Process Design Decisions and Project Economics

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Module - 4
Design of Separation Processes
Lecture - 22
Tutorial - Part II
(Design of Distillation Column)

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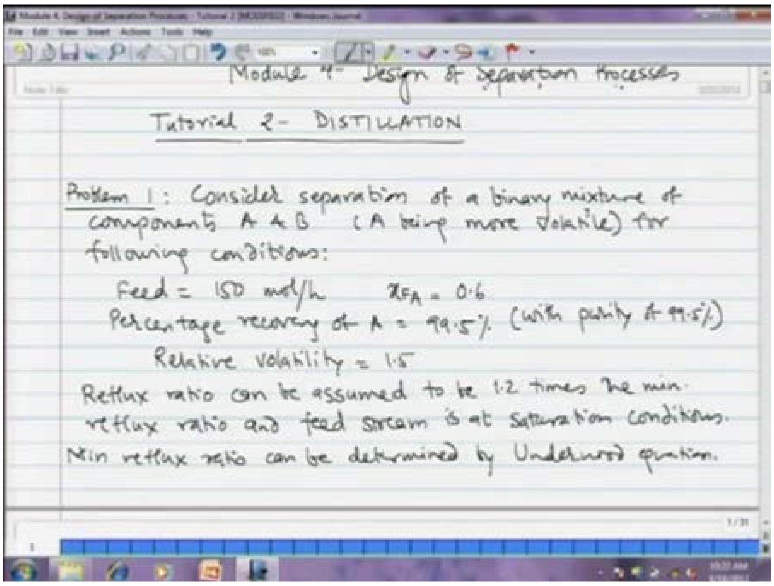

Module 4- Design of Separation Processes

Tutorial 2- DISTILLATION

Problem 1: Consider separation of a binary mixture of components A & B (A being more volatile) for following conditions:

Feed = 150 mol/h $x_{F_A} = 0.6$

Percentage recovery of A = 99.5% (with purity of 99.5%)

Relative volatility = 1.5

Reflux ratio can be assumed to be 1.2 times the min. reflux ratio and feed stream is at saturation conditions. Min reflux ratio can be determined by Underwood equation.

Welcome, we are now in the second tutorial of module 4 that is design of separation processes. And today we shall see four problems related to the distillation operation. First problem statement appears on your screen now. Consider separation of a binary mixture of components A and B, A being more volatile for following conditions. Feed 150 mole per hour, the composition of feed yields mole fraction of A is 0.6, the more volatile component is 60 percent.

Percentage recovery of A desired is 99.5 percent with purity of 99.5 percent. The relative volatility is 1.5, reflux ratio can be assumed to be 1.2 times the minimum reflux ratio which is optimum condition for reflux ratio and feed stream can be assumed to be at saturation condition, the minimum reflux ratio can be determined by Underwood's equation.

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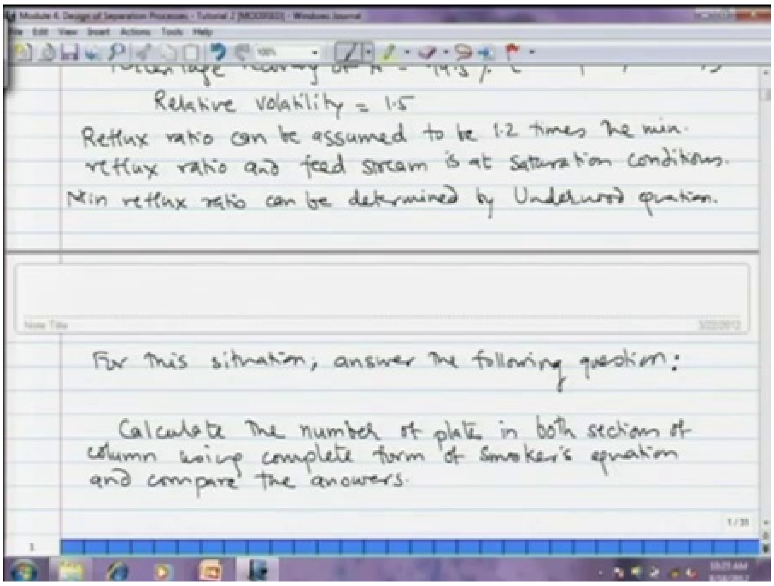

Relative volatility = 1.5

Reflux ratio can be assumed to be 1.2 times the min. reflux ratio and feed stream is at saturation conditions.

Min reflux ratio can be determined by Underwood equation.

For this situation, answer the following question:

Calculate the number of plate in both section of column using complete form of Smoker's equation and compare the answers.

And for this situation we have to calculate the number of plates in both sections of the column, rectification section as well as stripping section using the complete form of Smoker equation. And then we have to compare the answers of the rectification section plates and stripping section plates. So, we start our solution.

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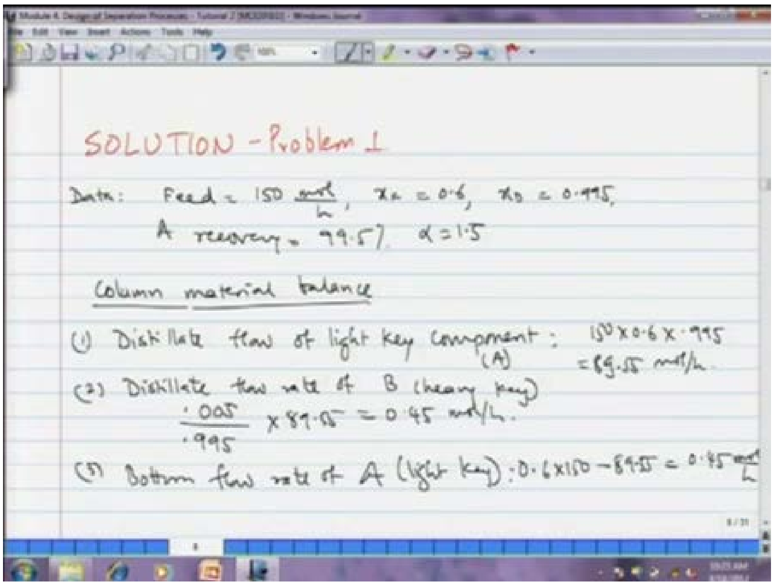

SOLUTION - Problem 1

Data: Feed = 150 $\frac{mol}{h}$, $x_F = 0.6$, $x_D = 0.995$,

A recovery = 99.5 %, $\alpha = 1.5$

Column material balance

(1) Distillate flow of light key component (A) : $150 \times 0.6 \times .995 = 89.55$ mol/h.

(2) Distillate flow rate of B (heavy key)

$$\frac{.005}{.995} \times 89.55 = 0.45 \text{ mol/h.}$$

(3) Bottom flow rate of A (light key) : $0.6 \times 150 - 89.55 = 0.45 \frac{mol}{h}$

The data given is feed, total feed is 150 per hour, mole fraction of more volatile component is 60 percent, x D the mole fraction of A, the more volatile component in distillate is 99.5 because purity is 99.5. So, x D is 0.995, percentage recovery of A is

99.5 percent, alpha equal to 1.5. So, to start with we do the complete material balance across distillation column.

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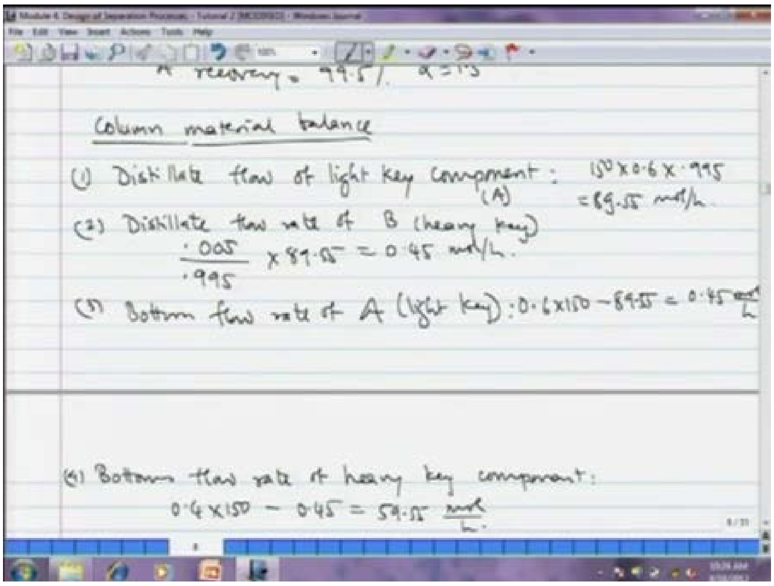

A recovery = 99.5 % $\alpha = 1.5$

Column material balance

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(2) Distillate flow rate of B (heavy key)

$$\frac{.005}{.995} \times 89.55 = 0.45 \text{ mol/h.}$$

(3) Bottom flow rate of A (light key) : $0.6 \times 150 - 89.55 = 0.45 \frac{\text{mol}}{\text{h}}$

(4) Bottom flow rate of heavy key component:

$$0.4 \times 150 - 0.45 = 59.55 \frac{\text{mol}}{\text{h}}.$$

The distillate flow rate of light key component that is A is 150 times, 150 mole per hour into 0.6 that is the total feed of A and we have to recover 99.5 percent of that in the distillate. So, the distillate flow rate of light key component is 150 mole per hour into 0.6 that is x of A into 0.995 that is percentage recovery and we get 89.55 mole per hour as the answer. Then 99.5 percent is the purity of A in distillate. So, 0.005 percent is obviously B. So, the distillate flow rate of B the heavy key component can be determined straight forwardly as 0.005 divided by 0.995 into 89.55 that is 0.45 mole per hour.

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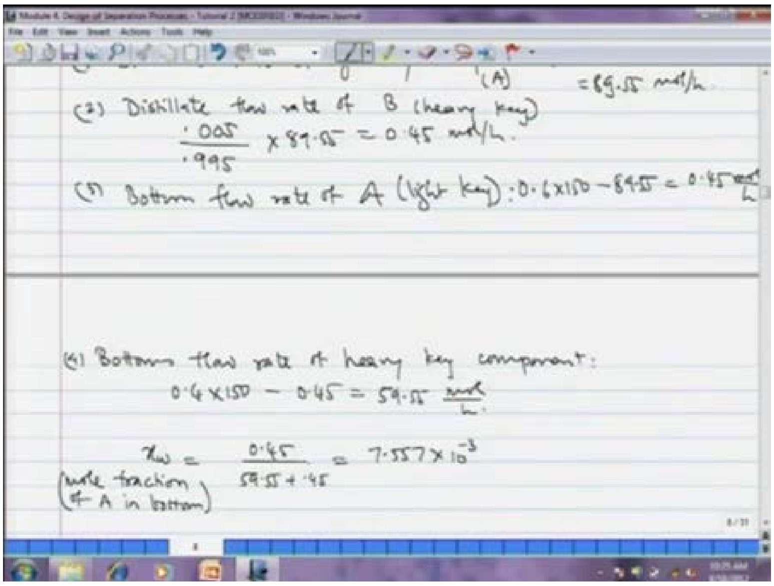

(A) $= 89.55$ mol/h

(4) Distillate flow rate of B (heavy key)

$$\frac{.005}{.995} \times 89.55 = 0.45 \text{ mol/h}$$

(5) Bottom flow rate of A (light key) : $0.6 \times 150 - 89.55 = 0.45 \frac{\text{mol}}{\text{h}}$

(6) Bottoms flow rate of heavy key component:

$$0.4 \times 150 - 0.45 = 59.55 \frac{\text{mol}}{\text{h}}$$

$$x_w = \frac{0.45}{59.55 + .45} = 7.557 \times 10^{-3}$$

(mole fraction of A in bottom)

The bottom flow rate of A that is light key component is total feed A that is 0.6 into 150 mole per hour minus 89.55 that is 0.45 mole per hour.

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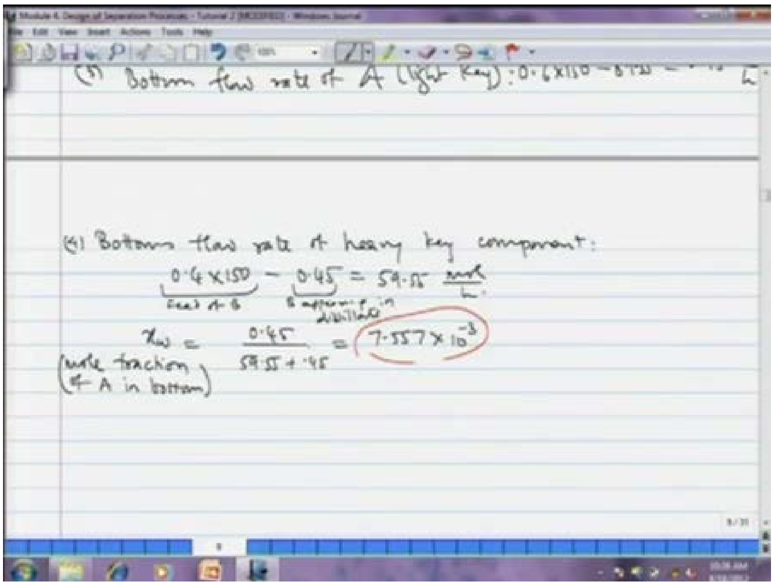

(5) Bottom flow rate of A (light key) : $0.6 \times 150 - 89.55 = 0.45 \frac{\text{mol}}{\text{h}}$

(6) Bottoms flow rate of heavy key component:

$$\underbrace{0.4 \times 150}_{\text{feed of B}} - \underbrace{0.45}_{\text{B appearing in distillate}} = 59.55 \frac{\text{mol}}{\text{h}}$$

$$x_w = \frac{0.45}{59.55 + .45} = \boxed{7.557 \times 10^{-3}}$$

(mole fraction of A in bottom)

The bottom flow rate of heavy key component is the feed of B 0.4 into 150, this is the total feed of B minus the B appearing in distillate and the remaining fraction goes to bottom 59.55 mole per hour. The mole fraction of A in bottom can then be calculated as x w is equal to 0.45, the amount of A in bottom divided by the total bottom flow rate 59.55 plus 0.45 and then x w that is the mole fraction of more volatile component in bottom comes out to be 7.557 into 10 to power minus 3. So, this way we have done the basic material balance across the column.

