Process Design Decisions and Project Economics
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Module - 4
Design of Separation Processes
Lecture - 21
Tutorial – Part I
Design of Absorption Column

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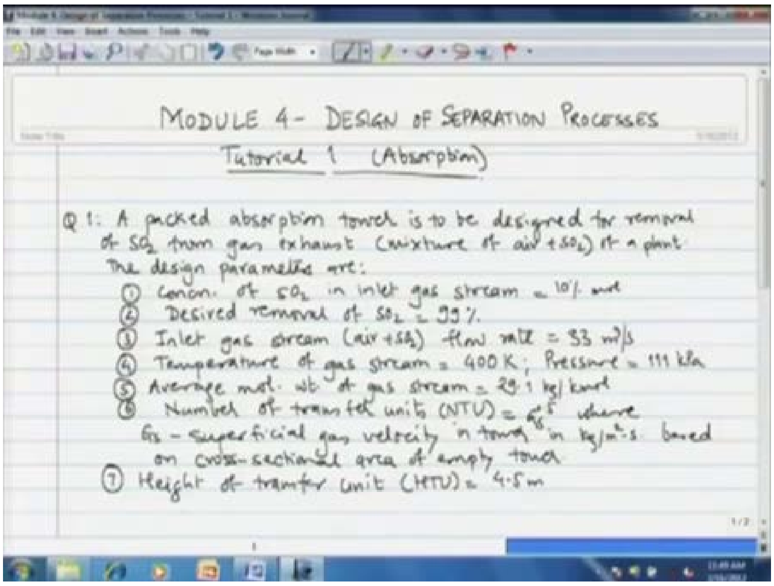

MODULE 4- DESIGN OF SEPARATION PROCESSES

Tutorial 1 (Absorption)

Q 1: A packed absorption tower is to be designed for removal of SO_2 from gas exhaust (mixture of air + SO_2) at a plant. The design parameters are:

① Concn. of SO_2 in inlet gas stream = 10% mol
② Desired removal of SO_2 = 99%.
③ Inlet gas stream (air + SO_2) flow rate = 53 m^3/s
④ Temperature of gas stream = 400 K; Pressure = 111 kPa
⑤ Average mol. wt. of gas stream = 29.1 kg/kmol
⑥ Number of transfer units (NTU) = $G_s^{.5}$ where G_s - superficial gas velocity in tower in $kg/m^2 \cdot s$ based on cross-sectional area of empty tower
⑦ Height of transfer unit (HTU) = 4.5 m

Welcome, today we shall have the first tutorial of module 4, that is design of separation processes. In the theory lectures we have seen various kind of separation process applied in chemical industry, distillation, absorption, adsorption, crystallization, liquid liquid extraction, membrane process, etcetera. We also saw under what criteria processes are selected, and now in the tutorial we shall see some problems based on this processes. Among all processes that are applied the most common processes, which are applied almost in every chemical industry may be there medium scale or large scale or absorption and distillation. So, in the tutorial we shall mainly focus on these two processes, so today we shall have the tutorial on absorption.

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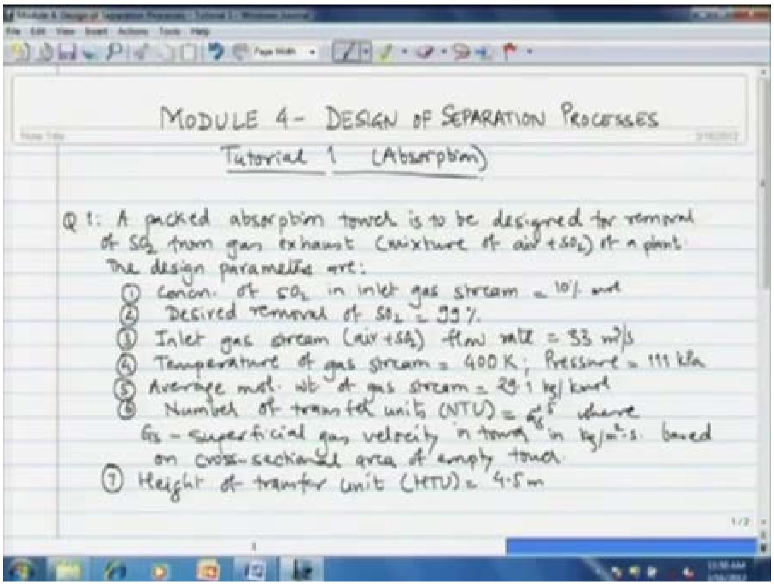

MODULE 4- DESIGN OF SEPARATION PROCESSES

Tutorial 1 (Absorption)

Q 1: A packed absorption tower is to be designed for removal of SO_2 from gas exhaust (mixture of air + SO_2) at a plant. The design parameters are:

① Concn. of SO_2 in inlet gas stream = 10% and
② Desired removal of SO_2 = 99%.
③ Inlet gas stream (air + SO_2) flow rate = 33 m^3/s
④ Temperature of gas stream = 400 K; Pressure = 111 kPa
⑤ Average mol. wt. of gas stream = 29.1 kg/kmol
⑥ Number of transfer units (NTU) = $G_s^{.5}$ where G_s - superficial gas velocity in tower in $kg/m^2 \cdot s$ based on cross-sectional area of empty tower
⑦ Height of transfer unit (HTU) = 4.5 m

Now, let us see the first problem of the tutorial, the problem statement is on your screen a packed absorption tower is to be designed for removal of sulfur dioxide from gas exhaust or mixture of air and S o 2 of a plant. The design parameters are concentration of sulfur dioxide, in inlet gas stream 10 mole percent. The desired removal of S o 2 is 99 percent. Inlet gas stream flow rate which is air plus S o 2 is 33 meter cube per second. Temperature of the gas stream is 400 Kelvin pressure of 111 kilo Pascals. Average molecular weight of gas stream is 29.1 k g per kilo mole. Number of transfer in its NTU or a function of the superficial velocity, through superficial gas velocity, through the tower G s and NTU is equal to G s raise to 0.5. Now, this superficial gas velocity is based on cross sectional area of empty tower. The height of transfer unit HTU is 4.5 meters.

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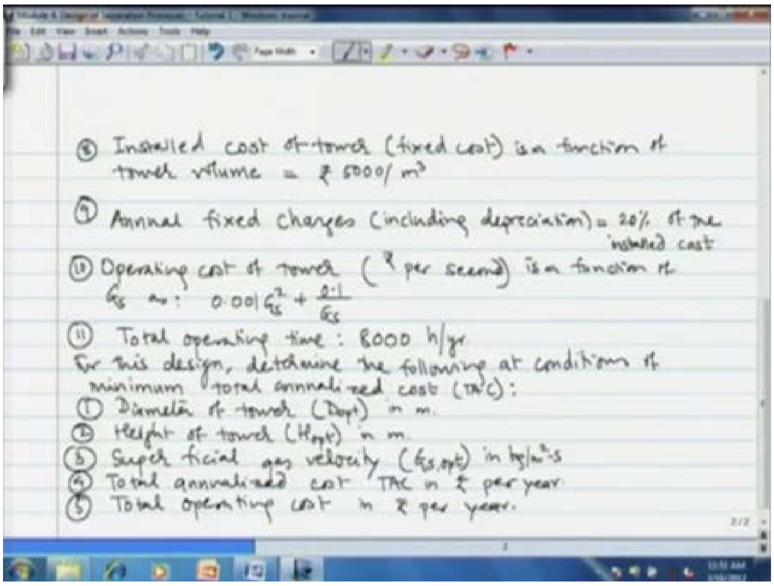

⑧ Installed cost of tower (fixed cost) is a function of tower volume = ₹ 5000/ m^3

⑨ Annual fixed charges (including depreciation) = 20% of the installed cost

⑩ Operating cost of tower (₹ per second) is a function of G_s as: $0.001 G_s^2 + \frac{0.1}{G_s}$

⑪ Total operating time : 8000 h/yr

For this design, determine the following at conditions of minimum total annualized cost (TAC):

① Diameter of tower (D_{opt}) in m.
② Height of tower (H_{opt}) in m.
③ Superficial gas velocity ($G_{s,opt}$) in kg/m²s
④ Total annualized cost TAC in ₹ per year
⑤ Total operating cost in ₹ per year.

