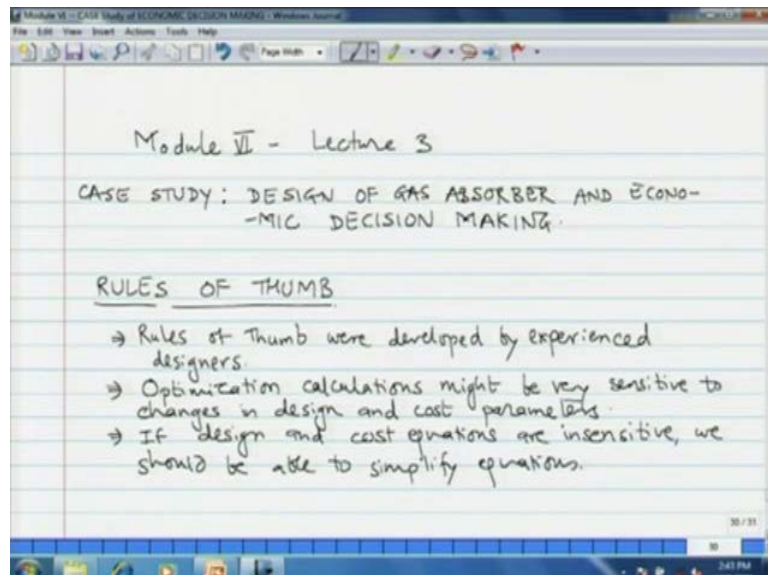


**Process Design Decisions and Project Economics**  
**Prof. Dr. V.S. Moholkar**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Guwahati**

**Module - 6**  
**Economic Decision Making: Case Study of a Gas Absorber**  
**Lecture - 31**  
**Rules of Thumb and Their Limitations and Tutorial**

Welcome, we are in the 3rd lecture of this module of Case Study of design of a Gas Absorber and Economic Decision Making for the Design. We have already seen the synthesis of flow sheet then we saw the process alternatives, then in the last lecture we tried to design the main unit of the absorber; that is the absorber column itself, using some simplified equation. We started with Kremser equation and then we tried to simplify it and then we saw that we lose very little of accuracy, but we simplify calculations to a great deal by making modifications of the Kremser equation. That kind of approach can be adopted to all design processes, so we shall see again some other case studies later. In this lecture we will try to focus on the rules of thumb of the process design.

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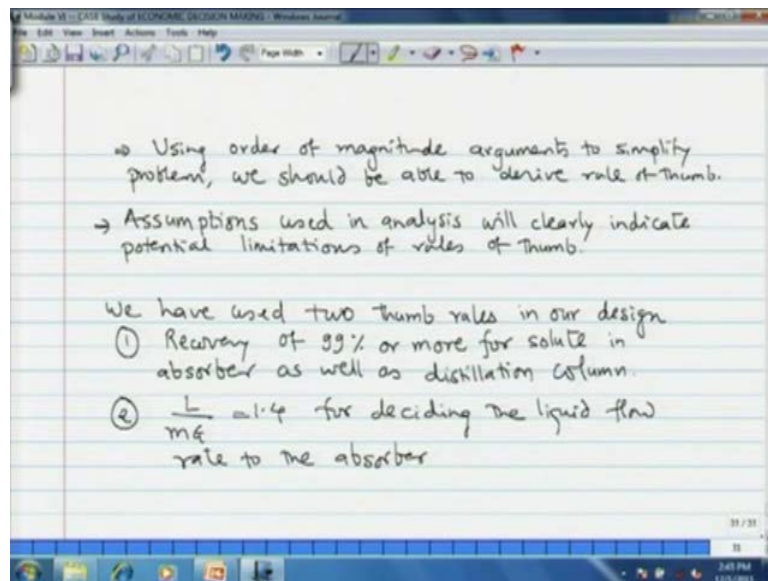


While designing the gas absorber, we made use of 2 thumb rules; that is recovery of 99 percent and above for acetone. And second was the liquid flow rate to the absorber; that is  $1 \text{ by m g equal to } 1.4$  these were the 2 thumb rules that we used. Now, how these

thumb rules have come? Thumb rules are usually developed by experienced designers. So, suppose there is a designer who has already designed 10 15 20 absorbers and has found that the most optimum design was at  $L$  by  $mG$  equal to 1.4 or close to 1.4 and for solute recovery of higher than 99 percent.

Then that is how we started with that experience for the next absorber but now several software's are available in which we can do very large parameter study and therefore, you can easily develop rules of thumb using this detailed simulations. However, in some cases optimization calculations might be very sensitive to changes in design and cost parameters. Like whatever, design parameter that we have used if the cost of operation or the capital cost is very sensitive then the rules of thumb may not be that applicable. However, if the designing cost equations are relatively insensitive we should be able to simplify equations as we did in the Kremser equation case and then we can develop some rules of thumb.

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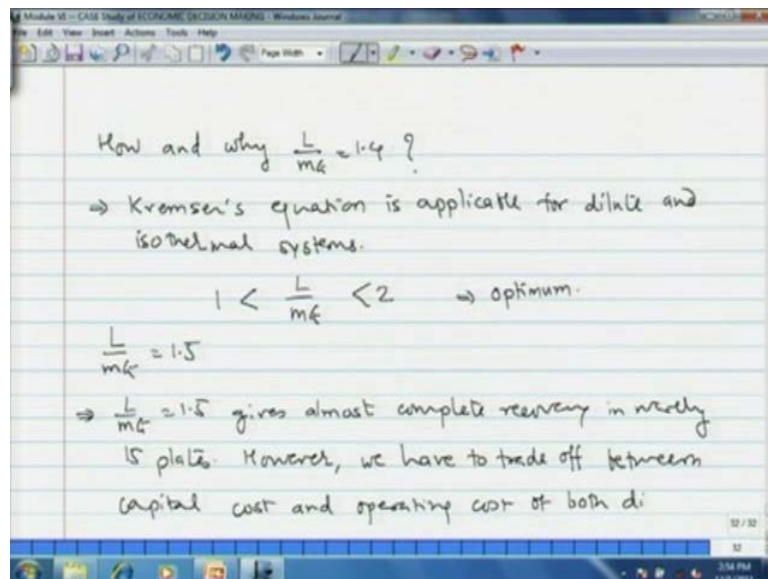


Now, use of order of magnitude arguments to simplify problem should be able to give us some rules of thumb, that we have seen. Like for example, in distillation column there is a rule of thumb that reflux ratio should be 1.2 times the minimum reflux ratio. This again has come out of order of magnitude calculation, the assumptions used in the analysis will clearly indicate the potential limitations of the rules of thumb. Now what we will do is

that we will see as what is the basis for this particular thumb rule that we have used in the design of absorber.