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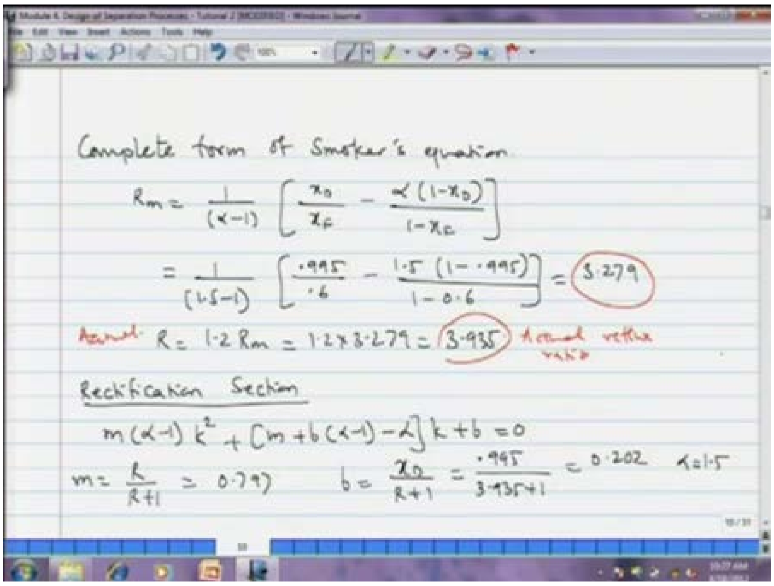

Complete form of Smoker's equation

$$R_m = \frac{1}{(\alpha-1)}\left[\frac{x_D}{x_F} - \frac{\alpha(1-x_D)}{1-x_F}\right]$$

$$= \frac{1}{(1.5-1)}\left[\frac{.995}{.6} - \frac{1.5(1-.995)}{1-0.6}\right] = \boxed{3.279}$$

Actual $R = 1.2 R_m = 1.2 \times 3.279 = \boxed{3.935}$ Actual reflux ratio

Rectification Section

$$m(\alpha-1)k^2 + [m + b(\alpha-1) - \alpha]k + b = 0$$

$$m = \frac{R}{R+1} = 0.797 \qquad b = \frac{x_D}{R+1} = \frac{.995}{3.935+1} = 0.202 \qquad \alpha = 1.5$$

Now, we start our calculations for number of plates. First of all we have to calculate the reflux ratio and we have been given that the reflux ratio is 1.5 times sorry 1.2 times the minimum reflux ratio. We use the Underwood's equation for minimum reflux ratio. R m is equal to 1 divided by alpha minus 1 into the bracket x D by x F minus alpha into 1 minus x D divided by 1 minus x F. And now we substitute all values that we have alpha equal to 1.5, x D equal to 0.995, x F equal to 0.6.

These are the values that we are putting in and then reflux ratio can be calculated. Minimum reflux ratio is calculated as 3.279, the actual reflux ratio R is 1.5 time sorry 1.2 times the minimum reflux ratio. So, 1.2 into 3.279 that is 3.935 that is actual reflux ratio. Now, having done this we have to solve the quadratic equation for the rectification section.

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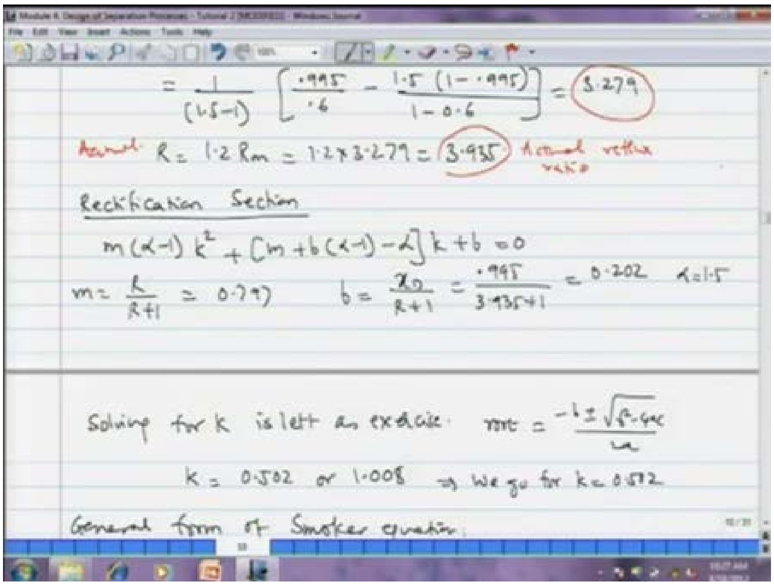

$$= \frac{1}{(1.5-1)}\left[\frac{.995}{.6} - \frac{1.5\,(1-.995)}{1-0.6}\right] = \boxed{3.279}$$

Actual $R = 1.2\,R_m = 1.2 \times 3.279 = \boxed{3.935}$ Actual reflux ratio

Rectification Section

$$m(\alpha-1)\,k^2 + [m + b(\alpha-1) - \alpha]\,k + b = 0$$

$$m = \frac{R}{R+1} = 0.797 \qquad b = \frac{x_D}{R+1} = \frac{.995}{3.935+1} = 0.202 \qquad \alpha = 1.5$$

Solving for k is left as exercise. $\text{root} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$k = 0.502$ or 1.008 ⇒ We go for $k = 0.502$

General form of Smoker equation:

This was derived in the previous class. The equation is m into alpha minus 1 k into k square plus m plus b into alpha minus 1 minus alpha into k plus b equal to 0. m is the slope of the operating line. Now, we are dealing with rectification section. So, the slope is R divided by R plus 1 and now putting value of R as 3.935 we calculate the value of slope as 0.797, the b is essentially the y intercept of the rectification line, rectification operating line that is x D by R plus 1. We have again values of all of these x D and R. So, 0.9 x D equal to 0.995, R equal to 3.935 after substituting these two values we calculate value of b as 0.202.

We have been already given value of alpha is 1.5. So, putting all these values here we can calculate, we can solve this quadratic equation and calculate values of k that I leave you as exercise. You can use the standard formula for a a x square plus b x plus c type of equation and the root is minus b plus minus b square minus Underwood b square minus 4 a c divided by 2 a. So, putting all these values you can calculate, you can get two values of k as 0.502 and 1.008. Now, k is a, is a mole fraction. So, we have to discard the value greater than 1 as absent and we go for the other value of k 0.502.

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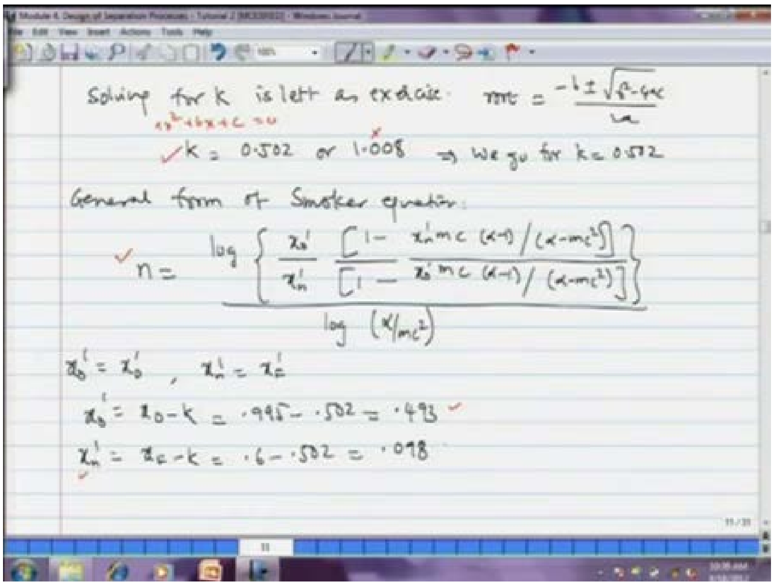

Solving for k is left as exercise · root $= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$ax^2+bx+c=0$

✓ $k = 0.502$ or $\overset{\times}{1.008}$ ⇒ we go for $k = 0.502$

General form of Smoker equation:

✓ $$n = \frac{\log\left\{\frac{x_0'}{x_n'}\frac{\left[1 - x_n' mc(\alpha-1)/(\alpha - mc^2)\right]}{\left[1 - x_0' mc(\alpha-1)/(\alpha - mc^2)\right]}\right\}}{\log(\alpha/mc^2)}$$

$x_0' = x_D'$, $x_n' = x_F'$

$x_0' = x_D - k = .995 - .502 = .493$ ✓

$x_n' = x_F - k = .6 - .502 = .098$

Then the general form of Smoker equation was derived in previous class as follows n is equal to log into x 0 dash divided by x n dash into bracket 1 minus x n dash into m into c into alpha minus 1 divided by alpha minus m c square, and again that bracket divided by 1 minus x 0 dash into m into c into alpha minus 1 divided by alpha minus m c square and the whole thing divided by log alpha divided by m c square.

Now, these dash are essentially the shifted coordinates. Now, when we apply the general, this general form of Smoker equation for rectification section then x 0 dash becomes x D dash, x n dash becomes x F dash and x 0 dash is essentially x, x 0 minus k, but x 0 is x B. So, we put value of 0.995 minus k value of k that we just derived 0.502 and that x 0 dash becomes 0.493. Similarly, we can get x n dash. x n dash is x n minus k, but x n is equal to x F. So, we put the value of x F as 0.6 minus 0.502 the value of k and then x n dash is 0.098.

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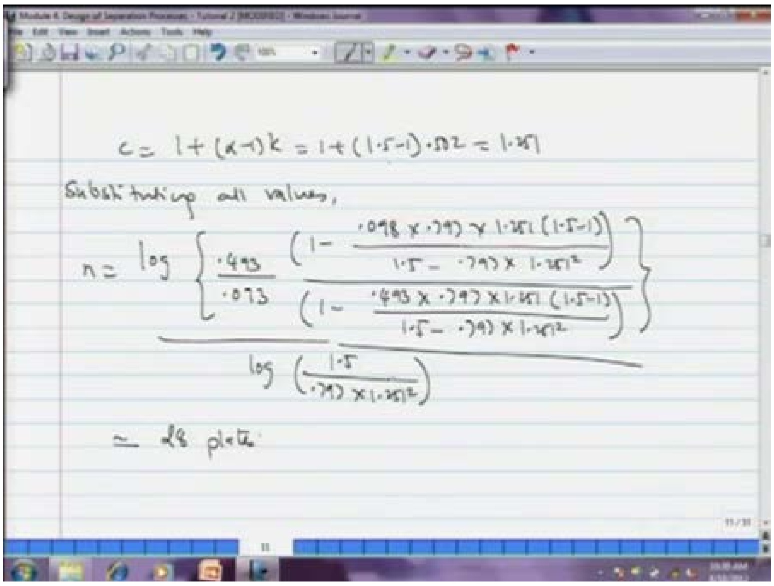

$$c = 1 + (\alpha - 1)k = 1 + (1.5 - 1)\,.502 = 1.251$$

Substituting all values,

$$n = \frac{\log\left\{\frac{.493}{.093}\,\frac{\left(1 - \frac{.098 \times .797 \times 1.251\,(1.5-1)}{1.5 - .797 \times 1.251^2}\right)}{\left(1 - \frac{.493 \times .797 \times 1.251\,(1.5-1)}{1.5 - .797 \times 1.251^2}\right)}\right\}}{\log\left(\frac{1.5}{.797 \times 1.251^2}\right)}$$

$\simeq$ 28 plate

The value of c can be found as follows. c is 1 plus alpha minus 1 into k. We have alpha equal to 1.5, k equal to 0.502. So, putting these two values you can get value of c as 1.251 and now we have all the values. We substitute those in the general form of equation n log x n dash divided by x F dash, x 0 dash. So, 0.493 x 0 dash divided by x n dash. So, x 0 dash is 0.493 divided by 0.093 and all other values that you see on the screen. And this you can solve on a simple calculator and the value that you get is 28 plates.

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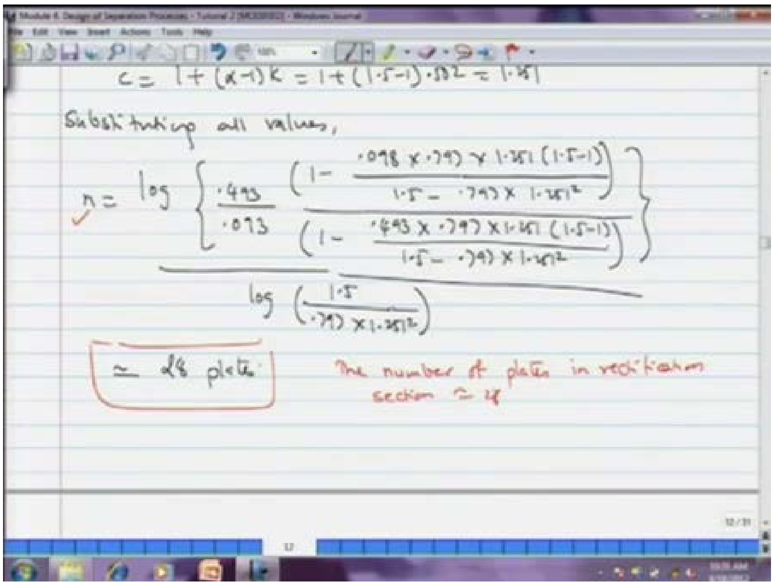

$$c = 1 + (\alpha - 1)k = 1 + (1.5 - 1)\,.502 = 1.251$$

Substituting all values,

$$n = \frac{\log\left\{\frac{.493}{.093}\,\frac{\left(1 - \frac{.098 \times .797 \times 1.251\,(1.5-1)}{1.5 - .797 \times 1.251^2}\right)}{\left(1 - \frac{.493 \times .797 \times 1.251\,(1.5-1)}{1.5 - .797 \times 1.251^2}\right)}\right\}}{\log\left(\frac{1.5}{.797 \times 1.251^2}\right)}$$

$\boxed{\simeq 28 \text{ plate}}$

The number of plates in rectification section $\simeq$ 28

So, you have the first answer the number of plates in rectification section are 28.

