Mass Transfer Operations I

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Module - 1
Diffusion Mass Transfer
Lecture - 5
Steady state molecular diffusion in fluids
Part 2

Welcome to the fifth lecture on diffusion mass transfer, which is module 1. This lecture, we will cover the steady state molecular diffusion through variable area.

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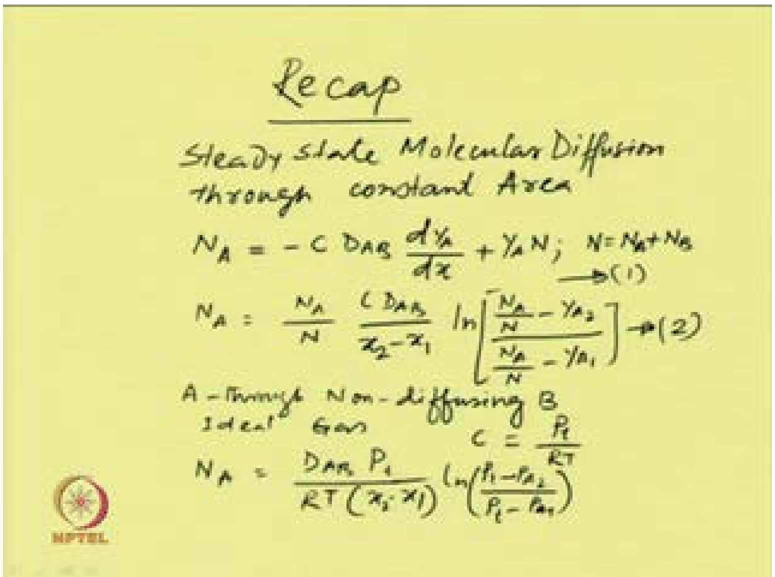

Before going to this lecture, let us have a quick recap on our previous lecture. In the previous lecture, we have considered steady state molecular diffusion through constant area, constant area. Here we have seen from the basic equations of flux of a component A is equal to C D AB d Y A dx in the x direction plus Y A N, where N is equal to for two component system N A plus N B. So this is equation number 1, and then we have seen the flux of A will be equal to N A by N C D AB. Between the two different points x 1 and x 2 will be this distance ln N A by N minus Y A 2 divided by N A by N minus Y A 1.

So this is equation number 2, and if we consider the case of diffusion of a particular component A through non-diffusing B diffusing B component; that means the B is stagnant in that case, and also if we consider ideal gas mixture; ideal gas in that case, this C we can write P t total pressure by RT.

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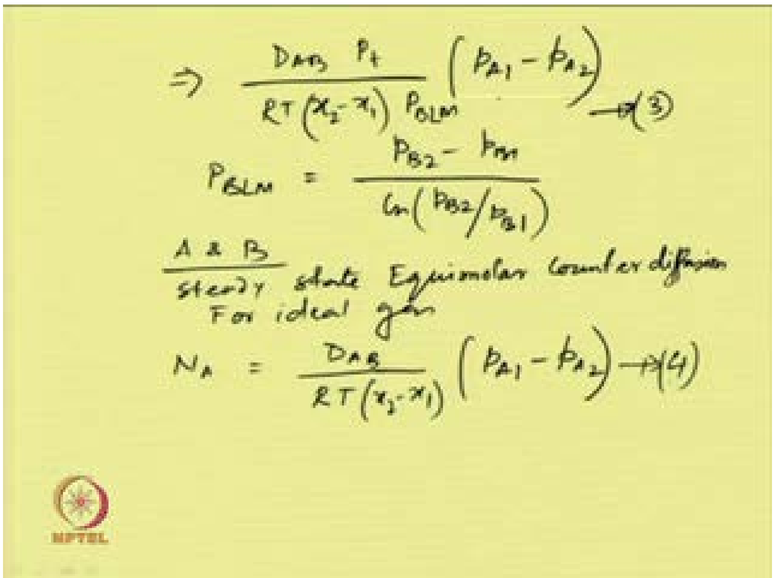

So that, we can write the flux equation N A will be D AB P t by RT (x 2 minus x 1) ln P t minus p A 2 by P t minus p A 1, or which is we can write D AB P t by RT (x 2 minus x 1) into P BLM (p A 1 minus p A 2), and this P BLM is equal to p B 2 minus p B 1 pressure difference between the two points divided by ln (p B 2 by p B 1); this is at the partial pressure difference.

So, this is the flux equations of a component A through non-diffusing B, and also we have seen the steady state equimolar counter diffusion. If between component A and B, the diffusion is steady state equimolar counter diffusion. So in this case for ideal gas, we have seen N A the flux is equal to D AB by RT (x 2 minus x 1) into the partial pressure difference (p A 1 plus p A 2). So these are the quick review of our previous class, where we have considered the diffusion is occurring through the constant area of the system.

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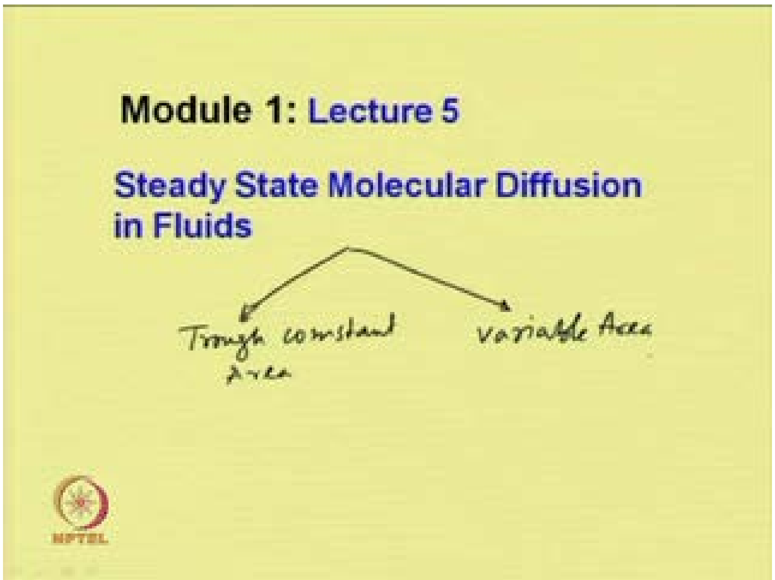

So today, we will also discuss the steady state molecular diffusion in fluids. As we said these has two separate classifications; one is through constant area and this is through variable area.