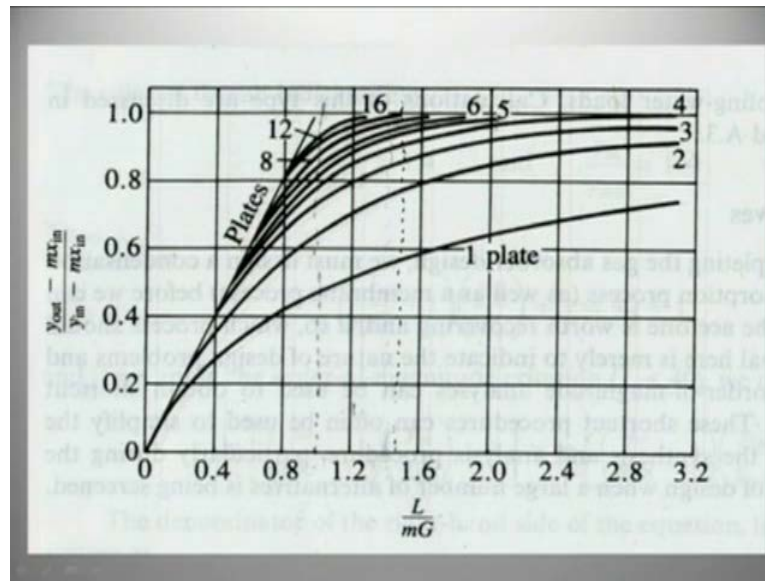
As I just mentioned we have used 2 thumb rules, recovery of 99 percent or more solute and  $L$  by  $mG$  equal to 1.4. Now let us see the validity of these thumb rules, as what is the basis for these thumb rule I would like to specifically mention that Kremser equation is mainly applicable for dilute and isothermal system where, both operating line and equilibrium line are both operating curve and equilibrium curve are essentially straight lines. That is when the system is dilute or the concentration of solution is dilute and the operation is isothermal.

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Let us see how we have come at  $L$  by  $mG$  equal to 1.4 thumb rule. I would like to note the point I just mentioned Kremser equation is applicable for dilute and isothermal systems. Let us see a plot of this Kremser equation.

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What you see on the screen now, is a lot of the Kremser equation. On the x axis you have  $L$  by  $mG$  on the y axis you have the parameter  $y$  out minus  $m x$  in divided by  $y$  in minus  $m x$  in which is essentially the percentage of recovery. Now, we want to recover as much as acetone as possible, now let us say that we pickup  $L$  by  $mG$  equal to 1, which is particular basically this line which I am now showing  $L$  by  $mG$  equal to 1.

Now, you can see that we cannot get close to the complete recovery that is  $y$  out minus  $m x$  in divided by  $y$  in minus  $m x$  in equal to 1 even with infinite number of plates. What you see here, is the parameter is number of plates and the percentage recovery for  $L$  by  $mG$  equal to 1.4 and you can see here that even with the infinite number of plates which is this particular tangent if  $L$  by  $mG$  equal to 1, I cannot get complete recovery.

Now, complete recovery is desirable when the solute is very valuable or if the solute is highly toxic for example, hydrogen cyanide. So, in these 2 case, we would like to have as high recovery as possible. And therefore,  $L$  by  $mG$  equal to 1 or less than 1 is never going to give us that kind of recovery, desired recovery. Now, let us say that we double the ratio  $L$  by  $mG$  equal to 2, now here we see that the follow this line which I am showing now  $L$  by  $mG$  equal to 1.

We see that we get very large recovery or almost complete recovery within just five or six plates; however, the solvent flow rate in such case will be very high for a given gas flow rate. Now, if solvent flow rate is high then the load and distribution column will

increase remember, that when we are optimizing a flow sheet our attempt should be optimized the complete flow sheet and not particular equipment now, equipment are interconnected. So, optimization of 1 equipment may de optimize another equipment.

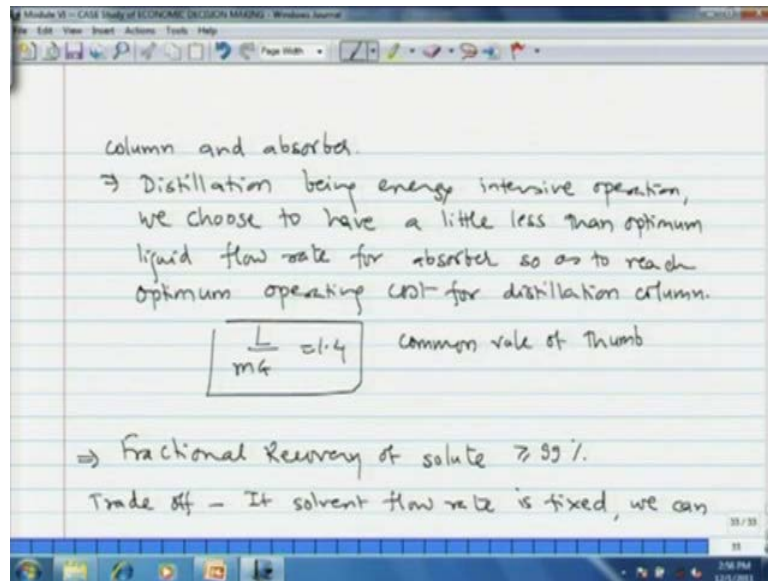
So, we have to have a sort of a global view of the flow sheet while optimizing the flow sheet. Therefore, if I use  $l$  by  $m$   $g$  equal to 2 I know that I am going to get very high recovery or almost complete recovery in just 5 to 6 plates; that means, a very short column whose cost will be very, very small, when we talk of investment of cores of rupees for process plant if the absorber is costing me few 1000 rupees then it is almost negligible.

