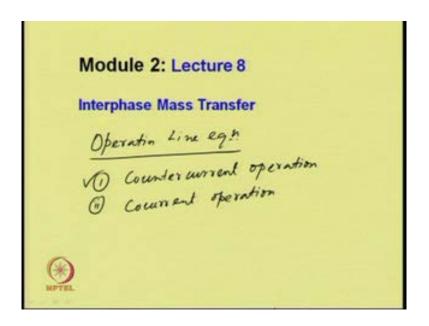
Mass Transfer Operations l Prof. Bishnupada Mandal Department of Chemical Engineering Indian Institute of Technology, Guwahati

Module - 2 Mass Transfer Coefficients Lecture - 8 Interphase Mass Transfer and Mass Transfer Theories Part 3

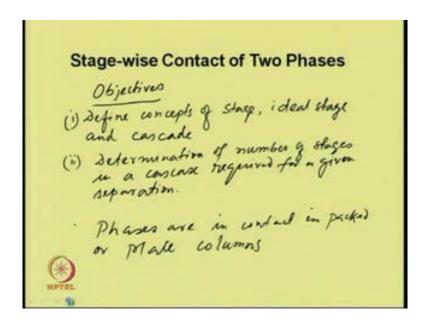
Welcome to the eighth lecture of the module two. The module two is on mass transfer coefficient; so we are discussing the interphase mass transfer. We will continue in this lecture the interphase mass transfer. In the previous lecture, we have considered the material balances in a two phase contact equipment to obtain the operating line, operating line equation.

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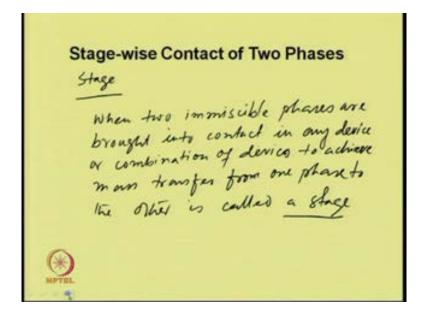
And this, we have discussed for two cases. One is countercurrent operation and second one is cocurrent operation. And we have seen that among these two operations countercurrent and cocurrent, the countercurrent is preferred because the ever is the driving force for mass transfer in case of the countercurrent operation, is more compared to cocurrent operations. So, it is favorable for mass transfer operations, the countercurrent mode.

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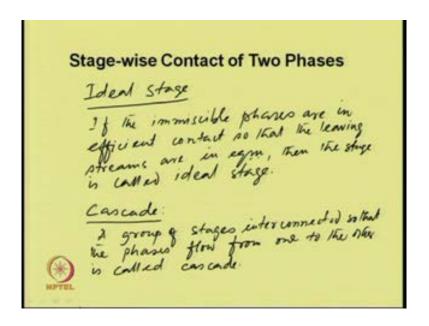
Today we will discuss the stage-wise contact of the two phases. So, here our objective is to define concept concepts of stage, ideal stage and cascade. And secondly, determination of number of stages in a cascade required for a given separation. In general in a stage wise contact operations, the liquid are fed at the top of column and the gas fed at the bottom of the column and they are in contact with each other and the phase flows in opposite direction. It is generally in a tray or packed column. Phases are in contact in packed or plate columns.

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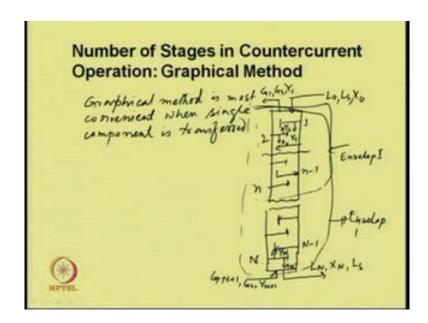
Then, what is stage? When two immiscible phases are brought into contact in any device or combination of devices to achieve mass transfer from one phase to the other, and it is called a stage.

