Mass Transfer Operations I Prof. Bishnupada Mandal Department of Chemical Engineering Indian Institute of Technology, Guwahati

Module - 1 Diffusion Mass Transfer Lecture - 4 Steady State Molecular Diffusion in Fluids Part 1

Welcome to the fourth lecture of module one. This lecture will be on steady state molecular diffusion in fluids under stagnant and laminar flow conditions.

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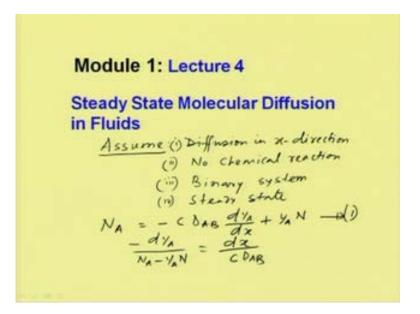
JAN = (-) DAR dea $\begin{aligned} \mathcal{D}_{AB} &= \mathcal{D}_{BA} \\ \frac{\partial \zeta_{A}}{\partial t} &= \partial_{AB} \left(\frac{\partial \zeta_{A}}{\partial x^{2}} + \frac{\partial^{2} \zeta_{A}}{\partial y^{2}} + \frac{\partial^{2} \zeta_{A}}{\partial z^{2}} \right) \end{aligned}$ Applicable : Soleds - Less frequently or limited situations

So, before proceeding to this lecture we will have some recap on the previous lecture. In the previous lecture, we have discussed fixed first law of diffusion, where for onedimensional diffusion in a particular direction x, we can write the molar flux J A, x is equal to minus D AB dC A dx; dC A is the concentration gradient and dx is the distance and D AB is the diffusion coefficient. The negative sign indicates the drop in concentration towards the directions of diffusion. And then we have seen, for an ideal gas mixture the diffusion coefficient between component A and B, we call the mutual diffusion coefficients are equal and we can write D AB is equal to D BA. And then we have discussed the unsteady state diffusion, where rate of diffusion changes with respect to time. And we have derived the governing equations containing unsteady state diffusion, and governing equations are del C A del t is equal to D AB del C A del x 2 plus del 2 C A del y 2 plus del 2 C A del Z 2 and this is known as fixed second law, and this is frequently applicable in solids and less frequently or limited situations in fluid.

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So, today we will start with steady state molecular diffusion in fluids and in this case there are stagnant conditions and laminar flow condition. And also, this can occur in two different geometry, one is constant area, constant area and variable area. So, today we will discuss only on constant area and later we will continue with the variable area. (Refer Slide Time: 04:27)



So, for constant area let us assume diffusion in x-direction, no chemical reaction, binary system and under steady state condition. So, then for diffusion of particular species A we can write N A is equal to minus C D AB d Y A dx plus Y A N. Now, if we separate the variables we can write minus d Y A by N A minus Y A N is equal to dx divided by C D AB.

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Steady State Molecular Diffusion Through a Constant Area Gas Mix : if constant P&T Then C & DAR => constant BC's at $x = x_1$ $y_A = y_{A1}$ at $x = x_2$ $y_A = y_{A2}$ $\int_{y_{A_1}}^{y_{A_2}} \frac{-dy_{A_1}}{N_A - y_A N} = \int_{x_1}^{x_2} \frac{dx}{c b_{AB}}$ $det N_{A} - Y_{A}N = \overline{z} \Rightarrow -dY_{A}N = d\overline{z} \Rightarrow d$ $\int_{Z_{1}}^{Z_{1}} \frac{d\overline{z}}{\overline{z}} = \frac{1}{c} \int_{X_{1}}^{X_{2}} d\overline{z}$

Now, consider mixture of gases, gas mixture, which is at constant pressure and temperature. Then, the concentration and diffusion coefficient D AB, these are constant

and independent of position. So, we can write the boundary condition as, at x is equal to x 1, the mole fraction of component 1 will be Y A1; at x is equal to x 2, the mole function of component 2, Y A2. So, using these boundary conditions if we integrate the previous equations, so we can write Y A1 Y A2 minus d Y A by N A minus Y A N is equal to integral x 1 to x 2 dx divided by C D A B. So, this is equation number 2.

