Mass Transfer Operations - I
Prof. Bishnupada Mandal
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Module - 2
Mass Transfer Co-efficient
Lecture - 3
Mass transfer co-efficient in Laminar Flow Condition

Welcome to the third lecture of module 2. The module 2 is on mass transfer coefficients. In this lecture, we will consider the mass transfer coefficient in laminar flow conditions. So, before going to this lecture, let us have a quick recap of our previous lecture, what we have discussed.

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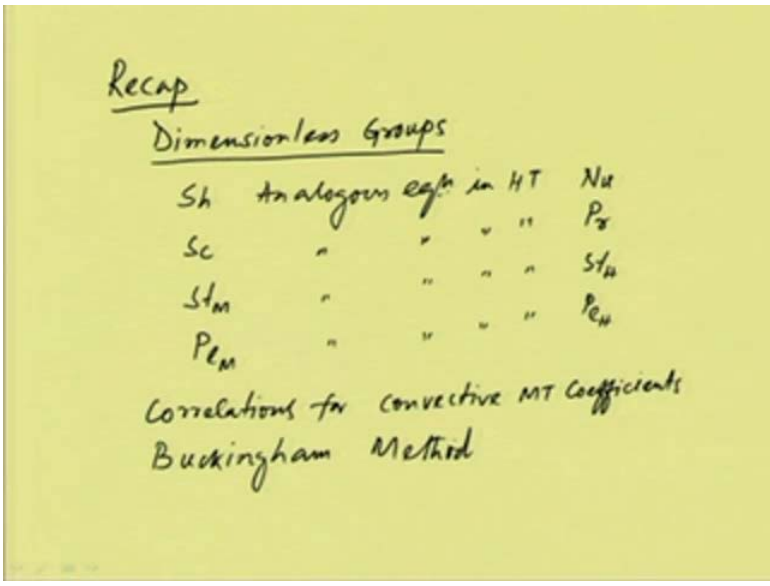

In our previous lecture, we have discussed dimensionless groups; in this, particularly, we have discussed Sherwood number, which is analogous equation in heat transfer is Nusselt number and Schmidt number; analogous equation in heat transfer is Prandtl number; Stanton number, for mass transfer; and similarly, in heat transfer, it is Stanton number for heat transfer; and then Peclet number for mass transfer and analogous equation in heat transfer is also Peclet number.

We have also discussed how to obtain the correlations for mass transfer coefficients, the correlations for convective mass transfer coefficients. Here, we have particularly discussed Buckingham method to obtain correlations for different systems. So, in this lecture, we will consider mass transfer coefficient in laminar flow. Practically, we do not need to discuss the mass transfer coefficient for laminar flow, because whatever we have discussed in module 1, which is the molecular diffusion mass transfer, that will be prevailed in case of laminar flow conditions.

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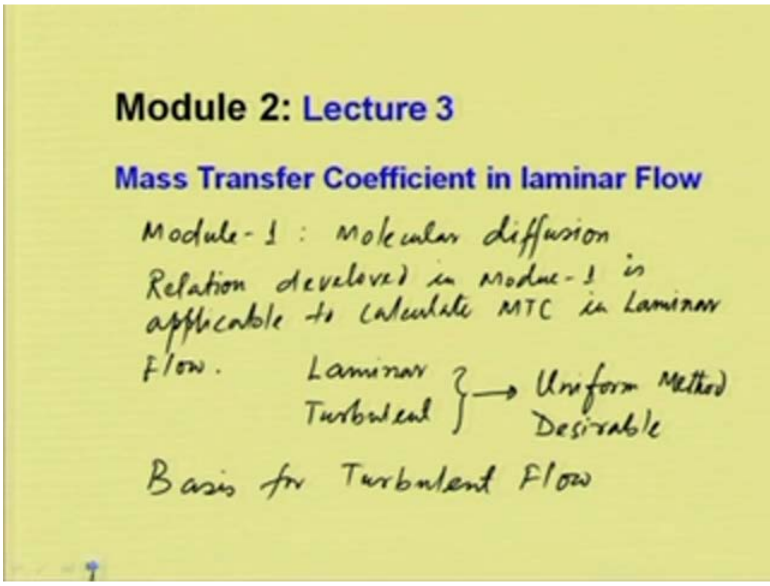

In module 1, we have discussed molecular diffusion and the relation developed in module 1 is applicable to calculate mass transfer coefficient in laminar flow. But a uniform method is desirable for the laminar and turbulent flow conditions; for laminar and turbulent, a uniform method is desirable. So, we will consider a simple case, and then, this lecture will make the basis for turbulent flow.