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So, what is then ideal stage? If the immiscible phases are in efficient contact in a phase and the leaving stream are in equilibrium, so, then we call the stage is ideal stage. If the immiscible phases are in efficient contact so that, the leaving streams are in equilibrium in that stage, then the stage is called ideal stage. If a group of stages are interconnected, so that the stages are moved from one stage to the other, then we call that cascade. So cascade; a group of stages interconnected so that, the phases flow from one to the other is called cascade. So cascade, in most of the cases are required because in one stage it may not be possible to obtain the desired separation. So, in a cascade we can achieve depending on our separation requirement, we can decide the number of the stages required for the given separation and so cascade is effective in a particular separation.

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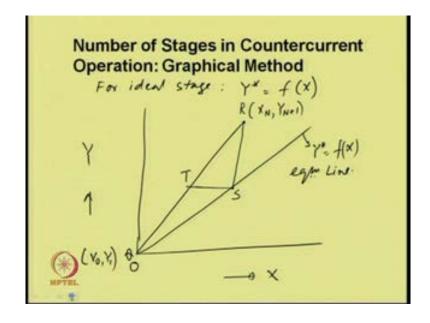
Now, we will consider how to determine these number of stages in countercurrent operation. First, we will consider the graphical method and then we will consider analytical method. So, graphical method is most convenient when single component is transferred. Now, consider our earlier columns. We will consider tray columns.

So, gas inlet at the bottom, and we consider bottom. It is N stage. This is stage number N, this is N minus 1 and this is the liquid in at the top of the tower. Like the previous conventions, this is L 0, L s and X 0. And, this is stage number one and gas out from stage one. So, this will be G 1, G s and Y 1. So, here is X 1 and the liquid from here is Y 2, this is 2, and this is the liquid coming from the stage X 2 and suppose this is the N minus the 1 th tray and this is n th tray, small n, or any arbitrary location and the gas in from G N plus 1 and G s and Y N plus 1. And liquid coming out from this stage, which is N tray is L N, X N and L s. So, it is X N and going out is Y N.

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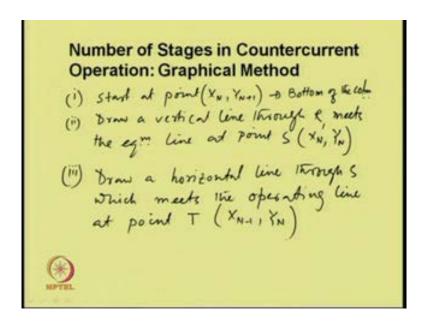
So, if you do the material balance over the envelope one and this is the overall envelope say two. And, if you do the material balance we can write, for n trays it will be G s Y n plus 1 minus Y 1 will be equal to L s X n minus X 0. This is the equation of operating line. Similarly, overall mass balance for a particular component in envelope two we can write, G s Y N plus 1 minus Y 1 will be L s X N minus X 0. So, this is also equation of straight line passing through points (X 0, Y 1) and (X N, Y N plus 1).

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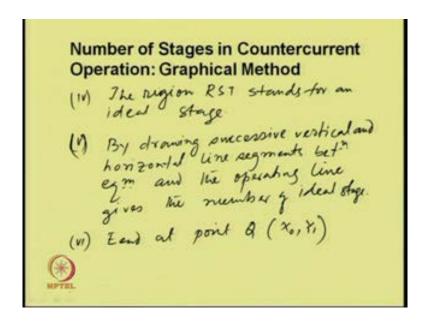
Now, if these plates are ideal plate or ideal stage, now if these plates are ideal plate or ideal stage, then Y star will be equal to function of X. Now, we can plot the operating line and the equilibrium line. Suppose this is, Y star is equal to function of X, this is the equilibrium line. And, the operating line mass transfer from mass transfer from gas to the liquid, suppose this is our point which is R and it is X N, Y N plus 1. And, this is X and this is Y, this is 0 and this is point Q. Q R is the operating line and this point is, Q point is basically X 0 and Y 1. Now to obtain the number of stages or plates using the graphical method, we can proceed step by step. From point R, we can draw a vertical line which is crossing equilibrium line at point S and then we can draw a horizontal line from the equilibrium line. So, this is at T.

