

So, for single time investment compounding factor is 1 plus i to the power N discounting factor is 1 plus i to the power minus N for investment with periodic instalment, the compounding factor is 1 plus i to the power N minus 1 by i, the discounting factor is i divided by 1 plus i to the power N minus 1.

Now derivation of this i leave as home work to you have to work with the same series as, we did for deriving the compounding factor and for capital recovery factor that is the comparison between single time investment and periodic installment is the compounding factor is i into 1 plus i to the power N divided by 1 plus i to the power N minus 1.And the discounting is the reciprocal of this 1 plus i to the power N minus 1 i into minus i to the power N, 1 plus i to the power N minus 1 divided by i into 1 plus i to the power N.

So, this is the simple economic analysis of different investment and earnings the with respect to certain interest, now in the next lecture we shall see the profitability natures of a chemical project and then we shall also see as how we can make use of the formulae or analysis, that we have done in this lecture for estimation of the profitability of the process.