Mass Transfer Operations I
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Module - 2
Mass Transfer Coefficient
Lecture - 4
Boundary Layer Theory and Film Theory in mass transfer

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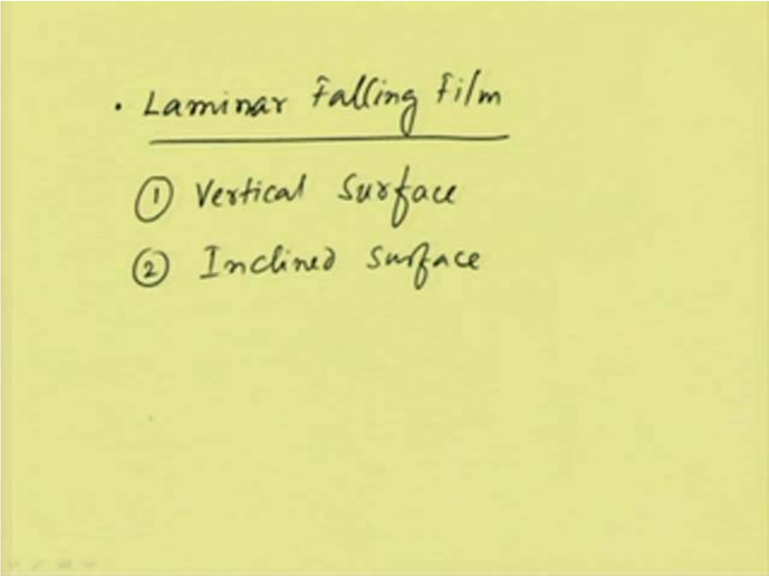

Welcome to the fourth lecture of module 2 which is on mass transfer coefficients. In the previous lecture, we have discussed mass transfer coefficient in laminar falling film. In this case, we have considered V different geometry; one is vertical surface and second one is inclined surface. In both the cases, we have discussed how to calculate the mass transfer coefficient and the flux for a particular system.

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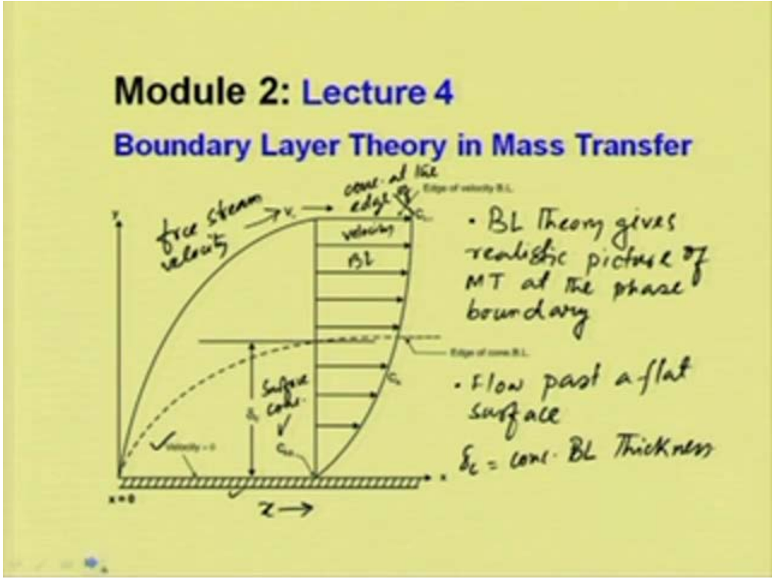

Today, we will discuss the boundary layer theory in mass transfer and also the mass transfer coefficients in turbulent flow. So, first let us consider boundary layer theory in mass transfer. So, boundary layer theory gives a realistic picture, boundary layer theory gives realistic picture of mass transfer at the phase boundary. So, let us consider flow past a flat surface. Here, the V infinity is the free stream velocity; C s is the surface concentration and C A infinity is the concentration at the edge of velocity boundary layer, velocity boundary layer and delta C is the concentration boundary layer thickness; this delta C is the concentration boundary layer thickness. So, the flow is going from x is equal to 0 and it is increasing; x is increasing towards the surface and this is the concentration and velocity profile; and the velocity at the surface is 0; there is no slip condition.

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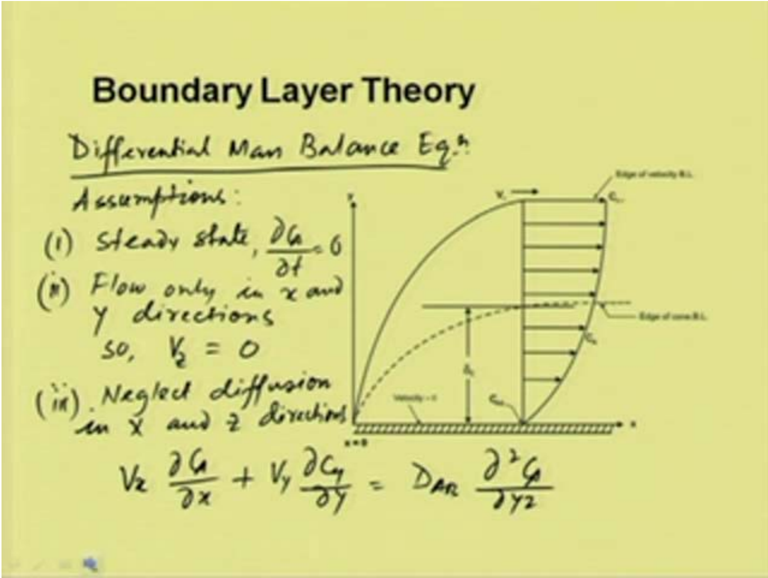

So, let us consider the differential mass balance equation and then, simplify the equations; differential. So, we know the continuity equation. Now, let us take the following assumptions. One is steady state; so, del C A del t will be 0. Flow only in x and y directions; so, V z should be 0; and third, neglect diffusion in x and z directions. So, using this assumptions, the differential mass balance equations will reduce to V x del C A del x plus V y del C y del y is equal to D AB del 2 C A del y 2.