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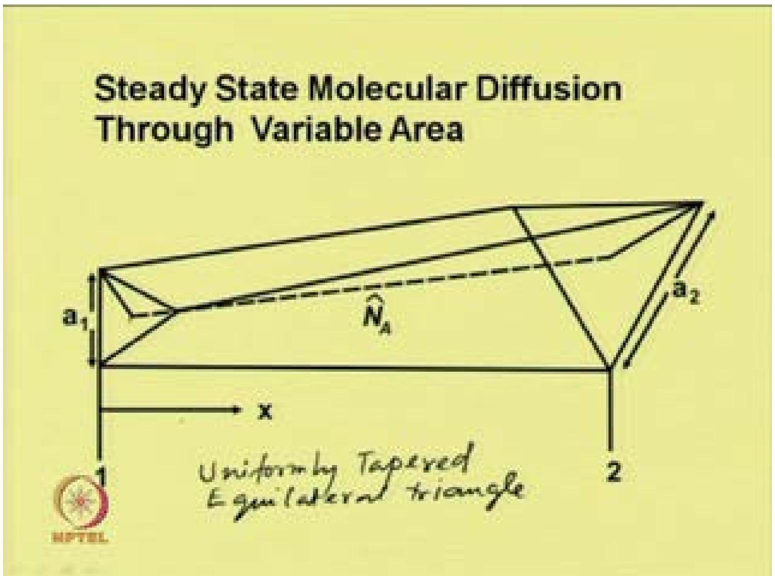

So now, consider steady state molecular diffusion of component A through equilateral triangle, which is uniformly tapered uniformly tapered equilateral triangle, where the diffusion this is point one. From this plane it is diffusing to point a 2, where the N A cap is the diffusion rate of component A, and the cross-sectional area is varying along the length of the conduit. If we consider this is point 1 and this is point 2, the length of the side of the triangle is a 1 at point 1 and a 2 at point 2. So, the relations between this side of the triangle, which is gradually increasing from the left hand to the right hand we can write.

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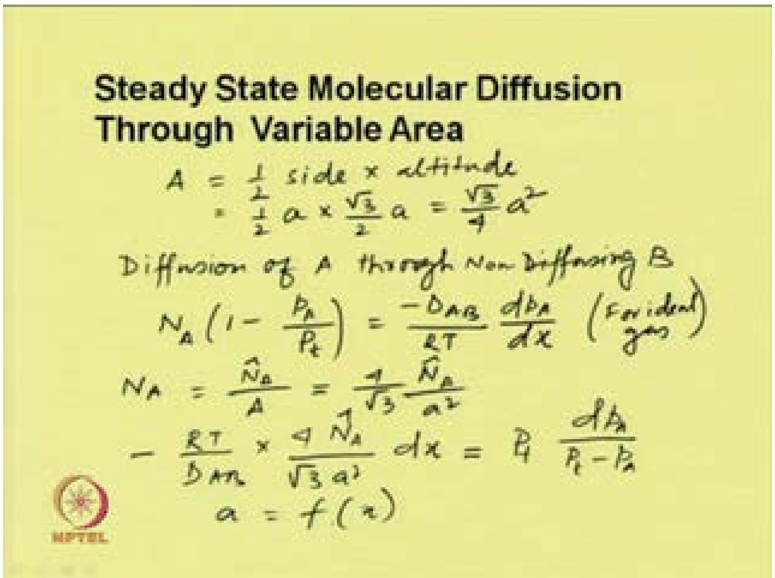

So, area will be half into side into altitude. So, which is equal to its side is a. It will be a into height will be for equilateral triangle it will be root 3 by 2 a. So, it will be root 3 by 4 a square. Now if we return back to the Fick's law diffusion of A through non-diffusing B. So in this case, the governing equations of the flux N A into (1 minus partial pressure of A by P t) total pressure is equal to minus D AB by RT d P A by dx for ideal gas. Now if we consider as N A is the molar flux.

So, N A is equal to N A cap by area, which is equal to 4 by if we substitute area over here, 4 by root 3 N A cap by a square, and if we substitute this this one over here N A. So, it will be minus RT by D AB into 4 N A cap divided by root 3 a square dx, will be equal to P t total pressure into d p A partial pressure of a divided by P t minus p A. So before we integrate this equations, we have to impose the limit where a is function of x. Since a is function of (a) x, the size of the triangle will continuously change.