The installed cost of tower, the fixed cost is a function of tower volume as rupees 5000 per meter cube. The annual fix charges including depreciation are 20 percent of the installed cost. Operating cost of the tower given in rupees per second is a function of again the superficial gas velocity, G s as 0.001,G s square plus 0.1 by G s. The total operating time is estimated at as estimated at 8000 hour per year. For this design we have to determine the following parameters at conditions of minimum total annualize cost. First the diameter of the tower D optimum, the height of the tower H optimum, superficial gas velocity at optimum condition G s opt and total annualize cost TSC in rupees per year and total operating cost again in rupees per year.

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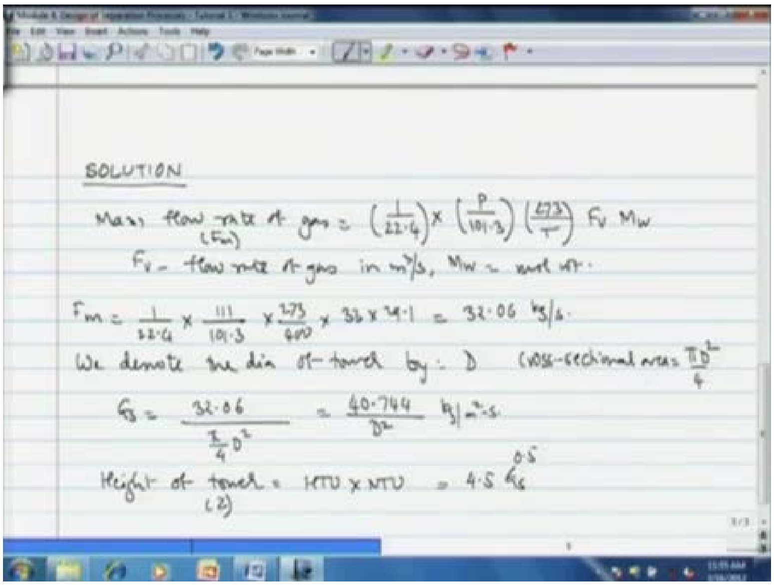

SOLUTION

Mass flow rate of gas (F_m) = $\left(\frac{1}{22.4}\right) \times \left(\frac{P}{101.3}\right) \left(\frac{273}{T}\right) F_v M_w$

F_v - flow rate of gas in m³/s, M_w = mol wt.

$F_m = \frac{1}{22.4} \times \frac{111}{101.3} \times \frac{273}{400} \times 33 \times 29.1 = 32.06$ kg/s.

We denote the dia of tower by : D   cross-sectional area = $\frac{\pi D^2}{4}$

$G_s = \frac{32.06}{\frac{\pi}{4} D^2} = \frac{40.744}{D^2}$ kg/m²·s.

Height of tower (Z) = HTU × NTU = $4.5\, G_s^{0.5}$

So, we start the solution, we have been given the flow rate of gas in meter cube per second. We first convert it into k g per meter square per second, the superficial gas velocity. The mass flow rate of gas is for simple relation we assume ideal gas law to follow 22.4 meters is the volume of 1 gram mole of gas at s t p condition 101.3 kilo pascal and 273 kelvin temperature. F v is the flow rate of gas in meter cube per second and M w is the molecular weight. So, we substitute all values here, the mass flow rate of the gas that we denote by F m, F m is equal to 1 by 22.4 into 111 by 101.3 into 273 by 400 into 33 into 29.1 and this turns out to be 32.06 k g per second.

We denote the diameter of the tower by letter capital D, so the cross sectional area is pi D square by 4 and then G s becomes 32.06 divided by pi by 4 D square or after simplification this becomes 40.744 by D square k g per meter square per second the height of the tower. Height of the tower that we denote by letter Z is HTU height of transfer unit into number of transfer unit NTU, HTU has been given to us as 4.5 meter NTU number of transfer units or the function of G s, G s raise to 0.5. So, the total height of the tower Z is 4.5 into G s raise to 0.5.

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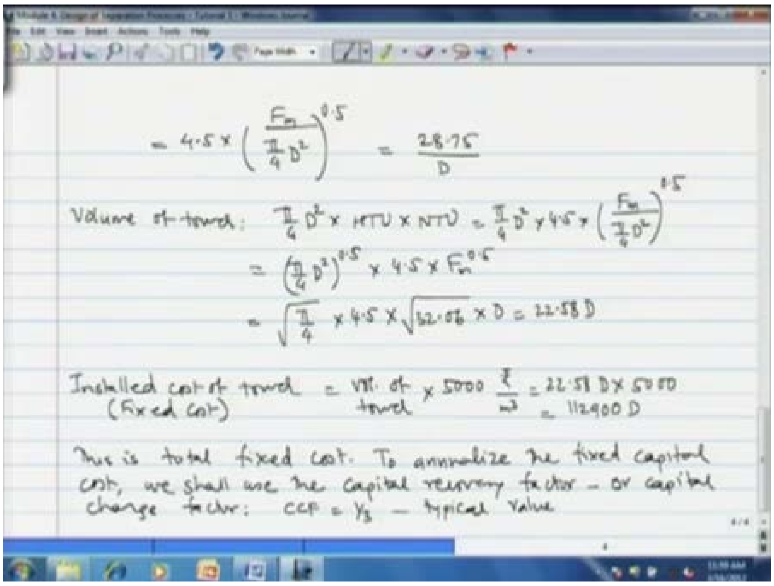

$$= 4.5 \times \left(\frac{F_m}{\frac{\pi}{4}D^2}\right)^{0.5} = \frac{28.75}{D}$$

Volume of tower: $\frac{\pi}{4}D^2 \times HTU \times NTU = \frac{\pi}{4}D^2 \times 4.5 \times \left(\frac{F_m}{\frac{\pi}{4}D^2}\right)^{0.5}$

$$= \left(\frac{\pi}{4}D^2\right)^{0.5} \times 4.5 \times F_m^{0.5}$$

$$= \sqrt{\frac{\pi}{4}} \times 4.5 \times \sqrt{32.06} \times D = 22.58\,D$$

Installed cost of tower (Fixed cost) $=$ Vol. of tower $\times 5000 \frac{₹}{m^3} = 22.58\,D \times 5000 = 112900\,D$

This is total fixed cost. To annualize the fixed capital cost, we shall use the capital recovery factor – or capital charge factor: CCF $= 1/3$ – typical value

We now substitute the expression that we derived for G s just now 4.5 into F m by pi by 4 D square rest to 0.5 and after substituting for F m and simplifying, we get this as 28.75 divided by D. So, that is the height of the tower in terms of diameter of the tower. Now the volume of the tower, volume of the tower is cross sectional area into the height which we write in terms of HTU and NTU. So, this is pi by 4 D square into 4.5 into F m by pi by 4 D square plus 2.5 and now after substituting all values we get this and then after simplification it gives 22.58 into D, that is the volume of the tower in terms of diameter. The installed cost of tower or fixed cost is volume of tower multiplied by the unit cost 5000 rupees per meter cube, so 22.58 into D into 5000 that turns out to be 112900 D this is the total annualized capital cost, sorry this is the total fixed cost.

Now, we have to calculate the total annualized capital cost and therefore, we have to annualize this particular installed fixed cost. So, to annualize the fixed capital cost we shall use the capital recovery factor is some books have also called as capital charge factor and a typical value of this factor is 1 by 3. Capital charge factor is the acronym that we are going to use CCF and typical value of this is 1 by 3, we shall deal with this things in greater detail in module 8 when we shall see the project economics. There are always two types of cost associated with every equipment, the first cost is the fixed cost or the capital cost for the equipment and second is the operating cost.