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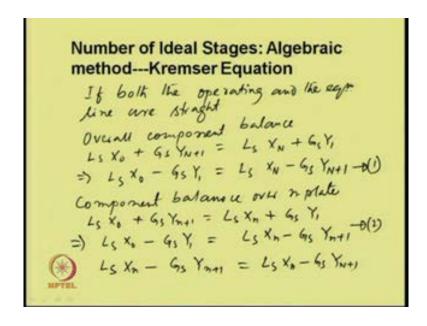
So, the first stage starts at point X N, Y N plus 1. That is the bottom of the column. And then, draw a vertical line through R which meets the equilibrium line at point S, which is X N and Y N. Now third step is to draw a horizontal line through S which meets the operating line at point T, which is X N minus 1, Y N. So, this is the first three steps we followed. And, we obtained R, S, T.

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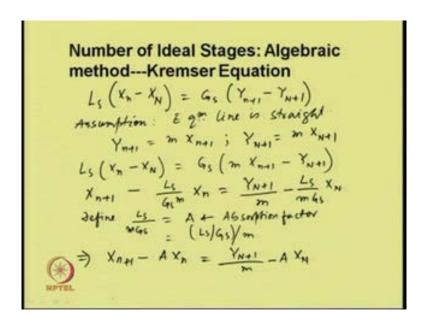
So, this R S T stands for an ideal plate. So, the region R S T stands for an ideal stage. Now, we can proceed successfully drawing the vertical line and the horizontal line. So, this is N, this will be N minus 1. Similarly, we can proceed. This way we can obtain the total number of stages required. So, by drawing successive vertical and horizontal line segments between equilibrium and the operating line, gives the number of ideal stage. And, this operations will end at point Q, which is X 0, Y 1. So, this is the graphical method of solutions to obtain the number of ideal stages.

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Now, we will discuss the number of ideal stages; how we can determine by algebraic method and which is commonly known as the Kremser equation. In this case if both the operating and the equilibrium line are straight, then we can be able to obtain the number of ideal stage by algebraic method. Doing the same component balance, referring the same figure, we can do the overall component balance. That is, L s X 0 plus G s Y N plus 1 is equal to L s X N plus G s Y 1. And, from this we can write L s X 0 minus G s Y 1 will be equal to L s X N minus G s Y N plus 1. Similarly for component balance, say this is equation 1 over n plate. We can obtain L s X 0 plus G s Y n plus 1 is equal to L s X n plus G S Y 1. And, from this we can obtain L s X 0 minus G s Y 1 would be equal to L s X n minus G s Y n plus 1. So, if we compare between 1 and 2 we will get L s X n minus G s Y n plus 1 will be equal to L s X n minus G s Y n plus 1 will be equal to L s X n minus G s Y n plus 1.

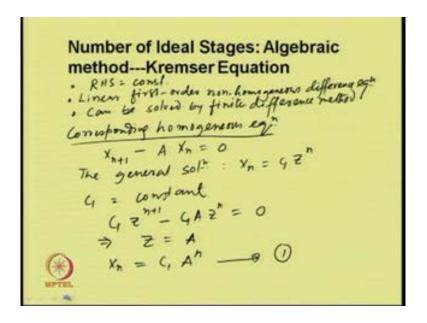
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Now from this, we can write L s into X n minus capital X N is equal to G s Y n plus 1 minus Y N plus 1. As per our assumptions, equilibrium line is straight. So, you can write Y n plus 1 is equals to m X n plus 1 and Y N plus 1 will be m X N plus 1. Now, if we substitute in this equation, it will be L s X n minus X N is equal to G s m X n plus 1 minus Y N plus 1. After rearrangement, we can write X n plus 1 minus L s by G s m X n is equals to Y N plus 1 by m minus L s by m G s into X N. So, if we define L s by m G s is equal to A which is known as absorption factor, it is the ratio of L s by G s divided by m. Ratio of the slope of the operating line to the equilibrium line, then we can write this

above equation will be X n plus 1 minus A X n is equal to Y N plus 1 by m minus A X capital N.