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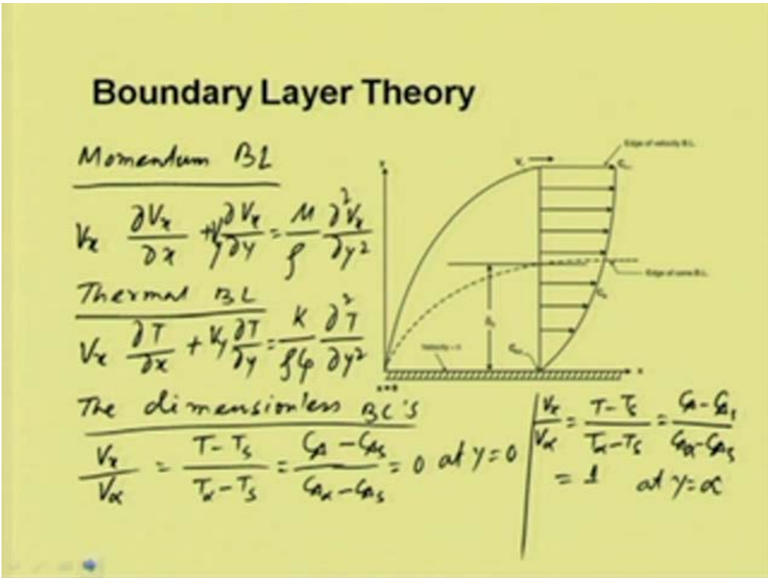

Now, the momentum boundary layer is also similar; momentum boundary layer will be also similar kind. So, V x del V x del x plus V y del V x del y is equal to mu by rho viscosity and the density, del 2 V x del y 2. So, then, the thermal boundary layer equations is also similar, which is V x del T del x, V y del T del y is equal to K, thermal conductivity, divided by rho, specific heat, C P, heat capacity, del 2 T del y 2. Now, the dimensionless boundary conditions are V C D x by V infinity would be T minus T s, surface temperature, by T infinity minus T s, will be equal to C A minus C A s, surface concentration, by C A infinity minus C A s, will be equal to 0, at y equal to 0. Similarly, V x by V infinity will be T minus T s by T infinity minus T s will be equal to C A minus C A s by C A infinity minus C A s, will be equal to 1, at y equal to infinity. So, these are the boundary conditions and the equations of the Blasius solutions can be applicable to solve these convective mass transfer problems.

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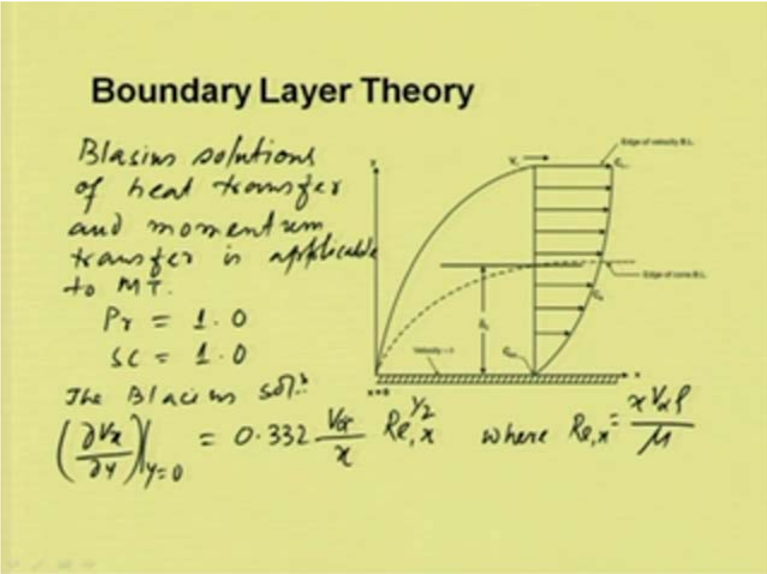

Blasius solution, solutions of heat transfer and momentum transfer is applicable to mass transfer. So, if we apply, for heat transfer, this solution is applicable when Prandtl number equal to 1. So, in this case, when Schmidt number for mass transfer is equal to 1, this is also applicable. For momentum transfer, the Blasius solutions can be written as del V x del y at y is equal to 0 is 0.332 V infinity by x Reynolds number x to the power half, where Re x is equal to x V infinity rho by mu.

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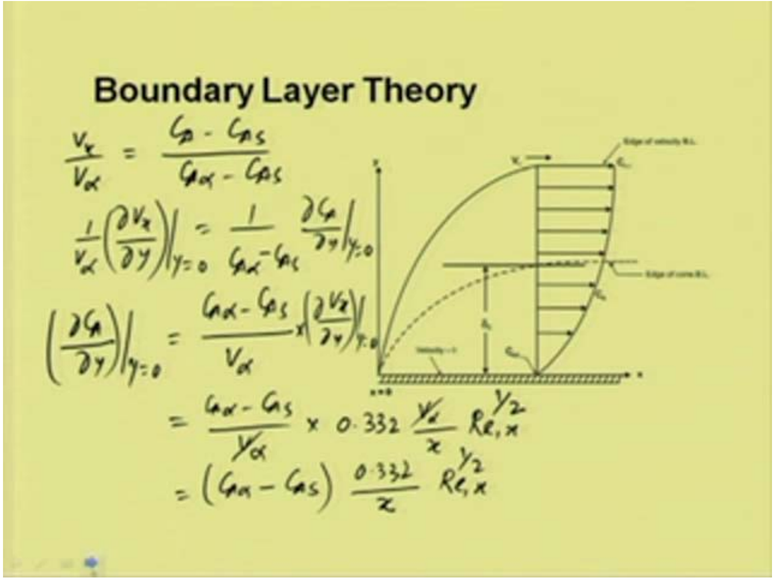

So, now, from the previous boundary conditions, we can write, V x minus V infinity is equal to C A minus C A s by C A infinity minus C A s. So, now, if you differentiate this equation, so, it will be, 1 by V infinity del V x del y at y equal to 0, would be 1 by C A infinity minus C A s del C A del y at y equal to 0. So, del C A del y at y equal to 0 will be equal to C A infinity minus C A s by V infinity into, we know the value of del V x del y at y equal to 0, that is, del V x del y at y equal to 0; now, substituting the value C A infinity minus C A s, V infinity into 0.332 V infinity by x Re x to the power half. So, this will be equal to C A infinity minus C A s into 0.332 by x, this will cancelled out; so, Reynolds number x to the power half.