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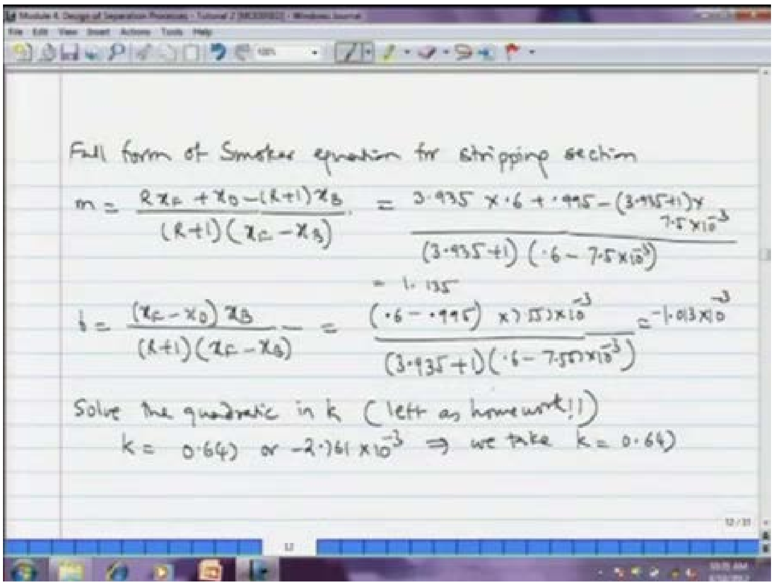

Full form of Smoker equation for stripping section

$$m = \frac{R x_F + x_D - (R+1) x_B}{(R+1)(x_F - x_B)} = \frac{3.935 \times .6 + .995 - (3.935+1) \times 7.5 \times 10^{-3}}{(3.935+1)(.6 - 7.5 \times 10^{-3})}$$

$$= 1.135$$

$$b = \frac{(x_F - x_D) x_B}{(R+1)(x_F - x_B)} = \frac{(.6 - .995) \times 7.557 \times 10^{-3}}{(3.935+1)(.6 - 7.557 \times 10^{-3})} = -1.013 \times 10^{-3}$$

Solve the quadratic in k (left as homework!!)

$k = 0.647$ or -2.761×10^{-3} $\Rightarrow$ we take $k = 0.647$

Then we move on to solving the Smoker equation for stripping section. Here the slope of operating line m is R into x F plus x D minus R plus 1 into x B divided by R plus 1 into x F minus x B. Now, this value you have already learnt the derivation of this particular equation, you have already covered in the course of mass transfer. I request you to revise the equations of operating line that will help you in understanding this. This equation is also given; this derivation of these equations is also given in the standard books like mass transfer operation by Treybal and unit operations by McCabe and Smith.

So, what I have done is that I have directly taken these values from these books. You can always go back to the book and see the derivation. So, slope of, the slope of operating line of stripping section is that m is equal to R x F into plus x D etcetera. The intercept is x F minus x D into x B divided by R plus 1 into x F minus x B. Now, these are there is some confusion about notations. Some books have used x w as the notation for mole fraction of more volatile component or light key component bottoms and some books have used x b as notation for the same.

These essentially are the same. x B is equal to x w. So, we have all the values. We have R equal to 3.935, x F equal to 0.6, x D 0.995, R equal to again 3.935, x B equal to 7.5 into 10 to power minus 3. So, after putting all these values we get the slope of operating line of stripping section as 1.135 and the y intercept as minus 1.013 into 10 to power minus 3. And again we have to go exactly the same way. We have to solve the quadratic for k, the equation that we solved for rectification section exactly same equation has to be solved.

So, that I am leaving you as a homework. You have all the values. Again, you use the standard formula for quadratic equation a x square plus b x plus c and then determine these values of k. Now, one value of k is essentially mole fraction. So, it has to be greater than 0 and less than 1. We have one value that falls between 0 and 1, 0.647 and the other value is less than 1 minus 2.761 into 10 to power minus 3. So, this value is discarded as up surd. We take the other value and proceeding exactly the same way.

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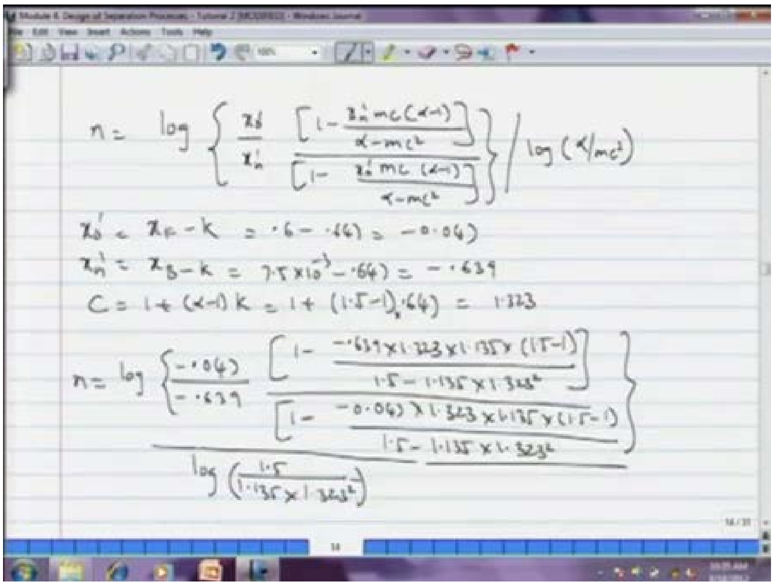

$$n = \log\left\{\frac{x_0'}{x_n'}\,\frac{\left[1-\frac{x_n' mC(\alpha-1)}{\alpha-mC^2}\right]}{\left[1-\frac{x_0' mC(\alpha-1)}{\alpha-mC^2}\right]}\right\}\Big/\log\left(\alpha/mC^2\right)$$

$$x_0' = x_F - k = .6 - .647 = -0.047$$

$$x_n' = x_B - k = 7.5\times10^{-3} - .647 = -.639$$

$$C = 1 + (\alpha-1)k = 1 + (1.5-1)\times.647 = 1.323$$

$$n = \frac{\log\left\{\frac{-.047}{-.639}\,\frac{\left[1-\frac{-.639\times1.323\times1.135\times(1.5-1)}{1.5-1.135\times1.323^2}\right]}{\left[1-\frac{-0.047\times1.323\times1.135\times(1.5-1)}{1.5-1.135\times1.323^2}\right]}\right\}}{\log\left(\frac{1.5}{1.135\times1.323^2}\right)}$$

But now remember here x 0 dash; this is the lower section of the column. So, x 0 dash is x f minus k because we are solving between the feed point and the bottom point. So, x 0 dash is x F minus k, x n dash is x B minus k. So, x n is essentially x B and x 0 is x F. And this is the only difference. Rest of the procedure is exactly the same, x 0 dash is x F minus k. So, we put x F equal to 0.6 minus 0.647. So, that gives minus 0.047, x n dash is x B minus k, so 7.5 into 10 to power minus 3 that is minus 0.64 value of k.

So, that is minus 0.639, value of C is 1 plus alpha minus 1 into k. Again, we put all values and get value of C as 1.323. Then we substitute all these values in the general form of Smoker equation, log x 0 dash divided by x n dash and then the brackets divided by log alpha divided by m C square and then the entire calculation is on your screen.

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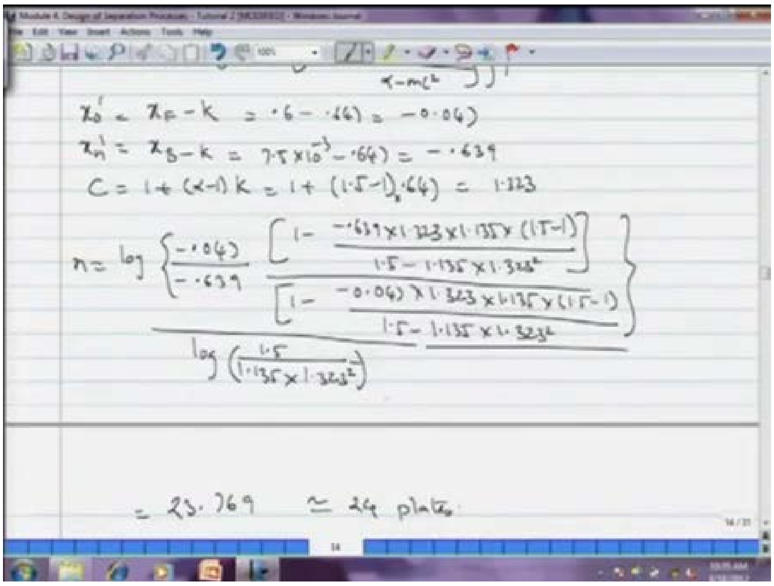

$x_0' = x_F - k = \cdot 6 - \cdot 647 = -0\cdot 047$

$x_n' = x_B - k = 7.5\times 10^{-3} - \cdot 647 = -\cdot 639$

$C = 1 + (\alpha - 1)k = 1 + (1.5-1)\cdot 647 = 1.323$

$$n = \frac{\log\left\{\dfrac{-\cdot 047}{-\cdot 639}\dfrac{\left[1 - \dfrac{-\cdot 639\times 1.323\times 1.135\times (1.5-1)}{1.5 - 1.135\times 1.323^2}\right]}{\left[1 - \dfrac{-0\cdot 047\times 1.323\times 1.135\times (1.5-1)}{1.5 - 1.135\times 1.323^2}\right]}\right\}}{\log\left(\dfrac{1.5}{1.135\times 1.323^2}\right)}$$

$= 23.769 \simeq 24$ plates

You can solve it in on any simple calculator to get the number of plates as 23.679. So, that is adjusted to the next integer as 24 plates.

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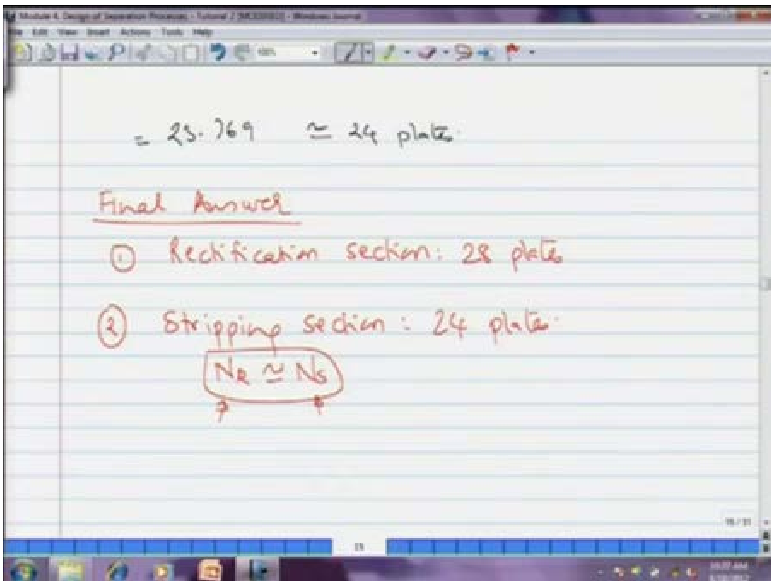

$= 23.769 \simeq 24$ plates

Final Answer

① Rectification section: 28 plates

② Stripping section: 24 plates

$\boxed{N_R \simeq N_S}$

So, we have the final answer as rectification section has 28 plates and the stripping section has 24 plates. Now, typically for a closed value mixture where alpha is anywhere between 1.2 to 1.5, 1.6 this is the answer that we are likely to get. That the number of plates in rectification and stripping section are more or less the same. Here you have only difference of 4 plates, but among if you see the percentage difference like 4 divided by

24 is only about 15 percent. So, you have, you can, you have a generalized relation as N R more or less equal to, not equal to, but more or less equal to N S, number of plates in rectification section or number of more or less equal to number of plates in stripping section. So, that completes the first problem of today’s session.

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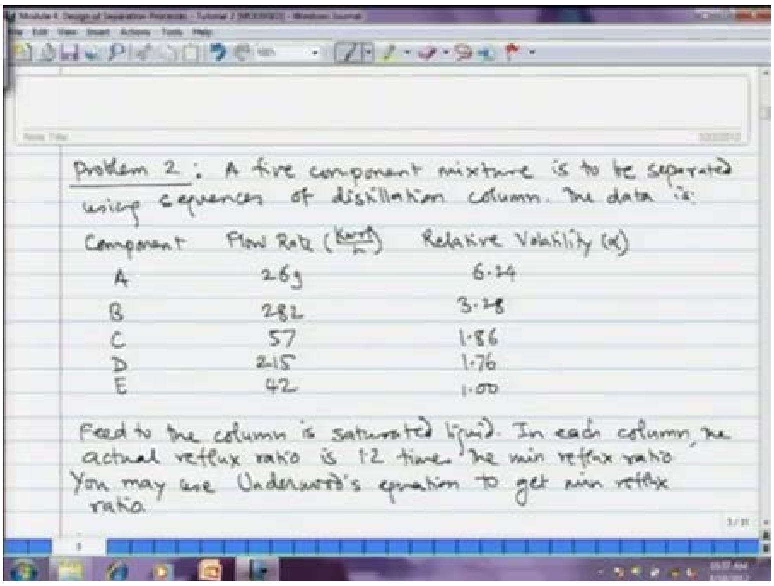

Problem 2: A five component mixture is to be separated using sequences of distillation column. The data is:

Component	Flow Rate ($\frac{Kmol}{h}$)	Relative Volatility (α)
A	269	6.24
B	282	3.28
C	57	1.86
D	215	1.76
E	42	1.00

Feed to the column is saturated liquid. In each column, the actual reflux ratio is 1.2 times the min reflux ratio. You may use Underwood's equation to get min reflux ratio.