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So, in this equation right hand side is constant. And, this is linear first-order non-homogeneous difference equation. This can be solved by finite difference method. So, we can write the corresponding homogeneous equation; X n plus 1 minus A X n is equal to 0. The general solution of this is X n will be equal to C 1 into Z to the power n. C 1 is constant. If we substitute this equation in the original equation, we can write C 1 Z to the power n plus 1 minus C 1 A Z to the power n is equal to 0. And from this, we can write Z is equal to A. So, the solution is X n will be equal to C 1 A to the power n.

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Number of Ideal Stages: Algebraic method---Kremser Equation

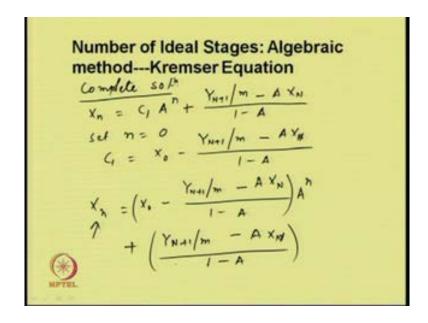
The particular solt in
$$X_n = C_2$$
, $G = a$ constant

 $C_2 - AC_2 = \frac{Y_{N+1}}{m} - AX_N$

$$C_2 = \frac{Y_{N+1}/m - AX_N}{1 - A}$$

Now, since the right hand side is constant, the particular solution is X n is C 2; where C 2 is a constant. Now, if we substitute both these things in the original equation, we have C 2 minus A C 2 will be equal to Y N plus 1 divided by m minus A X N. So, from this we can obtain C 2 will be equal to Y N plus 1 divided by m minus A X N divided by 1 minus A.

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So, the complete solution we can write, X n will be equal to C 1 A to the power n plus Y N plus 1 divided by m minus A X N divided by 1 minus A. Now if we set n equal to 0,

then C 1 will be X 0 minus Y N plus 1 by m minus A X N divided by 1 minus A. Therefore, X n will be X 0 minus Y N plus 1 divided by m minus A X N divided by 1 minus A into A to the power n plus Y N plus 1 by m minus A X N divided by 1 minus A. So, this is a very useful equation to generate concentration X n at any stage, if we know the terminal concentrations.

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Number of Ideal Stages: Algebraic method...Kremser Equation

Man Transfu
$$\eta$$
 solute for gents liquid

Case 1: $A \neq 1$

$$\frac{Y_{N+1} - Y_1}{Y_{N+1} - mX_0} = \frac{A^{N+1} - A}{A^{N+1} - 1}$$
 $N = \frac{105}{105} \left[\left(\frac{Y_{N+1} - mX_0}{Y_1 - mX_0} \right) \left(1 - \frac{1}{A} \right) + \frac{1}{A} \right]$

Case II: $A = 1$

$$\frac{Y_{N+1} - Y_1}{Y_1 - mX_0} = \frac{Y_{N+1} - Y_1}{Y_{N+1} - mX_0} = \frac{N}{N+1}$$

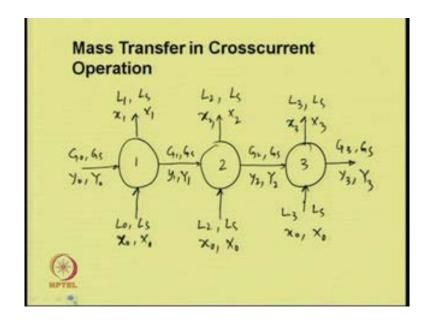
Where $\frac{Y_{N+1} - Y_1}{Y_1 - mX_0} = \frac{N}{N+1}$

Now, putting in this equation n is equal to N plus 1, we can obtain several forms. First, we will consider mass transfer of solute transfer of solute from gas to liquid, say in case of absorption. So in this case, case 1; A, absorption factor not equal to 1. We can write Y N plus 1 minus Y 1 divided by Y N plus 1 minus m X 0 will be equal to A to the power N plus 1 minus A divided by A to the power N plus 1 minus 1. And, the number of stages N we can write, log of Y N plus 1 minus m X 0 divided by Y 1 minus m X 0 into 1 minus 1 by A plus 1 by A divided by log A. So in case 2, A equal to 1; in this case, we can write Y N plus 1 minus Y 1 divided by Y N plus 1 minus m X 0 will be N by N plus 1 and N will be; we can write N will be Y N plus 1 minus Y 1 divided by Y 1 minus m X 0.