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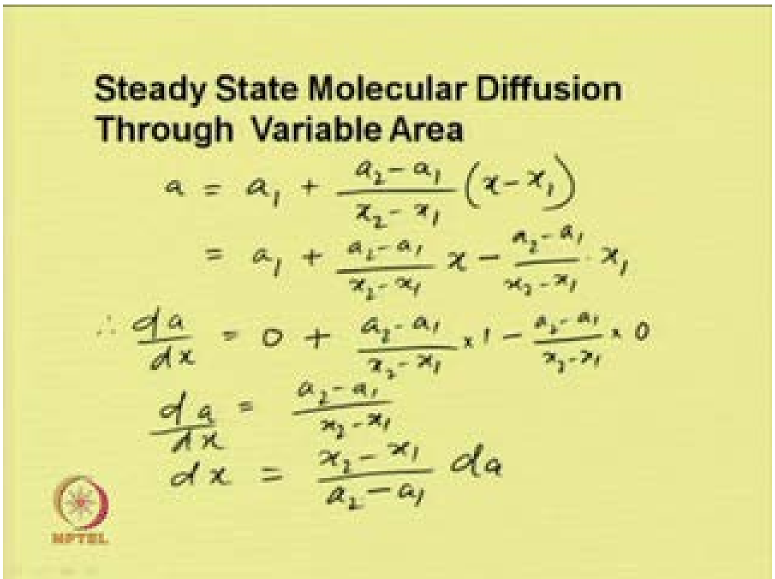

So, we can have the relation a will be a 1 at 0.1 plus a 2 minus a 1 by x 2 minus x 1 into (x minus x 1). So, this is the relations how a is changing with x for the equilateral triangular shape. So, in this case we can write this will be a 1 plus a 2 minus a 1 by x 2 minus x 1 into x minus a 2 minus a 1 by x 2 minus x 1 into x 1.

Now if we differentiate, this will be da by dx. So, since this is constant; so this will be 0 plus a 2 minus a 1 into by x 2 minus x 1 into 1 minus a 2 minus a 1 by x 2 minus x 1 into 0. So, this will be a 2 minus a 1 by x 2 minus x 1. So, we can write d a by dx is this. So, dx we can write x 2 minus1 divided by a 2 minus a 1 da. So, if we substitute in our previous relation, say this is equation 1.

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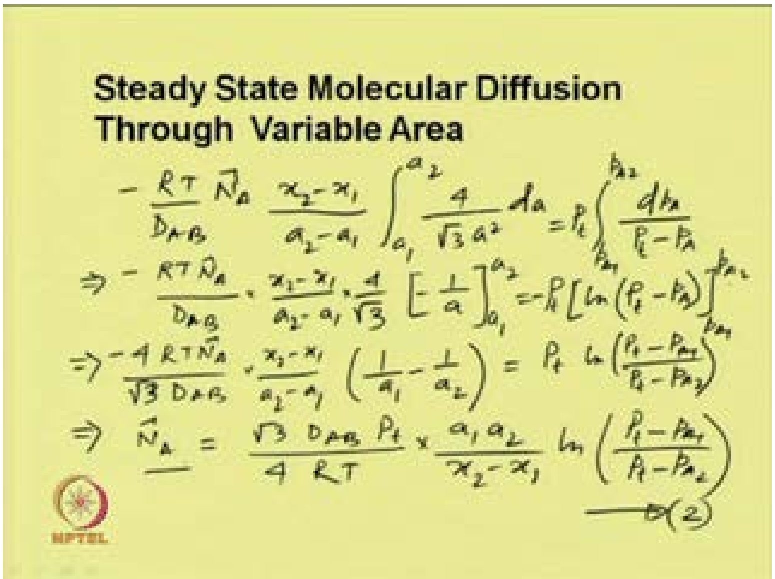

If we substitute here, the relations between a and x in this limit we can have RT by D AB N A cap x 2 minus x 1 by a 2 minus a 1 integral a 1 to a 2 4 by root 3 a square da, which is equal to P t integral partial pressure of a at 0.1 partial pressure of a at 0.2 d p A by P t minus P A. Now if we integrate, we can get minus RT N A cap by D AB into x 2 minus x 1 by a 2 minus a 1 into 4 by root 3 and this integration of these will be minus 1 by a with the limit a 1 to a 2 will be equal to minus P t into l n (P t minus p A) into and limit is p A 1 to p A 2.

So, if we simplify this one then we will have 4 minus 4 RT N A cap by root 3 D AB into x 2 minus x 1 by a 2 minus a 1 into (1 by a 1 minus 1 by a 2). So, will be equal to P t ln (P t minus p A 1 by P t minus p A 2); N A cap is the diffusion rate will be equal to root of bar 3 D AB into P t by 4 RT into, a 2 minus a 1 will cancel out; so it will be a 1 a 2 by x 2 minus x 1 ln (P t minus p A 1 by P t minus p A 2). So, this is the molar flow rate equations of component A through the variable area of equilateral triangular cross section.

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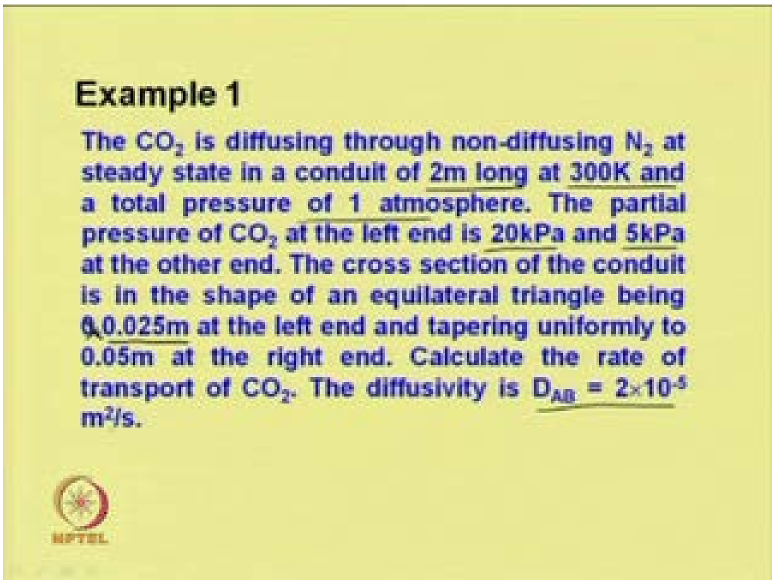

If we consider a simple example like we consider before the CO 2 is diffusing through non-diffusing nitrogen at steady state in conduit of 2 meter long and at 300 Kelvin and 1 atmosphere pressure and the partial pressure at the left end is 20 kilo Pascal and at the right end is 5 kilo Pascal. The cross section of the conduit is equilateral triangle of side 0.025 meter at the left end and tapering uniformly to 0.05 meter at the right end. So, we have to calculate the rate of transport of CO 2, and the diffusivity between A and B; that is CO 2 into through nitrogen is given 2 into 10 to the power minus 5 meter square per second.

