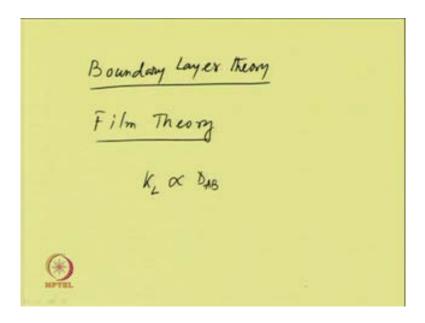
Mass Transfer Operations I Prof. Bishnupada Mandal Department of Chemical Engineering Indian Institute of Technology, Guwahati

Module - 2 Mass Transfer Coefficients Lecture - 5 Mass Transfer Coefficients in Turbulent Flow

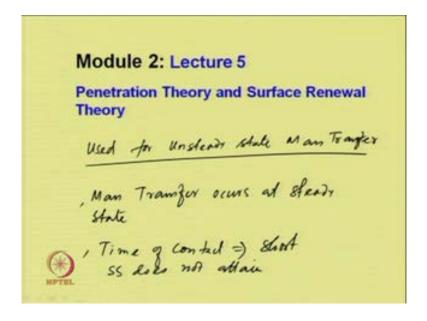
Welcome to fifth lecture of module two. The module 2 is on mass transfer co-efficient. So, in this lecture, we will consider mass transfer co-efficient in turbulent flow conditions.

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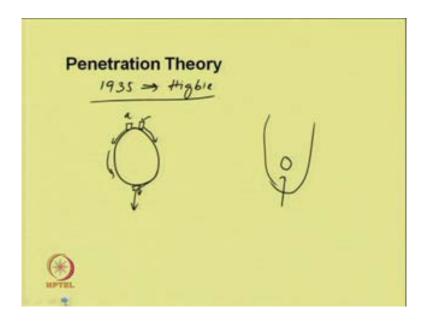
So, in the previous lecture we have discussed boundary layer theory, which governs the basis for mass transfer in turbulent flow conditions. And also we have introduced that in turbulent conditions, how to calculate the mass transfer coefficient using film theory. And in film theory, we have seen liquid phase mass transfer coefficient is directly proportional to the diffusion coefficient of the component. And we have discussed a simple problem, how to calculate the film thickness using film theory?

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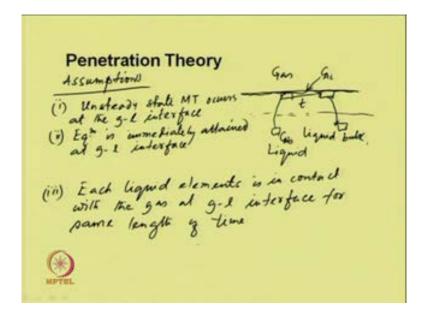
In this lecture, we will discuss penetration theory, and surface renewal theory. This theories are very important for the cases, where the mass transfer coefficient cannot be calculated using the different other theories or mass transfer correlations. So, this theory a gives a simplistic estimation of the mass transfer co-efficient and flux for turbulent flow conditions. This is used for unsteady state mass transfer, in case of film theory, we assume that there will be a concentration profile, which will obtain at steady state condition and mass transfer will follow at steady state. In many situations, where the gas liquid contact times are very sort, so that the time required to attain the steady state may not arise time of contact is short. So, steady state condition does not attain. So, in this case we have to use some realistic theory, which assumes unsteady state condition and mass transfer in turbulent conditions.

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So, one of such theory is penetration theory and in 1935 Higbie, he assumes that a pool of liquid in which a bubble is rising say this is the bubble, and an element of liquid which is swept through the surface, say element liquid a is swept on the surface and it remains in contact during its travelled through its diameter of the bubble; and finally the bubble detach at the bottom. So, the time of contact is the time, the length travelled by the element of the liquid about its diameter; diameter of the bubble.