Let N A minus Y A N is equal to z. So, then we can write minus d Y A N will be dz and hence, d minus d Y A would be equal to d z by N. So, if we substitute this over here, with the change of limit we can write, z 1 to z 2 dz by z 1 by N will be equal to 1 by C D AB integral x 1 to x 2 dx.

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Steady State Molecular Diffusion Through a Constant Area $\frac{1}{N} \ln \frac{z_{2}}{z_{1}} = \frac{1}{c} \frac{1}{D_{AR}} (x_{2} - x_{1})$ $\Rightarrow \ln \left[\frac{N_{A} - Y_{A}N}{N_{A} - Y_{A}N} \right] = \frac{N}{c} \frac{1}{D_{AR}} (x_{2} - x_{1})$ $\Rightarrow \ln \left[\frac{M}{N} \left(\frac{N_{A}}{N} - Y_{A}\right) \right] = \frac{N}{c} \frac{1}{c} \frac{$

So, now, upon integration (()) write 1 by N ln z 2 by z 1 is equal to 1 by C D AB x 2 minus x 1. So, if we substitute the limit z 1, then we can write ln N A minus Y A 2 N divided by N A minus Y A1 N would be equal to N by C D AB into x 2 minus x 1. So, again, we can write ln N into N A by N minus Y A2 divided by N N A by N minus Y A1 is equal to N by C D AB into x 2 minus X 1.

And if we rearrange this one we can write, 1 is equal to C D AB by x 2 minus x 1 and 1 by N ln N A by N minus Y A 2 divided by N A by N minus Y A1. So, if we multiplied both sides by N A, so these equations will become N A and N A by N this is equation three. So, this is the final form of the steady state molecular diffusion of a component A to a constant area.

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Steady State Diffusion Through Non-Diffusing Component constan = = N= NA

And then if we use these equations for a particular case, steady state diffusion through non-diffusing components, so consider A is diffusing and B is non-diffusing. In this case since B is non-diffusing, so flux for B component, N B will be 0 and the flux for component A will be constant. Now, if we go back to the governing equation we can write, N A is equal to N A by N C D AB by x 2 minus x 1 ln N A by N minus Y A2 by N A by N minus Y A1.

Now, since N is equal to N A plus N B and N B is equal to 0, in this case, so N will be N A. So, this term N A by N will be equal to N A by N A is equal to 1. So, now, this equation will become N A is equal to C D AB by x 2 minus x 1 ln. So, this term becomes 1, this becomes 1 and this becomes 1. So, this will be 1 minus Y A 2 by 1 minus Y A 1. So, this is equation number 4.

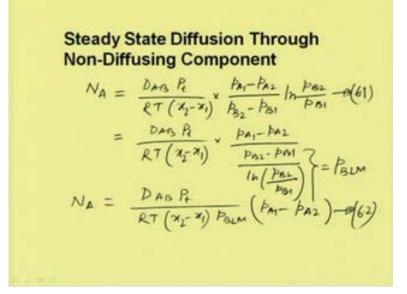
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Steady State Diffusion Through Non-Diffusing Component Ideal Gas: $c = \frac{R}{RT}, y_A = \frac{A}{R}$ $N_A = \frac{D_{AB} R}{RT(x_2 - x_1)} l_m \left[\frac{R}{A} - \frac{R}{A_3}\right] - \theta(5)$ $P_{A} + \dot{P}_{B} = P_{t}$ $P_{t} - \dot{P}_{A_{2}} = \dot{P}_{B_{2}}, \quad P_{t} - \dot{P}_{A_{1}} = \dot{P}_{B_{1}}$ PA + PB = Pt PAY - PAZ = PBZ- PBI $\Rightarrow \frac{P_{A_1} - P_{A_2}}{P_{B_2} - P_{B_1}} = 1$

Now, consider the system is ideal gas. So, in this case we can write, the concentration is equal to total pressure P t divided by RT; mole fractions Y A, we can write, partial pressure of component A by total pressure. So, in terms of partial pressure we can write the earlier equations, N A is equal to D AB P t by RT x 2 minus x 1 into ln P t minus p A2, partial pressure of component A active, divided by total pressure minus partial pressure of component A at location 1. So, p A1 and p A2 are the partial pressure at location 2 and 1.