However, I have to take care of the distillation column that is in the line with absorber. I am going to recover the solute from the solvent of the absorber using the distillation column and  $l$  by  $m$   $g$  equal to 2 is going to create intensive load on that column. I will have high vapor flow rate, high re boiler heat load. So, that will make my distillation column highly expensive therefore, we have to find something in between we have to go for a value (Refer Slide Time: 04:16) which is between 1 and 2 that could be our optimum value.

Now, if I put  $l$  by  $m$   $g$  equal to 1.5 which is exactly half way, then again if you go back to the plot you will see 1.5 is somewhere here, you will see that you are going to get almost complete recovery within about 16 17 plates. So, therefore, we have a tradeoff between decrease in the number of plates which will reduce the capital cost and increase in the operating cost of the distillation column by putting liquid flow rate.

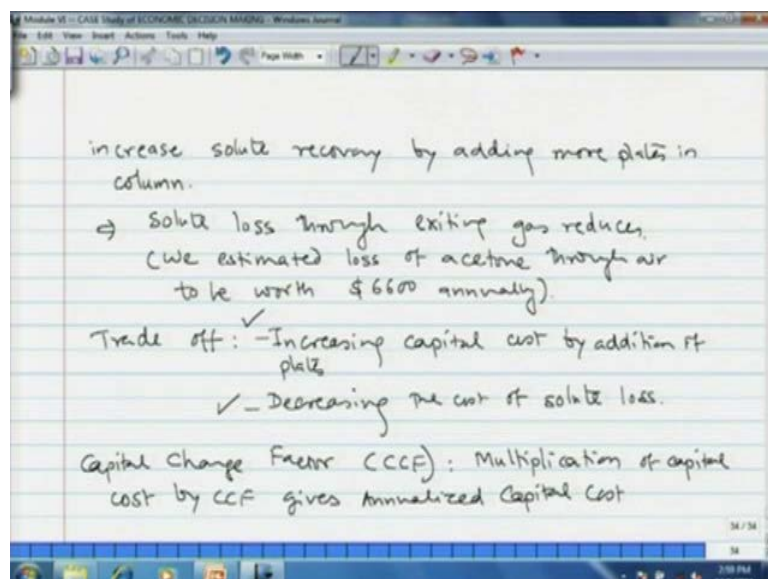
However, distillation operation is energy intensive, therefore we would like to have a slightly lesser liquid flow rate than  $l$  by  $m$   $g$  equal to 1.5. Therefore, in order to optimize the operating and capital cost of both distillation column and absorber together. We choose a little less than 1.5  $l$  by  $m$   $g$  that is could be 1.4. So, that point we note  $l$  by  $m$   $g$  equal to 1.5 gives almost complete recovery in nearly 15 plates. However, we have to trade of between capital cost and operating cost of both distillation column and absorber.

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If I put more liquid the number of plates will go down but the load on distillation column goes up. And distillation being an energy intensive operation we, choose to have a little less than optimum liquid flow rate for absorber. So, as to reach optimum operating cost for distillation column and therefore, we choose  $L$  by  $mG$  equal to 1.4. So, this is a common rule of thumb for the absorber design. Now, the second thumb rule that we have used is fractional recovery of solute greater than 99 percent second thumb rule that we have used. Now, again here there is a trade off if solvent flow rate is fixed we can increase the solute recovery by adding more plates in the column.

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If solute recovery increases then solute loss through exiting gas reduces. In previous lecture we have already estimated the loss of acetone through exiting air to be about worth dollar 6000. So, that point I repeat again we estimated loss of acetone through air to be worth dollar 6600 annually. So, if the solute recovery increases; obviously, this loss decreases. So, that trade off is like increasing the capital cost by addition of plates and decreasing the cost of solute loss.

We will try to form a sort of an objective function that will give us the total cost one of the this cost like capital cost is a single time cost and the other cost is recurring cost. So, how do we bring the two cost on the same platform in the previous module of project economics we have already seen several such methods which bring the two cost on same platform. One of them was the use of Capital Charge Factor or what we had abbreviated as CCF. So, if you multiply the capital cost by CCF gives the annualized capital cost, and we shall use the exactly the same technique for formation of the objective function.

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$$\text{TAC (Total Annualized Cost)} = \frac{\$}{\text{yr}} = C_s \left( \frac{\$}{\text{mole}} \right) \times \left( G \frac{y_{\text{out}}}{h} \right) \times 8150 \frac{h}{\text{yr}} + C_N \left[ \frac{\$}{\text{plate-yr}} \right] \times N \text{ (plates)}$$

objective function

$$N = 6 \log \frac{y_{\text{in}}}{y_{\text{out}}} - 2 \quad \left. \vphantom{N} \right\} \text{Simplified Kremser's equation.}$$

$$\text{TAC} = 8150 C_s G y_{\text{in}} \left( \frac{y_{\text{out}}}{y_{\text{in}}} \right) + C_N \left( 6 \log \frac{y_{\text{in}}}{y_{\text{out}}} - 2 \right)$$

$$\frac{\partial \text{TAC}}{\partial \left( \frac{y_{\text{out}}}{y_{\text{in}}} \right)} = 0 = 8150 C_s G y_{\text{in}} - \frac{6 C_N}{\left( \frac{y_{\text{out}}}{y_{\text{in}}} \right)}$$

$$\frac{y_{\text{out}}}{y_{\text{in}}} = \frac{6 C_N}{8150 C_s G y_{\text{in}}}$$

So, let us say that a Total Annualized Cost that we denote by TAC units of dollar per year is equal to the cost of solute  $C_s$  dollar per mole into the loss of solute  $G y$  out that is mole per hour into the total number of hours of operation per year 8150. So, this is what the total loss is of the solute and then the cost of the plate but it is an annualized cost. So, dollar per plate per year into the number of plates  $N$  is the annualized capital cost.

So, this is the objective function that we are going to optimize this objective function gives the total annualized cost in terms of the fractional recovery. For N we substitute 6 into log y in by y out minus 2 the expression that we derived in previous lecture that is simplified kremser equation and then we write total annualized cost as 8150 into C s cost of solute G y in G is the molar flow rate of carrier gas. Now, it in the original objective function the loss of solute was written as G into y out which we rewrite as g into y in into y out by y in.