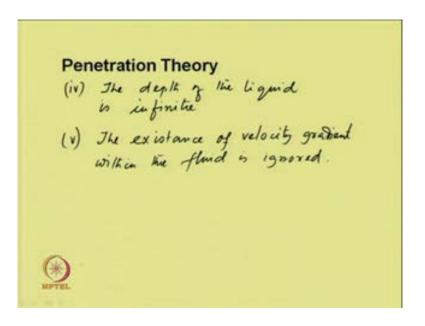
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Using the similar phenomena Higbie, assume that in case of turbulent flow conditions, if we considered this is the gas liquid interface, this is gas and this is liquid and the liquid element are travelled at a d during turbulent flow conditions, and say this is the liquid element it will come to the surface, and it travels to its diameter and remains expose to the surface at t time, and then it returned back to the liquid bulk. So, this is the liquid bulk and the concentration of component A in the liquid is a C AB the bulk concentration at the surface the concentration of component A is C Ai that is the interfacial concentration of the component, and the time of exposure is t.

So, it assumes unsteady state mass transfer occurs at the gas liquid interface, and the equilibrium is immediately attend at the gas liquid interface, and then each liquid element is in contact with the gas at gas liquid interface for same length of time. This means that every element will travel from the bulk to the gas liquid interface, and they remains in the interface at t amount of time and each element will have same length of time, and that will be again replaced by the fresh element of the liquid.

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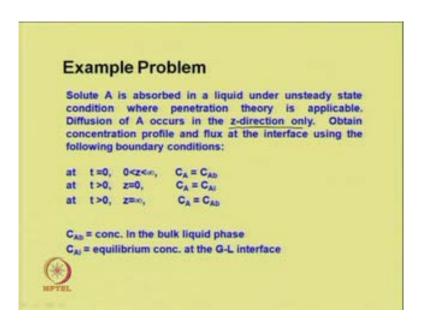


And the fourth 1 is the depth of the liquid liquid is infinite, this means that the concentration at the interface is C Ai and immediately to that it will remain as bulk concentration C AB.

So, at other side of the element it also remains as bulk concentration, and this is possible when the contact time between gas and liquid is very short. So, that it will not reach the

interfacial concentration that is not reach to the bulk concentration, and immediately after that it remains bulk concentration. And the existence of the velocity gradient existence of velocity gradient within the fluid is ignored. So, at every case the fluid elements are travelling or moving at the same rate.

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Now with these assumptions, let us considered a simple case example problem where solute A is absorbed in a liquid under unsteady state conditions, where penetration theory is applicable. Diffusion of a occurs in the z direction only in 1 dimension obtain concentration profile and flux at the interface using the following boundary conditions, at t is equal to 0, z greater than 0 less than infinity, C A will be C Ab the bulk concentration, C Ab is the concentration in the liquid bulk at t is equal to 0; that is z is equal to 0 that is at the interface, the concentration of solute A will be interfacial concentration of solute A at t greater than 0, z will be equal to a infinity. So, at infinite death C A will be bulk concentration.

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Solution

Fick's 2nd Law derived in Module-1

Technics to
$$\frac{\partial^2 G}{\partial t} = \delta_{AB} \frac{\partial^2 G}{\partial z^2}$$

We can write cone in terms of deviation

Variables

 $C = G - G_{AB}$
 $\frac{\partial C}{\partial z} = \frac{\partial G}{\partial z^2} = \frac{\partial^2 G}{\partial z^2}$
 $\frac{\partial C}{\partial z} = \frac{\partial G}{\partial z} = \frac{\partial C}{\partial z} = \frac{\partial C}{\partial z^2}$
 $\frac{\partial C}{\partial z} = \delta_{AB} = \frac{\partial C}{\partial z} = \delta_{AB} = 0$

Wherea

We can write cone in terms of deviation

 $C = G - G_{AB} = \frac{\partial C}{\partial z^2} = \frac{\partial C}{\partial z^2} = \frac{\partial C}{\partial z^2} = 0$