And for binary gas mixtures we know, that p A plus p B, at any location will be total pressure, so that we can write P t minus p A2 will p B2. Similarly, P t minus p A1 is p B1 and p A1 minus p A2 will be p B2 minus p B1. From this we can write, p A1 minus p A2, partial pressure at location 1 and 2 for component A divided by partial pressure of component B at location 2 minus partial pressure of, component, component B at location 1 will be equal to 1.

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So, this if we substitute in this relation equation 5, so now if we substitute this P t minus p A2 by p B2 and P t minus p A1 by p B1 and then if we multiply it, this equations by this ratio, which is 1, we will essentially have N A is equal to D AB P t by RT x 2 minus x 1 into p A1 minus p A2 by p B2 minus p B1 ln p B2 by p B1. So, this is equation 6.1.

Now, if we rearrange this equation D AB P t divided by R T x 2 minus x 1 into p A1 minus p A2 divided by p B2 minus p B1 divided by ln p B2 by p B1. So, this term, essentially, we called the (()) partial pressure difference, so we can write p BLM. Then, we can write d flux N A will be D AB P t, total pressure, divided by RT x 2 minus x 1 p BLM into the partial pressure difference p A1 minus p A2. So, this is equation 6.2.

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Example 1 CO₂ is diffusing through non diffusing air under steady state conditions at a total pressure of 1 atmosphere and temperature 300K. The partial pressure of CO2 is 20kPa at one point and 5kPa at other point. The distance between the points is 5cm. Calculate the flux of CO2. Given that at 300K and at 1 atm, D_{CO2-air}= 2 x 10-5 m²/s. $\frac{\text{Solution}}{N_{co_2}} = \frac{\text{Assume} : \text{Ideal Gas}}{\frac{\text{Comp Air = B}}{\text{RT}(x_2^{-x_1})}} \left(\frac{P_{co_2, 5}}{P_{BM}} \left(\frac{P_{co_2, 5}}{P_{box}} \right) \right)$

Now, consider carbon dioxide is diffusing through non-diffusing air under steady state conditions at a total pressure of 1 atmosphere and temperature 300 Kelvin. The partial pressure of CO 2 is 20 kilo Pascal at one point and 5 kilo Pascal at other point. The distance is given between the two points is 5 centimeter and we have to calculate the flux of CO 2. The diffusivity at a particular condition is given CO 2, 2 into 10 to the power minus 5 meter square per second.

Let us assume ideal gas and component, B, air, air, component air is considered as B. So, we can write the flux equations of component A to non-diffusing B. We can write flux of CO 2 is equal to D CO 2 air divided by RT x 2 minus x 1 P t by P BLM into p CO 2 at 1 minus p CO 2 at 2. The data, which is given is diffusion coefficient, total pressure, R and T are known, distance between the two points are known, partial pressure were given, we have to calculate P BLM.