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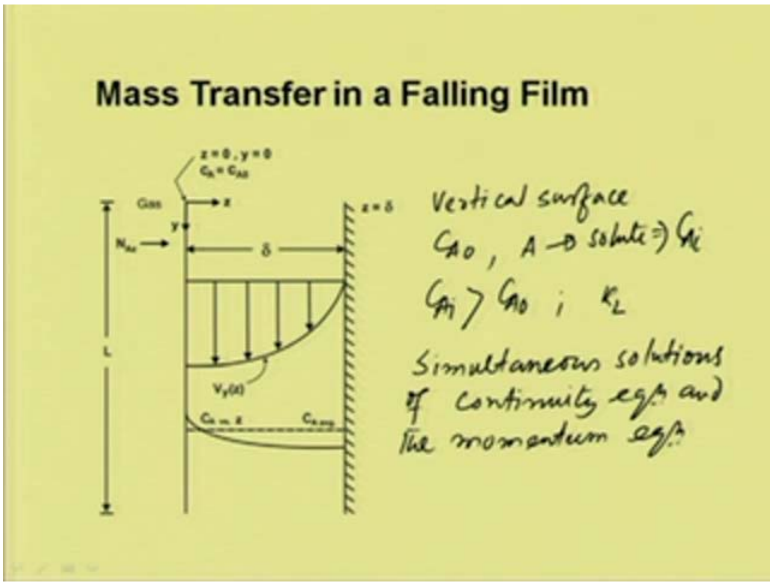

Now, let us consider a mass transfer in a thin falling film. Consider a vertical surface; this is vertical surface and where the liquid is falling from the top; and it is at initial concentration C A naught and at the interface, the concentration of the solute A is C A i, which is equilibrium with the partial pressure of A in the gas phase; and C A i is greater than C A naught, so that, the solute is dissolved in the liquid phase. And, the liquid phase mass transfer coefficient is K L. So, this problem requires the simultaneous solutions of continuity equation and the momentum equations, that is, the Navier-Stokes equation, with some simplifying assumptions.

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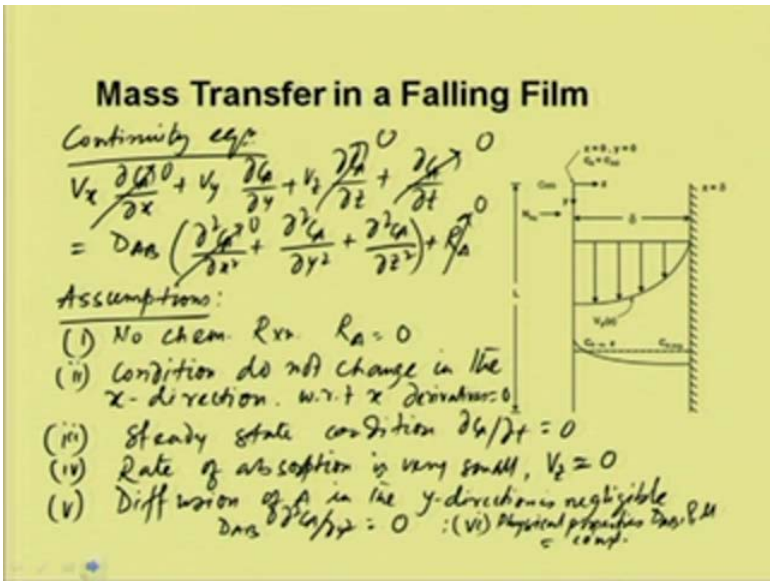

Now, let us consider the equation of continuity, which we have derived in case of unsteady state mass transfer, that is, continuity equation V x del C A del x, plus V y del C A del y, plus V z del C A del z, plus del C A del t will be equal to D AB del 2 C A del x 2, plus del 2 C A del y 2, plus del 2 C A del z 2, plus R A, the rate of reaction. Now, for this case, following assumptions can be made. No chemical reaction; so, R A will be 0. Conditions do not change in the x direction. So, all derivatives with respect to x, with respect to x, all derivatives will be 0. So, this will be 0; this will be 0. And, steady state conditions prevail; so, this will be 0; steady state condition; del C A del t will be 0. The rate of absorption of gas is very small. So, V z is essentially 0. So, in this case, this will be 0; and, diffusion of A in the y direction is negligible. So, in these conditions, D AB del 2 C A del y 2 will be 0. So, using this, and all other physical properties and chemical properties, that is, D AB, rho and mu, these are constant. So, using these assumptions, this equation reduces to the following form.

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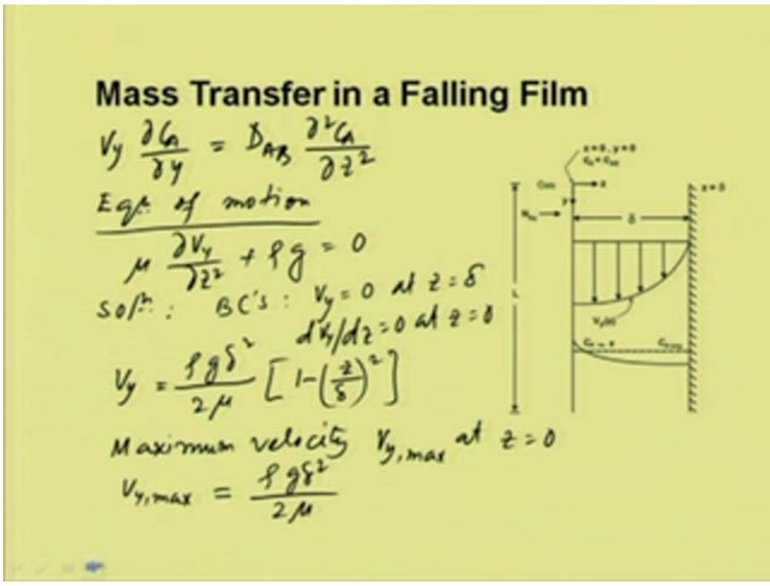