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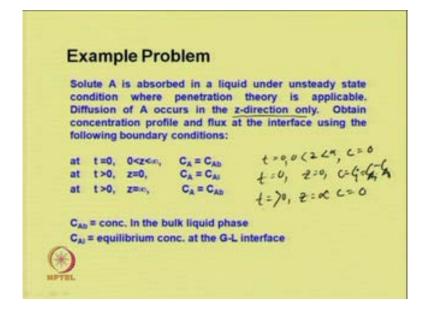
We can write $G_{AB} = M_{AB} = 0$

The since $G_{AB} = 0$

The sinc

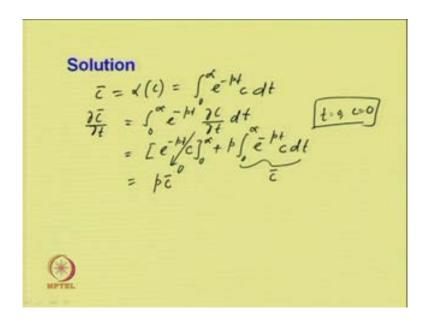
So, if we considered the Fick's second law for unsteady state diffusion derived in module 1 with the avop assumptions reduces to del C A del t is equal to D A B del 2 C A del z 2. So, we can write concentrations in terms of deviation variable, that is C is equal to C A minus C A bulk. So then we can write del C del t will be equal to del C A del t and del 2 C del z 2 del 2 C A del z 2. Since, C Ab as you can see from the boundary conditions with respect to time and space is constant. This equations reduces to del C del t is equal to D A B del 2 C del z 2.

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Then the boundary conditions which is given this will be reduce to t at t equal to 0, z less than infinity less than 0, C will be 0, t greater than 0, at z equal to 0, C will be interfacial concentration C i; that is equal to C Ai minus C A b and at t greater than 0 at z equal to infinity, C will be 0. So, with these boundary conditions these equations may be conveniently solved using Laplace transform.

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C bar, if we say Laplace transform of C we can write integral 0 to infinity e to the power minus p t C dt, then we can write del C bar del t will be equal to integral 0 to infinity e to the power minus p t del C del t dt. Now, putting the limit this will be equal to e to the power minus p t C 0 to infinity plus p integral 0 to infinity e to the power minus p t C dt. We know that when t equal to 0, C equal to 0 from the boundary condition. So, this will be 0 and it will be p and this part will be C bar, so p into C bar.

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Solution
$$\frac{\partial^{2}C}{\partial z^{2}} = \frac{\partial^{2}C}{\partial z^{2}}$$

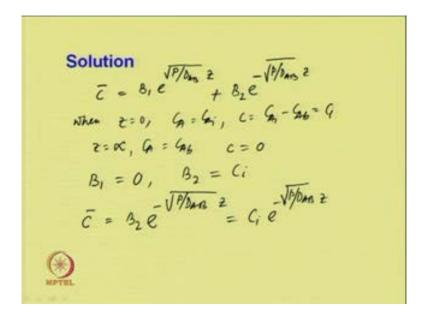
$$\beta \overline{C} = \partial_{AB} \frac{\partial^{2}C}{\partial z^{2}}$$

$$\Rightarrow \frac{\partial^{2}C}{\partial z^{2}} - \frac{f^{2}C}{\partial_{AB}} = 0$$

$$0DE \text{ in } \overline{C}$$

Now, using the Laplace operator we know that del 2 C del z 2 will be equal to del 2 C bar del z 2. So, if we take the Laplace transform for both sides this part will be p C bar will be equal to D A B del 2 C bar del z 2. So, this will lead to del 2 C bar del z 2 it minus p by D A B C bar will be equal to 0.

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So, this is an ordinary differential equation of C bar, and the solutions of this equation is C bar will be equal to B 1 e to the power root over P by D A B z plus B 2 e to the power minus root over P by D A B z. We know that when z equal to 0, C A will be C Ai and C

will be C Ai minus C A b is equal to C i. And when z equal to infinity, C A will be C A bulk, and C will be 0.

So, using this boundary conditions we can obtain B 1 is equal to 0, and B 2 is equal to C i. So, we can write the solution C bar will be equal to B 2 e to the power minus root over P by D A B into z is equal to C i e to the power minus root over P by D A B into z.