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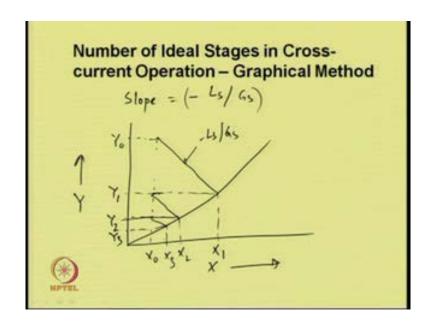
Similarly, mass transfer from liquid liquid to gas. We can write case 1; A not equals 1. It will be X 0 minus X N divided by X 0 minus Y N plus 1 by m will be equal to 1 by A to the power N plus 1 minus 1 by A divided by 1 by A to the power N plus 1 minus 1. And, we can write N is equal to log of X 0 minus Y N plus 1 divided by m divided by X N minus Y N plus 1 divided by m into 1 minus A plus A. So, this is divided by log 1 by A. Now for case 2, A equals 1; we can write X 0 minus X N divided by X 0 minus Y N plus 1 by m will be N by N plus 1 and N is equal to X 0 minus X N divided by X N minus Y N plus 1 by m. So, these are the different cases for absorption and striping in particular. We can get the number of stages and the concentration in different stages we can obtain.

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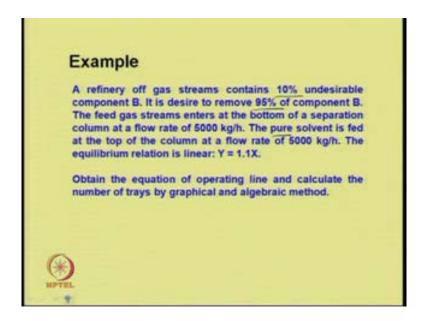
Now, if we consider mass transfer in crosscurrent operations; so, we will consider three stages connected in series. And, in which the gas is flowing from one stage to the other. And in each case, phase liquid is introduced. They may be of the same initial concentration or in baring concentration. Let us consider same initial concentrations of the liquid surface, so this is G 0, G s, y 0, Y capital 0 and L 0, L s, x 0, capital X 0 and liquid leaving L 1, L s, x 1, capital X 1; this is G 1, G s. This is one, this is two and this is three. And y 1, capital Y 1. This is L 2, L s, x 0, capital X 0 and this is leaving X, L 2, L s, x 2, capital X 2 and L 3, L s, x 0, capital X 0 and leaving at L s, L 3, L s, x 3, capital X 3. And, this gas is G 2, G s, y 2 capital Y 2 and this is G 3, G s, y 3, capital Y 3.

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So similar way if we do the mass balance, we will obtain the slope of the operating line is minus L s by G s. So, this is similar to the cocurrent operation; the slope of the operating line, that is, in individual stages in the crosscurrent operation the stages are in cocurrent in nature. So, we can plot the equilibrium line and operating line X-Y diagram. Suppose, this is the equilibrium line and this is the inlet point or this is X 0 and this is Y 0, so with a slope of minus L s by G s, it context to the equilibrium line and then we can go horizontally and this will give you Y 1. And from this, with the same slope we can obtain this line. So, this will give again Y 2. And, from this we can obtain Y 3. So, this will be X 3, this will be X 2 and this will be X 1. So, this way we can obtain the number of stages required in case of crosscurrent operations.