V y del C A del y will be D AB del 2 C A del z 2. So, the equations of motion under these conditions will be reduced to V mu, which is viscosity, to del V y del z square plus rho g is equal to 0. And, the solutions of this equation with the conditions, boundary conditions, we can use V y is equal to 0, at z equal to delta; that is, at the surface; and d V y d z is 0, at z is equal to 0. So, the solution is well known and we can write V y is equal to rho g delta square by twice mu into 1 minus z by delta square. Now, the maximum velocity occurs at the surface. So, the maximum velocity V y max will be at z equal to 0; that is, at the gas liquid interface. So, V y max, putting these conditions in this equation, V y max will be rho g delta square by twice mu.

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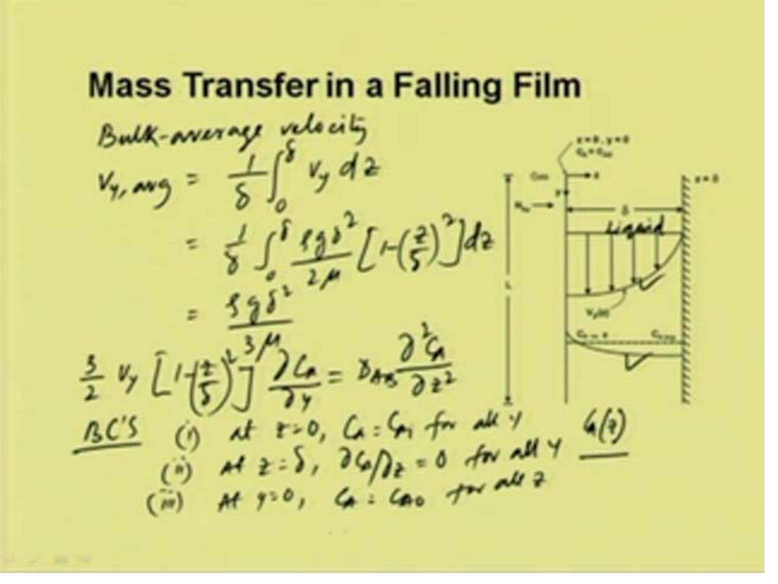

Now, the bulk average velocity can be obtained, V y average will be equal to 1 by delta, delta is the thickness of the liquid film, this is liquid film, integral 0 to delta V y d z. So, V y, we have obtained, V y equations, rho g delta square by twice mu into 1 minus z by delta square. So, if we include that, integral 1 by delta, integral 0 to delta, rho g delta square by twice mu into 1 minus z by delta square d z. So, integrating, this will be equal to rho g delta square by thrice mu. And, the film thickness, we can write, delta will be 3 mu V y average divided by rho g to the power half.

Now, if we substitute the velocity profile in the continuity equations, we have 3 by 2 V y into 1 minus z by delta square into del C A del y, will be D AB del 2 C A del z 2. So, these equations can be solved with the following boundary conditions. One is, at z equal to 0, C A will be C A i, for all y. Secondly, at z is equal to delta, del C A del z will be 0, for all y; that is, no diffusion takes place in the solid wall; and third, at y equal to 0, C A will be C A naught, for all z. So, with this boundary conditions, if you solve these equations, the concentration profile C A z, function of z, is shown with respect to length, this can be, look like this. This is concentration profile and this is the velocity profile. It is a result of infinite series solutions.

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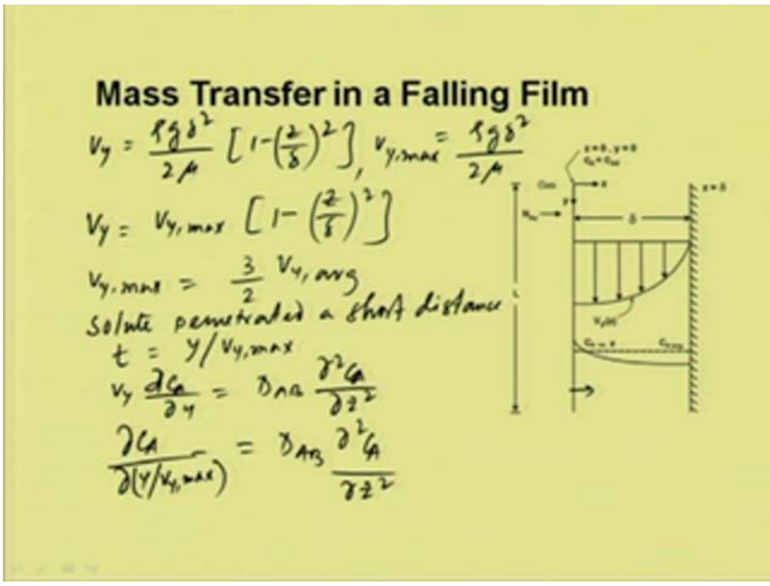