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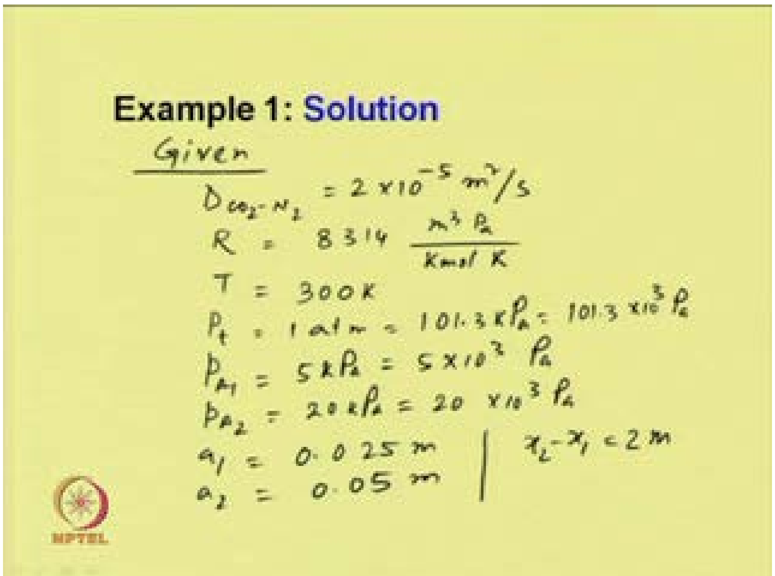

So now, let us consider the values which are given. The diffusion coefficient of CO 2 into nitrogen is given is 2 into 10 to the power minus 5 meter square per second. R is known to us; 8314 meter cube Pascal by Kmol Kelvin. T the temperate of the system is 300 K. Pressure P t is given is 1 atmosphere, which is equal to 101.3 kilo Pascal is equal 101.3 into 10 to the power 3 Pascal. Partial pressure p A 1 is given is 5 kilo Pascal, which is 5 into 10 to the power 3 Pascal. p A 2 is also given 20 kilo Pascal, which is 20 into 10 to the power 3 Pascal, and the a 1 is 0.025 meter; a 2 is 0.05 meter. The side of the triangle and the distance x 2 minus x 1 is given as 2 meter.

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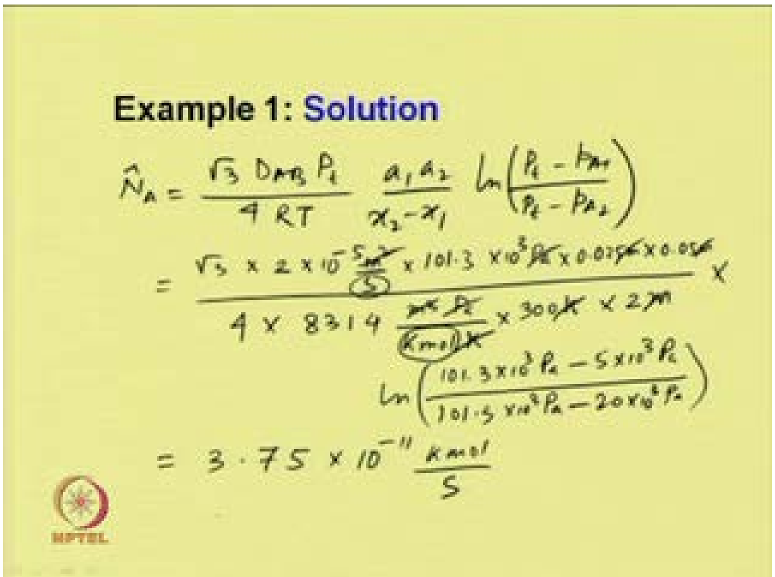

So, now if we substitute the values which are given N A cap will be root 3 D AB P t by four RT a 1 a 2 by x 2 minus x 1 ln (P t minus p A 1 by P t minus p A 2). So, if we substitute the values which is root 3 into 2 into 10 to the power minus 5 meter square per second into 101.3 into 10 to the power 3 Pascal total pressure and then into the a 1 a 2 which is 0.025 meter into 0.05 meter; this divided by 4 into 8314 meter cube Pascal divided by Kmol Kelvin temperature 300 Kelvin into 2 meter multiplied by ln (101.3 into 10 to the power 3 Pascal minus 5 into 10 to the power 3 Pascal divided by 101.3 10 to the power 3 Pascal total pressure minus 20 into 10 to the power 3 Pascal).

So, if we solve this is equal to 3.75 into 10 to the power minus 11 Kmol per second. So, if we look into the unit balance, the pressure term will cancelled; this this will cancelled out and Kelvin will cancel. So, you will have Kilo mol per Kilo mol per second. So, this is the molar fluoride or of component A through non-diffusing B.