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Let us take a simple example. A refinery off gas streams contains ten percent undesirable component B. It is desirable to remove 95 percent of component B. The feed gas streams enters at the bottom of a separation column at a flow rate of 5000 kilogram per hour. The pure solvent is fed at the top of the column at a flow rate of 5000 kilogram per hour. The equilibrium relation is linear: Y is equal to 1.1 X. Obtain the equation of operating line and calculate the number of trays by graphical and algebraic method. That we can see this is a countercurrent operation and the concentration of solute is to be removed is given as 10 percent and we have to remove 95 percent of component B, which is undesirable. And both the liquid and the gas streams, their flow rates are given.

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Solution

Feed com = 10%; On shale free burss

45 = 5000 (+0.1) = 4500 kg/h

Man 3 solute endering = 5000 x 61 = 500 kg/h

45% removal of component B

Feed Cont. Yn+1 = 0.11

Exit Lone = 0.111-0.111 x 0.95 = 0.006

Solvend in preve Xo = 0

Make at balane: G3 (Yn+1-Y1)=L5 (Xn-X1)

4500 (0.111-0.006) = 5000 (X1-0)
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So, the feed concentration is given. It is equal to 10 percent. We can write on solute free basis. We can obtain G s will be 5000, which is G into 1 minus 0.1, ten percent, is 4500 kilogram per hour. Mass of solute entering is 5000 into 0.1 is equal to 500 kilogram per hour. And, 95 percent removal of component B; so, feed concentration Y N plus 1 will be 0.1 divided by 0.9, which is 0.111 and exit concentration would be point 0.111 minus 0.111 into 0.95, which is equal to 0.006. The solvent is pure. So, X 0 is 0. So, the material balance G s will be Y N plus 1 minus Y 1 is L s X N minus X 0. So putting the values, 4500 into 0.111 minus 0.006 is equal to 5000 X 1 minus 0. So, you can get X1 is 0.0945. So, X 1 is known to us.

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Solution

Gs (
$$Y_{n+1} - Y_1$$
) = L_5 ($X_N - X_2$); $X_2 = 0$

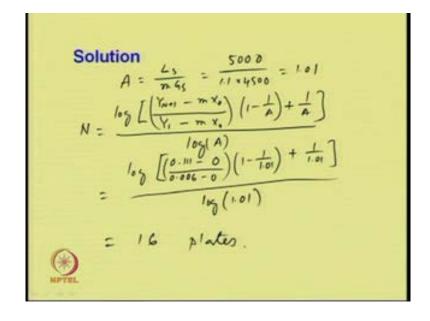
=) 4560 ($Y_{n+1} - 0.006$) = 5000 Y_n

=) $(Y_{n+1} = 1.111 \times Y_n + 0.006)$ openhing line

 L_7 $(X_1 - Y_2) = 1.1 \times 1000$ $(X_1 - Y_2) = 1.1 \times 1000$ $(X_1 - Y_2) = 1.1 \times 1000$ $(X_1 - Y_2) = 1.1 \times 1000$

Now if we do the material balance over the envelope two, so G s as considering in previous figure Y n plus 1 minus Y 1 would be L s X N minus X 0 and X 0 is 0. So, from this, we can get 4500 Y n plus 1 minus 0.006 will be 5000 into X n. So, from this we can get Y n plus 1 will be 1.111 X n plus 0.006. This is the equation of operating line operating line; so we know the equation of operating line and we know the equation Y is equal to 1.1 X with a slope 1.1. So, we know the slope of the equilibrium line. This is equilibrium line. Then, we can graphically obtain like previous method. We can graphically obtain the number of stages by plotting a vertical and the horizontal line.

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Now, if we use the Kremser equation, which is A is equal to L s by m G s, which is L s is 5000 by 1.1 into 4500, which is 1.01. Now, N if we substitute, which is log Y N plus 1 minus m X 0 divided by Y 1 minus m X 0 into 1 minus 1 by A plus 1 by A divided by log A. Now, if we substitute, it will be log of 0.111 minus 0 divided by 0.006 minus 0 into 1 minus 1 by 1.01 plus 1 by 1.01 divided by log 1.01. So, this will give around 16 number of plates.

Thank You.