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Solution

$$\beta_2 = \int_0^{\infty} (G_{hj} - G_{h}) e^{-pt} dt$$

$$= \frac{1}{p} (G_{hj} - G_{h}) e^{-pt} dt$$

$$\overline{c} = \frac{1}{p} (G_{hj} - G_{h}) e^{-pt} dt$$

$$\overline{c} = \frac{1}{p} (G_{hj} - G_{h}) e^{-pt} dt$$

$$Taking inverse (aplace)$$

$$c = G_h - G_{hs} = (G_h - G_{h}) e^{-pt} (\frac{2}{2\sqrt{D_{hs}}t})$$

$$e^{-pt} = e^{-pt} e^{-pt} e^{-pt} dt$$

$$e^{-pt} = e^{-pt} e^{-pt} e^{-pt} e^{-pt} dt$$

$$e^{-pt} = G_{hj} - G_{hj} e^{-pt} dt$$

$$e^{-pt} = G_{hj} e^{-pt} dt$$

So, now B 2 is equal to integral 0 to infinity C Ai minus C A b e to the power minus p t dt which is equal to 1 by p C Ai minus C A b. So, then we can write C bar is equal to 1 by p C Ai minus C A b e to the power minus root over p by D AB z. So, taking inverse transform we can obtain C will be equal to C A minus C Ab is equal to C Ai minus C Ab into e r f c - complimentary error function into z by 2 root over D AB t. We know the definition e r f x is the error function of a x e r f c x is complimentary error function of x.

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Solution

$$exf(x) = \frac{2}{\sqrt{n}} \int_{x}^{\infty} e^{-x^{2}} dx$$

$$\int_{0}^{\infty} e^{-x^{2}} dx = \frac{\sqrt{n}}{2}, exf(x) = 0 \text{ so to } 1 \text{ to } 0$$

$$\int_{0}^{\infty} e^{-x^{2}} dx = \frac{\sqrt{n}}{2}, exf(x) = 0 \text{ so to } 1 \text{ to } 0$$

$$\int_{0}^{\infty} e^{-x^{2}} dx = \frac{\sqrt{n}}{2} \left[\frac{\sqrt{n}}{\sqrt{n}} \int_{0}^{\infty} e^{-x^{2}/4} \ln \frac{1}{n} \right]$$

$$= -\left(\frac{\sqrt{n} - \sqrt{n}}{\sqrt{n}} \right) \frac{\sqrt{n}}{\sqrt{n}} \int_{0}^{\infty} e^{-x^{2}/4} \ln \frac{1}{n}$$

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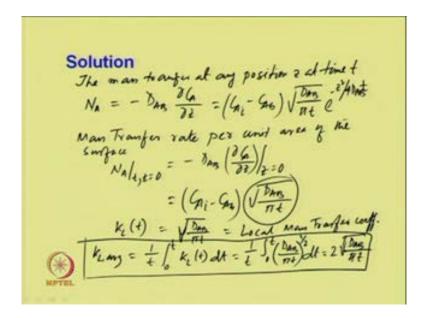
$$= -\left(\frac{\sqrt{n} - \sqrt{n}}{\sqrt{n}} \right) \frac{\sqrt{n}}{\sqrt{n}} \int_{0}^{\infty} e^{-x^{2}/4} \ln \frac{1}{n}$$

$$= -\left(\frac{\sqrt{n} - \sqrt{n}}{\sqrt{n}} \right) \frac{\sqrt{n}}{\sqrt{n}} \int_{0}^{\infty} e^{-x^{2}/4} \ln \frac{1}{n}$$

So, by definition we can write e r f C complimentary error function of x is equal to 2 by root pi integral x to infinity e to the power minus x square dx, and since integral 0 to infinity e to the power minus x square dx is equal to root pi by 2 e r complimentary error function goes to 1 to 0 when x varies from 0 to infinity. So, the concentration gradient we can obtain by the differentiation of the previous equations, these equations we can differentiate and obtain the concentration profile, so that is 1 by C A minus C A b del C A del z will be equal to del del z of 2 by root pi integral z by 2 root-over D AB t from this to infinity e to the power minus z square divided by four D AB t d of z divided by 2 root over D AB t.