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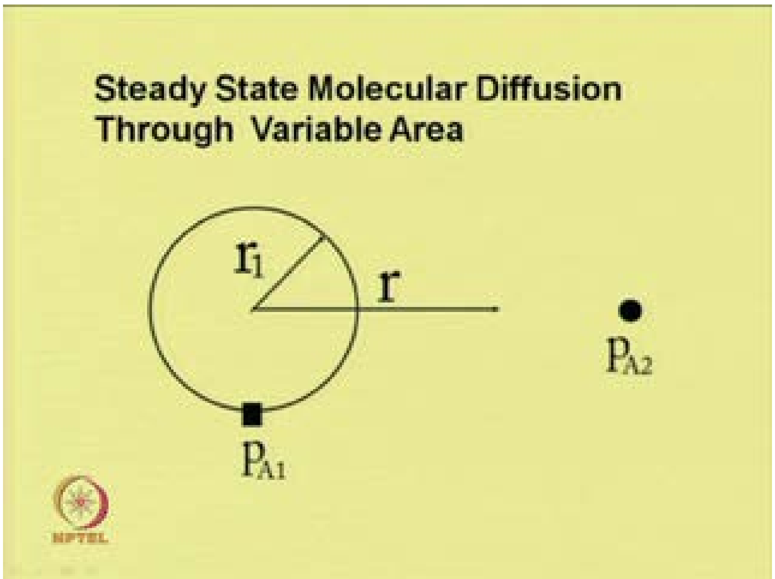

Let us consider another example. The evaporation of a drop of liquids occurs and area continuously gets changed; or if we have a naphthalene ball and diameter of the ball is reduced and suddenly the area and its flux gets changed. So we will consider this, where at 0.1 the partial pressure is p A 1; at 0.2 partial pressure of component A is P 2, and the radius is changing. So the initial radius is r A 1; this is on the surface of the ball.

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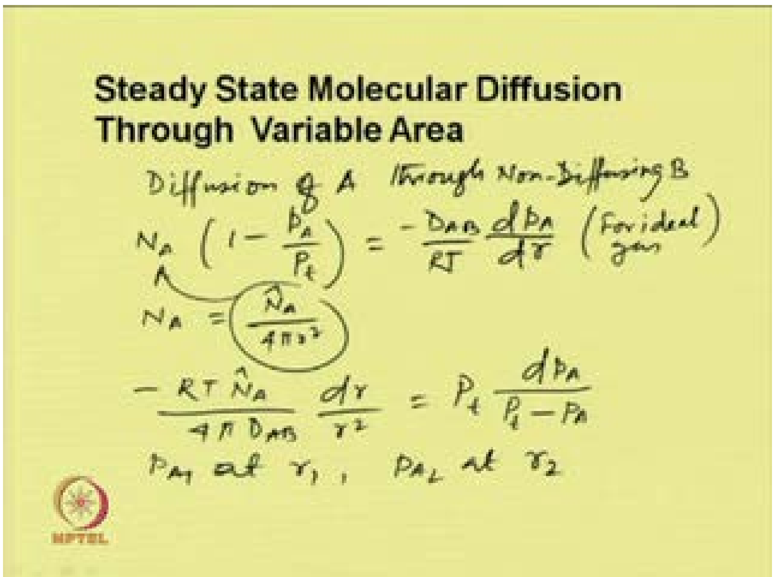

So if we apply the Fick's law of diffusion, consider the case diffusion of A through non-diffusing to B. So in this case, we know the flux is related with the partial pressure p A by P t the total pressure is equal to minus D AB RT d P A d r for ideal gas. ideal gas N A we can write the flux N A cap by 4 pi r square, which is the area. So now, we can have the relation between this two we can substitute this N A over here.

So, we will essentially have minus RT N A cap by 4 pi D AB dr by d r square is equal to P t d P A by P t minus p A. So now, if we include the limit partial pressure of A 1 at (r a) r 1 and partial pressure of P A 2 at r 2.

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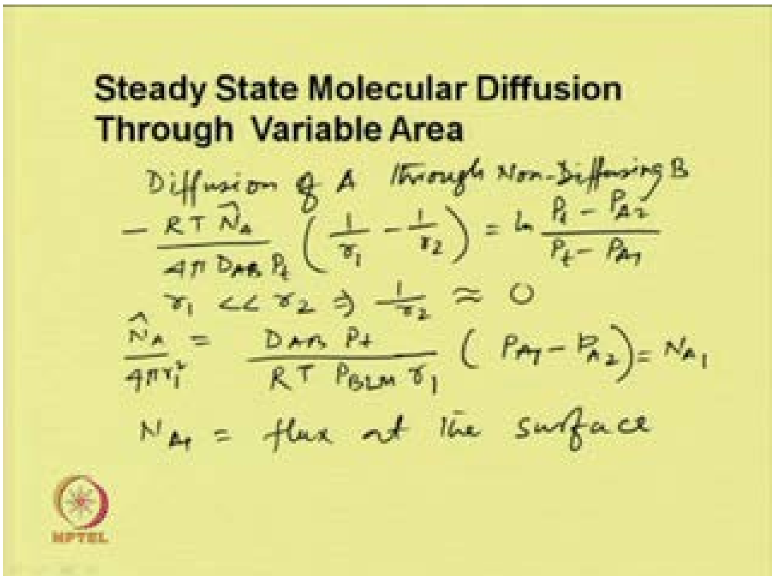

If we include this limit and integrate the equation this will be minus RT N A cap by 4 pi D AB into P t (1 by r 1 minus 1 by r 2) is equal to ln P t minus p A 2 by pt minus p A 1. So, if we consider r 1 is very, very less than r 2 then 1 by r 2 approximately equal to 0. So, if we substitute this then N A cap is equal to D AB P t by RT p BLM into r 1 (p A 1 minus p A 2) is divided by 4 pi r 1 square. So, this is nothing but N A 1; N A 1 is the flux at the surface.