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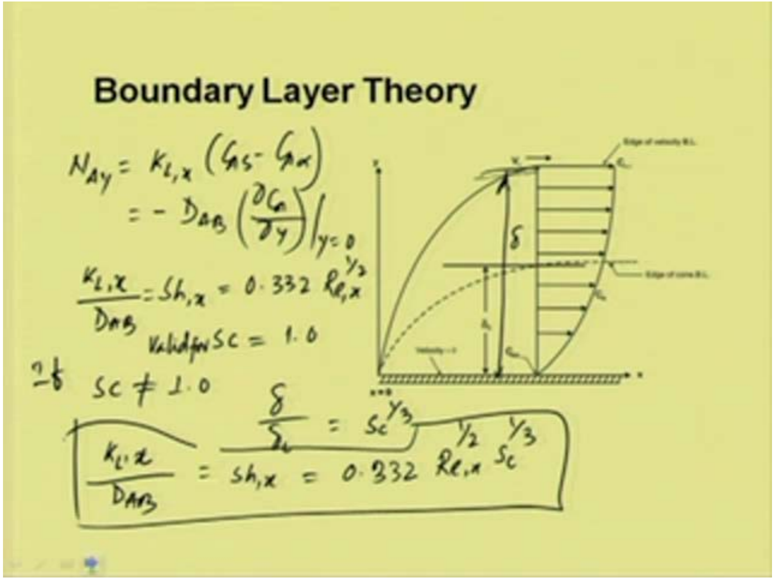

Now, if you consider for convective mass transfer, the fix equations, we can write the flux equation N A Y would be K L x C A s minus C A infinity, which is equal to minus D A B del C A del y at y equal to 0. So, now putting this value K L x by D A B equal to Sherwood number x will be 0.332 Reynolds number x to the power half. And, this is valid when Schmidt number equal to 1. Now, if Schmidt number not equal to 1, in that case, the relations between the thickness of the concentration boundary layer and thickness of the velocity boundary layer can be derived as delta. So, delta, is basically the, this is delta by delta C is Schmidt number to the power one-third. In this case, we can write the local convective mass transfer coefficient K L x divided by D A B equal to Sherwood number x, is equal to 0.332 Reynolds x to the power half, Schmidt to the power one-third. So, this is for the case when Schmidt number is not equal to 1.

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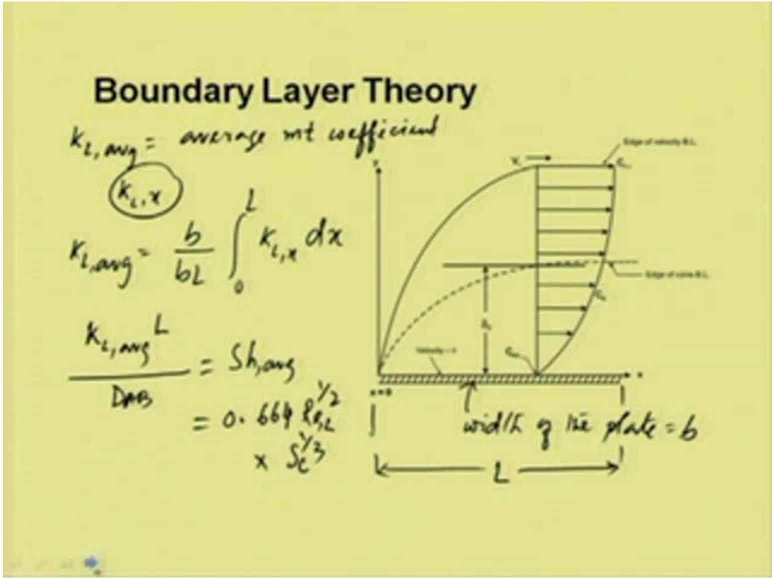

Now, the average mass transfer coefficients we can obtain K L average, we can obtain by integration of the equations of K L x with the, with the boundary conditions 0 to l. So, K L average will be equal to B by, B is width of the plate, by B L; so, the length is L; so, B L integral 0 to L K L x D x. So, the result is K L average into L divided by D A B is equal to Sherwood number average is equal to 0.664 Re L to the power half into Schmidt number to the power one-third.

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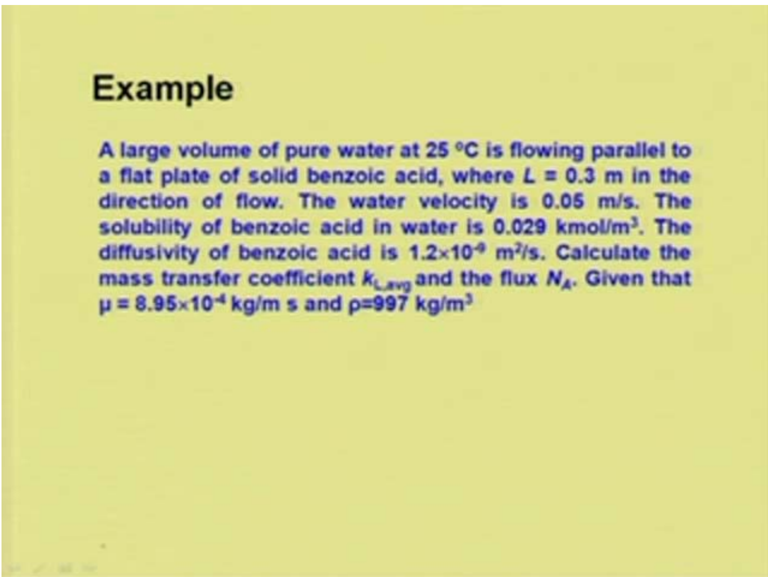

Let us consider an example. A large volume of pure water at 25 degree centigrade is flowing parallel to a flat plate of solid benzoic acid, where L is equal to 0.3 meter. In the direction of flow, the water velocity is 0.05 meter per second. The solubility of benzoic acid in water is 0.029 kilo mol per meter cube. The diffusivity of benzoic acid is given. Calculate the mass transfer coefficient and the flux, given that, the viscosity and the density of the water at that condition.