And the annualized cost of plates. And now we differentiate this with respect to the recovery y out by y in and then equated to 0. What we see is 8150 into C s into G into y in minus 6 into C n divided by y out y in. And then if we rearrange we get the optimum fractional recovery at which the total annualized cost is minimum that turns out to be 6 into C n divided by 8150 into C s into G into y in. We put some typical values in this expression.

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$$C_s = \frac{\$15.5}{\text{mole}} \quad G_{y_{in}} = 10 \frac{\text{mole}}{\text{h}} \quad C_N = \frac{\$2250}{3} = \$750/\text{yr}$$

$$\boxed{CCF = \frac{1}{3}}$$

$$\frac{y_{out}}{y_{in}} = \frac{6 \times 850}{8150 \times 15.5 \times 10} = 0.004 \Rightarrow 99.6\% \text{ recovery}$$

Sensitivity Analysis: Suppose any cost number on RHS doubles, then also we are close to 0.01  $\approx$  99% recovery

The optimum values of design parameters are relatively insensitive to cost.

Let us say the cost of solute is dollar 15.5 per mole total flow rate of solute into the absorber G into y in is 10 mole per hour the cost of plate is 2250 dollars per plate. But, annualized cost is this divided by 3, now y 3 because we have considered capital charge factor as 1 by 3. Now the how capital charge factor is equal to 1 by 3 this derivation has already been covered in the module of project economics. So, I would request you to revise it.

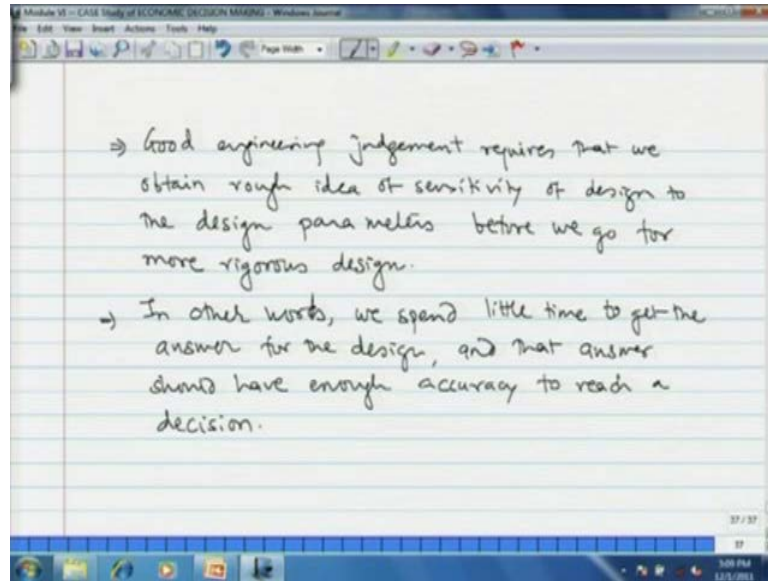


So, dollar 2250 by, so that is that comes out to be something like dollar 580 per year. So, this is a annualized cost of the plate, if you put these numbers and then try to calculate the optimum fractional recovery you will see that  $y_{out}$  by  $y_{in}$  is 0.004 which essential means or implies that 99.6 percent recovery and this justifies our thumb rule. Let us see how sensitive is this particular fractional recovery to the cost, sensitivity analysis. Suppose if any of the numbers that is on right hand side any of the cost number that is on right hand side doubles.

Then also we are close to 0.01 which means 99 percent recovery. So, try substituting instead of 850 here may be 1000 or for 15.5 substitute 20 or 25 whatever number. No matter which number cost number that you double you are very close to 99 percent or the value of  $y_{out}$  by  $y_{in}$  turns out to be 0.01. This means that the optimum values of parameter, optimum values of design parameters are relatively insensitive to cost or in other words the total cost of operation of a process is relatively insensitive to the design parameter.

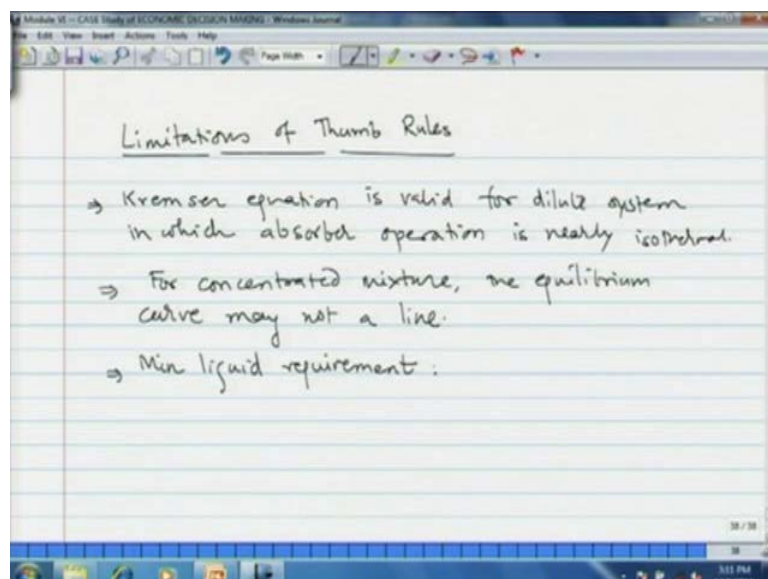
And this is a characteristic of a large number of design problems, that solutions are often very insensitive to the physical property data of the functional form of design equation or the design parameter such as, heat transfer coefficient cost data etcetera. Therefore, good engineering parameters, good engineering judgment requires that we obtained some idea of the sensitive, sensitivity of the solution before we go further with the analysis with the design.

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So, that point you note that a good engineering judgment requires that we obtained some idea or rough idea of the sensitivity of solution or of design to the design parameters. Before, we go for more rigorous design. That is in other words we spend as little time as possible to get the answer for the design and we want that answer to have close enough accuracy that the decision we are facing here. So, that is the discussion on rules of thumb. Question comes as whether rules of thumb should always be applied or will there be any limitation, under what circumstances the rules of thumb may not be applicable. So, that point we discuss limitations of rules of thumb.