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Example 1: Solution $D_{10_2-air} = 2 \times 10^{-5} \text{m}^2/\text{s}$ $P_t = 1 \text{ atm} = 101.3 \text{ kB} = 1013 \times 10^{5} \text{R}$ T = 300 K, x1-x, = 5 cm = 0.05 m Puors = 20 x R = 20,000 Pe PW2,2 = 5xPa = 5000Pa R = 8314 Pa m3 / Kmolk PB.s = P+ - Pw.s = (1013-20) & Pa= \$1.3th \$ B.2 = Pt - Proz. 2 = (101.3-5) x Pa= 763 x Pa PELM = PB,2-PB1 = A6-3-813 AR= 88-54 RR In PB2/PAY In (963/813) = 88590 R

So, let us consider D CO 2 air, which is 2 into 10 to the power minus 5 meter square per second; P t, total pressure is given 1 atmosphere, which is 101.3 kilo Pascal, which is 1.013 10 to the power 5 Pascal; t is given 300 Kelvin; x 2 minus x 1, the distance between the two points is 5 centimeter, which is 0.05 meter; partial pressure of CO 2 at 0.2 is 20 kilopascal is equal to 20000 Pascal; partial pressure of CO 2 at 0.2 is given 5 kilopascal, which is 5000 Pascal and R is known to us, 8314 in SI unit Pascal meter cube per K mole Kelvin.

And then we have to calculate the P BM, p B, 1 is P t minus partial pressure of CO 2 at 0.1, which is equal to 101.3 minus 20 kilopascal, which is equal to 81.3 kilopascal; p B, 2, P t minus p CO 2 at point 2, which is 101.3 minus 5 kilo Pascal, which is 96.3 kilo Pascal. So, putting this value we can calculate P BLM, which is p B, 2 minus p B, 1 by ln p B2 by P B1. So, putting the values, 96.3 minus 81.3 divided by ln 96.3 by 81.3 kilo Pascal is 88.59 kilo Pascal, so which is equal to 88590 Pascal.

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Example 1: Solution $N_{co_2} = \frac{2 \times 10^5 \text{ m}^3/\text{s} \times 1.013 \times 10^5 \text{Fa}}{\frac{8 \times 10^3}{\text{km}^{-1} \text{k}} \times 300 \text{k} \times 0.05 \text{m} \times 88590 \text{Fa}} = 2.75 \times 10^{-6} \frac{\text{km}^{-1}}{\text{m}^2 \text{s}}$

So, now, if we substitute in the flux equation, N CO 2 we can write, this is diffusion coefficient, 2 into 10 to the power minus 5 meter square per second. And then the total pressure, 1.013 into 10 to the power 5 Pascal divided by R, 8314 Pascal meter cube per K mole Kelvin into 300 Kelvin. The distance 0.005 meter into (()) partial pressure difference is of component B, 88590 Pascal into 20000 partial pressure of component A at point one minus 5000 Pascal. So, this will cancel out and this will cancel out. So, if we calculate, the flux will be 2.75 into 10 to the power minus 6 K mole per meter square second.

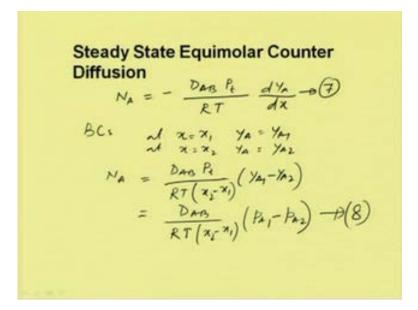
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Steady State Equimolar Counter Diffusion $N_{A} = - c D_{AB} \frac{dY_{A}}{dx} + Y_{A} \mathcal{A}^{T} - o(1)$ NA = - NB = constant $N = N_A + N_B = 0$ $N_A = -c D_{AB} \frac{dy_0}{dx}$ Assume ideal Gas: $C = \frac{R}{87}$

Now, consider the other case where steady state equimolar counter diffusion is occurring. If there is an equimolar counter current diffusion for our governing equations, as we know, N A is equal to minus C D AB dY A dx plus Y A N. This is equation number 1, as we have discussed earlier.

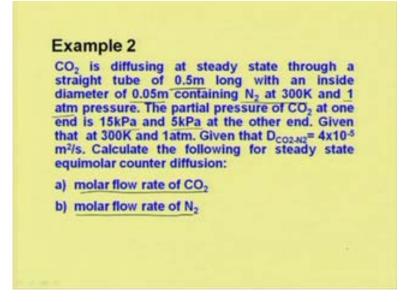
And for equimolar counter current diffusion, N A is equal to minus N B N is equal to constant. So, N, which is equal to N A plus N B, would be equal to 0. Since this is 0, so we can write N A is equal to minus C D AB dY A dx. Assume ideal gas, if we assume ideal gas, let C is equal to total pressure by RT.