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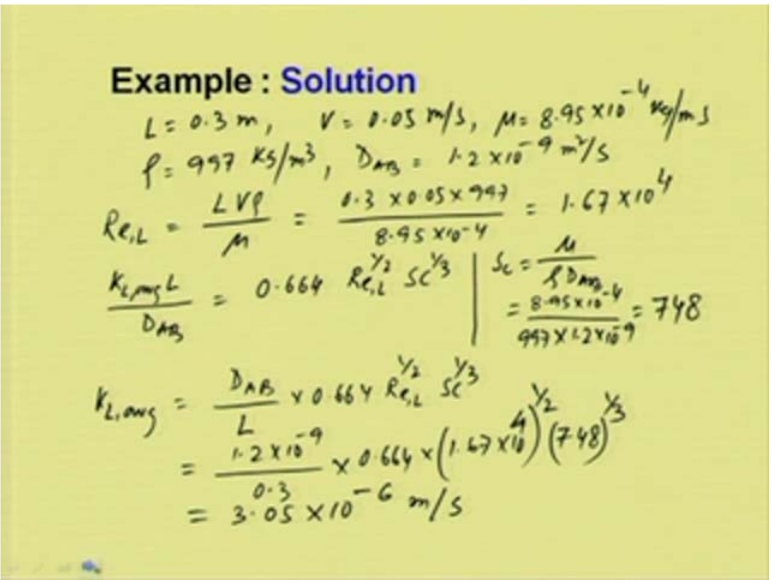

The data which are given, the length is 0.3 meter; then, velocity V is given, which is 0.05 meter per second. Then, viscosity is given, which is 8.95 into 10 to the power minus 4 kg per meter second. The data for density is 997 kg per meter cube. D AB is 1.2 into 10 to the power minus 9 meter square per second. So, we can calculate Re L is equal to L V rho by mu, which is equal to 0.3 into 0.05 into 997 divided by 8.95 into 10 to the power minus 4. This is equal to 1.67 into 10 to the power 4. This is the Reynolds number, and we know that, K L average into L by D A B is equal to 0.664 Reynolds number to the power half and Schmidt number to the power one-third. So, Schmidt number, we need to calculate. The Schmidt number for this case is Sc is equal to mu by rho D A B; so, which is equal to 8.95 into 10 to the power minus 4 divided by 997 into 1.2 into 10 to the power minus 9, which is equal to 748. So, we know the Schmidt number and the Reynolds number. Diffusivity value is given. L is known. So, K L average is equal to D A B by L into 0.664 Re to the power half Schmidt number to the power one-third. So, this is equal to 1.2 into 10 to the power minus 9 divided by 0.3 into 0.664 into 1.67 into 10 to the power 4 to the power half into 748 to the power one-third. So, this will be equal to 3.05 into 10 to the power minus 6 meter per second. So, K L average we have calculated.

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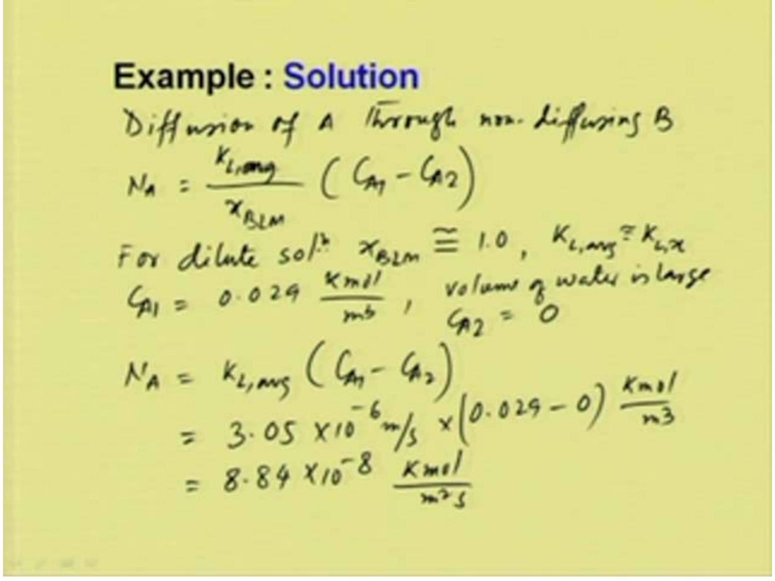

Now, the diffusion of A through non diffusing B, through non diffusing B, we can write flux is equal to K L average divided by x B L M into C A 1 minus C A 2. So, for dilute solution x B L M is approximately equal to 1.0 and K L average is approximately equal to K L x; and C A 1 is given 0.029 k mole per meter cube. Since the volume of water is large, so, C A 2 is equal to 0. So, we can write N A is equal to K L average C A 1 minus C A 2. Now, putting the values 3.05 into 10 to the power minus 6 meter per second into 0.029 minus 0 k mole per meter cube, so, it will be equal to 8.84 into 10 to the power minus 8 k mole per meter square second.