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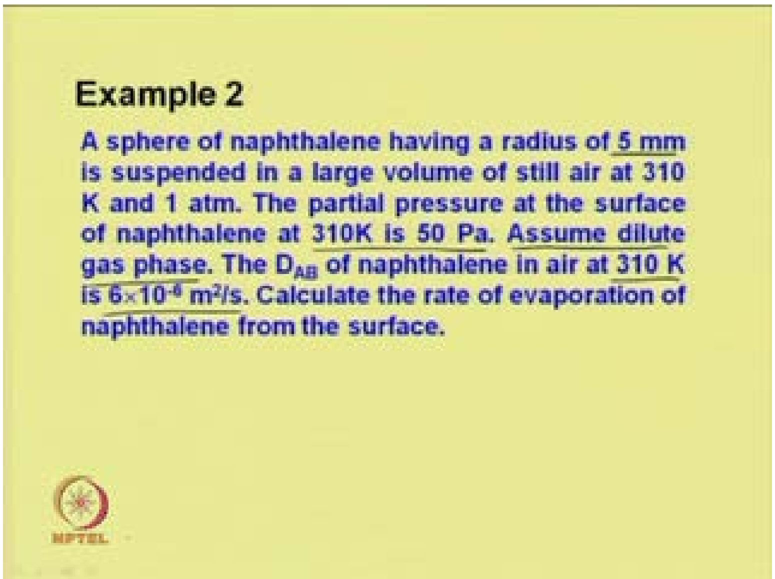

Now let us have the example. A sphere of naphthalene ball having a radius of 5 millimeter is suspended in a large volume of still air at 310 Kelvin and 1 atmosphere pressure. The partial pressure at the surface of naphthalene at 300 K is 50 Pascal. Assume dilute gas phase. So, and the D AB of naphthalene in air at 300 Kelvin is given 6 into 10 to the power minus 6 meter square per second. Calculate the rate of evaporation of naphthalene from the surface.

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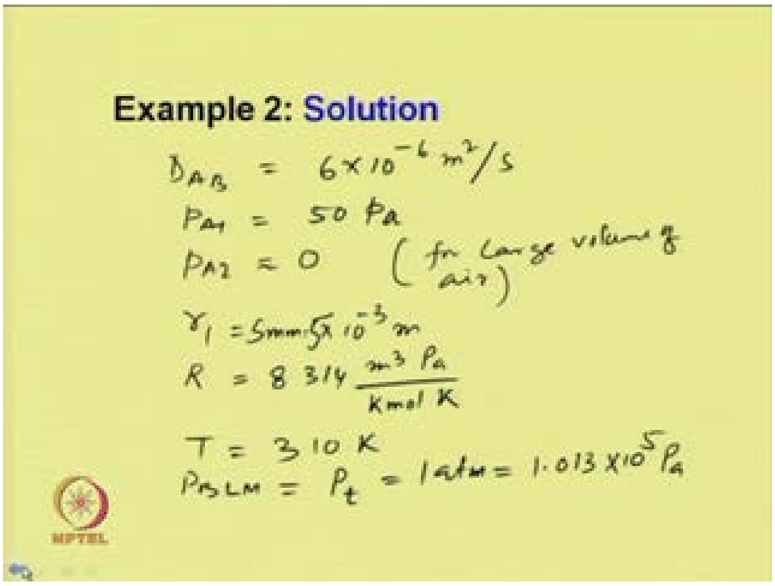

Now the data which are given is D AB is 6 into 10 to the power of minus 6 meter square per second. Partial pressure p A 1 is given is 50 Pascal. p A 2 we can, since this is large volume of air we can consider approximately equal to 0 (for large volume of air), and r 1 is given is 5 millimeter, which is 5 into 10 to the power minus 3 meter; equal to 5 into this, and R is known to us. R is 8314 meter cube Pascal by Kmol Kelvin. T is given 310 Kelvin, and P BLM since it is very dilute solution we can considered as P t total pressure, which is equal to 1 atmosphere is equal to 1.013 into 10 to the power 5 Pascal.

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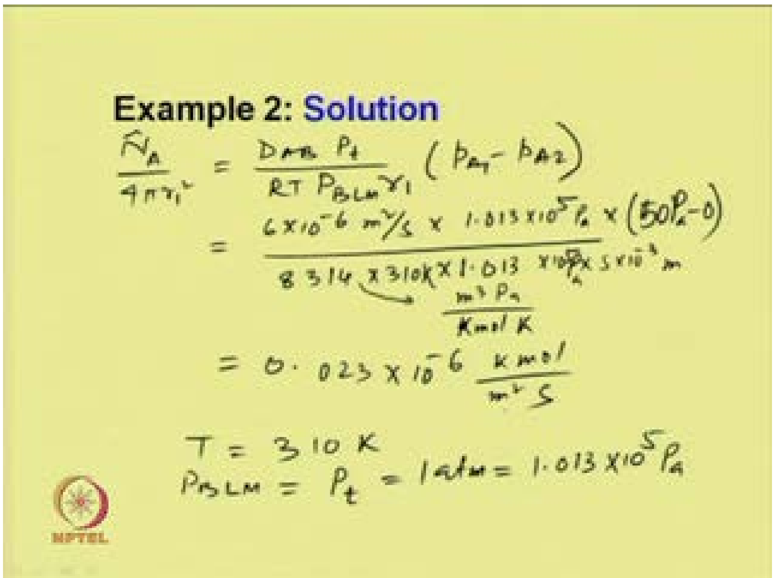

So now, if we substitute these values in our governing equations flux N A cap by 4 pi r 1 square will be equal to D AB P t by RT P BLM into r 1 into the partial pressure difference (p A 1 minus p A 2). So p A 1 is 50, which is given and p A 2 is 0 we are assuming for large volume of air.

So, if we substitute that values, which is 6 into 10 to the power minus 6 meter square per second; P t is 1.013 into 10 to the power 5 Pascal into (50 Pascal minus 0) divided by 8314 into 310 into 1013 1.013 into 10 to the power 5 into 5 into 10 to the power minus 3; this is meter, this is Pascal, and this is Kelvin, and this is meter cube Pascal Kmol Kelvin. So now, after substitution of this we will have this is 2.023 0.023 into 10 to the power minus 6 Kmol per meter square second.