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So, this equations will be N A, is equal to minus D AB P t by RT dY A dx. So, this is equation number 7. Now, if we use the boundary conditions, at x is equal to x 1, Y A will be Y A1; at x is equal to x 2, Y A will be Y A 2. So, using this boundary condition we can write, flux will be equal to D AB P t by RT x 2 minus x 1 into Y A1 minus Y A2. And hence, we can write, D AB by RT x 2 minus x 1. If it multiplied by the total pressure, this will be partial pressure p A1 minus p A2 of component A. So, this is equation 8.

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Now, let us have an example where the equimolar counter current diffusion is occurring. Consider the similar gas as we have considered in the example 1, CO 2 is diffusing at steady state through a straight tube of 0.5 meter long. The distance is given with an inside diameter 0.05 meter, which contains the nitrogen gas at 300 Kelvin and 1 atmosphere pressure.

The partial pressure is given at two points, at one point 15 kilopascal and other point is 5 kilopascal and the diffusion coefficient, d CO 2 nitrogen is given 4 into 10 to the power of minus 5 meter square per second. And we have to calculate molar flow rate of component A, that is, CO 2 and the molar flow rate of component B, that is N 2.

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Example 2: Solution Assume ideal Gos Dur N2 (Dur N2) DW3-N = 4×10 m/s Pw2.1 = 15 + Pa= 15,000 Pa buint = 51 Pa = 5000 Pa R = 8314 Pa 9 × 10 5 mms × (15.000-5000) Pa \$514 Par-45

Let us assume ideal gas and then we know the flux equation N CO 2 will be equal to D CO 2 into nitrate to the nitrogen, RT x 2 minus x 1 into partial pressure of CO 2 at point 1 minus partial pressure of CO 2 at point 2. So, the data, which are given, diffusion coefficient of CO 2 nitrogen, 4 into 10 to the power minus 5 meter square per second pressure is given; P t, which is 1 atmosphere, 101.3 kilopascal, which is equal to 1013 into 10 to the power 5 Pascal and temperature is given, which is 300 Kelvin. And the distance, x 2 minus x 1, between two points is 0.5 meter. The partial pressure of CO 2 at point 1 is 15 kilo Pascal, which is 15000 Pascal and the partial pressure of CO 2 at point 2 is 5 kilo Pascal, which is 5000 Pascal and R is given, R is known to us, 8314 Pascal meter cube K mole Kelvin.

Now, if we substitute in this equation, the flux N CO 2 will be equal to diffusivity, 4 into 10 to the power minus 5 meter square per second and RT, 8314 Pascal meter cube per K mole Kelvin into 300 Kelvin into 0.5 meter multiplied by the partial pressure difference, 15000 minus 5000 Pascal. So, if we calculate, this will be equal to 3.21 into 10 to the power minus 7 K mole per meter square second. So, this is the flux of CO 2.

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Example 2: Solution Molar flow rate of COL = Neg XA Internal dia $(D_i) = 0.05 \text{ m}$ $A = \frac{\pi D_i^2}{4} = \frac{\pi}{4} (0.05)^2 \text{ m}^2 = 1.96 \times 10^3 \text{ m}^2$ Molan flow rate of Co_L $= 3.21 \times 10^7 \frac{\text{xmol}}{\text{ym}^2 \text{ s}} \times 1.91 \times 10^3 \text{ m}^2$ $= 6.29 \times 10^{-10} \frac{\text{km} \Omega}{5}$

We have to calculate the molar flow rate of CO 2. So, to calculate the molar flow rate, the molar flow rate is equal to flux of CO 2 multiplied by the cross-sectional area of the tube which is A. Internal diameter is given, say, D i is 0.05 meter and then we can calculate the cross-sectional area, A is equal to pi D i square by 4 is equal to pi by 4 0.05 square meter square, which is equal to 1.96 into 10 to the power minus 3 meter square. So, the molar flow rate, CO 2, is equal to, flux we have obtained, N, earlier which is 3.21 into 10 to the power minus 7 K mole per meter square second.