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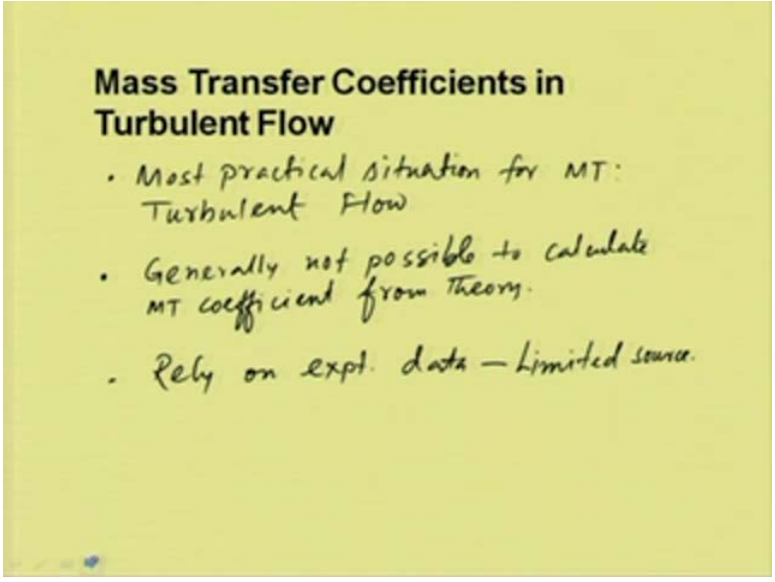

So now, we will start mass transfer coefficient in turbulent flow. In turbulent flow is the most practical situation for mass transfer, most practical situation is the turbulent flow. And, for turbulent flow, it is generally, not possible to calculate, generally, not possible to calculate mass transfer coefficient from theory. So, we have to rely on experimental data and these experimental data are of limited source.

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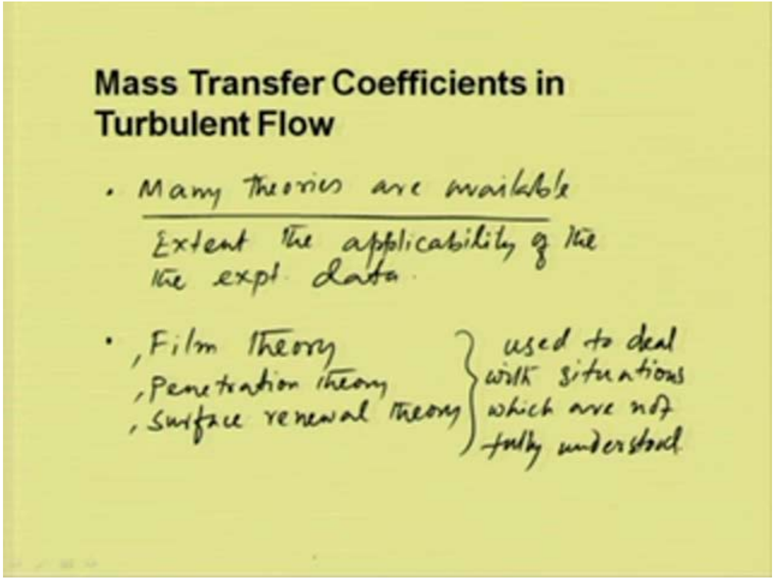

So, we need to extend the applicability of this experimental data and for this, there are many theories; many theories are available, which extend the applicability of the experimental data. Some theories of this kind are the film theories, penetration theory, surface renewal theory and some other theories. These theories generally used to deal with the situations which are not fully understood. So, for these cases, these theories are helpful to calculate the mass transfer coefficient and to understand the system clearly. Now, let us consider a very simple, simple theory which is film theory.

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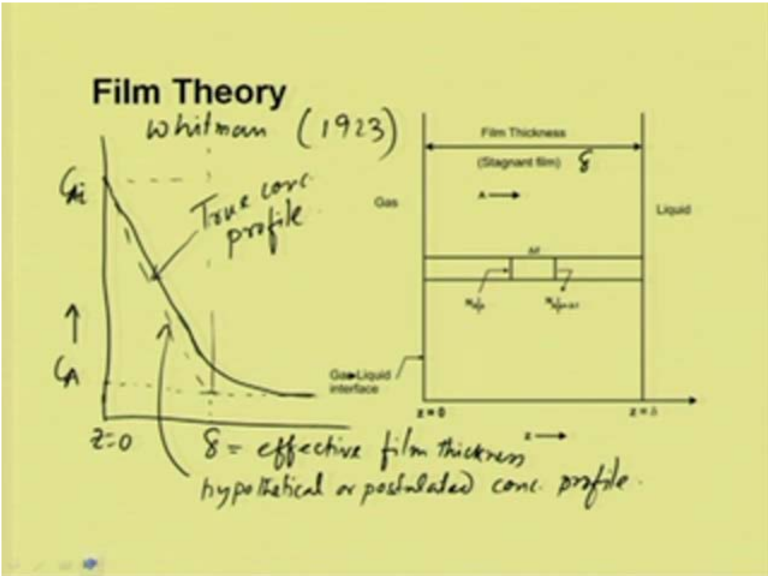

We will consider a gas liquid film and this is the gas liquid interface; and thickness of the film is delta, which is mentioned at delta. And, the concentration profile for this case looks like…So, the interfacial concentration, if you define by C A i, then the concentration profile is linear and this is the concentration and this is z, z equal to 0 and this is z equal to delta and this is the hypothetical concentration profile for the, hypothetical, or postulated concentration profile… And, the true concentration profile will look like… So, this is the true concentration profile and this thickness we called the effective film thickness. Now, this film theory is first proposed by Whitman in 1923. So, it is known Whitman film theory as well.