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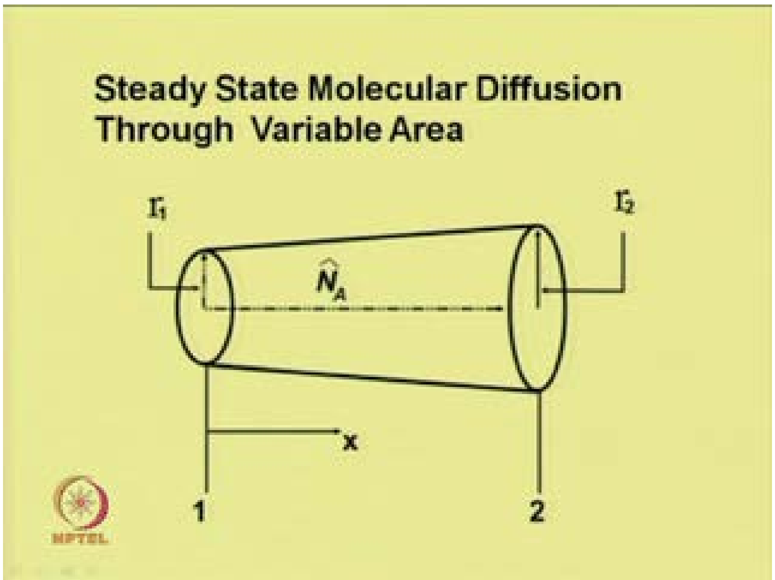

Now, we will consider steady state molecular diffusion through another geometry which is uniformly tapered, cylindrical in nature, and that point 1 the radius of the cylinder is r 1 and at point 2 the radius is r 2.

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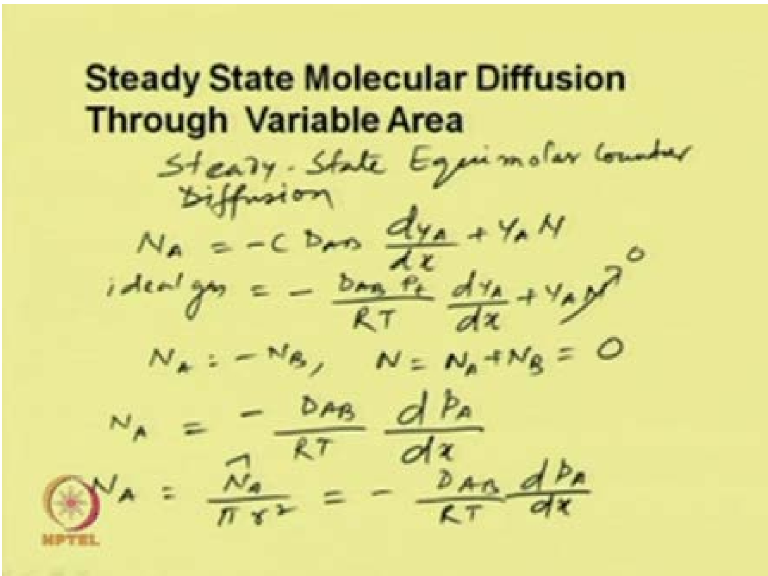

In this case, we will consider steady state equimolar counter diffusion. steady state equimolar counter diffusion As we know, the flux is minus C D AB d Y A dx plus Y A N. So for equimolar counter diffusion and if we consider ideal gas, then this will be minus D AB P t by RT d Y A by dx plus Y A N.

Now for two component system and equimolar counter diffusion N A will be minus N B and N is N A plus N B, and which is equal to 0. So then this term will be 0, and we have the flux N A will be minus D AB by RT to d P A dx; and also N A we can write N A cap by pi r square, and this is equal to minus D AB by RT d P A by dx.

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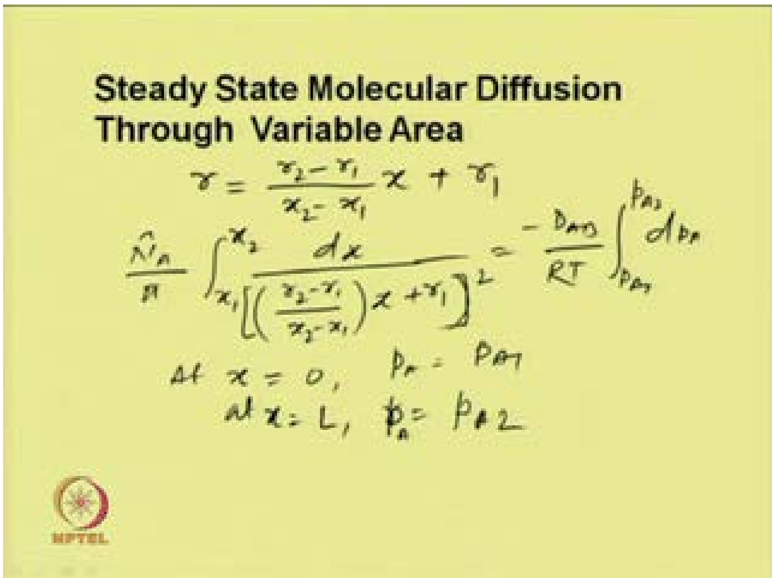

If we use the geometry, then we can have a relations between r is equal to r 2 minus r 1 by x 2 minus x 1 x plus r 1. So, this is the change of radius along the distance; along the length of the cylinder. We can replace this r in these equations and we can then differentiate. So, it will be N A cap by pi integral x 1 to x 2 dx by (r 2 minus r 1) by (x 2 minus x 1) into x (by) plus r 1 square is equal to minus D AB by RT integral p A 1 p A 2 d P A. If we integrate with the limit at x is equal to 0, p A is p A 1 at x is equal to L, p A is p A 2.