The fixed or capital cost is a single time cost and operating cost is a recurring cost. So, when we try to find out the total annualized capital cost we have to convert the single time cost at fixed cost, the fixed capital cost to a recurring basis cost and then this particular factor is kept in charge factory is used for multiplication for converting the capital cost to annualize cost ok. So, we shall see these things in greater detail in module 8 for time being you can take these values for granted.

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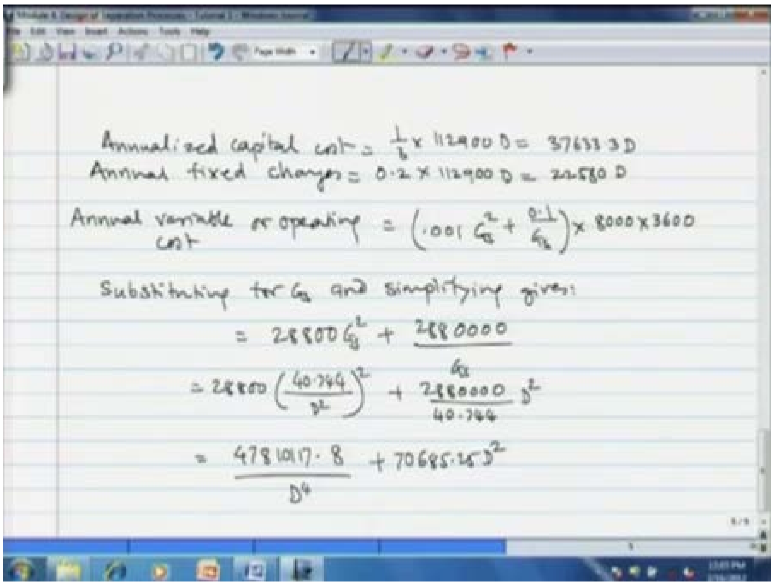

Annualized capital cost $= \frac{1}{3} \times 112900\,D = 37633.3\,D$

Annual fixed charges $= 0.2 \times 112900\,D = 22580\,D$

Annual variable or operating cost $= \left(.001\,G_s^2 + \frac{0.1}{G_s}\right) \times 8000 \times 3600$

Substituting for G_s and simplifying gives:

$$= 28800\,G_s^2 + \frac{2880000}{G_s}$$

$$= 28800\left(\frac{40.744}{D^2}\right)^2 + \frac{2880000}{40.744}\,D^2$$

$$= \frac{4781017.8}{D^4} + 70685.25\,D^2$$

So, the annualize capital cost is 1 by 3 of 112900 D that is 37633 into D. The annual fixed charges are 20 percent of the capital cost, so 0.2 into 112900 D that is 22580 D. The annual variable or operating cost is 0.001 G s square plus 0.01 by G s this is rupees per second, we are given 8000 hours of operations so into 8000 into 3600.

So, now you substitute for G s here, substituting for G s and simplifying is G s, G s we have already determined here as 28.75 divided by or no G s is determined as 40.744 divided by D square. So, that value you have to substitute here and then simplify and giving you the final answer simplification and living as an exercise 28800 G s square plus 2880000 into G s and then so that is the analyzed, so annual variable in operating cost.

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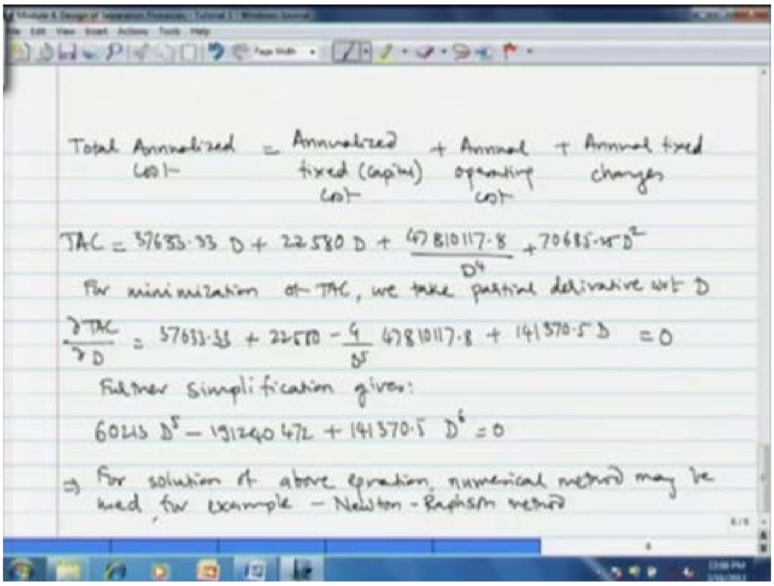

Total Annualized cost = Annualized fixed (capital) cost + Annual operating cost + Annual fixed charges

$$TAC = 37633.33\,D + 22580\,D + \frac{47810117.8}{D^4} + 70685.25\,D^2$$

For minimization of TAC, we take partial derivative wrt D

$$\frac{\partial TAC}{\partial D} = 37633.33 + 22580 - \frac{4}{D^5}\,47810117.8 + 141370.5\,D = 0$$

Further simplification gives:

$$60213\,D^5 - 191240472 + 141370.5\,D^6 = 0$$

⇒ For solution of above equation, numerical method may be used, for example – Newton – Raphson method

Now, the total annualized cost is the sum of annualized capital cost or annualized fixed cost plus the annual operating cost plus the annual fixed charges. Now, we substitute for all three cost that we have just derived total, annualized cost is 37633.33 into D plus 22580 into D plus the variable cost that we just determine 47810117.8 by D raise to 4 plus70685.25 D square. Now, for minimization of TAC, we take partial derivative with respect to D. So, D TAC by DD so this turns to be 37633.33plus22580 minus 4 by D raise to 5 into 47810117.8 plus 141370.5 into D and this we have to equate to 0.

Then further simplification gives 60213 D raise to 5 minus 191240472 plus 141370.5 D to the power 6 D equal to 0. Now, we have a basically polynomial in terms of D we have to find out it is roots, so the best way is to use a numerical method, the simplest method is a Newton Raphson method. For solution of above equation numerical method may be used. For example, the simplest method is Newton Raphson method, Newton Raphson method sub rooting is available in all simple softwares like mat lab or poly math or maple. So, we can very easily put this equation give a guess, let us say guess of 1 or 2 or 3 D is equal to 1 or 2 or 3 and then you can get a solution.

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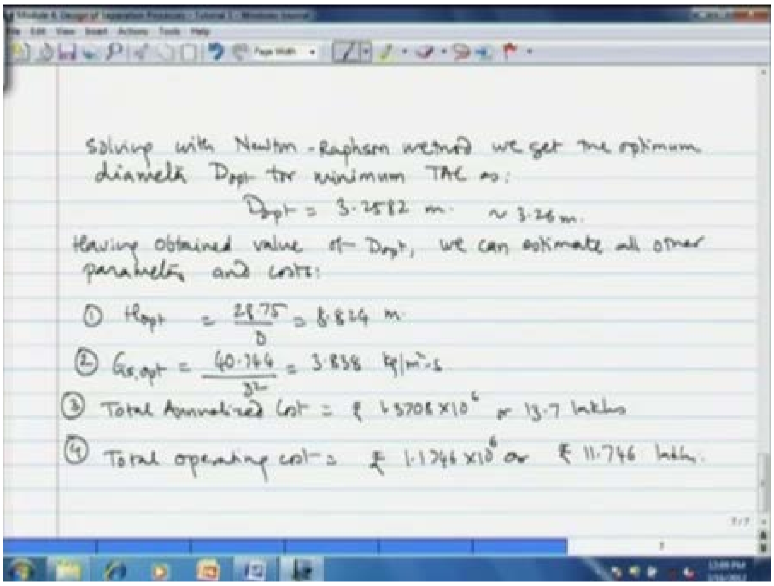

Solving with Newton - Raphson method we get the optimum diameter D_{opt} for minimum TAC as:

$$D_{opt} = 3.2582 \text{ m} \quad \sim 3.26 \text{ m}$$

Having obtained value of D_{opt}, we can estimate all other parameters and costs:

① $H_{opt} = \frac{28.75}{D} = 8.824$ m

② $G_{s,opt} = \frac{40.744}{D^2} = 3.838$ kg/m^{2}s

③ Total Annualized Cost = ₹ 1.3708×10^6 or 13.7 lakhs

④ Total operating cost = ₹ 1.1746×10^6 or ₹ 11.746 lakhs.