We know that, V y is equal to rho g delta square by twice mu into 1 minus z by delta square. And, V y max, when z equal to 0, that is, at the surface, is rho g delta square by twice mu. So, if you divide the ratio V y by V y max, so, we can write, V y will be V y max into 1 minus z by delta square. So, V y max, again, will be 3 by 2 V y average. We have calculated V y average. So, the relations between V y max and V y average will be this. Now, if we considered, the solute penetrated a short distance. So, if it is short distance from the surface, so, then, we can assume, the velocity will remain nearly maximum velocity. So, the time t will be y by V y max. So, the continuity equations we have derived, V y d del C A del y will be D AB del 2 C A del z 2; we can write del C A del y by V y max; it will be maximum velocity, nearly will be equal to D AB del 2 C A del z 2.

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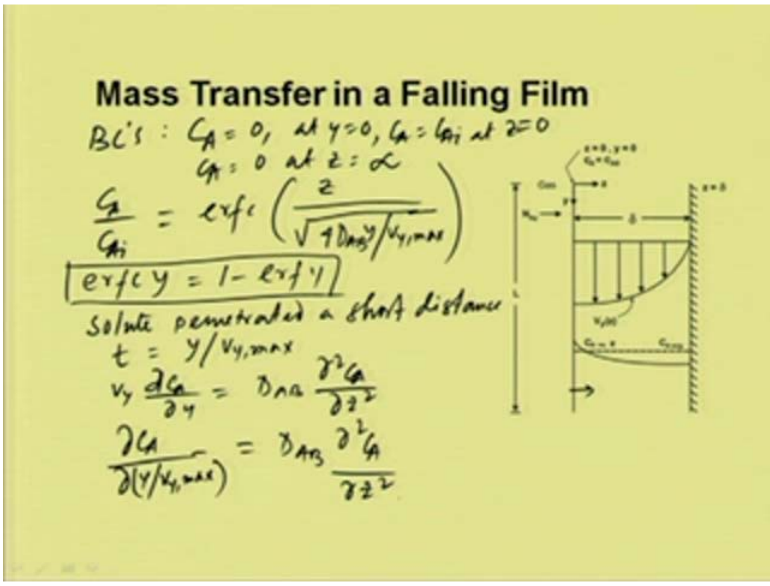

Now, if you use the boundary conditions C A is equal to 0, at y equal to 0 and C A will be C A i, at interfacial concentration, at z equal to 0, and C A will be 0, at z equal to infinity. So, if you integrate this equation, we can obtain C A by C A i will be equal to complimentary error function e r f C z by root over 4 D AB y by V y max. So, the definition of complimentary error function is e r f C y is equal to 1 minus error function of y. So, this is a tabulated function; from here, we can obtain the values.

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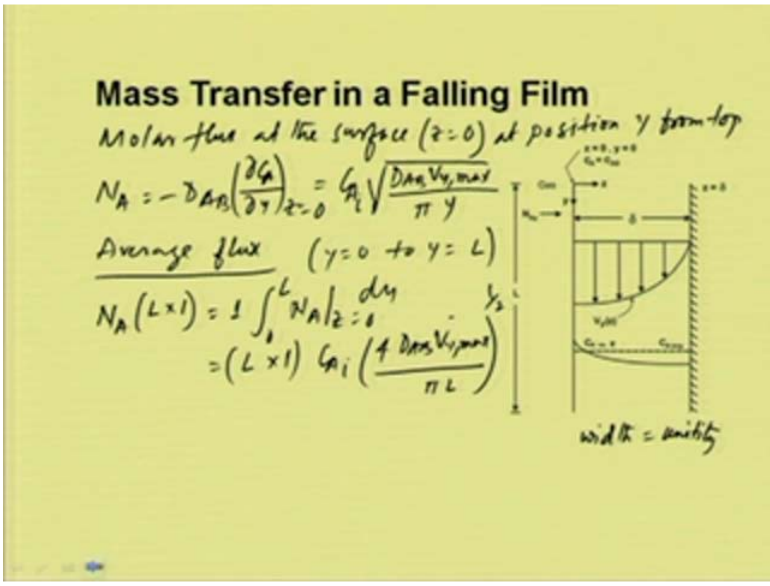

Now, the molar flux at the surface, that is, at z is equal to 0 and at position y from top, we can obtain N A is equal to minus D AB del C A del y, at z equal to 0, which is equal to C A root over D AB V y max divided by 5 y. And, the average flux over the entire surface can be written as N A entire surface at y equal to 0 to y equal to L; this is length of the film. So, N A into L into unit width, width of the film is unity, is equal to 1, integral 0 to L, N A at z equal to 0 d y. So, this will be equal to 1, integral 0 to L, C A i into D AB V y max divided by 5 to the power half into 1 by y half d y. So, if we integrate these equations, it will be equal to L into 1 C A i 4 D AB V y max by 5 L to the power half. So, this is the flux equation.