We now see the second problem. This problem is about determination of the total vapor flow rate in different sequences of distillation and we use the Porter and Momoh formula for it. I read out the problem statement for you. A five component mixture is to be separated using sequences of distillation column. The data is as given is occurring on the, on your screen. We have five component A, B, C, D, E; A being the lightest component and E being the heaviest component.

The flow rates are also given, flow rate of A is 269 kilo mole per hour, B is 282 kilo mole per hour, C is 57 kilo mole per hour, D is 215 kilo mole per hour and E is 42 kilo mole per hour. Relative volatility is defined with respect to the heaviest component. E, E has the relative volatility of 1.76. C has 1.86. B has 3.28 and A has a relative volatility of 6.24 with respect to E. Feed to, in the column, all columns in the sequence is saturated liquid feeds. In each column the actual reflux ratio can be taken as 1.2 times the minimum reflux ratio which is optimum condition. Again, like last problem we can use Underwood’s equation to get the minimum reflux ratio.

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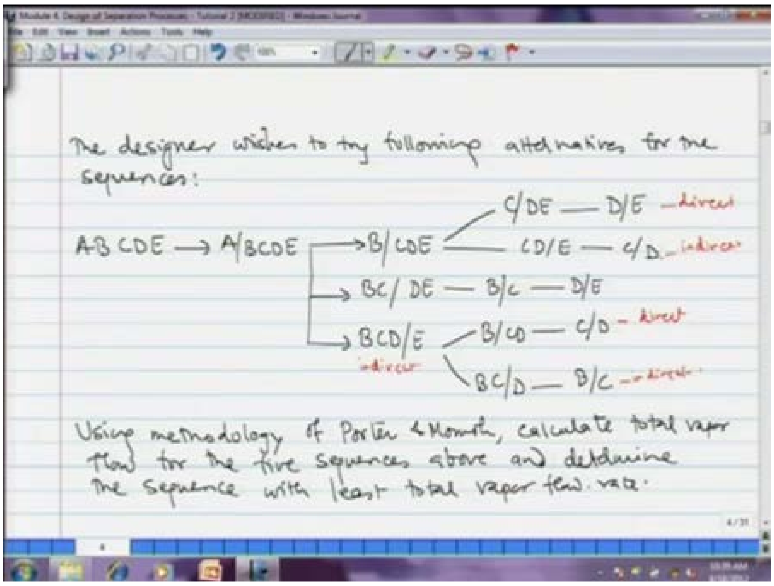

The designer wishes to try following alternatives for the sequences:

Using methodology of Porter & Momoh, calculate total vapor flow for the five sequences above and determine the sequence with least total vapor flow rate.

And then we have to compare five different sequences for the separation of this mixture in terms of the total vapor flow rate. These sequences now appear on your screen. A B C D E, going first to separate A, A from B C D E and the quaternary mixture of B C D E is split in direct and indirect sequence in both ways; first B being separated from C D E, then C being separated from D E. D and then split of D E. So, this is direct sequence. Then the other way round.

First is up to separation of B direct sequence and C D is separated in indirect sequence. Then we have equimolar split between B C D E and B C and D E and then the binary mixtures of B C and D E are split separately and once again here we have first an indirect sequence, indirect sequence of separation of E from B C D mixture, ternary mixture and the ternary mixture is spilt again in direct and indirect sequence.

So, we have to compare all these options, sequence options in terms of the total vapor flow rate and we have to use the methodology of Porter and Momoh. In the previous lectures I have told you the advantage of method of Porter and Momoh because it is explicit. If you do the same with Underwood's equation FUG method, Fenske Underwood Gilliland method, you have to first solve the equation for value of theta which is a variable.

Now, if that time you will get a polynomial which you have to solve iteratively and then you have to pick up that value of theta that lies between the relative volatility of heavy key and light key component. And substitute in the next equation for R minimum to get R minimum, but that is rather rigorous calculation because it is implicit. You have to do iterative calculation.

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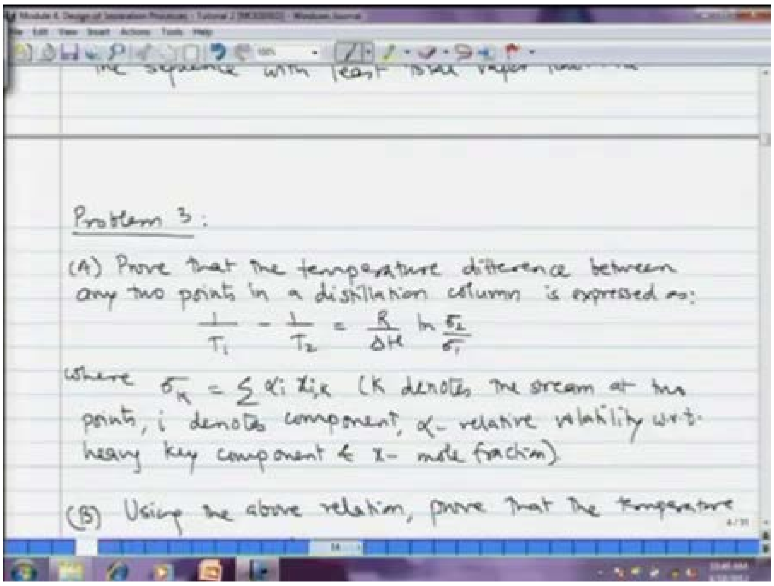

the sequence with least total vapor

Problem 3:

(A) Prove that the temperature difference between any two points in a distillation column is expressed as:

$$\frac{1}{T_1} - \frac{1}{T_2} = \frac{R}{\Delta H} \ln \frac{\sigma_2}{\sigma_1}$$

where $\sigma_K = \sum \alpha_i x_{iK}$ (K denotes the stream at the points, i denotes component, α- relative volatility w.r.t. heavy key component & x- mole fraction)

(B) Using the above relation, prove that the temperature

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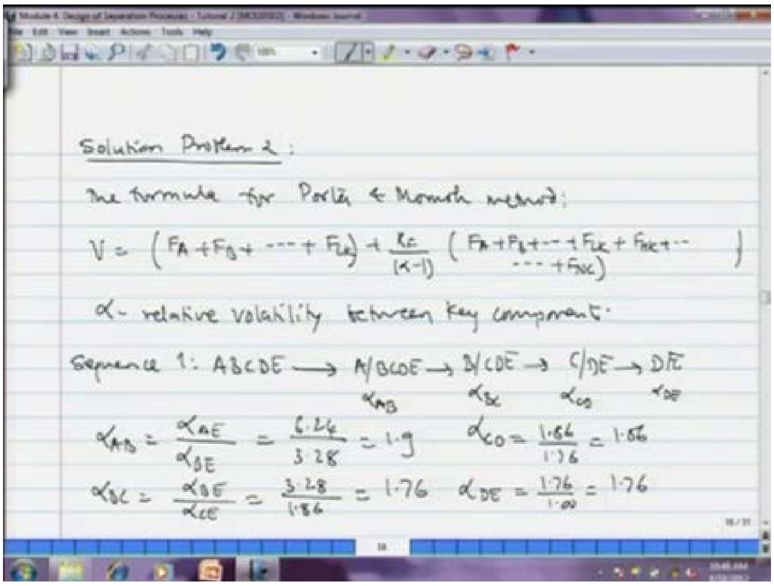

Solution Problem 2:

The formula for Porter & Momoh method:

$$V = (F_A + F_B + \cdots + F_{LK}) + \frac{R_c}{(\alpha - 1)}\left(F_A + F_B + \cdots + F_{LK} + F_{HK} + \cdots \cdots + F_{NC}\right)$$

α- relative volatility between key component.

$$\text{Sequence 1: } ABCDE \longrightarrow \underset{\alpha_{AB}}{A/BCDE} \rightarrow \underset{\alpha_{BC}}{B/CDE} \rightarrow \underset{\alpha_{CD}}{C/DE} \rightarrow \underset{\alpha_{DE}}{D/E}$$

$$\alpha_{AB} = \frac{\alpha_{AE}}{\alpha_{BE}} = \frac{6.24}{3.28} = 1.9 \qquad \alpha_{CD} = \frac{1.66}{1.76} = 1.56$$

$$\alpha_{BC} = \frac{\alpha_{BE}}{\alpha_{CE}} = \frac{3.28}{1.86} = 1.76 \qquad \alpha_{DE} = \frac{1.76}{1.00} = 1.76$$

So, Porter and Momoh have provided method which is explicit. Now, we solve the, we start the solution.

The formula for Porter and Momoh method is V the vapor flow rate is equal to F A plus F B plus F L K. Now, these are the flow rates of different components. F L K is the flow rate of light key component into R F divided by alpha minus 1 into the total flow rate, F A plus F B up to F L K and F H K heavy key up to F N C the total number of components. Alpha is a relative volatility between the key components and we have been already told that the ratio of actual reflux to minimum reflux is 1.2.

So, value of R F is 1.2. We have been given relative volatilities with respect to E the heaviest component, but when we are splitting between the different components like we are splitting here between A and B. So, we have to calculate alpha A B. The relative volatility between adjacent components that we can do by taking ratio of the components, adjacent components with respect to E, like we have to take a ratio of relative volatility of adjacent components with respect to E.

So, alpha A B is calculated as alpha A E divided by alpha B E, alpha A is 6.24, alpha B is 3.28. So, alpha A B becomes 1.9. Similarly, alpha B C is alpha B E divided by alpha C E. That becomes 3.28 divided by 1.86, so 1.76. Alpha C D E is alpha C E divided by alpha D E that is 1.86 divided by 1.76 that is 1.06 and alpha D E is 1.76 that is straight forward. So, for the first column in the first sequence we have distillate as A F A and then we have the other components.

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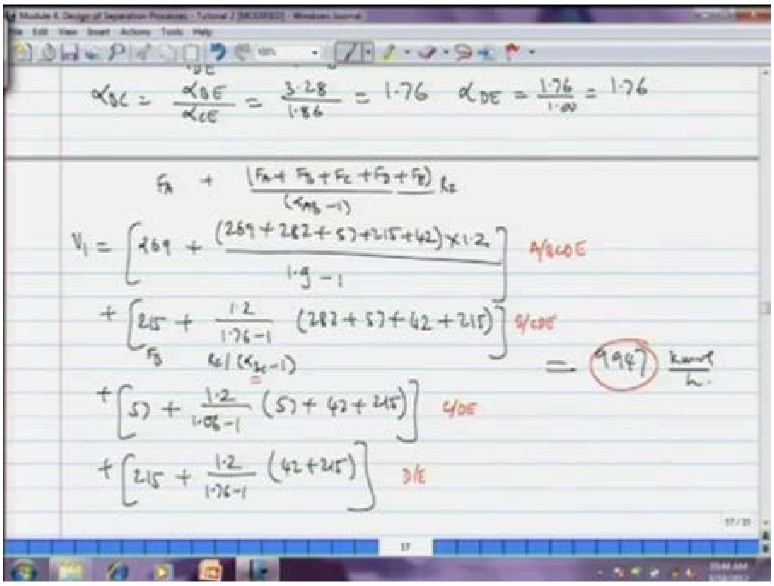

$$\alpha_{BC} = \frac{\alpha_{BE}}{\alpha_{CE}} = \frac{3.28}{1.86} = 1.76 \qquad \alpha_{DE} = \frac{1.76}{1.00} = 1.76$$

$$F_A + \frac{(F_A + F_B + F_C + F_D + F_E)\,R_F}{(\alpha_{AB} - 1)}$$

$$V_1 = \left[269 + \frac{(269 + 282 + 57 + 215 + 42) \times 1.2}{1.9 - 1}\right] \quad A/BCDE$$

$$+ \left[\underset{F_B}{215} + \underset{R_F/(\alpha_{BC}-1)}{\frac{1.2}{1.76 - 1}}\,(282 + 57 + 42 + 215)\right] \quad B/CDE \qquad = \boxed{9947}\ \frac{\text{kmol}}{\text{h}}$$

$$+ \left[57 + \frac{1.2}{1.06 - 1}\,(57 + 42 + 215)\right] \quad C/DE$$

$$+ \left[215 + \frac{1.2}{1.76 - 1}\,(42 + 215)\right] \quad D/E$$

So, the first column of first sequence that splits A from B C D E mixture you can substitute all values F A 269 F B, F C, F D and F E into 1.2 that is R F divided by alpha A B minus 1, so 1.9 minus 1. Then the second column of first sequence separates B from

C D E mixture, but here you are splitting between B and C. So, R F divided by alpha minus 1 you have to use value of alpha B C. Remember, this thing very thoroughly that you, whenever you are splitting between adjacent components you have to take the relative volatility between those components and not the volatility which has been given to you.

The volatility which has been given to you is with respect to the heavy component, heaviest component. So, F B plus 1.2 divided by alpha B C minus 1 into the other component F B, F C, F D, F E and similarly, you go ahead doing all these things, third column that splits C from D E mixture and the final that splits D E. And after substituting values you can calculate these numbers on a simple calculator and you have to add up the values of vapor flow rates of all the four columns that are there.

First column separating A from B C D E mixture, second column separating B from C D E mixture, third column separating C from C D E mixture, and final column splitting the binary mixture of D E. And this you can do on a simple calculator and get the answer as 9947 kilo mole per hour. I have given the direct answer, I suggest you to recalculate and check for correction, so that about the first sequence.