So, I have done it and then I have solved it with Newton Raphson method, solving with Newton Raphson method we get the optimum diameter D opt for minimum total annualized cost as 3.2582 meters. Now, I have taken four decimals for accuracy you can even terminate at two, so 3.26 meters and or 3.3 meters so forth. Now, having obtained value of D opt we can estimate all cost and other parameters and now I am giving the answers. First is H opt height of the tower at optimum condition, that is 28.75 divided by D, so substituting D we get this as 8.824 meters. Then G s opt it is 40.744 by D square and again putting D equal to 3.2582 this comes out to be 3.838 k g per meter square per second.

The total annualized cost turns out to be rupees 1.3708 into 10 to power 6 or 13.7 lakhs and the total operating cost or a total variable cost turns out to be rupees 1.1746 into 10 to power 6 or rupees 11.746 lakhs, so that completes the solution to the problem. Now, we shall see a theoretical problem related to absorption, we often use Kremsers equation for obtaining the number of plates in an absorption tower, however Kremsers equation has its own limitation. So, we shall see as under what circumstances we can use Kremsers equation or in other words we shall derive a criteria for use of Kremsers equations, so that is out next problem.

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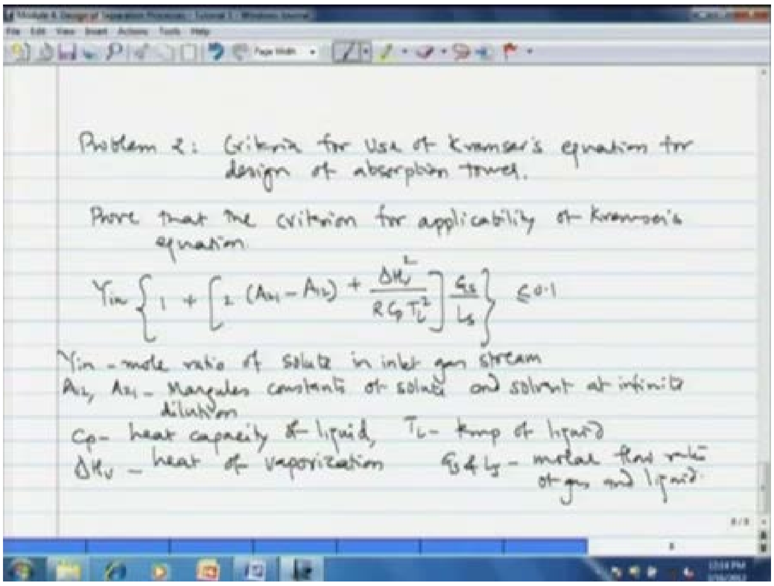

Problem 2: Criteria for use of Kremser's equation for design of absorption tower.

Prove that the criterion for applicability of Kremser's equation

$$Y_{in}\left\{1+\left[2(A_{21}-A_{12})+\frac{\Delta H_v^2}{R C_p T_L^2}\right]\frac{G_s}{L_s}\right\} \le 0.1$$

Y_{in} - mole ratio of solute in inlet gas stream

A_{12}, A_{21} - Margules constants of solute and solvent at infinite dilution

C_p - heat capacity of liquid, T_L - temp of liquid

ΔH_v - heat of vaporization G_s & L_s - molar flow rate of gas and liquid

We have to prove that the criterion for applicability of Kremsers equation is given by the following expression Y in into 1 plus 2 into A 21 minus A 1 2 into or a plus delta H v square divided by R C p T l square bracket complete into G s by L s and this has to be less than r equal to 0.1 this is the criteria. Now, let us see various notation Y in is the mole ratio of solute in the inlet gas stream A 1 2 and A 2 1 are the Margules constants of solute and solvent at infinite dilution. Then C p is the heat capacity of liquid, T l is a temperature of liquid, delta H v is the heat of vaporization and G s and L s are the flow rates of gas and liquid, molar flow rates of gas and liquid, so we have to develop this criteria.

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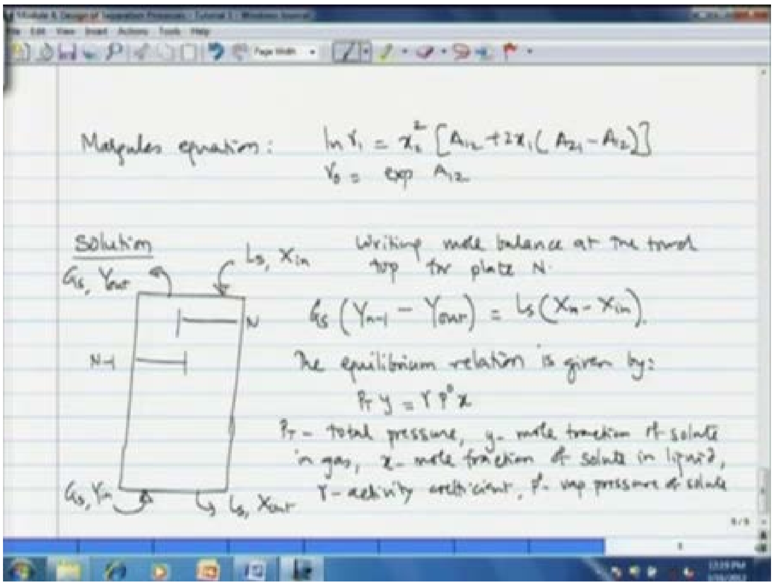
Margules equation: $\ln \gamma_1 = x_2^2 [A_{12} + 2x_1 (A_{21} - A_{12})]$

$\gamma_0 = \exp A_{12}$

Solution

Writing mole balance at the tower top for plate N

$G_s (Y_{n-1} - Y_{out}) = L_s (X_n - X_{in})$.

The equilibrium relation is given by:

$P_T y = \gamma p^0 x$

P_T - total pressure, y - mole fraction of solute in gas, x - mole fraction of solute in liquid, γ - activity coefficient, p^0 - vap pressure of solute

We are given the Margules equation for activity coefficient, Margules equation is l n gamma 1, 1 denote solute to denote solvent x 2 square into A 1 2 plus 2 x 1 into A 21 minus A 1 2 and gamma naught that is activity coefficient at infinite dilution is exponential A 1 2, so we start our solution. We first draw a simple schematic of an absorption tower, a tower having N plates in that flow rate of gas G s with mole ratio X in outlet flow rate of G s carrier gas not the total gas flow rate that is the carrier gas. G s into X out is the mole ratio in the outgoing gas, l is the total flow rate that comes in carrier flow, solvent flow rate then X in oh sorry, here we are using notation Y for gas mole fractions. So, the solute mole ratio in the incoming gas is Y in, capital Y in the solute mole ratio in the outgoing gas is capital Y out so please make this change.

For liquid where we using the letter X for mole ratio X in is the mole ratio of solute in the incoming liquid and X out is the mole ratio of solute in outgoing liquid. Now, if we write a balance for the top of the tower, writing mole balance at the tower top which means we include the first only the first two plates, then we can write a simple balance G s into Y n minus 1 minus Y out is equal to l s into X n minus X in, this is the balance for plate n.

Now, the equilibrium relation is given by P t into y is equal to gamma into p naught into x, P t is the total pressure of the tower, y is the mole fraction of solute in gas, x is the mole fraction of solute in liquid, gamma is the activity coefficient. We are not taking to account the ((Refer Time: 29:16)) here, because the pressure is relatively small, so we can take P the coefficient as 1 and P naught this is vapor pressure of solute ok.