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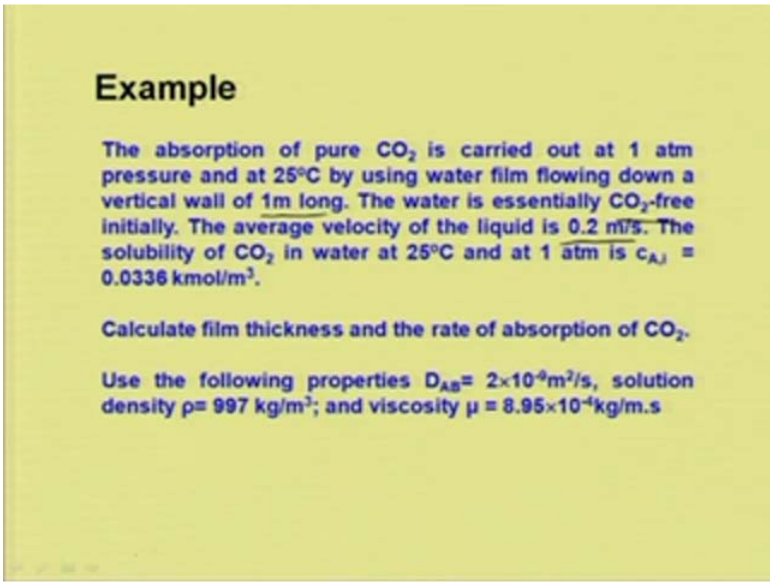

Now, let us consider a simple example, where, in a vertical surface, the absorption of pure CO 2 is carried out at 1 atmosphere pressure, and at 25 degree centigrade by using water film falling down the surface; and the length of the surface is 1 meter. The water, initially CO 2 free; and the average velocity of the liquid is given, 0.2 meter per second. The solubility of CO 2 in water at temperature and pressure conditions is given, 0.0336 kilo mole per meter cube. Calculate the film thickness and the rate of the absorption of CO 2; and, the physical, chemical properties, diffusivity, density and viscosity are given.

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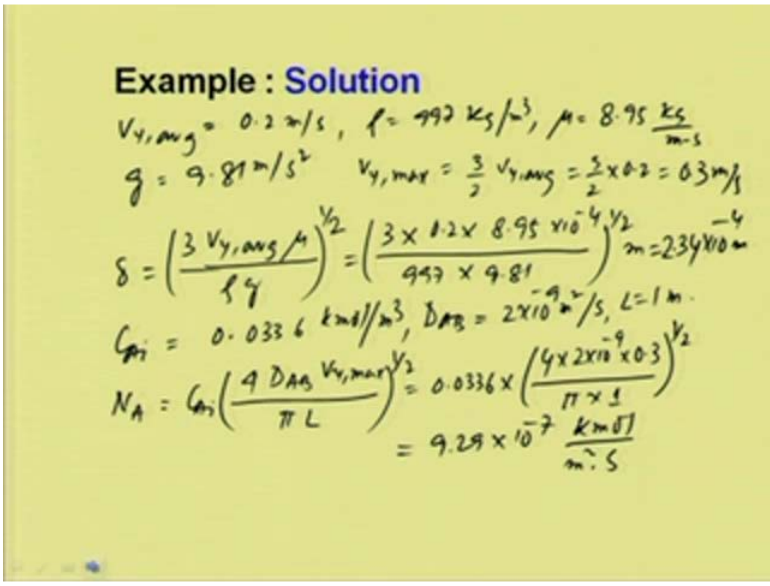

Given that, V y average is 0.2 meter per second, rho is given, 997 kg per meter cubed; viscosity is given, 8.95 kg per meter second; and, g is 9.81 meter per second square; and V y max, we can calculate, 3 by 2 V y average, which is equal to 3 by 2 into 0.2, is equal to 0.3 meter per second. Now, we can calculate the film thickness delta, is equal to 3 V y average mu by rho g to the power half; so, this will be equal to 3 into 0.2 into 8.95 into 10 to the power minus 4, divided by 997 into 9.81 to the power half meter, which is equal to 2.34 into 10 to the power minus 4 meter. Now, C A i, the interfacial concentration is given, 0.0336 k mole per meter cube; D AB is equal to 2 into 10 to the power minus 9 meter square per second; L is 1 meter. So, if you substitute, flux N A will be C A i 4 D AB V y max by 5 into L to the power half, is equal to 0.0336 into 4 into 2 into 10 to the power minus 9 into 0.3, divided by 5 into 1 to the power half, which is equal to 9.29 into 10 to the power minus 7 k mole per meter square second.