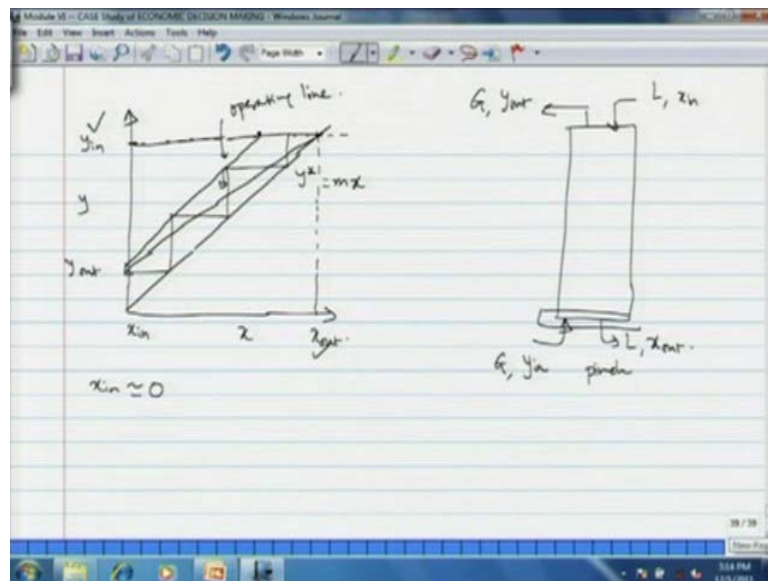
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I would like to repeat one point that I have stated earlier that Kremser equation is valid for dilute system in which the absorber operation is nearly isothermic; that means, the relatively small amount of solute is absorbed in relatively large amount of solvent. So, that the heat of dissolution that is liberated with the absorption of solute does not increase the temperature of the solvent significantly. This is possible only when the amount of solute that is dissolving is small.

If the amount of solute is large then the temperature of solvent increases as the liquid flows down the absorber. And then the equilibrium characteristic also change, in that case the equilibrium curve is not likely to be a straight line as in case of Kremser equation. So, that point we note that for concentrated mixtures the equilibrium curve may not be aligned. Now, how do we determine the minimum liquid that is required for a particular operation you might recall these kinds of graphs that you made in mass transfer one.

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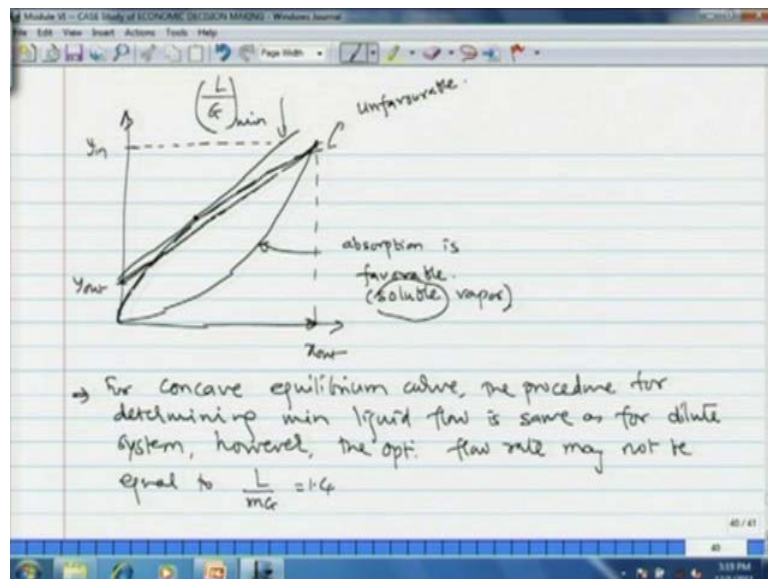


Suppose you have a counter current absorber where  $G$  is the gas that is going in carrier gas with the mole fraction  $y_{in}$  of the solute the same gas comes out with mole fraction  $y_{out}$  the liquid that goes in is  $L$  with mole fraction of solute  $x_{in}$  and liquid that comes out is again  $L$  with mole fraction  $x_{out}$ . And then you have the equilibrium line as  $y^* = mx$ . Now, given that you have to reduce certain mole fraction of solute from  $y_{in}$  to  $y_{out}$  you are suppose to calculate the minimum liquid requirement.

So, what you do is essentially you try to get the 2 points  $y$  in and  $y$  out on the  $y$  axis from the  $y$  in you draw a straight line that cuts the equilibrium curve and from this point you draw line joining point  $y$  out and  $x$  out this point is  $x$  out  $x$  in as shown here that  $x$  in is 0. Now, the slope of this line will give you the minimum solvent that is required for absorption. Now, when you use minimum solvent then you have a pinch at the bottom of the tower; that means, the outgoing liquid is always in equilibrium with the incoming gas.

The point  $G$  in and  $y$   $G$  sorry the point  $y$  in and  $x$  out indicates the bottom of this particular absorber. You will always choose a liquid flow rate that is higher than the minimum which gives the operating line, and then you would like to make out the number of plates or number of trays that are required for a particular operation. This is what procedure you have followed while determining the number of plates incase of dilute and isothermal systems.

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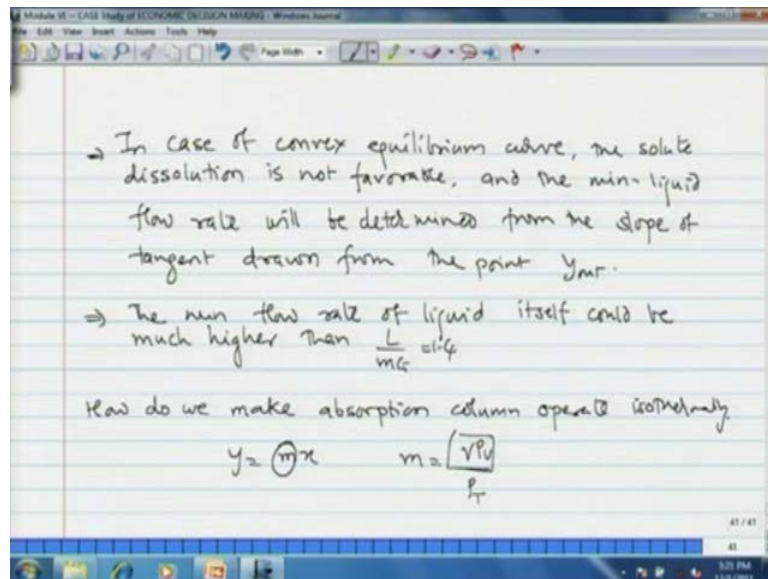


However incase of concentrated system the curve the equilibrium curve itself will be either parabolic or the reverse. This kind of curve will be obtained if the absorption is favorable which means the amount the solute for soluble gas soluble vapor and this is for unfavorable. Now, in such cases to have the same reduction of concentration of a particular solute you will find the minimum liquid flow rate using this procedure that you

will draw a line horizontal line that will cut the equilibrium curve at point y in x out and then you join the line.