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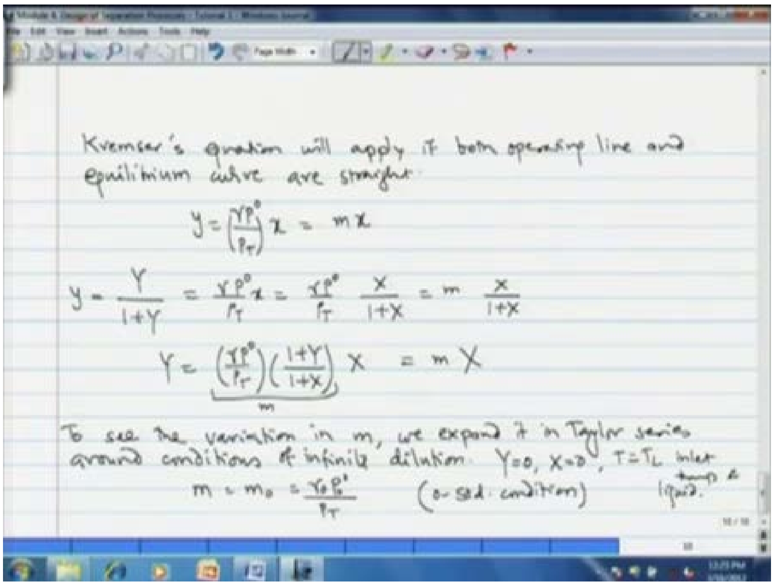

Kremser's quation will apply if both operating line and equilibrium curve are straight.

$$y = \left(\frac{\gamma P^\circ}{P_T}\right) x = m x$$

$$y = \frac{Y}{1+Y} = \frac{\gamma P^\circ}{P_T} x = \frac{\gamma P^\circ}{P_T} \frac{X}{1+X} = m \frac{X}{1+X}$$

$$Y = \underbrace{\left(\frac{\gamma P^\circ}{P_T}\right)\left(\frac{1+Y}{1+X}\right)}_{m} X = m X$$

To see the variation in m, we expand it in Taylor series around conditions of infinite dilution $Y=0$, $X=0$, $T=T_L$ inlet temp of liquid.

$$m = m_0 = \frac{\gamma_0 P_0^\circ}{P_T}$$ (0-sd. condition)

Now, Kremsers equation we will apply if both operating line and equilibrium line or equilibrium curve say are straight. We write the equilibrium relation, we rearrange the equilibrium relation that we have written as y is equal to gamma P naught by P t into x and this particular thing gamma P naught by P t in bracket is denoted by letter m, so y is equal to m x is our equilibrium relation. The relation between mole ratio and mole fraction, mole fraction denoted by small y mole, ratio denoted by capital Y is this small y is equal to capital Y divided by 1 plus capital Y and this is equal to gamma P naught by P t into x, then x also written in terms of capital X and then we rearrange this.

Now, what we have here is the modified m for mole ratio, when equilibrium relation is defined in terms of mole ratio the m the constant of the equilibrium is gamma P naught by P t into 1 plus Y divided by 1 plus X. Now, you see the variation in m, we expand it in a Taylor series around conditions of infinite dilution, as long as m remains constant the equilibrium curve will be a line and Kremsers equation will apply.

So, we shall see as what are the criteria for constancy of m and conditions of infinite dilution are Y equal to 0, capital Y equal to 0, capital X equal to 0 and T equal to T l the inlet temperature of liquid. And at these conditions, at the conditions of infinite dilution we write m is equal to m naught it is equal to gamma naught P naught naught by P T, where the subscript not denotes standard condition or reference condition.

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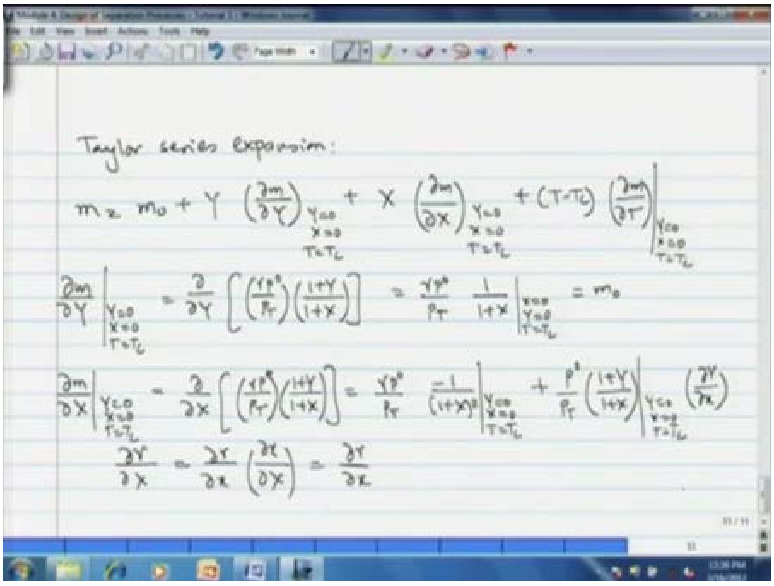

Taylor series expansion:

$$m = m_0 + Y\left(\frac{\partial m}{\partial Y}\right)_{\substack{Y=0\\X=0\\T=T_L}} + X\left(\frac{\partial m}{\partial X}\right)_{\substack{Y=0\\X=0\\T=T_L}} + (T-T_L)\left(\frac{\partial m}{\partial T}\right)\Bigg|_{\substack{Y=0\\X=0\\T=T_L}}$$

$$\frac{\partial m}{\partial Y}\Bigg|_{\substack{Y=0\\X=0\\T=T_L}} = \frac{\partial}{\partial Y}\left[\left(\frac{\gamma P^0}{P_T}\right)\left(\frac{1+Y}{1+X}\right)\right] = \frac{\gamma P^0}{P_T}\,\frac{1}{1+X}\Bigg|_{\substack{X=0\\Y=0\\T=T_L}} = m_0$$

$$\frac{\partial m}{\partial X}\Bigg|_{\substack{Y=0\\X=0\\T=T_L}} = \frac{\partial}{\partial X}\left[\left(\frac{\gamma P^0}{P_T}\right)\left(\frac{1+Y}{1+X}\right)\right] = \frac{\gamma P^0}{P_T}\,\frac{-1}{(1+X)^2}\Bigg|_{\substack{Y=0\\X=0\\T=T_L}} + \frac{P^0}{P_T}\left(\frac{1+Y}{1+X}\right)\Bigg|_{\substack{Y=0\\X=0\\T=T_L}}\left(\frac{\partial\gamma}{\partial x}\right)$$

$$\frac{\partial\gamma}{\partial X} = \frac{\partial\gamma}{\partial x}\left(\frac{\partial x}{\partial X}\right) = \frac{\partial\gamma}{\partial x}$$

So we now expand the Taylor series m is a function of three variables, capital X, capital Y and T. And now evaluate the partial differentials of that appear in this expansion. First dou m by dou Y at Y equal to 0, X equal to 0, T equal to TL is equal to dou by dou capital Y of gamma P naught by P T into 1 plus Y dou 1 plus X, this turns out to be gamma P naught by P T into 1 divided of 1 plus X at X equal to 0, Y equal to 0, T equal to TL and this is essentially m knot.

Then similarly, the second derivative dou m by dou X at Y equal to 0, X equal to 0, T equal to T L. Then here we substitute for m and then we get this. Now, here the activity coefficient is also a function of X, so we take variation of that as well. Now, dou gamma by dou capital X can be written using chain rule, as dou gamma by dou small x into dou capital X sorry, dou small x and dou capital X and then this is essentially equal to 1 the second thing, so dou gamma by dou small x.