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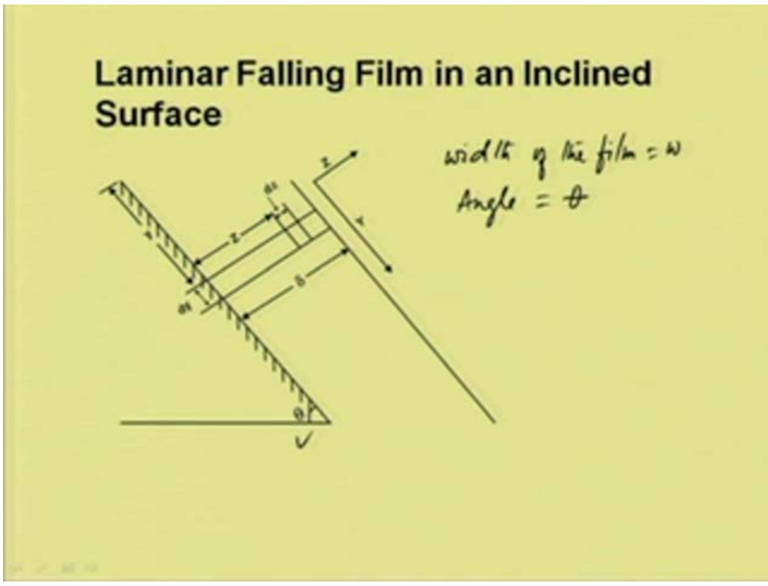

Now, let us consider another situation, where the laminar film is falling down a inclined surface. So, the depth of the liquid film is delta and width of the film is, is w, and the angle is theta.

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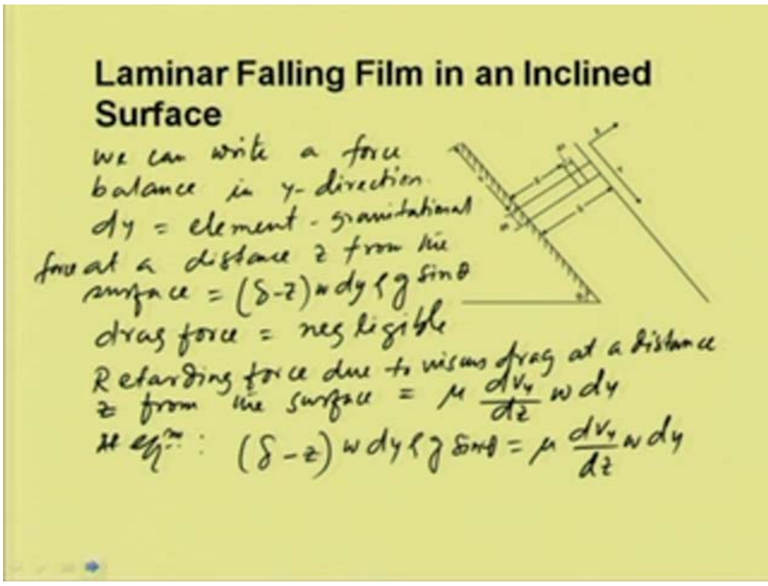

In this case, we can write a force balance in the y direction. Consider a element of fluid d y and the gravitational force acting on this element at distance z from the surface. So, we can write, this is, delta minus z w d y rho g sin theta; and, the drag force, if we considered negligible, then the retarding force due to viscous drag at z, at a distance z from the surface, we can write mu d V y d z w d y; V y is the fluid velocity. Now, at equilibrium, we can write, delta minus z w d y rho g sin theta will be equal to mu d V y d z w d y.

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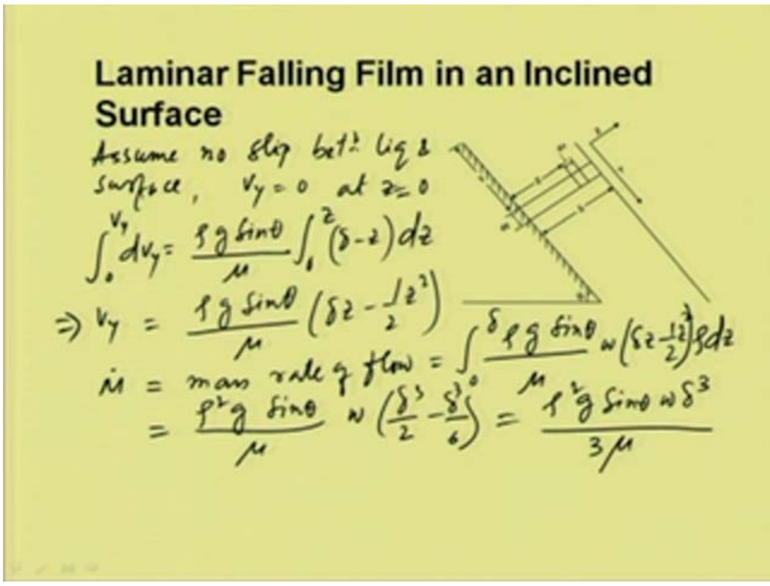

Now, if we assume no slip condition, no slip between liquid and surface, then, V y would be 0, at z equal to 0. So, we can integrate the earlier equations, integral 0 to V y d V y; this is equal to rho g sin theta by mu; integral 0 to z, delta minus z d z. So, V y will be equal to rho g sin theta divided by mu into delta z minus half z square. So, the mass rate of flow of the liquid down the surface, if we define M dash, which is equal to integral 0 to delta rho g sin theta, divided by mu into w delta z minus half z square rho d z. So, which is equal to rho square g sin theta by mu into w delta cube by 2 minus delta cube by 6, which is equal to rho square g sin theta w delta cube by thrice mu.