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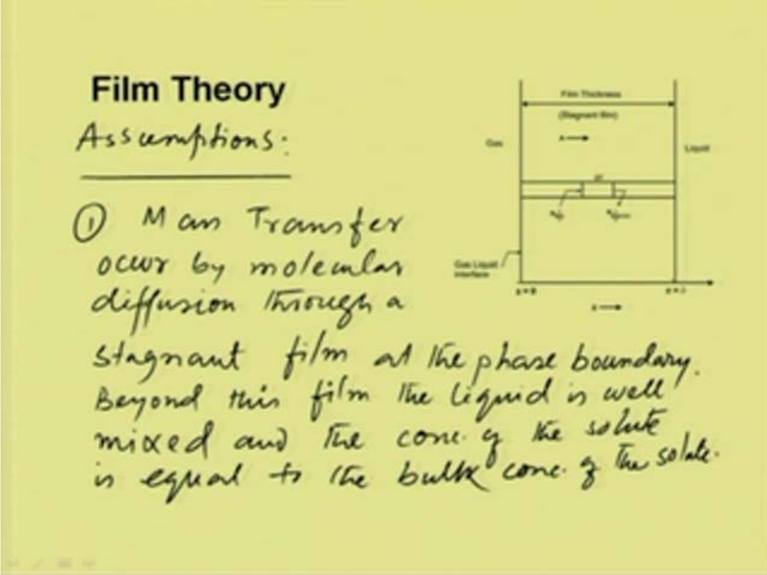

So, this theory, are based on certain assumptions. One is, mass transfer occur by molecular diffusion through a stagnant film at the phase boundary; that is, this one. Beyond this film, film, the liquid is well mixed and the concentration of the solute is equal to the bulk concentration of the solute.

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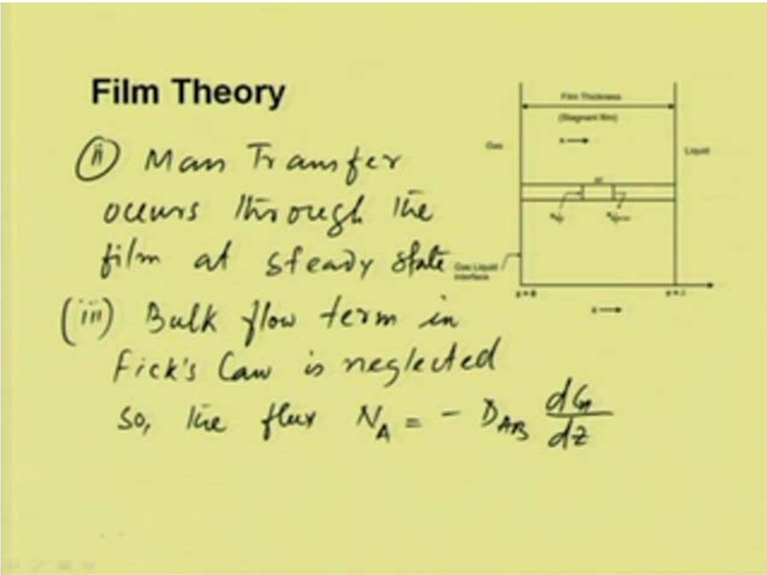

The second assumptions for this case, mass transfer occurs through the film at steady state. And third one is, bulk flow term in Fick’s Law is neglected, neglected. So, the flux can be represented as is equal to minus D A B d C A d z.

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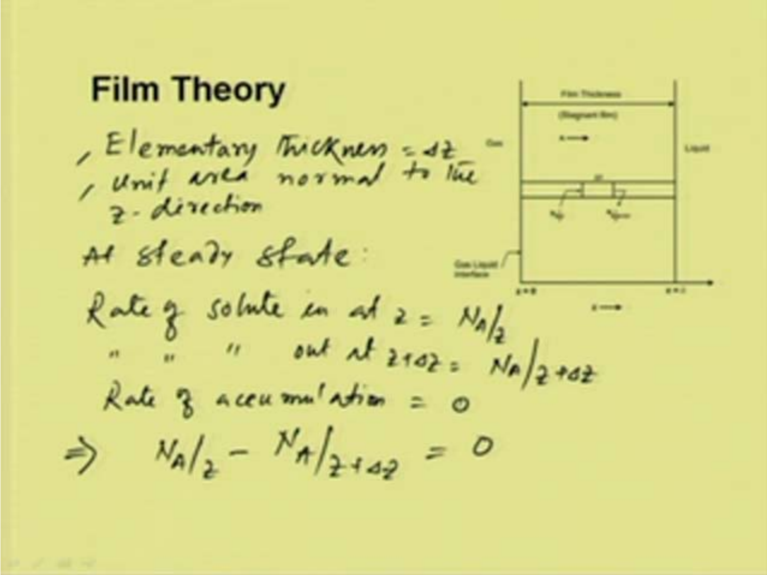

Now, let us consider a control element delta z, elementary thickness delta z and unit area, normal to the z direction, that is, the direction of mass transfer. So, at steady state, we can write rate of solute in, in at position z is equal to N A z; rate of solute out at z plus delta z is equal to N A at z plus delta z and rate of accumulation is equal to 0, at steady state. So, we can write the balance equations in N A at z minus N A at z plus delta z is equal to 0.

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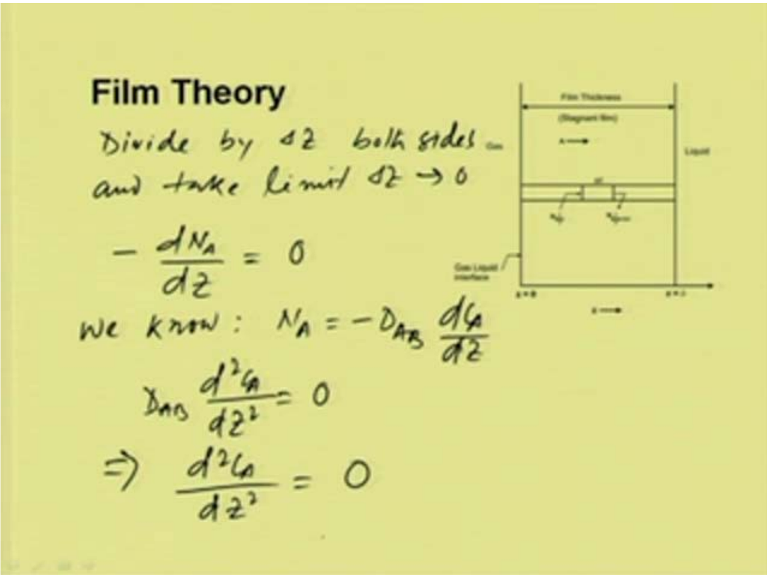