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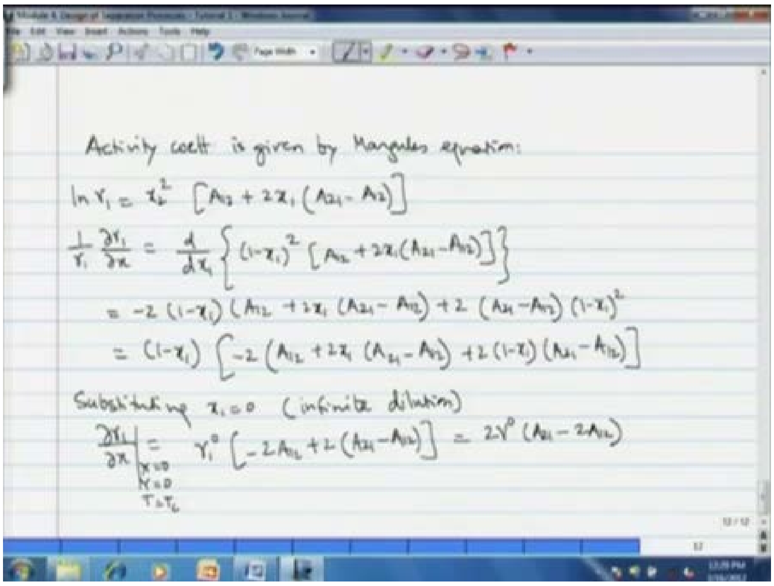

Activity coeff is given by Margules equation:

$$\ln \gamma_1 = x_2^2 \left[A_{12} + 2x_1 (A_{21} - A_{12})\right]$$

$$\frac{1}{\gamma_1}\frac{\partial \gamma_1}{\partial x} = \frac{d}{dx_1}\left\{(1-x_1)^2 \left[A_{12} + 2x_1(A_{21} - A_{12})\right]\right\}$$

$$= -2(1-x_1)(A_{12} + 2x_1(A_{21} - A_{12}) + 2(A_{21} - A_{12})(1-x_1)^2$$

$$= (1-x_1)\left[-2(A_{12} + 2x_1(A_{21} - A_{12}) + 2(1-x_1)(A_{21} - A_{12})\right]$$

Substituting $x_1 = 0$ (infinite dilution)

$$\left.\frac{\partial \gamma_1}{\partial x}\right|_{\substack{x=0\\Y=0\\T=T_L}} = \gamma_1^0\left[-2A_{12} + 2(A_{21} - A_{12})\right] = 2\gamma^0 (A_{21} - 2A_{12})$$

Now, activity coefficient is given by Margules equation and then we try to develop an expression for dou gamma by dou x using that. Margules equation is l n gamma 1 x 2 square A 1 2 plus 2 x 1 into A 2 1 minus A 1 2, then taking derivative with respect to x. Now, here we have to take with respect to x 1 because 1 denotes solute here we write x 2 square as 1 minus x 2 square. So, now simple expansion and simplification and let me take 1 minus x 1 common.

Now, we are evaluating this integral at conditions of infinite dilution, so we substitute x 1 equal to 0 and then we get dou gamma 1 by dou X is equal to at X equal to 0, Y equal to 0, T equal to T L, gamma 1 naught minus 2 A 1 2 plus 2 into A 2 1 minus A 1 2, this is equal to after simplification we can take this. And now we do not write 1 substitute here because we have only 1 solute, so the subscript 1 will drop and just write 2 gamma naught into A 2 1 minus 2 A 1 2. And now substituting for dou gamma 1 by dou X in the expression for dou m by dou X.

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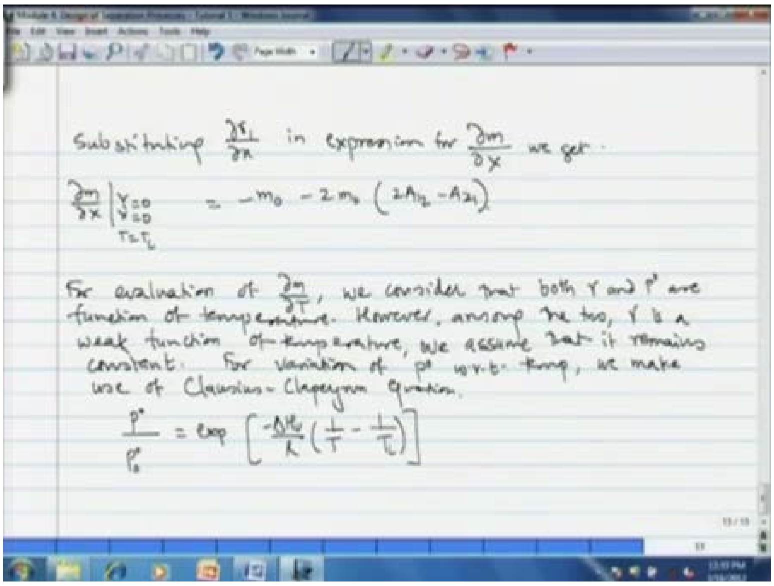

Substituting $\frac{\partial \gamma_1}{\partial x}$ in expression for $\frac{\partial m}{\partial x}$ we get.

$$\left.\frac{\partial m}{\partial x}\right|_{\substack{Y=0\\X=0\\T=T_L}} = -m_0 - 2m_0(2A_{12} - A_{21})$$

For evaluation of $\frac{\partial m}{\partial T}$, we consider that both γ and P^0 are function of temperature. However, among the two, γ is a weak function of temperature, we assume that it remains constant. For variation of P^0 w.r.t. temp, we make use of Clausius-Clapeyron equation.

$$\frac{P^0}{P^0_0} = \exp\left[\frac{-\Delta H_v}{R}\left(\frac{1}{T} - \frac{1}{T_L}\right)\right]$$

We get dou m by dou X at Y equal to 0, X equal to 0, T equal to T L is equal to minus m knot, minus 2 m naught into 2 times A 1 2 this A 2 1, so that completes the second partial derivative. Now, we have to evaluate the third partial derivative of m with respect to T, now for evaluating that partial derivative we consider that both gamma and P naught are function of temperature. However, among the two gamma is a weak function of temperature, for gas mixtures containing dilute concentrations of solute, the amount of heat that is liberated with dissolution of solute in the liquid is relatively small, so the temperature of the liquid does not vary by a large number may be by 5 or 10 degrees and for that small variation of temperature gamma can be assumed to be constant.

So, assuming or of knowing that gamma is a weak function of temperature, we assume that it remains constant. Now, for variation of P naught vapor pressure with respect to temperature, we make use of Clausius Clapeyron equation P naught divided by p naught naught that is at reference condition is equal to exponential minus delta H v by R the heat of vaporization into 1 by T minus 1 by T L. So, reference temperature we take as the inlet temperature of liquid.

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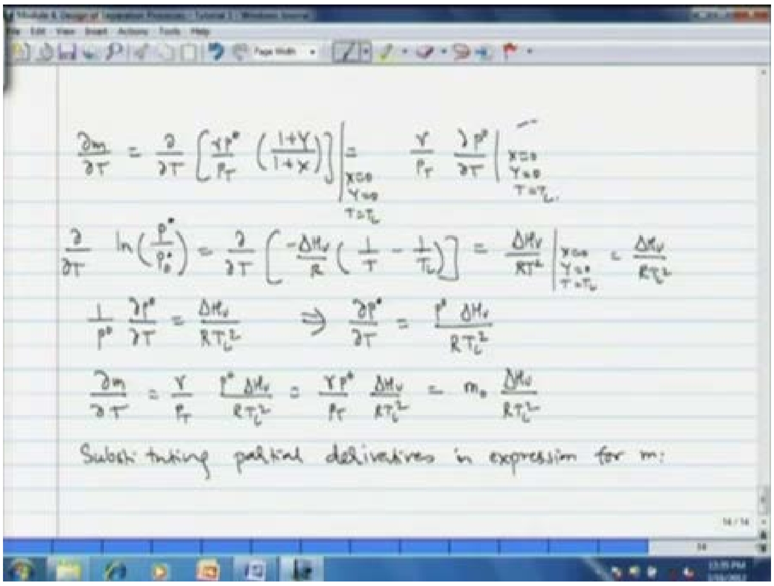