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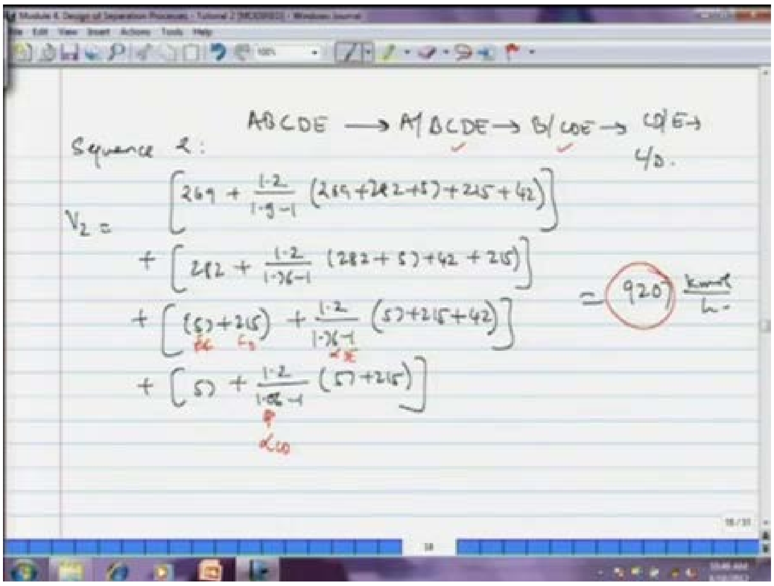

Sequence 2: ABCDE → A/BCDE → B/CDE → CD/E → C/D.

$$V_2 = \left[269 + \frac{1.2}{1.9-1}(269+282+57)+215+42\right] + \left[282 + \frac{1.2}{1.36-1}(282+57)+42+215\right] + \left[(\underset{F_C}{57}+\underset{F_D}{215}) + \frac{1.2}{\underset{\alpha_{DE}}{1.36-1}}(57+215+42)\right] + \left[57 + \frac{1.2}{\underset{\alpha_{CD}}{1.08-1}}(57+215)\right] = \boxed{920}\ \frac{\text{kmol}}{\text{h}}$$

In the second sequence we have direct separation direct sequence up to the C D E mixture and the C D E mixture is split in indirect sequence. Therefore, the first column, second column are essentially same as the previous one, previous sequence, exactly the same. A being separated, B being separated, however here you have separation of C D E mixture in indirect sequence, so here we have F D sorry F C plus F D in the third column and 1.2 into 1.76 minus 1 and then finally, here you have to use the value of alpha D E and here is alpha C D and then exactly same procedure. Putting all values you get final answer as 9207 kilo mole per hour. Now, the third sequence.

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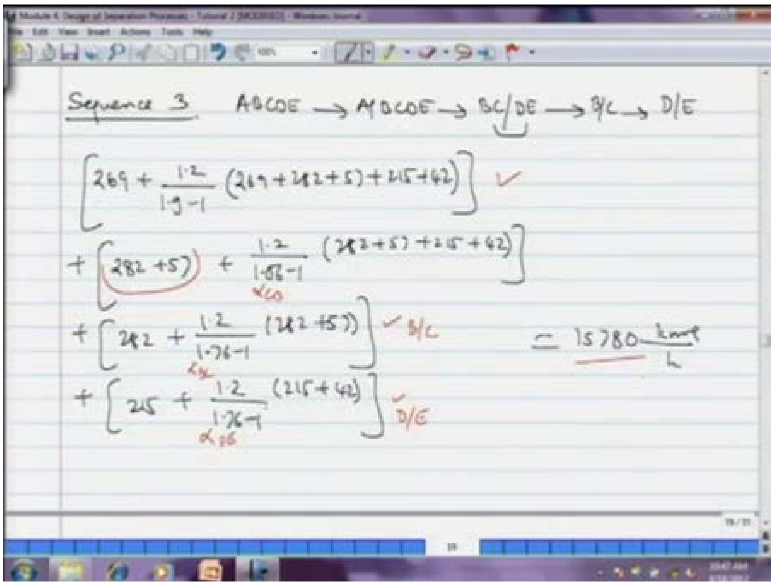

Sequence 3 ABCDE → A/BCDE → BC/DE → B/C → D/E

$$\left[269+\frac{1.2}{1.9-1}(269+282+5)+215+62)\right] \checkmark$$

$$+\left[(282+57)+\frac{1.2}{\underset{\alpha_{CD}}{1.58-1}}(282+5)+215+62)\right]$$

$$+\left[282+\frac{1.2}{\underset{\alpha_{BC}}{1.76-1}}(282+57)\right] \checkmark \text{B/C}$$

$$+\left[215+\frac{1.2}{\underset{\alpha_{DE}}{1.76-1}}(215+62)\right] \checkmark \text{D/E}$$

$$= \underline{15780}\ \frac{\text{kmol}}{\text{h}}$$

Here we have A B C D E mixture being spilt as A, B C D E under quaternary mixture of B C D E split into two binary mixtures B C, D E which are later on separated. First column is exactly the same as previous two sequences. For the other two columns, other three columns we have to do calculation. In the second column you have B C D E together appearing in the distillate and then here you have to use, you are splitting between C and D. So, you have to use value of C D, alpha C D in the second expression and then these are the two binary columns, binary mixture column. First column separating, so B C mixture; second column separating D E mixture.

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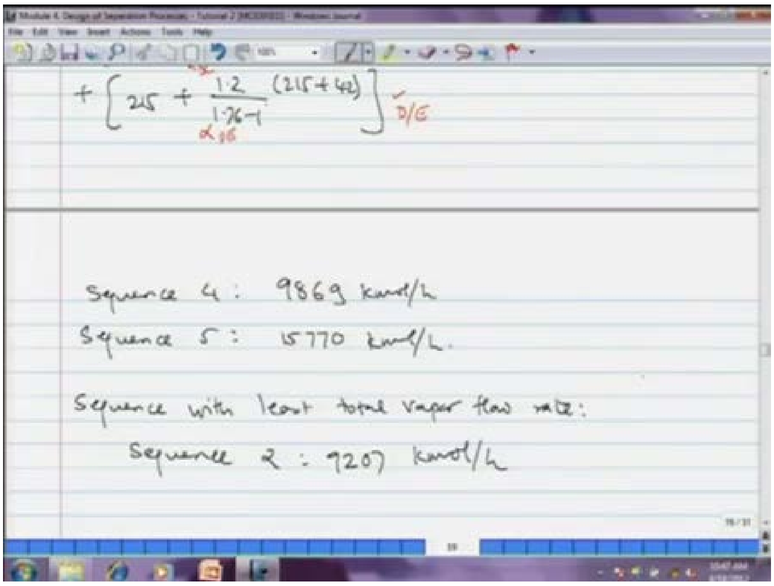

$$+\left[25+\frac{1.2}{\underset{\alpha_{DE}}{1.76-1}}(215+42)\right]_{D/E}$$

Sequence 4 : 9869 kmol/h

Sequence 5 : 15770 kmol/h.

Sequence with least total vapor flow rate:

Sequence 2 : 9207 kmol/h

And then you have to use here again value of alpha B C and alpha D E. And then you get final answer as 15780. Well sequence four and five I am leaving as an exercise for you because the calculations are rather straight forward. The only care you have to take is that whenever you are doing calculation for any column, use the value of alpha between the where you are splitting. So, if a column is splitting alpha, let us say quaternary mixture in B C D E then for that alpha you are splitting between essentially C and D.

So, you have to use alpha C D. If it is C D E means E being separated from C D mixture then you have to use alpha D E and so on and so forth. So, for the fourth and fifth sequence I am directly giving you the answer. Fourth sequence has total vapor flow rate of 9869 kilo mole per hour and sequence five has total vapor flow rate of 15770 kilo mole per hour. Now, if you compare the total vapor flow rates of all five sequences the first one we calculated as 9947, the second one we calculated as 9207, the third one we calculated as 15780 and fourth and fifth we have calculated as 9869 and 15770 kilo mole per hour.

The least total vapor flow rate among all of these is that of sequence two 9207 kilo mole per hour. So, we have the answer to the question that the sequence, the sequence with least total vapor flow rate is this A B C D E mixture being separated as A, B C D E. A being separated from quaternary mixture, and then B being separated from ternary mixture and ternary mixture of C D being split in indirect sequence. So, that completes the second problem of today’s session. Now, we shall see the third problem. The third problem is a theoretical problem. I will read out the statement.

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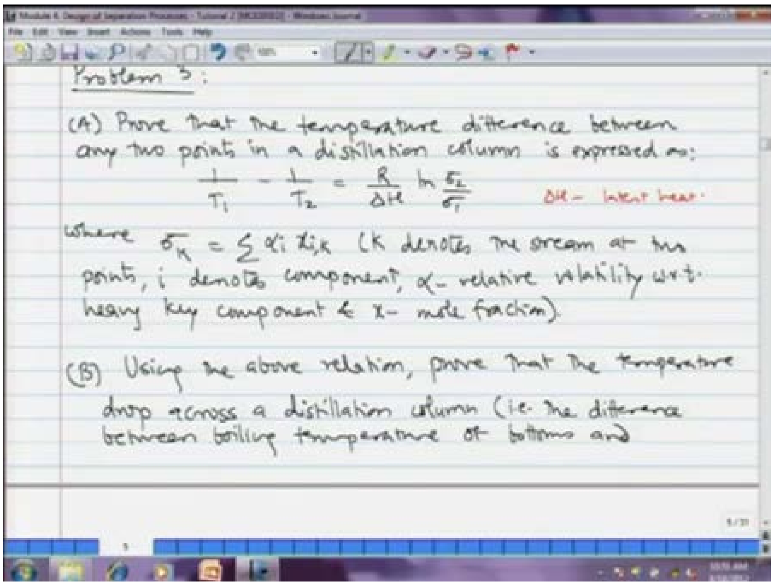

Problem 3:

(A) Prove that the temperature difference between any two points in a distillation column is expressed as:

$$\frac{1}{T_1} - \frac{1}{T_2} = \frac{R}{\Delta H} \ln \frac{\sigma_2}{\sigma_1}$$

ΔH – latent heat.

Where $\sigma_K = \sum \alpha_i x_{i,K}$ (K denotes the stream at two points, i denotes component, α – relative volatility w.r.t. heavy key component & x – mole fraction).

(B) Using the above relation, prove that the temperature drop across a distillation column (i.e. the difference between boiling temperature of bottoms and

First section of problem three is that prove that the temperature difference between any two points in a distillation column is expressed as 1 by T 1 minus 1 by T 2. 1 and 2 indicate the subscript indicate two points in a column is equal to R divided by delta H into l n sigma 2 by sigma 1. Delta H is the heat of vaporization or latent heat and sigma K is a variable defined as summation alpha i into x i K, K denotes the stream at two points, i denotes the component alpha relative volatility with respect to heavy key component and x is the mole fraction. So, K is a section of the column, alpha is the relative volatility of component i and x i is the mole fraction of that component. So, this is the first question that we have to answer. We have to derive the expression for temperature difference between any two points in distillation column.

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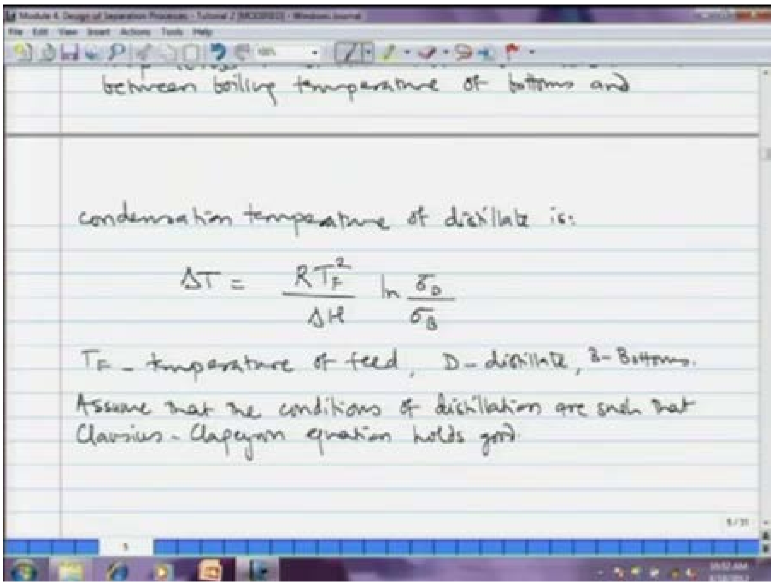

between boiling temperature of bottoms and

condensation temperature of distillate is:

$$\Delta T = \frac{R T_F^2}{\Delta H} \ln \frac{\sigma_D}{\sigma_B}$$

T_F - temperature of feed, D - distillate, B - Bottoms.

Assume that the conditions of distillation are such that Clausius - Clapeyron equation holds good

The second bit is about applying the same relation to the extreme sections of the column like bottoms and the condenser or re-boiler and condenser and then we have to determine the temperature drop across a distillation column. That is the difference between boiling temperatures of the bottom and condensation temperature of the distillate and that we have to prove as delta T equal to R into T F square divided by delta H into l n sigma D by sigma B.

Sigma is again a variable defined previously with in terms of relative volatility and mole fraction x i. T F is the temperature of the feed, D is the distillate, D denotes distillate, B denotes bottom, so that one and two are become D and B and the subscript one and two in the previous bit become D and B now. We can make an assumption that the conditions of the distillate, distillation are such that Clausius Clapeyron equation holds good. That means we are not having very high pressure distillation, we are having low to moderate pressure let us say atmospheric distillation and the mixture is relatively ideal. So, that the Clausius Clapeyron equation holds good.