So, therefore we will get del C A del z will be equal to minus C Ai minus C Ab 2 by root five 1 by root over D AB t into t e to the power minus z square by four D AB t. So, this will be equal to minus C Ai minus C Ab into 1 by root over pi D AB t into e to the power minus z square by four D AB t.

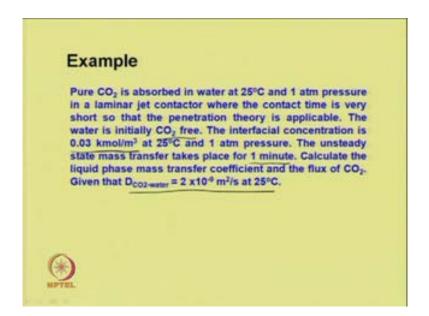
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The mass transfer at any position z at time t can be obtained as N A is equal to minus D AB del C A del z, if you substitute del C A del z obtained about here, then this will be equal to C Ai minus C A b root over D AB by pi t e to the power minus z square by 4 D AB t. The mass transfer rate per unit area of the surface per unit area of the surface can be written, that is N A at t z equal to 0 will be minus D AB del C A del z at z equal to 0, so if we put in this equation that z is equal to 0, this will be is equal to C Ai minus C A b into root over D AB by pi t, this is known as the local mass transfer coefficients.

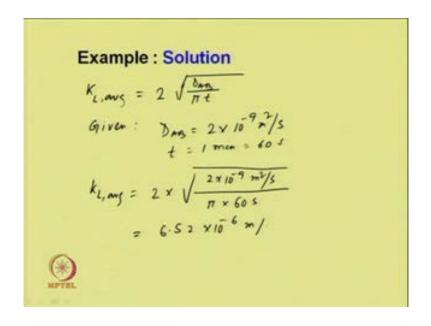
So, K L t is root over D AB by pi t. So, this is the local mass transfer coefficient. So, then K L average we can obtain 1 by t integral 0 to t K L t dt is equal to 1 by t integral 0 to t K L t is D AB by pi t to the power half into dt, which is equal to 2 into root over D AB by pi t. So, this is the average mass transfer coefficient obtained using penetration theory.

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Now, let us considered a simple example pure carbon dioxide is absorbed in water as we have discussed similarly problem earlier, at 25 degree centigrade and 1 atmosphere pressure in a laminar jet contactor, where the contact time is given very sorts. So, that the penetration theory is applicable, the water is initially CO 2, free interfacial concentration is given at particular temperature and pressure, unsteady state mass transfer takes place for 1 minute, calculate the liquid page mass transfer coefficient and the flux of CO 2, and the diffusion coefficient of CO 2 in water at 25 degree centigrade is given.

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So, according to penetration theory we know the average mass transfer coefficient K L average is equal to 2 into root over D AB by pi t, and given that D AB is equal to 2 into 10 to the power minus 9 meter square per second, t is equal to 1 minute which is 60 second.

So, we can calculate K L average will be 2 into root over 2 into 10 to the power minus 9 meter square per second divided by pi into 60 second. So, this will be equal to 6.52 into 10 to the power minus 6 meter per second.

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Example: Solution

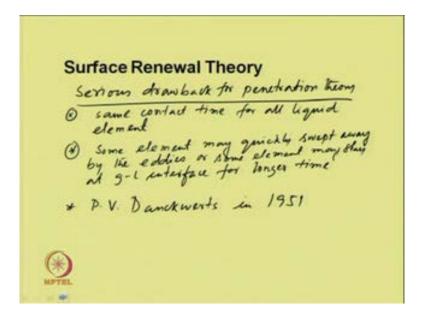
The rate of Absorbtion

$$N_A = K_{L,ang} \left(G_1 - G_2 \right)$$
 $G_1 = 0.03 \text{ kmB/m}^3$

Tritially water in G_1 : free $G_2 = 0$
 $N_A = 6.52 \times 10^{-6} \text{ m/s} \left(0.03 - 0 \right) \text{ kmB/m}^3$
 $= 1.96 \times 10^{-7} \frac{\text{kmol}}{\text{m}^2 \text{ s}}$