However, incase where the curve is not concave, but convex as I am showing now, the minimum liquid flow rate will be determined by the tangent to the curve from point y out. So, that point we not for concave equilibrium curve the procedure for determining minimum liquid flow rate is same as for dilute system. However, the minimum flow rate or the sorry the optimal flow rate may not be equal to l by m g equal to 1.4. It could be relatively less because of the solubility of the particular solute. You may get optimum design at l by m g into less than 1.4 if the equilibrium is favorable is concave incase of convex equilibrium curve the reverse is true.

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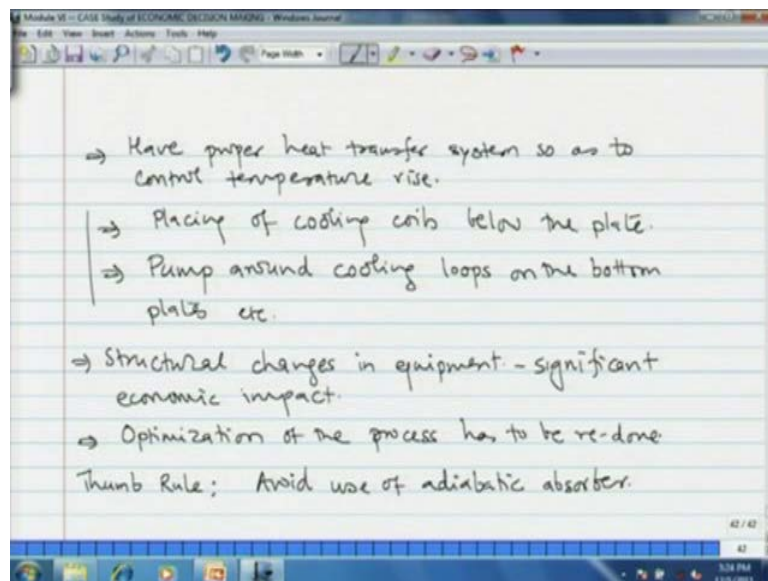
Incase of convex curve the solute dissolution is not favorable incase of convex equilibrium curve the solute dissolution is not favorable and the minimum liquid flow rate will be determined, from the slope of the tangent drawn from the point y out the slope will; obviously, be l by g minimum in both cases. Now this l by g minimum may be much higher than 1.4 because it has no relation whatsoever with l by m g equal to 1.4.

The minimum flow rate itself the minimum flow rate of liquid itself could be much higher than l by m g equal to 1.4. So, these are the rules the limitations of rules of thumb, you cannot apply rules of thumb every time you have to see the similarity of your system, the limitations of your system with the system for which the rule of thumb is

derived and if a similar if significant similarity exists then you can adopt the rule of thumb for designing your own system.

Now how do we make our system behave close to ideal or how do I make absorption column operate isothermally, if I look at the slope of the equilibrium curve  $m$  which is  $\gamma p_v$  by  $p_t$  that is what we derived in previous lecture. We see that the slope of the equilibrium curve is basically dependent on 2 parameters that is  $\gamma$  activity coefficient and  $p_v$  the vapor of pressure absolute at the temperature of operation. If I keep the temperature of operation constant then both  $\gamma$  and  $p_v$  are like to be constant and therefore, the equilibrium curve is likely to be a straight line for which I can apply the Kremser equation. Now, how do I make sure that my absorption column operates isothermally; obviously, I will have to take care of the heat transfer.

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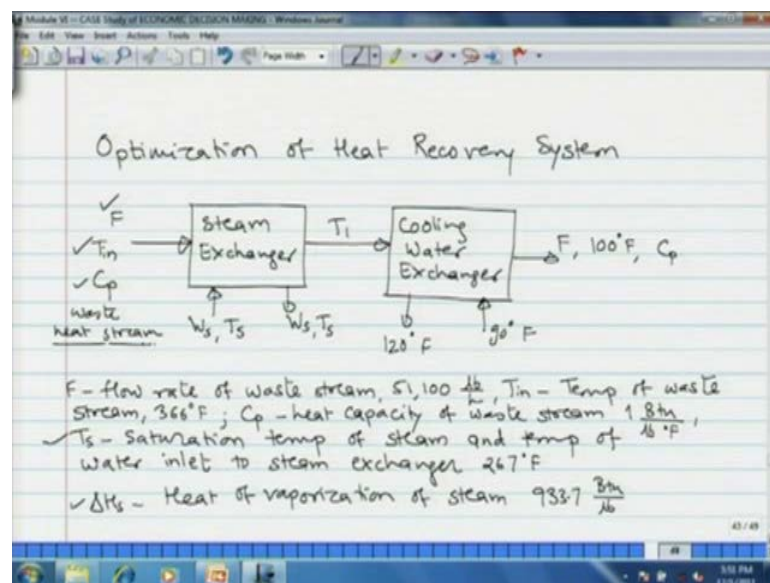


Have proper heat transfer system. So, as to control the temperature rise this could be like placing of cooling coils below the plate or pump around cooling loops at the bottom or 2 3 trays of the gas absorber. However, this leads structural changes in the equipment if I have an absorber and if I want to put cooling coils below each plate I will have to dismantle all the plates then put the cooling coil weld the cooling coil below and then reassemble the column.

So, this involves a significant structural change which has significant economic impact on the design and thereafter I will have to carry out optimization again after I do

structural changes. Therefore, another thumb rule that we can develop out of our own analysis for absorber is that avoid use of adiabatic absorber for make your absorber isothermal as much isothermal as possible. So, this was essentially the complete case study of economic decision making or finding the optimum values of parameters for a particular process unit. Now, we shall see the small example of similar type in which we will try to determine the optimum temperatures of the heat exchanger units. We shall see a small problem on optimization of a particular process unit taking into account total annualized cost or let us say economic optimization of process unit.