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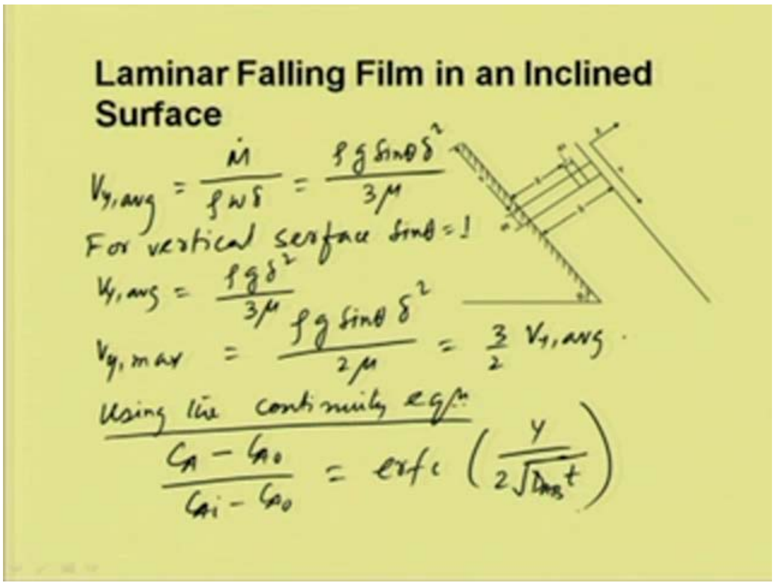

The average velocity V y average can be written as mass flow rate divided by rho w delta. So, if we substitute M dash, it will be rho g sin theta delta square by thrice mu. Now, for vertical surface, sin theta will be 1 and V y average will be equal to rho g delta square by thrice mu. The maximum velocity will be at the surface, that is, V y max; we can obtain from the earlier equations, rho g sin theta delta square by twice mu; and, which is equal to 3 by 2 V y average. The concentration profile can be obtained using the continuity equation. The concentration profile can be obtained as C A minus C A naught by C A i minus C A naught, will be complimentary error function y, divided by 2 into root over D AB into t.

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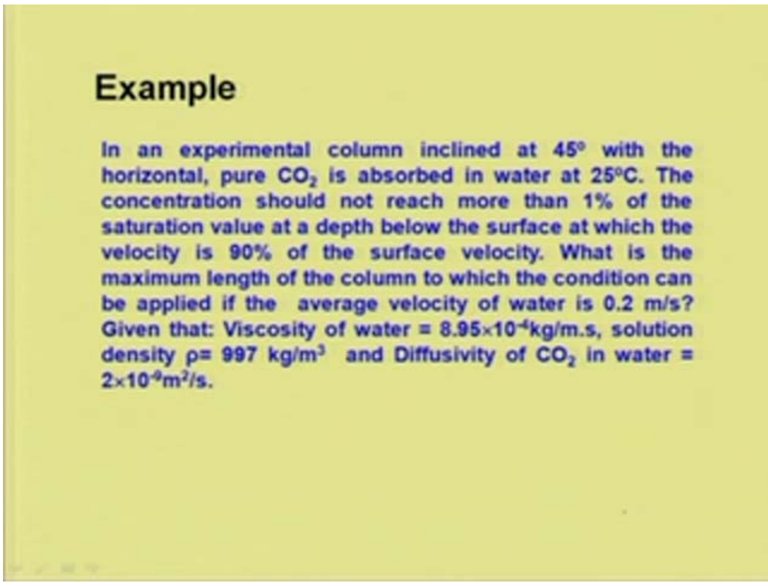

Now, let us consider a very simple example, where the surface is inclined at 45 degree with the horizontal surface and pure CO 2 is absorbed in water at 25 degree centigrade. The concentration should not reach more than 1 percent of the saturation value at a depth below the surface, at which the velocity is 90 percent of the surface velocity. What is the maximum length of the column to which the condition can be applied, if the average velocity of water is 0.2 meter per second; and, given that, all the physio-chemical properties like viscosity, density and diffusivity, which we have used for the earlier problem, is also given.