$$\frac{\partial m}{\partial T} = \frac{\partial}{\partial T}\left[\frac{\gamma P^{\circ}}{P_T}\left(\frac{1+Y}{1+X}\right)\right]\Bigg|_{\substack{X=0\\Y=0\\T=T_L}} = \frac{\gamma}{P_T}\frac{\partial P^{\circ}}{\partial T}\Bigg|_{\substack{X=0\\Y=0\\T=T_L}}$$

$$\frac{\partial}{\partial T}\ln\left(\frac{P^{\circ}}{P^{\circ}_0}\right) = \frac{\partial}{\partial T}\left[\frac{-\Delta H_v}{R}\left(\frac{1}{T}-\frac{1}{T_L}\right)\right] = \frac{\Delta H_v}{RT^2}\Bigg|_{\substack{X=0\\Y=0\\T=T_L}} = \frac{\Delta H_v}{RT_L^2}$$

$$\frac{1}{P^{\circ}}\frac{\partial P^{\circ}}{\partial T} = \frac{\Delta H_v}{RT_L^2} \Rightarrow \frac{\partial P^{\circ}}{\partial T} = \frac{P^{\circ}\Delta H_v}{RT_L^2}$$

$$\frac{\partial m}{\partial T} = \frac{\gamma}{P_T}\frac{P^{\circ}\Delta H_v}{RT_L^2} = \frac{\gamma P^{\circ}}{P_T}\frac{\Delta H_v}{RT_L^2} = m_0\frac{\Delta H_v}{RT_L^2}$$

Substituting partial derivatives in expression for m:

Now, with this we try to find out dou m by dou T, dou m by dou T is dou by dou T of gamma P naught by P T into 1 plus Y divided by 1 plus X at conditions X equal to 0, Y equal to 0, T equal to T L and then everything comes out. What we get is gamma P T gamma by P T into dou P naught by dou T at condition of X equal to 0, Y equal to 0, T equal to T L. Now, dou by dou T of l n P naught by P naught naught, we just take the derivative of the Clausius Clapeyron equation is dou by dou T of minus delta H v by R into 1 by T minus 1 by T L and then this is delta H v by R T square at X equal to 0, Y equal to 0, T equal to T L or delta H v by R T L square.

And the right hand side is nothing but 1 by P naught into dou P naught by dou T and this implies that dou P naught by dou T is equal to P naught into delta H v by R T L square. And that we put here so dou m by dou T is gamma by P T into P naught into delta H v by R T L square or gamma P naught by P T into delta H v by R T L square or simply m naught into delta H v by R T L square. Now, we substitute all partial derivatives, we now have a relationship of all variation, variation of m with respect to capital X, capital Y and capital T are the temperature, so substituting partial derivatives in expression for m.

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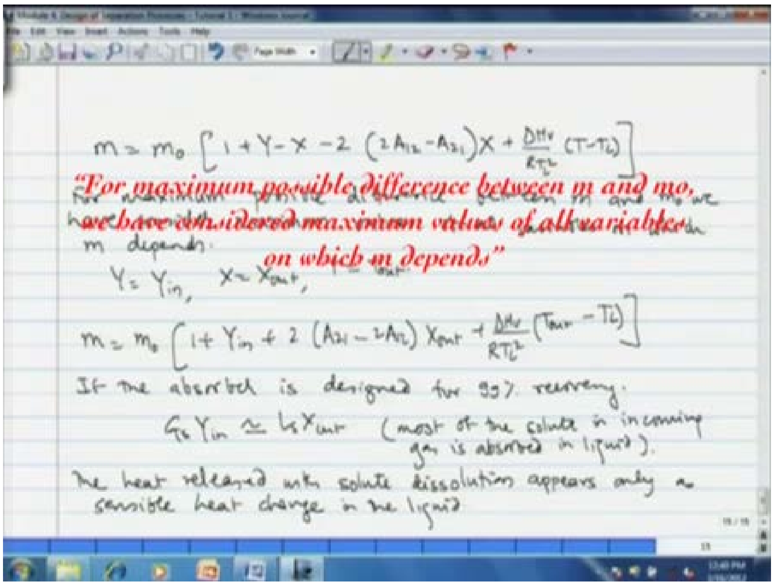

$$m = m_0 \left[1 + Y - X - 2\left(2A_{12} - A_{21}\right)X + \frac{\Delta H_V}{RT_L^2}(T - T_L)\right]$$

"For maximum possible difference between m and mo, we have considered maximum values of all variables on which m depends"

$Y = Y_{in}$, $X = X_{out}$,

$$m = m_0 \left[1 + Y_{in} + 2\left(A_{21} - 2A_{12}\right)X_{out} + \frac{\Delta H_V}{RT_L^2}(T_{out} - T_L)\right]$$

If the absorber is designed for 99% recovery.

$G_s Y_{in} \simeq L_s X_{out}$ (most of the solute in incoming gas is absorbed in liquid).

The heat released with solute dissolution appears only as sensible heat change in the liquid

We get m is equal to m naught into 1 plus Y minus X minus 2 into 2 A 1 2 minus A 2 1 here we take the negative sign outside and adjusted inside the bracket into X plus delta H v by R T L square into T minus T L this bracket complete. Now, for maximum possible difference between m the slope of equilibrium curve at any given condition and m naught which is the reference condition. We have to consider maximum values of all variable on which m depends, so we take Y is equal to Y in, X is equal to X out and T is equal to T out, we get the maximum values. The mole ratio of solute in gas is maximum by at inlet condition, the mole ratio of solute in liquid maximum at outgoing outlet condition and temperature of the liquid is maximum, is mostly different at exit conditions.

So, then m is equal to m naught into 1 plus Y in we substitute for X and Y these value minus, oh sorry now we take this as common 1 plus Y in plus 2 into A 2 1 minus 2 A 1 2 into X out plus delta H v by R T L square into T out minus T L. Now, if the absorber is designed for 99 percent recovery, then G s into Y in is same as L s into X out, most of the solute in incoming gas is absorbed in liquid and then heat that is released during this absorption, heat released with solute dissolution appears only as the sensible heat change in the liquid.

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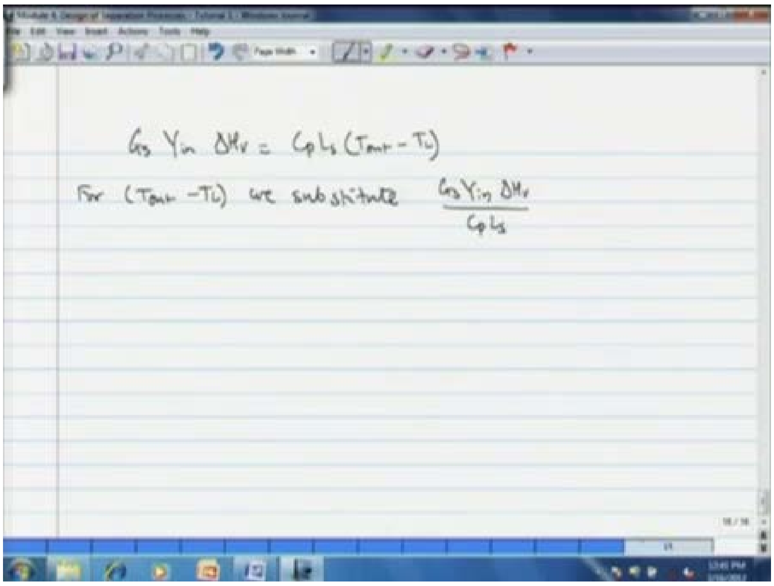

$$G_s Y_{in} \Delta H_v = C_p L_s (T_{out} - T_L)$$

For $(T_{out} - T_L)$ we substitute $\frac{G_s Y_{in} \Delta H_v}{C_p L_s}$

G s into Y in into delta H v is equal to C p into L s into T out minus T L and then we substitute for T out, for T out minus TL we substitute G s into Y in into delta H v by C p into L s here for this and for x out we substitute this G s into Y in by L s.