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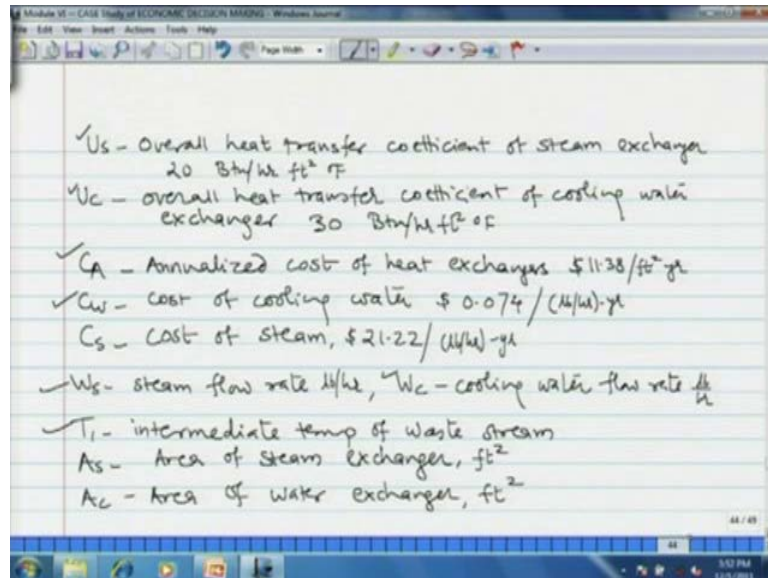


What you see on the screen now, is the heat recovery system waste heat stream enters a steam exchangers in which saturated water is fed and is converted into steam. And then the waste heat stream further goes to a cooling water exchanger where, cooling water is used to cool this stream hundred degrees. So, it is a system of two consecutive heat exchangers. Now, I have given here all the parameters as you see on the screen  $f$  is the flow rate of waste stream that is 51100 pound per hour.

$T_{in}$  is the temperature of waste stream that is 366 degrees fahrenheit,  $c_p$  is the heat capacity of waste stream that we assumed to be 1 b t u per pound per degree fahrenheit,  $t_s$  is the saturation temperature of steam and temperature of water inlet to the steam exchanger. That is exactly same as 267 degrees fahrenheit. So, there is only the face

change taking place in the steam exchanger with no super heating.  $\Delta h_s$  is a heat of vaporization of steam that is 933.7 Btu per pound.

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Then  $U_s$  is the overall heat transfer coefficient of steam exchanger taken to be 20 Btu per hour per foot square per degree Fahrenheit.  $U_c$  is the overall heat transfer coefficient of cooling water exchanger that is estimated as at 30 Btu per hour per foot square per degrees Fahrenheit.  $C_A$  is the annualized cost of heat exchangers that is estimated at 11 dollars and 30 cents per foot square per year.  $C_w$  is the cost of cooling water estimated at just above 7 cents 0.074 dollars per hour per year.

$C_s$  is the cost of steam taken to be 21 dollars and 22 cents pound per hour per year.  $W_s$  is the steam flow rate in pound per hour  $w_c$  is the cooling water flow rate in pound per hour.  $T_1$  is the intermediate temperature of the waste stream,  $A_s$  is the area of steam exchanger in feet square and  $A_c$  is the area of water exchanger or the sub cooler in feet square. Now, what we have to find out is the optimum intermediate temperature  $T_1$  for which the total annualized cost of the unit is minimum.



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SOLUTION

$$\sqrt{TAC = C_A A_s + C_A A_c + C_w W_c - C_s W_s}$$

For steam exchanger: heat given by waste stream = heat absorbed by rising stream.

$$Q_s = F C_p (T_h - T_c) = \frac{U_s A_s (T_h - T_c)}{\ln \left[ \frac{T_h - T_s}{T_c - T_s} \right]} = W_s \Delta H_s$$

Heat given up by waste stream = heat picked up by water

$$Q_c = F C_p (T_i - 100) = U_c A_c \frac{(T_i - 120) - (100 - 90)}{\ln \left[ \frac{T_i - 120}{T_i - 90} \right]} = W_c C_p (120 - 90)$$

heat given up by waste stream = heat absorbed by cooling water

Now, how we can write the total annualized cost of the unit. It is the annualized cost of the first exchanger, steam exchanger into the area of exchanger then the annualized cost of sub cooler into the area of sub cooler then the these are this is the annualized capital cost of the two exchangers. Then the operating cost  $C_w$  is the cost of water into the water flow rate and now, since the steam is generated into the system that is considered as a income from the system the steam can be utilized elsewhere.

So, the annual cost of steam we subtract from the total annualized cost. So, the objective function in the present case is this TAC that you see on the screen. Now, in the steam exchanger the heat given up by waste stream is picked up by the arising stream or the water stream we for simplicity we assume 100 percent efficiency. So, we can very quickly write the heat balance across the steam exchanger  $F$  into  $C_p$  into  $T_h$  minus  $T_c$  is the heat given by waste stream.

And that should be equal to  $U_s$  the overall heat transfer coefficient into the area into the  $\ln \left[ \frac{T_h - T_s}{T_c - T_s} \right]$  that you see on the screen. And that heat is picked up by water that is converted to steam. So,  $W_s$  into  $\Delta H_s$ , then the next exchanger here, again  $f$  into  $C_p$  into  $T_i$  minus 100 is the heat given up by waste stream the exit temperature of waste stream is taken to be 100 degrees fahrenheit. And again in terms of  $U_c$  and over all heat transfer coefficient it should be  $U_c$  into  $A_c$  into  $T_i$  minus 120 minus  $T_i$  minus 90 is this is the heat absorbed by cooling water.

The cooling water flow rate into the heat capacity into 120 minus 90 that is the temperature rise of cooling water. After having done the heat balances as you see for both exchangers  $Q_s$  and  $Q_c$ . What we tried to do is that we tried to get values of the 4 design variables the area of steam exchanger, area of sub cooler and the water flow rates in for both the systems in terms of the process parameters.