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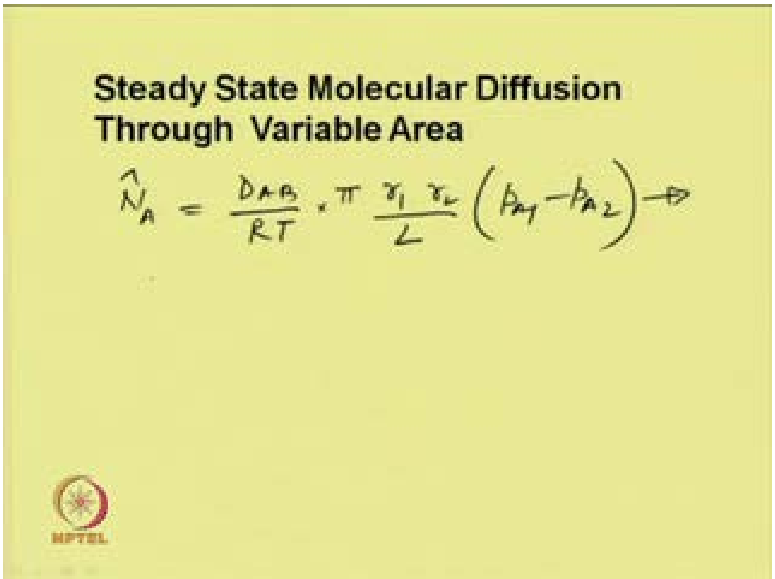

So, if we integrate this with the limits it will be N A cap will be D AB by RT into pi r 1 r 2 by L into (p A 1 minus p A 2). Like earlier example for equilateral triangle we can integrate in similar way, and we will have these diffusion rate equations of component A.

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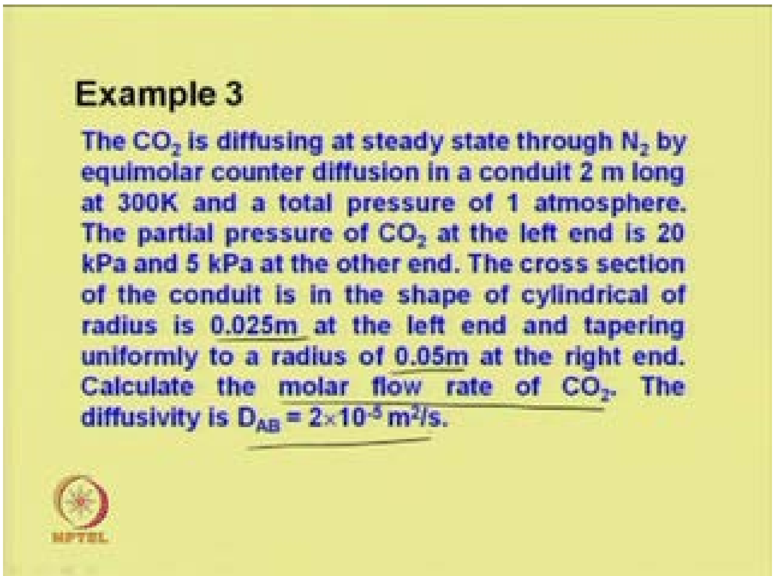

Now let us consider these examples where carbon dioxide is diffusing steady state through nitrogen by equimolar counter diffusion in the similar conduit of 2 meter length at 300 Kelvin and at 1 atmosphere pressure. The partial pressure at the left end is 20 kilo Pascal and 5 kilo Pascal at other end. The cross section of the conduit is in the shape of cylindrical of radius 0.025 meter and at the other end the radius is 0.05 meter. We have to calculate the molar flow rate of CO 2, and the diffusion coefficient is given.

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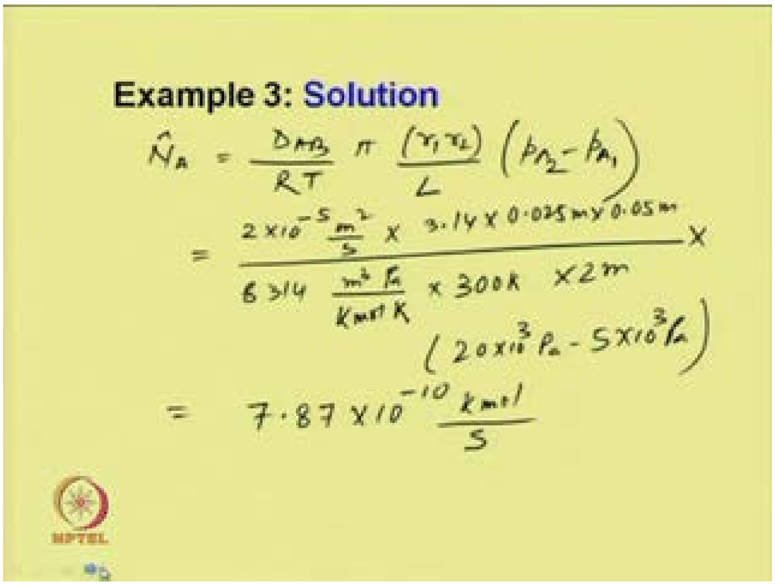

If we substitute in this equations N A cap is D AB by RT pi (r 1 r 2) by L (p A 2 minus p A 1). So, the value is given 2 into 10 to the power minus 5 meter square per second into 3.14 into 0.025 meter into 0.05 meter divided by 8314 meter cube Pascal by Kmol Kelvin into 300 K into 2 meter multiplied by (20 into 10 to the power 3 Pascal minus 5 into 10 to the power 3 Pascal. So, this will be equal to 7.87 into 10 to the power minus 10 Kmol per second.

So, this is end of our lecture 5, and in the next class we will consider the diffusion coefficient measurements. We will first consider the measurement of the diffusion coefficient in the gas phase, and then for the liquid phase, and then diffusion in solid phase.

Thank you.