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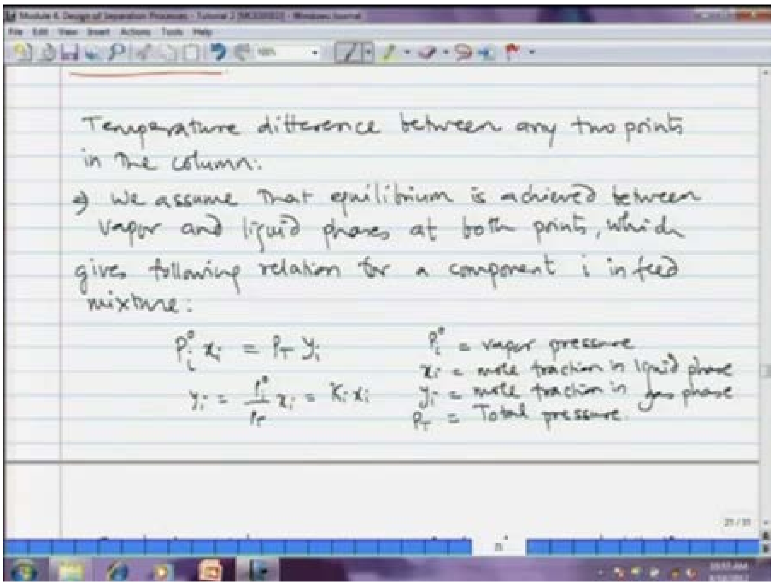

Temperature difference between any two points in the column:

⇒ We assume that equilibrium is achieved between vapor and liquid phases at both points, which gives following relation for a component i in feed mixture:

$$P_i^{\circ} x_i = P_T y_i$$

$$y_i = \frac{P_i^{\circ}}{P_T} x_i = K_i x_i$$

P_i° = vapor pressure
x_i = mole fraction in liquid phase
y_i = mole fraction in gas phase
P_T = Total pressure

Now, we start the solution. First of all we assume that equilibrium is achieved between vapor and liquid phases at both points one and two which gives the following relation for a component i in the feed mixture. This is the equilibrium relation. P i naught into x i is equal to P T into y i. P i naught is the vapor pressure, x i is the mole fraction of that component in liquid phase, y i is the mole fraction of that component in gas phase. Then we can rearrange that equation as y i is equal to P i naught by P T into x i and P i naught by P T this ratio, vapor pressure ratio to total pressure ratio is denoted as capital K i where K i is the equilibrium constant for that particular component.

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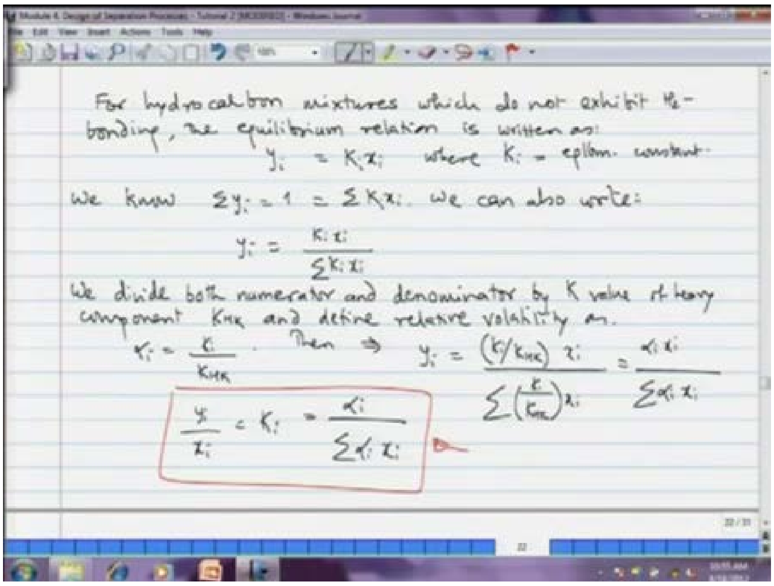

For hydrocarbon mixtures which do not exhibit H_2-bonding, the equilibrium relation is written as:

$y_i = K_i x_i$ where K_i = eqlbm. constant.

We know $\Sigma y_i = 1 = \Sigma K_i x_i$. We can also write:

$$y_i = \frac{K_i x_i}{\Sigma K_i x_i}$$

We divide both numerator and denominator by K value of heavy component K_{HK} and define relative volatility as.

$$\alpha_i = \frac{K_i}{K_{HK}} \quad \text{Then} \Rightarrow \quad y_i = \frac{(K_i/K_{HK})\, x_i}{\Sigma \left(\frac{K_i}{K_{HK}}\right) x_i} = \frac{\alpha_i x_i}{\Sigma \alpha_i x_i}$$

$$\boxed{\frac{y_i}{x_i} = K_i = \frac{\alpha_i}{\Sigma \alpha_i x_i}}$$

Now, before we procedure we have to derive a small relation. We are talking of mixture that are close boiling. So, these are very typical of hydrocarbon mixtures. For hydrocarbon mixtures which do not exhibit hydrogen bonding, the equilibrium relation is written as y i is equal to K i into x i which we just did, K i being the ratio of vapor pressure to total pressure. Summation y i is equal to 1 because y i is essentially the mole fraction of i th component.

So, if you add a mole fraction of all components, if you do summation over all i then it is obviously 1. This is equal to summation K i x i. We do apply summation sign both side. Then we can also write y i is equal to K i x i divided by summation K i x i because summation K i x i is essentially 1. So, K i x i divided by 1 is K i x i. Next, we divide both numerator and denominator of K value, denominator of that expression by K value of heavy component, heavy key component.

Now, as I said that relative volatility is always defined in terms of heavy key component as in the previous problem the heaviest component was E. So, we defined relative volatilities of all other components with respect to E. So, we do exactly the same thing here. So, alpha i is K i divided by K H K. The equilibrium constant of i th component divided by equilibrium constant of heavy key component, that we denote as alpha i. Then y i is equal to K i divided by K H K. K H K is a constant, so you can take it inside the summation sign also without any problem. K i divided by K H K into x i divided by summation K i by K H K into x i and then we replace K i by K H K as alpha i. So, we get y i is equal to alpha i x i divided by summation alpha i x i and y i divided by x i is K i. So, K i is equal to alpha i divided by summation alpha i x i. So, we use this relation for further derivation.

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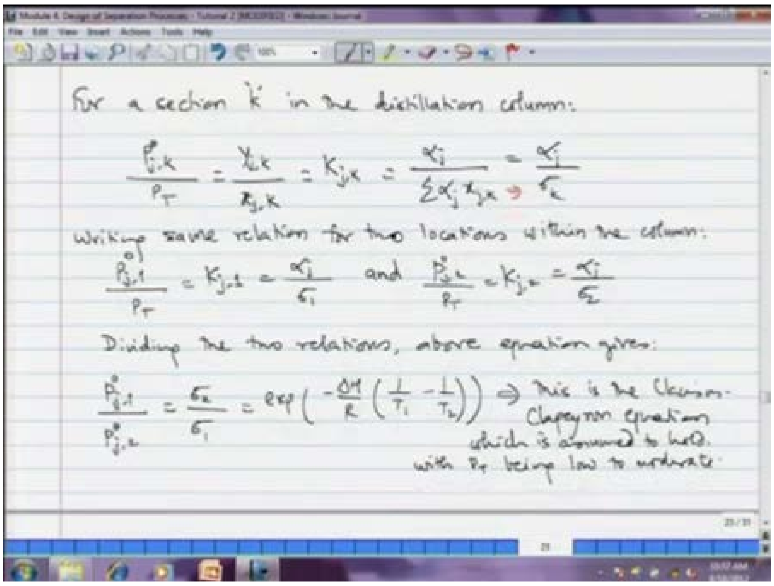

For a section 'K' in the distillation column:

$$\frac{P^0_{j,k}}{P_T} = \frac{y_{j,k}}{x_{j,k}} = K_{j,k} = \frac{\alpha_j}{\sum \alpha_j x_{j,k}} = \frac{\alpha_j}{\sigma_k}$$

Writing same relation for two locations within the column:

$$\frac{P^0_{j,1}}{P_T} = K_{j,1} = \frac{\alpha_j}{\sigma_1} \quad \text{and} \quad \frac{P^0_{j,2}}{P_T} = K_{j,2} = \frac{\alpha_j}{\sigma_2}$$

Dividing the two relations, above equation gives:

$$\frac{P^0_{j,1}}{P^0_{j,2}} = \frac{\sigma_2}{\sigma_1} = \exp\left(-\frac{\Delta H}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right) \Rightarrow$$

This is the Clausius-Clapeyron equation which is assumed to hold with P_T being low to moderate.

For a section K in a distillation column for a component j you can write P naught j K divided by P T which means the vapor pressure of j th component in K section divided by the total pressure is equal to y j K divided by x j K where y j is the vapor mole fraction of that component and x j is the liquid mole fraction of that component in that section is equal to K j K. And using the relation that we just derived K j K is can be written as alpha j divided by summation alpha j into x j K where K is that particular section.

And the denominator summation alpha j into x j K we write as sigma K. So, writing the same relation for two locations within the column one and two. So, K has now two values one and two, P naught j 1 divided by P T is equal to K j 1 that is equal to alpha j divided by sigma 1 and P naught j 2 divided by P T is equal to K j 2 that is equal to alpha j divided by sigma 2. Now, we take the ratio of these two relations and P T gets cancelled and so P naught j 1 divided by P naught j 2 is equal to sigma 2 divided by sigma 1.

The vapor pressure is now expressed in terms of Clausius Clapeyron equation. P naught j 1 divided by P naught that is one standard state is equal to exponential minus delta H R into 1, into 1 by T 1 minus 1 by T 0 where T 0 is the standard state, temperature of the standard state. Usually, it is a STP condition. So, we do the same thing for two locations, then T naught gets 1 by T naught gets cancelled because we add up the values of temperature and then we get P naught j 1 divided by P naught j 2 is equal to exponential minus delta H by R into 1 by T 1 divided by 1 by, then we take we convert exponential to l n.

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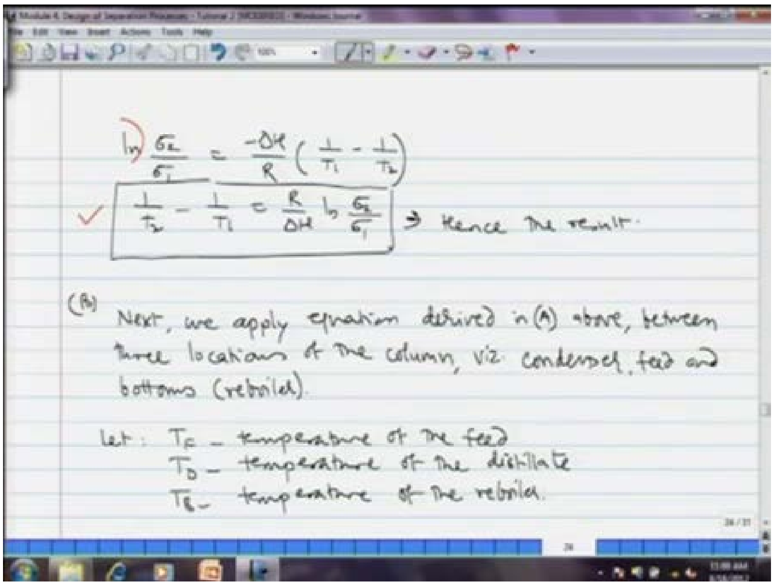

$$\ln \frac{\sigma_2}{\sigma_1} = \frac{-\Delta H}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

✓ $$\boxed{\frac{1}{T_2} - \frac{1}{T_1} = \frac{R}{\Delta H} \ln \frac{\sigma_2}{\sigma_1}} \Rightarrow \text{Hence the result.}$$

(B) Next, we apply equation derived in (A) above, between three locations of the column, viz. condenser, feed and bottoms (reboiler).

Let: T_F – temperature of the feed
T_D – temperature of the distillate
T_B – temperature of the reboiler.

So, we take l n of left hand side, l n sigma 2 by sigma 1 is minus delta H by R into 1 by T 1 minus 1 by T 2 and then you can reorganize this equation, rearrange this equation in terms of the temperature difference 1 by T 2 minus 1 by T 1 is equal to R by delta H into l n sigma 2 by sigma 1. So, we have derived the expression for temperature difference at any two points in a distillation column. Remember, this particular equation has several assumptions in it. In the first place it is for close boiling mixture, it is for the mixture which does not exhibit non ideal conditions like hydrogen bonding etcetera.

So, keep in mind the assumptions that we have made in derivation of this equation. This equation is very helpful because it can give us the idea as how the heat is being degraded in a distillation column. Whenever you have a close boiling mixture in distillation column the delta H is essentially the same, means delta H the latent heat for all components is more or less the same. So, in that way the total heat that you supply at the re-boiler more or less same heat is recovered in the condenser when the vapors condense, but what changes is the temperature.

You are supplying heat at the re-boiler at a much higher temperature and you are receiving more or less same amount of heat at a lower temperature. So, that is heat degradation across the distillation column and this formula is very handy formula to account for such degradation. Next, we apply the, this relation between the three locations in the column that is condenser feed and bottom because we have to now estimate the total heat degradation across a distillation column.

So, we denote T F as the temperature of the feed, T D as the temperature of the distillate and T B as the temperature of the re-boiler. Then we apply the above relation two times, first between feed and distillate and second between feed and bottoms. So, when we apply the relation between feed and distillate we get 1 by T F minus 1 by T D is equal to R by delta H into l n sigma F by sigma D. Now, if you reorganize this equation in terms of T D because we are interested in temperature of the distillate then you get minus R by delta H into l n sigma F by sigma D plus 1 by T F raised to minus 1. So, this is the first expression that we have, the expression for temperature on the distillate.