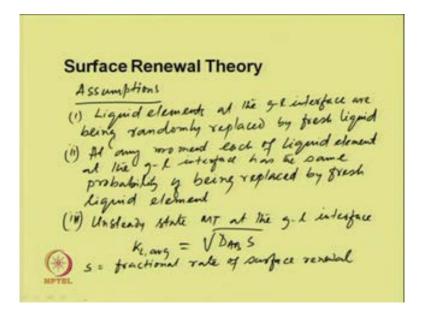
Now, we can calculate the flux or the rate of absorption, which is N A K L average into C Ai minus C Ab. C Ai is given 0.03 k mole per meter cube, and initially water is CO 2 free. So, C Ab is equal to 0. So, flux N A would be mass transfer coefficient meter per second into C Ai 0.03 minus 0 k mole per meter cube is equal to 1.96 into 10 to the power minus 7 k mole per meter square second.

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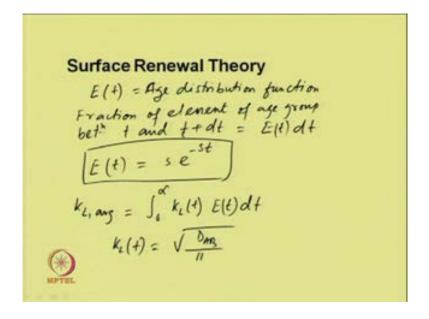
So, now consider another theory which is surface renewal theory, and one serious drawback for penetration theory is that. The assumption of equal contact time or same contact time for all a liquid elements - same contact time for all liquid element. So, it is highly or more likely more probable that there is a distribution of the age is at the gas liquid interface, due to turbulent flow conditions, the eddies may swept away the interfacial elements of fluid. So, they will leave the surface very quickly, and some element may stay at the surface for longer time, some elements may quickly swept away by the eddies or some element may stay at the gas liquid interface for longer time. So, there is a age distributions of the fluid elements in contact with the gas at the gas liquid interface. So, this way the situations was visualized by P V Danckwerts in 1951, that is what this theory is also known as Danckwerts surface renewal theory.

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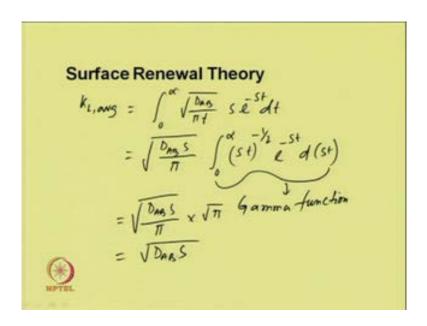
The important assumptions are assumptions: liquid elements at the gas liquid interface are being randomly replaced by fresh liquid from the bulk. At any moment each of liquid element at the gas liquid interface has the same probability of being replaced by fresh liquid element. And unsteady state mass transfer occurs mass transfer occurs at the gas liquid interface, so that the K L average can be obtained as root over D AB into S, where S is the fractional rate of surface renewal.

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We know that there will be a age distribution for this unsteady state mass transfer. So, if we considered E eth t is the age distribution function, then fraction of element of age group between t and t plus dt is equal to E t dt. So, the surface age distribution function E t can be obtained as S e to the power minus s t. So, this is the age distribution function for this case. Now, as we know K L average will be equal to integral 0 to infinity K L t E t dt. So, this k L t as we derived earlier is the point mass transfer coefficient which is root over D AB by pi t.

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So, the K L average is equal to integral 0 to infinity root over D AB by pi t S e to the power minus s t dt, which is equal to D AB S by pi root over integral 0 to infinity s t to the power minus half e to the power minus s t d st. So, this is known as gamma function. So, this is equal to root over D AB S by pi into root pi, so which is equal to root over D AB S.