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We find TAC in terms of process variables.

$$\checkmark A_s = \frac{F C_p}{U_s} \ln \left[ \frac{T_h - T_s}{T_1 - T_s} \right]$$

$$\checkmark A_c = \frac{F C_p}{U_c} \left( \frac{T_1 - 100}{T_1 - 130} \right) \ln \left[ \frac{T_1 - 120}{100 - 90} \right]$$

$$\checkmark W_c = \frac{F C_p (T_1 - 100)}{30} \quad \text{water flow rate to subcooler}$$

$$\checkmark W_s = \frac{F C_p (T_h - T_1)}{\Delta T_s} \quad \text{water flow rate to steam exchanger.}$$

We have rearranged the equations to get  $A_s$  in terms of  $F C_p U_s T_h T_1$  and  $T_s$  similar expression for  $A_c$  the area of sub cooler and then the 2 flow rates.  $W_c$  is the water flow rate to sub cooler and  $W_s$  is the water flow rate to steam exchanger. Now, having done this we substitute these values in the total annualized cost.

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Substituting these values in expression for TAC gives:

$$\begin{aligned} \text{TAC} = & C_A \frac{F_G}{U_c} \frac{(T_1 - 100)}{(T_1 - 30)} \ln \left[ \frac{T_1 - 120}{120 - 90} \right] \\ & + C_A \frac{F_G}{U_s} \ln \left[ \frac{T_m - T_s}{T_1 - T_s} \right] \\ & + C_w \frac{F_G}{30} (T_1 - 100) - C_s \frac{F_G}{\Delta H_s} (T_m - T_1) \end{aligned}$$

Optimum value of the intermediate temperature  $T_1$  for which the TAC of the heat recovery system will be minimum can be determined by:  $\frac{\partial(\text{TAC})}{\partial T_1} = 0$

And then we get an expression for total annualized cost. Now, here all the parameters on the right hand side are given except the  $T_1$  and that we treat as variable. So, optimum value of the intermediate temperature  $T_1$  for which the TAC of the heat recovery system will be minimum can be determined by, taking partial derivative of the objective function TAC with respect to  $T_1$ .

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If we make an approximation,  $(T_1 - 100) \sim (T_1 - 130)$ , the expression simplifies to:

$$\begin{aligned} \text{TAC} = & C_A \frac{F_G}{U_c} \ln \left[ \frac{T_1 - 120}{30} \right] + C_A \frac{F_G}{U_s} \ln \left[ \frac{T_m - T_s}{T_1 - T_s} \right] \\ & + C_w \frac{F_G}{30} (T_1 - 100) - C_s \frac{F_G}{\Delta H_s} (T_m - T_1) \end{aligned}$$

$$\frac{\partial \text{TAC}}{\partial T_1} = -\frac{C_A}{U_s (T_1 - T_s)} + \frac{C_A}{U_c (T_1 - 120)} + \frac{C_w}{30} + \frac{C_s}{\Delta H_s} = 0$$

Substituting various values gives:

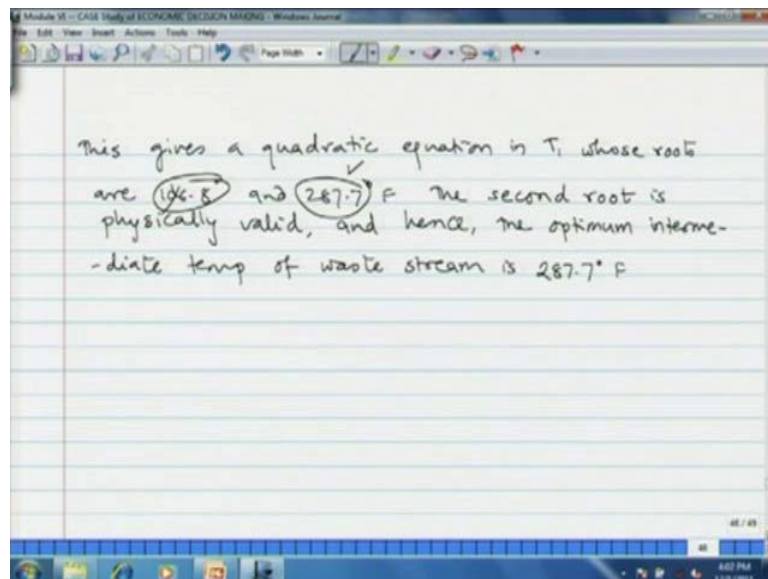
$$0 = \frac{-11.38}{30 (T_1 - 267)} + \frac{11.38}{30 (T_1 - 120)} + \frac{0.074}{30} + \frac{21.22}{33.7}$$

Now, here we make a small approximation that we assume that  $T_1$  minus 100 is more or less same as  $T_1$  minus 130. Now, this is possible if  $T_1$  value is sufficiently high of

course, this is an approximation to make objective function a simpler and with that approximation the modified objective function is  $C_a \ln \frac{T_1 - T_s}{T_1 - 120} + C_w \frac{T_1 - T_s}{\Delta H_s} + C_s \frac{T_1 - T_s}{\Delta H_s}$ . So, this particular the intermediate bracket is now, gone  $\ln \frac{T_1 - 120}{T_1 - 120}$  so on and so forth.

And now we take the partial derivative and then obtain an expression  $\frac{dC_a}{dT_1} = -\frac{C_a}{T_1 - 120} + \frac{C_w}{\Delta H_s} + \frac{C_s}{\Delta H_s}$ . And now we substitute values of all the parameters like  $C_a$  was 11.38 that we substitute here  $U_s$  was 20  $U_c$  was 30 then  $C_w$  was 0.074 and  $C_s$  was 21.22  $\Delta H_s$  was 933.7.

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Solution of this equation will give a quadratic in  $T_1$ . Now, those minor calculation I leave to you algebraic simplification and then if you take the routes of the 2 equation the 2 routes of that particular quadratic equation you will find that the routes are  $T_1$  equal to 106.8 and 287.7 degrees fahrenheit . Out of these two routes the 106.8 is physically not meaningful because the exiting cooling water from that particular exchanger is at 120 degrees fahrenheit . So, the inlet temperature of hot stream cannot be lesser than that.

So, we discard this value 106.8 and then we choose the value of 287.7 degrees fahrenheit. So, the total annualized cost of the heat exchanger unit will be optimum at an intermediate temperature of 287.7 degrees fahrenheit.