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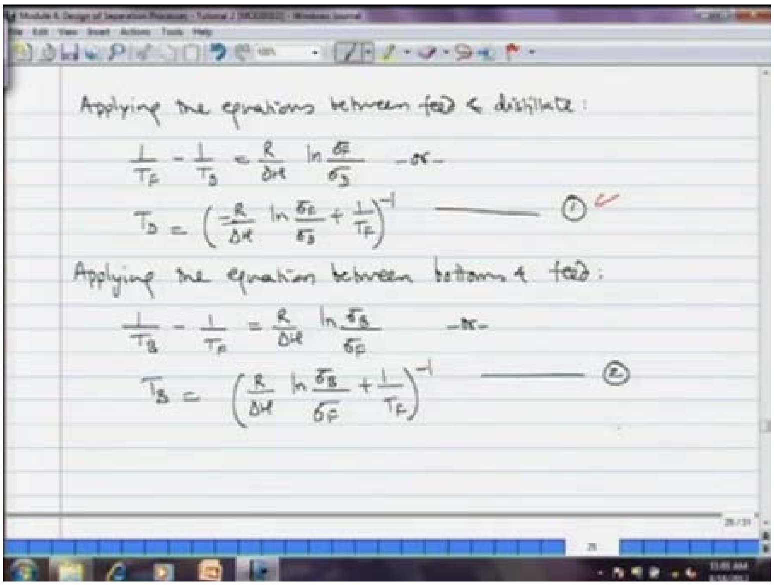

Applying the equations between feed & distillate:

$$\frac{1}{T_F} - \frac{1}{T_D} = \frac{R}{\Delta H} \ln \frac{\sigma_F}{\sigma_D} \quad \text{-or-}$$

$$T_D = \left(\frac{-R}{\Delta H} \ln \frac{\sigma_F}{\sigma_D} + \frac{1}{T_F} \right)^{-1} \text{———} ① ✓$$

Applying the equation between bottoms & feed:

$$\frac{1}{T_B} - \frac{1}{T_F} = \frac{R}{\Delta H} \ln \frac{\sigma_B}{\sigma_F} \quad \text{-or-}$$

$$T_B = \left(\frac{R}{\Delta H} \ln \frac{\sigma_B}{\sigma_F} + \frac{1}{T_F} \right)^{-1} \text{———} ②$$

Now, we apply the same general equation between bottoms and feed. So, 1 divided by T B minus 1 divided by T F. So, one and two becomes now B and F is equal to R by delta H into l n sigma B by sigma F and then we again make it explicit in terms of T B. T B is equal to R by delta H into l n sigma B by sigma F plus 1 by T F raised to minus 1. So, we have the second expression for T B. So, we have the expression for temperature of the distillate, temperature of the bottom.

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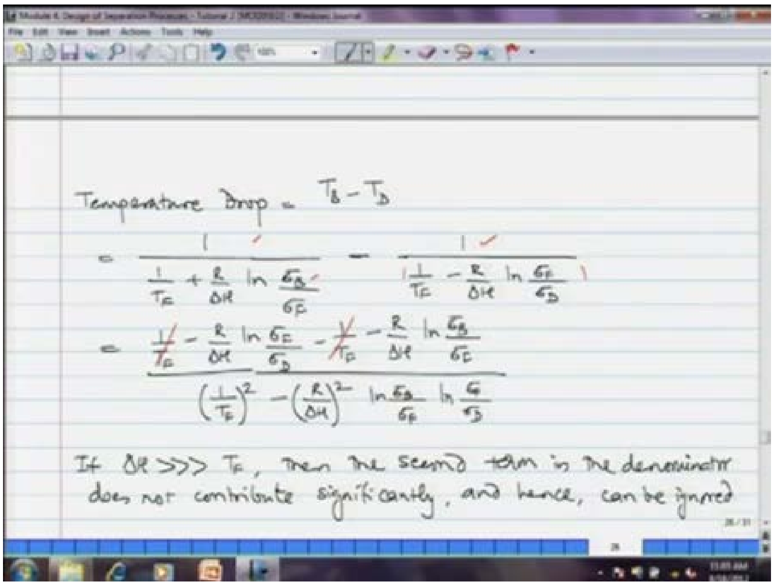

Temperature Drop = $T_B - T_D$

$$= \frac{1}{\frac{1}{T_F} + \frac{R}{\Delta H}\ln\frac{\sigma_B}{\sigma_F}} - \frac{1}{\left(\frac{1}{T_F} - \frac{R}{\Delta H}\ln\frac{\sigma_F}{\sigma_D}\right)}$$

$$= \frac{\cancel{\frac{1}{T_F}} - \frac{R}{\Delta H}\ln\frac{\sigma_F}{\sigma_D} - \cancel{\frac{1}{T_F}} - \frac{R}{\Delta H}\ln\frac{\sigma_B}{\sigma_F}}{\left(\frac{1}{T_F}\right)^2 - \left(\frac{R}{\Delta H}\right)^2 \ln\frac{\sigma_B}{\sigma_F}\ln\frac{\sigma}{\sigma_D}}$$

If $\Delta H >>> T_F$, then the second term in the denominator does not contribute significantly, and hence, can be ignored

And temperature drop across the column is T B minus T D as I just said you supply heat at a higher temperature in re-boiler and recover more or less same amount of heat at a lower temperature in the distillate, in the condenser that is distillate. So, the temperature drop is T B minus T D. We substitute now the values of T B and T D that we derived just now. 1 divided by 1 divided by T F plus R by delta H into l n sigma B by sigma F minus the expression for 1 divided by expression for T D, 1 divided by T F minus R by delta H into l n sigma F by sigma D and then we do the cross summation.

We multiply the numerator of the expression one by denominator of expression two and then vice versa. The numerator of expression two by denominator of expression one and then we multiply this thing. It is simple negation of fractions and then 1 by T F minus R by delta into l n sigma F by sigma D minus 1 by T F minus R by delta H into l n sigma B by sigma F. So, this cancels off, 1 by T F, 1 by T F goes off. When you multiply denominators then you have 1 by T F square minus R by delta square into l n sigma B by sigma F into l n sigma F by sigma D.

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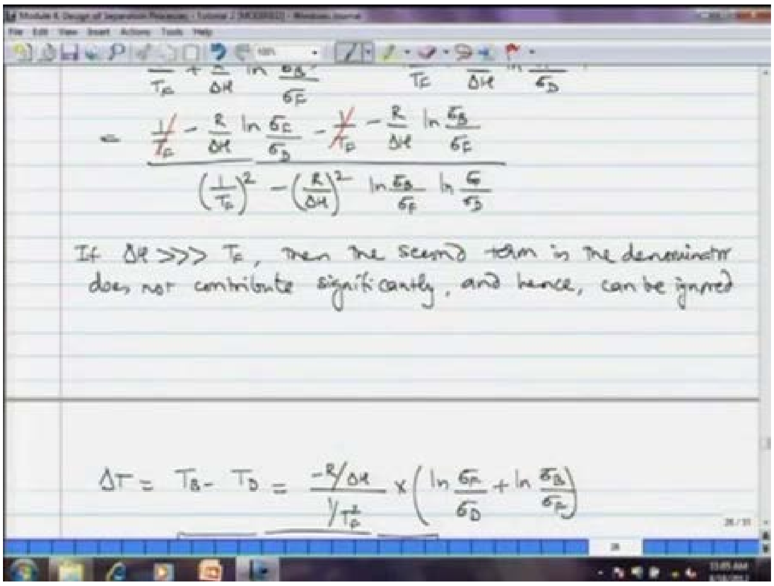

$$= \frac{\cancel{\frac{1}{T_F}} - \frac{R}{\Delta H}\ln\frac{\sigma_F}{\sigma_D} - \cancel{\frac{1}{T_F}} - \frac{R}{\Delta H}\ln\frac{\sigma_B}{\sigma_F}}{\left(\frac{1}{T_F}\right)^2 - \left(\frac{R}{\Delta H}\right)^2 \ln\frac{\sigma_B}{\sigma_F}\ln\frac{\sigma_F}{\sigma_D}}$$

If $\Delta H >>> T_F$, then the second term in the denominator does not contribute significantly, and hence, can be ignored

$$\Delta T = T_B - T_D = \frac{-R/\Delta H}{1/T_F^2} \times \left(\ln\frac{\sigma_F}{\sigma_D} + \ln\frac{\sigma_B}{\sigma_F}\right)$$

If you look at the two terms in the denominator 1 by T F square and R by delta i square into l n sigma B by sigma F into l n sigma F by sigma D. If you see the numerical values of this delta H is typically let us say 1500 kilo joule per kilo mole, 2000 kilo joule per kilo mole, R is 8.3 that R, R is the universal gas constant. So, it is 8.3 joule per mole per Kelvin. So, the R by delta H ratio is of the order of it is 8 divided by approximately 2000. So, it is 0.004 and then that squared.

So, you are going to have a very small number as the second expression of the denominator. So, R by delta i square into l n sigma B by sigma F into l n sigma F by sigma D is going to be very small as compared to 1 by T F square. What will be the temperature of the feed? It will be typically let us say 350 Kelvin, 320 Kelvin, 70 80 degrees approximately of that order. So, if 1 by T F square minus R by delta i square is going to be dominated by the first term because the second term is far smaller than the first term. And therefore, you can ignore this term.

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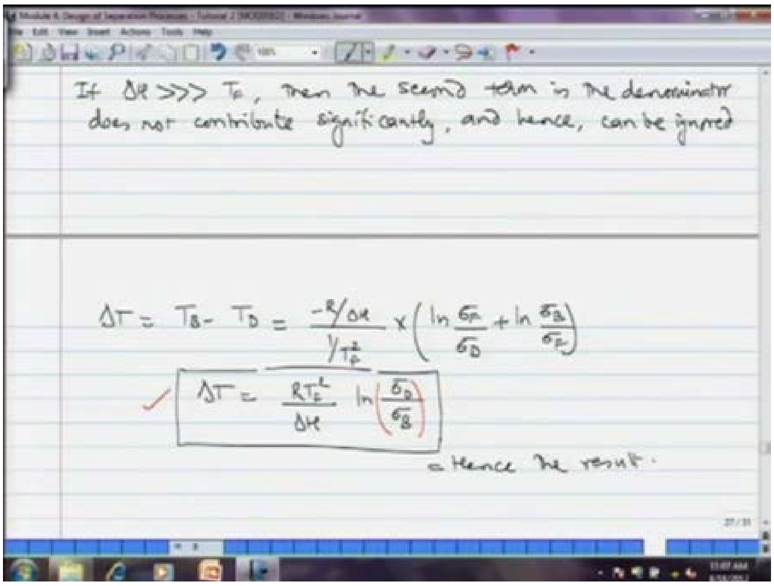

If $\Delta H >>> T_F$, then the second term in the denominator does not contribute significantly, and hence, can be ignored

$$\Delta T = T_B - T_D = \frac{-R/\Delta H}{1/T_F^2} \times \left(\ln \frac{\sigma_F}{\sigma_D} + \ln \frac{\sigma_B}{\sigma_F} \right)$$

$$✓ \boxed{\Delta T = \frac{RT_F^2}{\Delta H} \ln \left(\frac{\sigma_D}{\sigma_B} \right)}$$

= Hence the result.

Second term can be ignored, then what we are left with delta T is equal to T B minus T D is equal to R divided by delta H divided by 1 by T F square and then the summation l n sigma F by sigma D into l n sigma E B by sigma F, that you can multiply. And then you can have the final expression as delta T is equal to R into T F square divided by delta H into l n sigma D by sigma B. The negative sign is absorbed by changing the ratio.

The ratio inside the bracket is sigma B by sigma D that we take we reverse and then take care of the negative sign and this we have a very handy expression for the temperature drop across a distillation column. What we need to know to get delta T is just the temperature of the feed, the latent heat of the component, the average latent heat of all components, the distillate composition and the bottom composition and relative volatilities.

With this simple data you can have a very crucial parameter that is the temperature drop across a distillation column. Such expression is very handy for heat integration of the process where you want to integrate your distillation column with other process streams or you want to use, you have, you want to integrate two columns in the sequence. As I said in previous lecture that you can do that by varying pressures of the two columns. So, that the vapors of first column condensing the re-boiler of the second column. So, these kind of expressions are very handy for making such energy integration matches. Now, we move ahead to the next problem.

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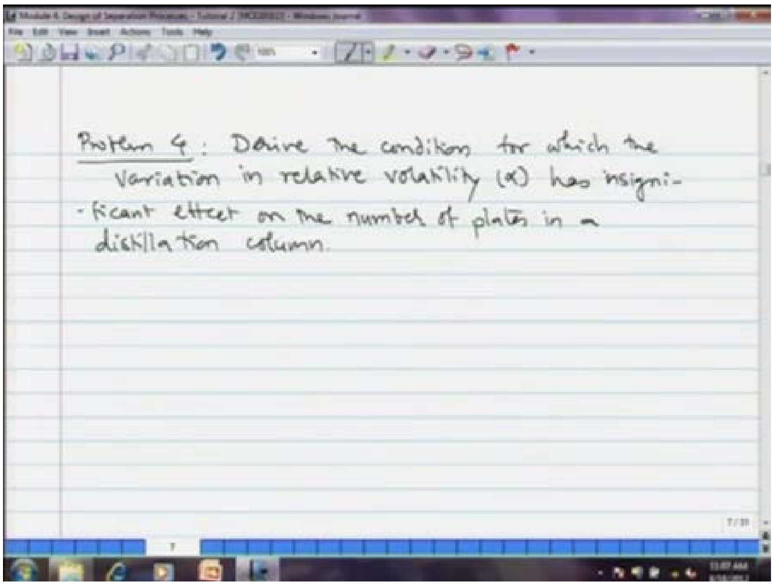

Problem 4: Derive the condition for which the variation in relative volatility (α) has insignificant effect on the number of plates in a distillation column.

The last problem of today's session problem four. This problem is also a theoretical problem, derive the condition for which variation in relative volatility alpha has insignificant effect on the number of plates in the distillation column. Whenever we do calculations like McCabe and Smith or FUG method we assume alpha to be constant. The relative volatility between components to be constant, this is the major assumption. But in many cases the relative volatility changes with temperature.