So, 3.21 into 10 to the power minus 7 K mole per meter square second multiplied by the cross-sectional area, 1.96 10 to the power minus 3 meter square. So, if we multiply, this will cancel out and then it will be 6.29 10 to the power minus 10 K mole per second.

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Example 2: Solution $N_{N_{2}} = \frac{D_{N_{3}} - \omega_{2}}{RT(n_{2} - n_{j})} \left(\frac{h_{N_{3}, j}}{h_{N_{3}, j}} - \frac{h_{N_{3}, 2}}{h_{N_{3}, j}} \right)$
$$\begin{split} \dot{p}_{N_{2},3} &= P_{t} - \dot{p}_{\omega_{1},1} = (1013 - 15) \ell_{0}^{L} \\ &= 863 \ell_{0}^{L} = 863 \ell_{0}^{L} = 863 \ell_{0}^{L} = 863 \ell_{0}^{L} \\ \dot{p}_{N_{2},2} &= l_{t} - \dot{p}_{\omega_{2},2} = (1013 - 5) \kappa f_{0}^{L} \\ &= 9413 \kappa f_{0}^{L} = 76300 f_{0}^{L} \\ \dot{p}_{N_{2},2} &= D_{N_{2}-CO_{2}} \\ \dot{p}_{N_{N_{2}}} &= -3 \cdot 21 \times 10^{-7} \frac{\kappa m \delta l}{m^{2} s} \end{split}$$

Similarly, for component B, that is, nitrogen we can calculate, N, nitrogen is equal to D N 2 CO 2 divided by RT x 2 minus x 1. Partial pressure of N 2 at point 1 minus partial pressure of N 2 at point 2, now p N 2 at point 1 will be P t minus p CO 2 point 1, which is 101.3 minus 15 kilo Pascal, which is equal to 86.3 kilo Pascal and 86300 Pascal; p N 2 at point 2 is P t minus p CO 2 at point 2 is 101.3 minus 5 kilo Pascal is equal to 96.3 kilo Pascal, which is equal to 96300 Pascal.

We know that for equimolar counter current diffusion, D CO 2 N 2 is equal to D N 2 CO 2, so we can use the same diffusion coefficient and calculate the flux N N 2. If we incorporate this data, so it will be minus 3.21 into 10 to the power minus 7 K mole per meter square second.

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Example 2: Solution Molan Flow rate 2N2 = - 3.21 × 107 kml × 1.96 × 10322 = - 6.29 × 10⁻¹⁰ Kml/ 5

Similarly, the molar flow rate of N 2 we can calculate, minus 3.21 into 10 to the power minus 7 K mole per meter square second into 1.96 into 10 to the power minus 3 meter square. So, it will be essentially same value, but with a negative sign, which indicates, that the flux is in opposite direction compared to the carbon dioxide, so K mole per second.

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Steady State Non-Equimolar **Counter Diffusion** Following rxn. 2A+B = 2C One mel 3 A diffusco to warde B Two moli 9 B " back is in opposite direction-NA = - NB/2

So, another type of diffusion occurs where the component are not equimolecular in nature. So, there are many situations, they are non-equimolar counter diffusion. Let us

consider the following reaction, twice A plus B for being twice C, where 1 mole of A diffuses towards, towards B, whereas 2 moles of B diffuses back or in opposite direction. So, in this case, the N A will be minus N B by 2. And we can derive the governing equations considering our basic equations. So, this is end of discussion of lecture 2 and in the next lecture we will consider the steady state diffusion through non-uniform geometries.

Thank you.