Now, if you divide this by delta z in both sides, and take limit delta z tends to 0; so, we will get minus d N A d z equal to 0. And, we know that, N A, as per the assumption, N A will be minus D A B d C A d z. So, substituting these values, it will be D A B d 2 C A d z 2 will be equal to 0. So, from this, we can write d 2 C A d z 2 is equal to 0. Now, if we integrate these equations with the boundary conditions…

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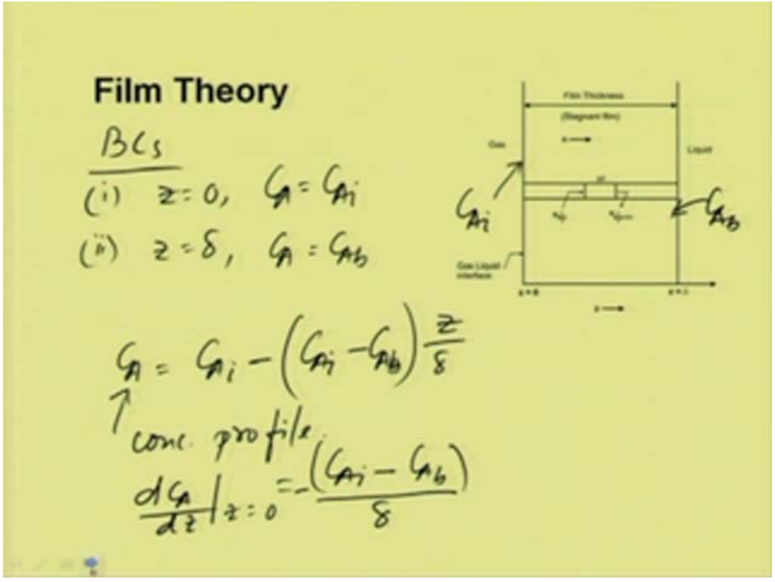

So, what are the boundary conditions, in this case? The first boundary conditions at z equal to 0, C A, we said, at the gas liquid interface C A i; So, C A will be C A i; and the second, at z equal to delta C A will be C A B; so, at the surface, so, C A B. So, if we integrate the equations, it will be C A, will be C A i minus C A i minus C A B z by delta. So, this is the concentration profile.

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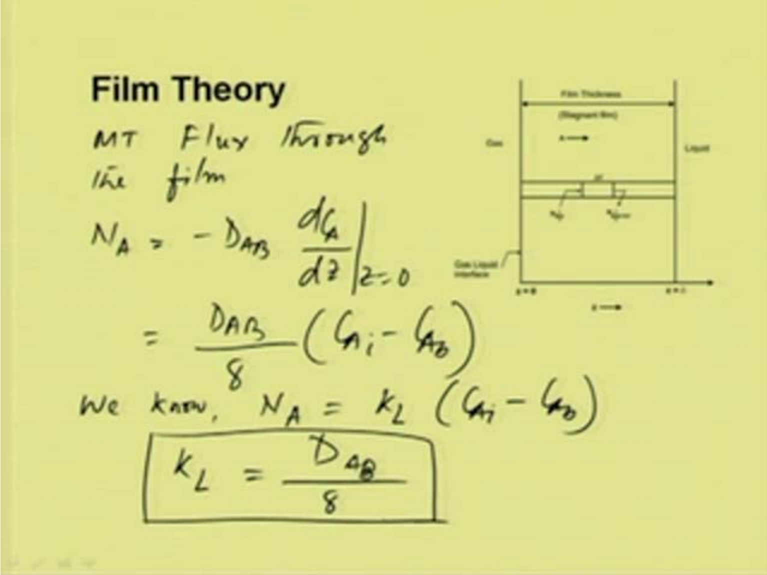

Now, at steady state, the mass transfer flux through the film is equal to N A minus D A B into d C A d z at z equal to 0. So, if we calculate d C A d z at z equal to 0 is equal to minus C A i minus C A B by delta. So, it will be D A B by delta C A i minus C A B, and we know that, N A will be K L C A i minus C A B. So, K L is equal to D A B by delta. So, this is the mass transfer coefficient which is directly proportional to diffusivity and inversely proportional to the film thickness.

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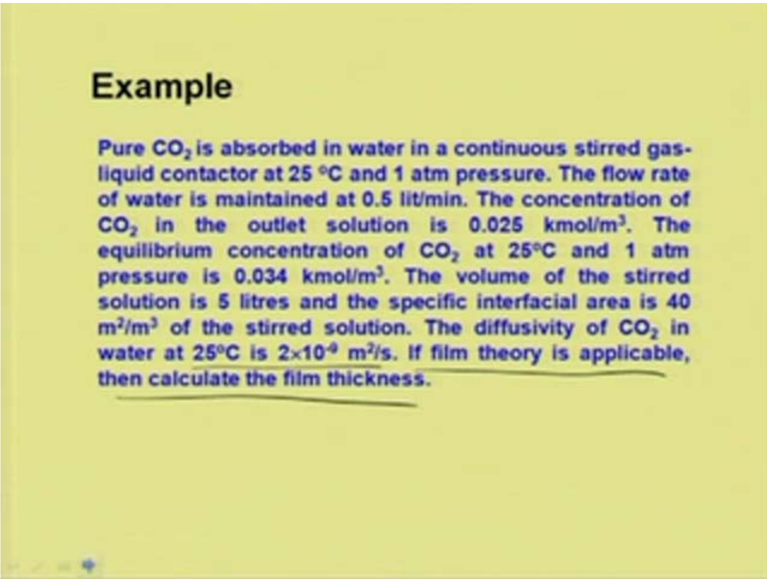

Now, let us consider a simple example. Pure CO 2 is absorbed in water in a continuous stirred gas liquid contactor at 25 degree centigrade at 1 atmosphere pressure. The flow rate of water is maintained at 0.5 litre per minute. The concentration of CO 2 at the outlet solution is 0.025 kilo mole per meter cube. The equilibrium concentration at 25 degree centigrade at 1 atmosphere pressure is 0.034 kilo mole per meter cube. The volume of the stirred contactor is given 5 litre and the specific interfacial area is 40 meter square per meter cube of stirred solution. The diffusivity of CO 2 in water at 25 degree centigrade is given. If the film theory is applicable, calculate the film thickness.