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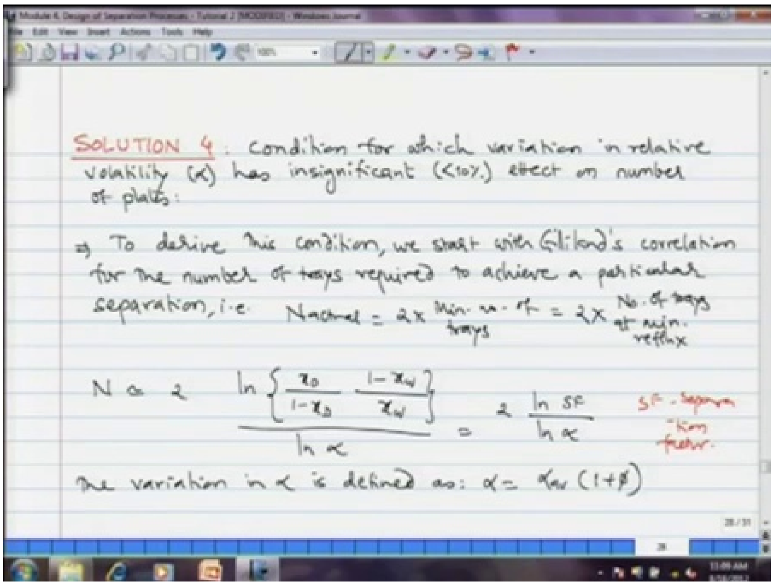

SOLUTION 4: Condition for which variation in relative volatility (α) has insignificant (<10%) effect on number of plates:

⇒ To derive this condition, we start with Gilliland's correlation for the number of trays required to achieve a particular separation, i.e. $N_{actual} = 2 \times$ Min. no. of trays $= 2 \times$ No. of trays at min. reflux

$$N \propto 2 \frac{\ln\left\{\frac{x_D}{1-x_D}\,\frac{1-x_W}{x_W}\right\}}{\ln\alpha} = 2\frac{\ln SF}{\ln\alpha}$$

SF - separation factor.

The variation in α is defined as: $\alpha = \alpha_{av}(1+\beta)$

And we have to now find out conditions under which such variation does not make much effect on the number of plates that we estimate using McCabe theory method or FUG method. So, we start our solution, condition for which variation in relative volatility has insignificant. Now, the word insignificant we quantifies less than 10 percent effect on the number of plates of a distillation column. To derive this condition we start with Gilliland correlation for the number of trays required to achieve a particular separation.

Gilliland plot says that typically the number of actual theoretical plates for separation of the mixture at conditions, optimum conditions of R equal to 1.2 times R minimum. That N actual are typically two times the minimum number of trays. Minimum number of trays result when you have a total reflux ratio and those number of trays you can calculate using Fenske's equation. That is one that is appearing on your screen now. Fenske's equation says that N plus 1, now that plus 1 stands for re-boiler.

We do not because we have, we are considering columns of 50 60 plates, so that one we ignore, one that re-boiler itself is a plate, so that we ignore. So, N is equal to l n, x D divided by 1 minus x D into 1 minus x ((Refer Time: 43:33)) x w divided by x w. Now, this product x D divided by 1 minus x D into 1 minus x w divided by x w is known as separation factor. This is popularly known as separation factor S F divided by l n alpha, where alpha is the relative volatility.

Now, we define the variation alpha as alpha average into 1 plus phi where phi is the variation. And according to Gilliland N is approximately 2 times l n x D divided by 1 minus x D into 1 minus x D divided by x w divided by l n, l n alpha. Alpha average we define as alpha T divided plus alpha B divided by 2. Alpha T is the relative volatility at the column top among the top most plates near condenser, alpha B is the relative volatility at temperature at the bottom of the column near the re-boiler.

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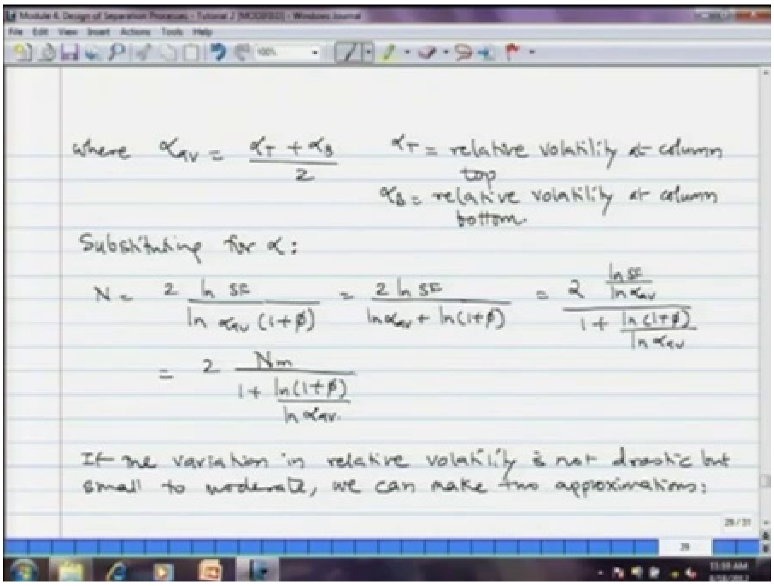

where $\alpha_{av} = \frac{\alpha_T + \alpha_B}{2}$

α_T = relative volatility at column top

α_B = relative volatility at column bottom.

Substituting for α:

$$N = \frac{2 \ln SF}{\ln \alpha_{av}(1+\phi)} = \frac{2 \ln SF}{\ln \alpha_{av} + \ln(1+\phi)} = 2\,\frac{\frac{\ln SF}{\ln \alpha_{av}}}{1 + \frac{\ln(1+\phi)}{\ln \alpha_{av}}}$$

$$= 2\,\frac{N_m}{1 + \frac{\ln(1+\phi)}{\ln \alpha_{av}}}$$

If the variation in relative volatility is not drastic but small to moderate, we can make two approximations:

And we substitute for alpha the expression that we just defined, alpha is equal to alpha a v into 1 plus phi. So, N becomes total number of theoretical plates becomes equal to 2 into l n S F separation factor divided by l n alpha a v into 1 plus phi, that we expand as 2 l n S F divided by alpha, divided by l n alpha a v plus l n 1 plus phi and then we divide the numerator and denominator by l n alpha a v. So, we have n is equal to 2 times N m the minimum reflux, minimum plates, minimum plates are l n S F divided by l n alpha v alpha a v divided by 1 plus l n 1 plus phi divided by 1 plus alpha a v.

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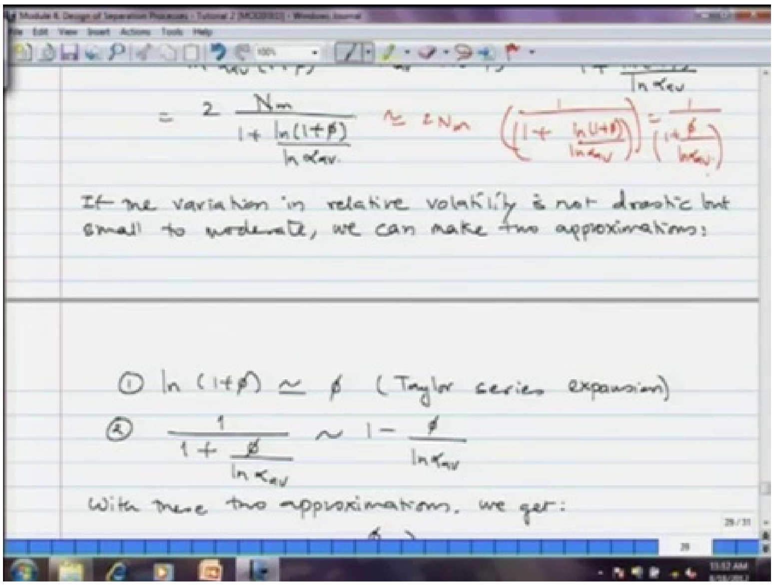

$$= 2\,\frac{N_m}{1 + \frac{\ln(1+\phi)}{\ln \alpha_{av}}} \simeq 2N_m \left(\frac{1}{1 + \frac{\ln(1+\phi)}{\ln \alpha_{av}}}\right) = \frac{1}{\left(1 + \frac{\phi}{\ln \alpha_{av}}\right)}$$

If the variation in relative volatility is not drastic but small to moderate, we can make two approximations:

① $\ln(1+\phi) \simeq \phi$ (Taylor series expansion)

② $\frac{1}{1 + \frac{\phi}{\ln \alpha_{av}}} \sim 1 - \frac{\phi}{\ln \alpha_{av}}$

With these two approximations, we get:

If the variation in relative volatility is not drastic, but small to moderate we can make two approximations for the second expression in the denominator. In the first place we can write l n 1 plus phi is approximately equal to phi as per Taylor series expansion and then the total fraction like 1 divided by 1 plus phi divided by l n alpha a v, that is we now separate like we write like two times N m into 1 divided by 1 plus l n 1 plus phi divided by l n alpha a v. Now, we have already approximated l n 1 plus 1 plus phi as phi and then what we get is this.

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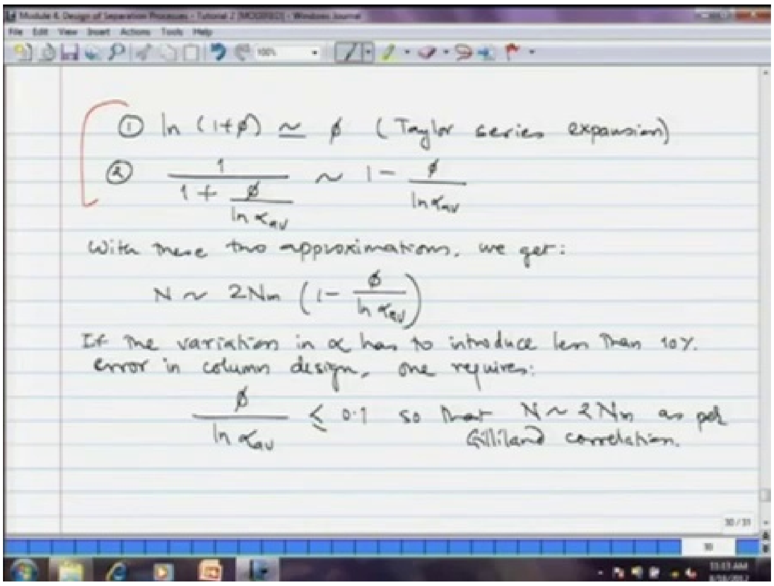

① $\ln(1+\phi) \simeq \phi$ (Taylor series expansion)

② $\dfrac{1}{1+\dfrac{\phi}{\ln\alpha_{av}}} \sim 1-\dfrac{\phi}{\ln\alpha_{av}}$

With these two approximations, we get:

$$N \sim 2N_m\left(1-\frac{\phi}{\ln\alpha_{av}}\right)$$

If the variation in α has to introduce less than 10% error in column design, one requires:

$\dfrac{\phi}{\ln\alpha_{av}} \leq 0.1$ so that $N \sim 2N_m$ as per Gilliland correlation.

So, the denominator of this expression can be written, can be expanded in terms of Taylor's series again as 1 minus phi divided by l n alpha a v. So, with these two approximations we get the total number of actual theoretical plates is equal to two times the minimum number of plates into 1 minus phi divided by l n alpha a v.

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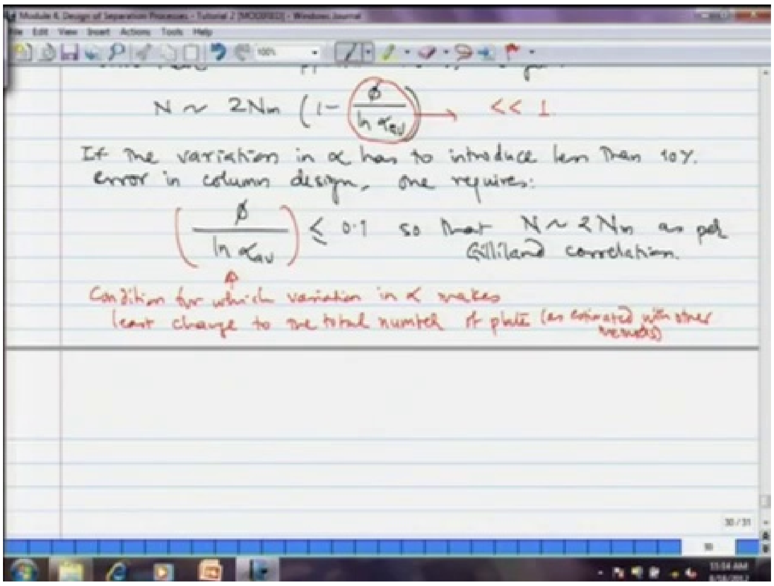

$$N \sim 2N_m \left(1 - \frac{\phi}{\ln \alpha_{av}}\right) \rightarrow \ll 1$$

If the variation in α has to introduce less than 10% error in column design, one requires:

$$\left(\frac{\phi}{\ln \alpha_{av}}\right) \leq 0.1$$ so that $N \sim 2N_m$ as per Gilliland correlation.

Condition for which variation in α makes least change to the total number of plate (as estimated with other methods)

Now, if the variation in phi has to introduce less than 10 percent error in the column design that means the number of plates, then the second expression has to be much smaller than 1. That means we can ignore this. If this second expression is far less than 1 then we can ignore this and then N becomes equal to 2 times N m which is the Gilliland expression and that will happen only when phi divided by l n alpha a v is the numerical value of this particular expression phi divided by l n alpha a v is less than or equal to 0.1.

So, that we get N is equal to 2 times N m as per Gilliland correlation. So, this the condition for which the variation in alpha makes least change to the total number of plates. Total number of plates as estimated with other methods like FUG methods. So, that completes the solution to the fourth problem. So, today we have seen four problems, two numerical and two theoretical problems related to the distillation operation. I have told you that distillation operation is one of the most common operation used in chemical industry. Therefore, learning distillation operation thoroughly is very essentially for chemical engineers. This completes the second tutorial of module 4.

Thank you.