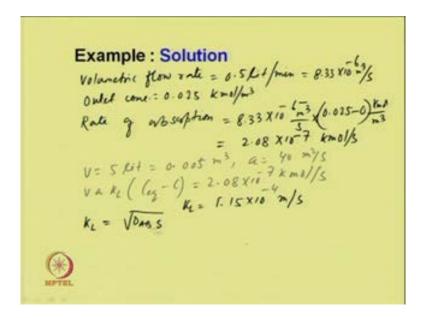
Example

Pure CO₂ is absorbed in water in a continuous stirred gasliquid contactor at 25 °C and 1 atm pressure. The flow rate of water is maintained at 0.5 lit/min. The concentration of CO₂ in the outlet solution is 0.025 kmol/m³. The equilibrium concentration of CO₂ at 25 °C and 1 atm pressure is 0.034 kmol/m³. The volume of the stirred solution is 5 litres and the specific interfacial area is 40 m²/m³ of the stirred solution. The diffusivity of CO₂ in water at 25 °C is 2×10 ° m²/s. If surface renewal theory is applicable, then calculate the fractional surface renewal rate.



Now, let us considered our earlier problem, which we have discussed in case of boundary layer theory pure CO 2 is absorbed in water in continues tear gas, liquid contactor at 25 degree centigrade and at 1 atmosphere pressure, the flow rate of water is maintain at 0.5 litre per minute, the concentration of CO 2 in the outlet solution in is 0.025 kilo mole per meter cube, the equilibrium concentration at 25 degree C n 1 at atmosphere presser is 0.034 kilo mole per meter cube. The volume of the stirred solution is 5 litre and the specific interfacial area is 40 meter square per meter cube of the stirred solution, the diffusivity of CO 2 in water at 25 degree centigrade is 2 into 10 to the minus 9 meter square per second. If the surface renewal theory is applicable then calculate the fractional surface renewal rate.

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Volumetric florid which is given which is equal to 0.5 litre meter per minute, so obtain this is equal to 8.33 into 10 to power minus 6 meter cube per second, and the outlet concentration is 0.025 k mole per meter cube. So, the steady state absorption rate absorption is 8.33 into 10 to the power minus 6 meter cube per second into 0.025 minus 0 kilo mole per meter cube. So, this is equal to 2.08 into 10 to the power minus 7 k mole per second. Now, v is given it is 5 litre, so which is equal to 0.005 meter cube, and interfacial area is 40 meter square per second.

So, we can calculate mass transfer coefficient v into a K L C equilibrium minus C is equal to 2.08 into 10 to the power minus 7 k mole per second. (()) here it is substitute k a b and equilibrium concentration is given, and C is known to us which is given C is 0.025 and this is 0.34. So, substituting these values we can calculate K L is equal to 1.15 into 10 to the power minus 4 meter per second. So, now according to surface renewal theory we know that K L is equal to root over D A B into S.

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Example: Solution

Volumetric flow rate = 0.5 Lt/min = 8.33 ×10 = 1/5

Order come: 0.025 kmol/m³

Rate q orbsorphion = 8.33 ×10
$$\frac{1}{3}$$
 × 0.015-0) km

Rate q orbsorphion = 2.08 ×10-7 kmol/s

 $K_L = 1.15 \times 10^{-4} \text{ m/s}$
 $K_L = 1.15 \times 10^{-4} \text{ m/s}$

So, K L we have calculated, K L is equal to 1.15 into 10 to the power minus 4 meter per second, and K L average is equal to root over D AB into S. So, S is equal to K L average K L average square divided by D AB. So, D AB in this case it is given D AB, the diffusion coefficient of CO 2 in water is 2 into 10 to the power minus 9 meter per second at 25 degree centigrade, so which is equal to 1.15 into 10 to the power minus 4 square divided by D AB 2 into 10 to the power minus 9, so this will be 6.61 second inverse. So, this means the surface is renewed for 6.61 times per second.

Thank you.