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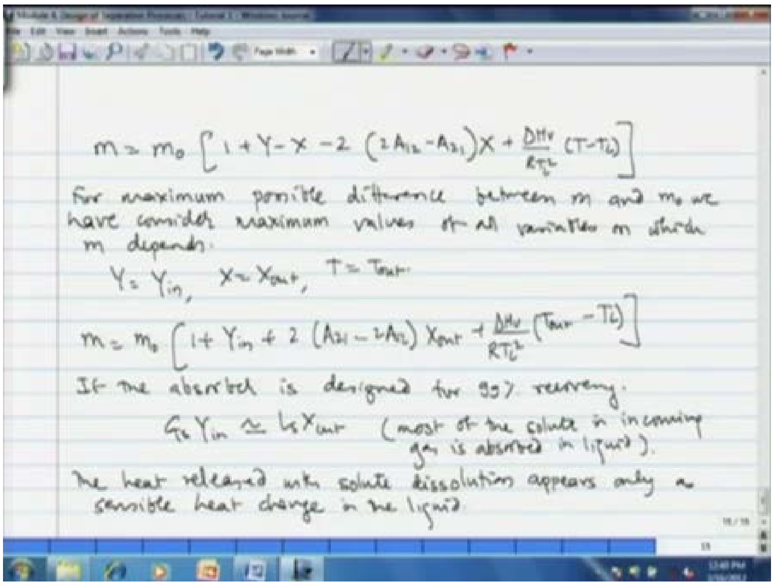

$$m = m_0 \left[1 + Y - X - 2(2A_{12} - A_{21})X + \frac{\Delta H_v}{RT_L^2}(T - T_L) \right]$$

For maximum possible difference between m and m_0 we have consider maximum values of all variables on which m depends.

$Y = Y_{in}$, $X = X_{out}$, $T = T_{out}$

$$m = m_0 \left[1 + Y_{in} + 2(A_{21} - 2A_{12}) X_{out} + \frac{\Delta H_v}{RT_L^2}(T_{out} - T_L) \right]$$

If the absorber is designed for 99% recovery.

$G_s Y_{in} \simeq L_s X_{out}$ (most of the solute in incoming gas is absorbed in liquid).

The heat released with solute dissolution appears only as sensible heat change in the liquid.

Heat that is released during this absorption, heat released with solute dissolution appears only as the sensible heat change in the liquid. So, G s into Y in into delta H v is equal to C p into L s into T out minus T L and then we substitute for T out, for T out minus T L we substitute G s into Y in into delta H v by C p into L s here for this and for X out we substitute this G s into Y in by L s.

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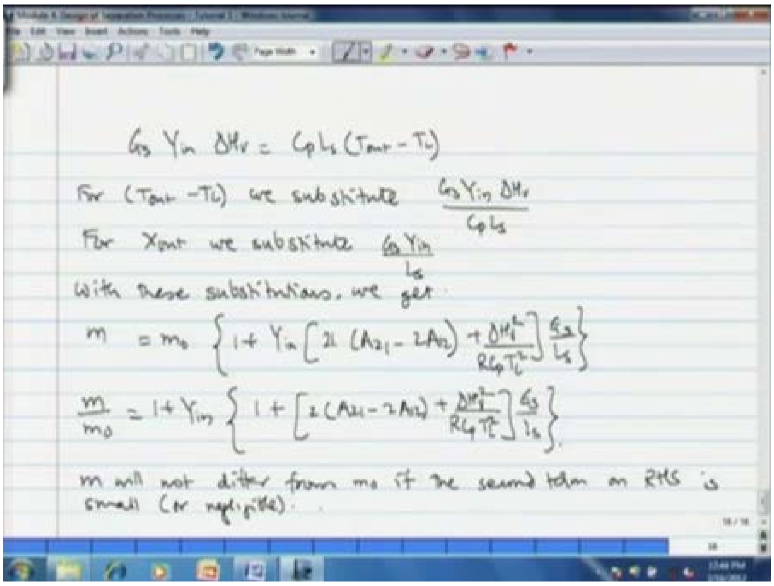

$$G_s Y_{in} \Delta H_v = C_p L_s (T_{out} - T_L)$$

For $(T_{out} - T_L)$ we substitute $\frac{G_s Y_{in} \Delta H_v}{C_p L_s}$

For X_{out} we substitute $\frac{G_s Y_{in}}{L_s}$

With these substitutions, we get

$$m = m_0 \left\{ 1 + Y_{in} \left[2(A_{21} - 2A_{12}) + \frac{\Delta H_v^2}{R C_p T_L^2} \right] \frac{G_s}{L_s} \right\}$$

$$\frac{m}{m_0} = 1 + Y_{in} \left\{ 1 + \left[2(A_{21} - 2A_{12}) + \frac{\Delta H_v^2}{R C_p T_L^2} \right] \frac{G_s}{L_s} \right\}.$$

m will not differ from m_0 if the second term on RHS is small (or negligible).

And with these substitutions we get m is equal to m naught into 1 plus Y in into the bracket 2 into A 2 1 minus 2 A 1 2 plus delta H v square divided by R C p T L square bracket complete into G s by L s or we take m by m naught equal to 1 plus Y in 2 into A 2 1 minus 2 A 2 1 plus delta H v square by R C p T L square or we rewrite this and is equal to m by m naught is equal to 1 plus Y in into 1 plus 2 into A 2 1 minus 2 A 1 2 plus delta H v square by R C p T l square bracket complete into G s by L s. Now, m will not differ from m knot, if the second term on right hand side is small in number a numerical value or negligible.

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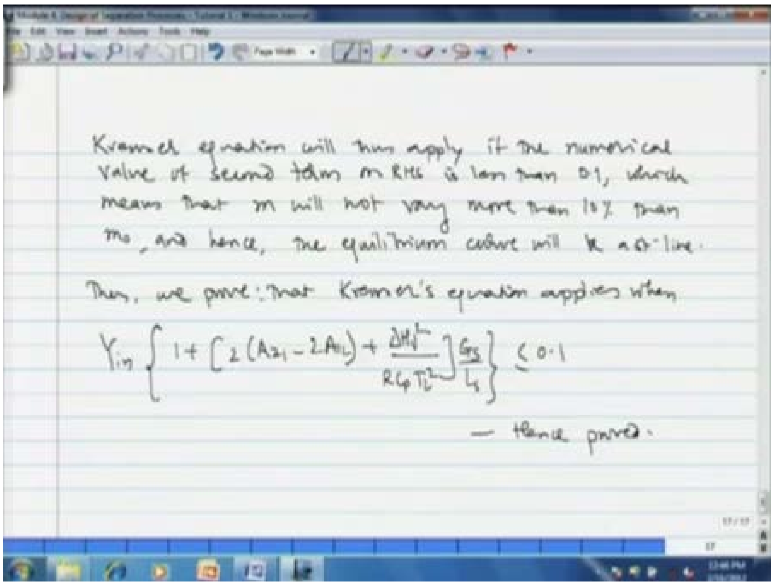

Kremser equation will thus apply if the numerical value of second term on RHS is less than 0.1, which means that m will not vary more than 10% than m_0, and hence, the equilibrium curve will be a st. line.

Thus, we prove that Kremser's equation applies when

$$Y_{in}\left\{1+\left[2(A_{21}-2A_{12})+\frac{\Delta H_v^2}{R C_p T_L^2}\right]\frac{G_s}{L_s}\right\} \le 0.1$$

— hence proved.

Therefore we can say that Kremsers equation will apply, if the numerical value of the second term on the right hand side is less than or equal to 0.1 that means m will not differ by more than 10 percent from m naught and therefore the equilibrium curve will be essentially a straight line, so that point we note. It means Kremser equation will thus apply if the numerical value of second term on RHS is less than 0.1, which means that m will not vary more than 10 percent than m naught and hence the equilibrium curve will be a straight line.

And thus we prove that Kremsers equation will apply, we prove that Kremsers equation applies when Y in into 1 plus 2 into A 2 1 minus 2 A 1 2 plus delta H v square by R C p T L square into G s by L s is less than or equal to 0.1 and that gives the solution. So, today we have seen two problems related to design of absorption column in the next tutorial we shall see problems related to distillation.