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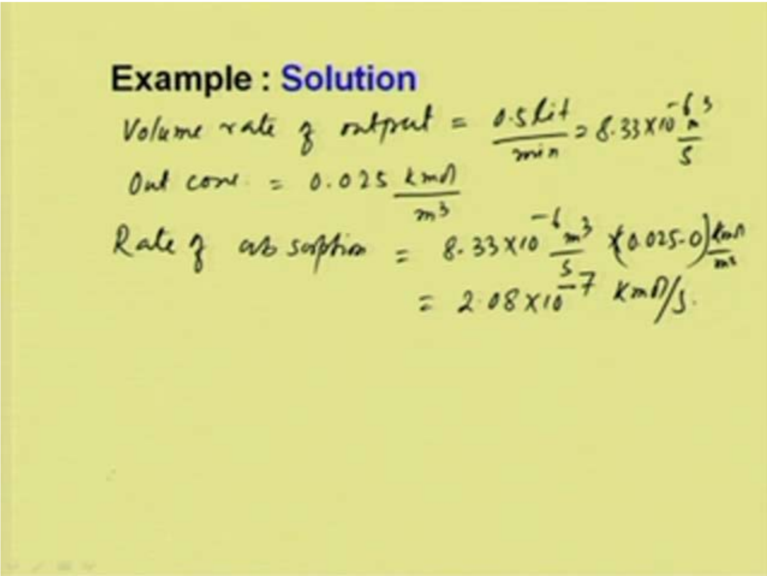

Now, we know, the volume rate of output, volume rate of output is equal to 0.5 litre per minute, which is equal to 8.33 into 10 to the power minus 6 meter cube per second. Now, the outlet concentration is given, is equal to 0.025 k mole per meter cube and we can calculate, the rate of absorption will be equal to 8.33 into 10 to the power minus 6 meter cube per second into 0.025 minus 0 k mole per meter cube, is equal to 2.08 into 10 to the power minus 7 k mol per second.

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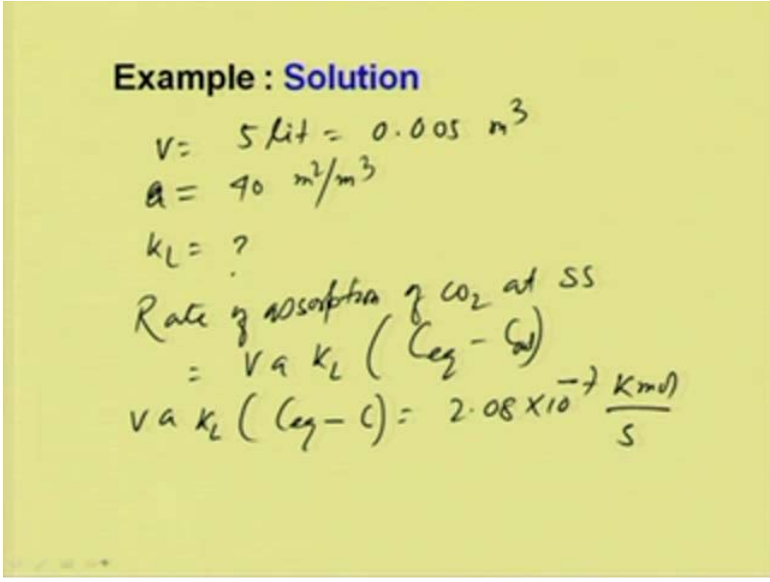

Now, volume is given; V is 5 litre, which is 0.005 meter cube; interfacial area is given, is 40 meter square per meter cube. We have to calculate K L. So, the rate of absorption, rate of absorption of CO 2 at steady state, we can calculate V a K L C equilibrium minus C outlet. So, V a K L C equilibrium minus C is equal to 2.08 into 10 to the power minus 7 k mol per second.

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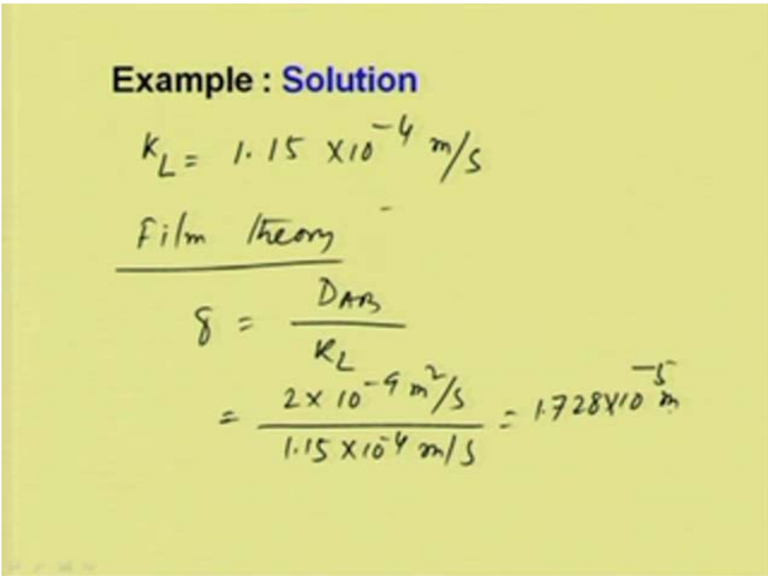

Now, from this K L, if we substitute, it will be 1.15 into 10 to the power minus 4 meter per second. According to film theory, we can calculate delta is equal to D A B by K L, so, which is equal to 2 into 10 to the power minus 9 meter square per second divided by 1.15 into 10 to the power minus 4 meter per second, which will be around 1.728 into 10 to the power minus 5 meter.

Thank you.