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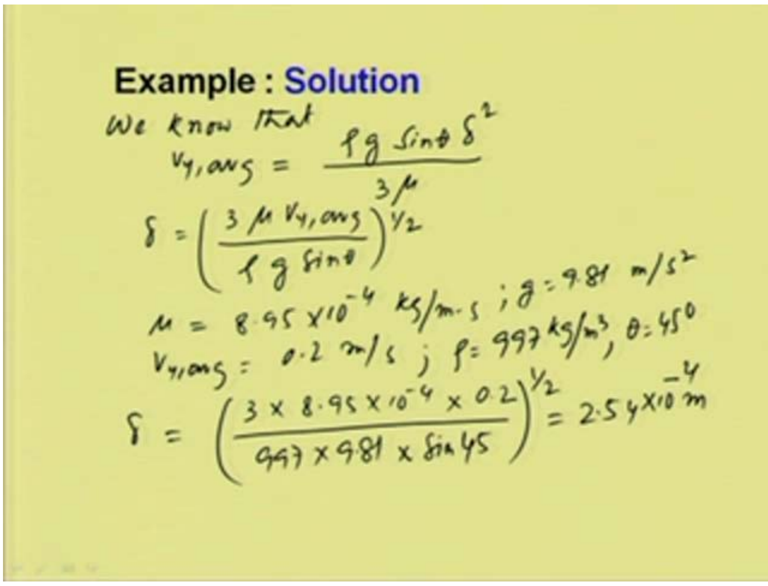

We know that, V y average will be rho g sin theta delta square divided by thrice mu. So, here, delta, we can write thrice mu, V y average divided by rho g sin theta to the power half. So, the values which are given is mu is 8.95 into 10 to the power minus 4 kg per meter second; V y average, 0.2 meter per second; rho is 997 kg per meter cube and theta is 45 degree; g is 9.81 meter per second square. Now, if we substitute this in this equation, so, delta will be 3 into 8.95 into 10 to the power minus 4 into 0.2 divided by 997 into 9.81 into sin 45 to the power half, which will be equal to 2.54 into 10 to the power minus 4 meter.

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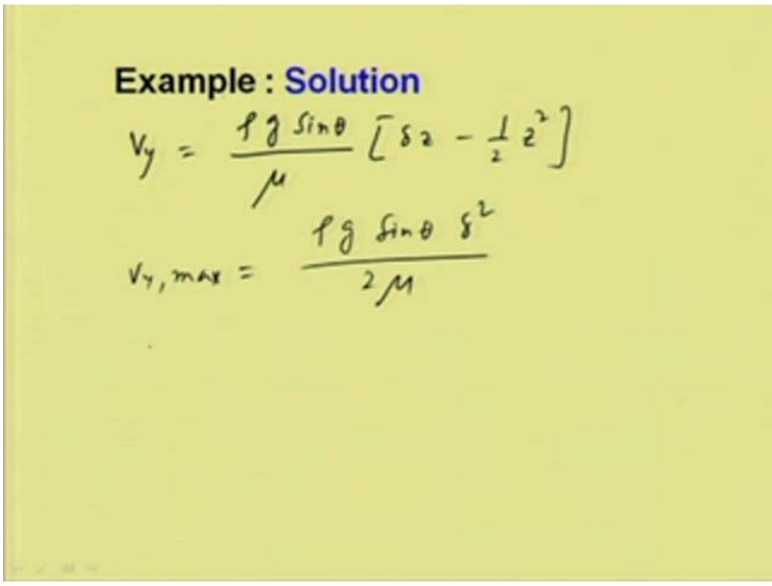

We know that, the velocity V y is equal to rho g sin theta divided by mu into delta z minus half z square; and V y max is equal to rho g sin theta delta square by twice mu.

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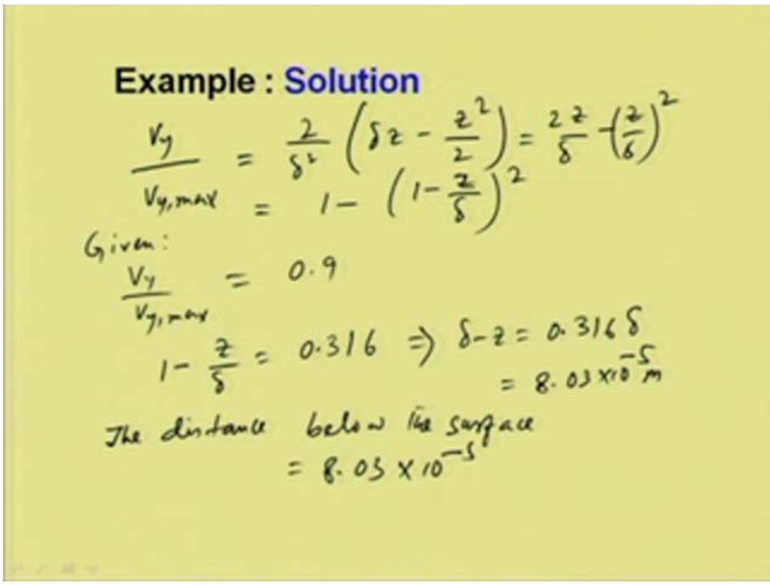

So, if we divide V y by V y max, we can write, 2 by delta square into delta z minus z square by 2, which is equal to 2 z by delta minus z by delta square, which is equal to 1 minus 1 minus z by delta whole square. And, given that, V y by V y max is 0.9, 90 percent of the surface velocity. So, if you substitute in this equation, 1 minus z by delta would be equal to 0.316; and from this, we can write delta minus z is equal to 0.316 delta, which is equal to 8.03 into 10 to the power minus 5 meter. So, the distance below the surface is equal to 8.03 into 10 to the power minus 5 meter.

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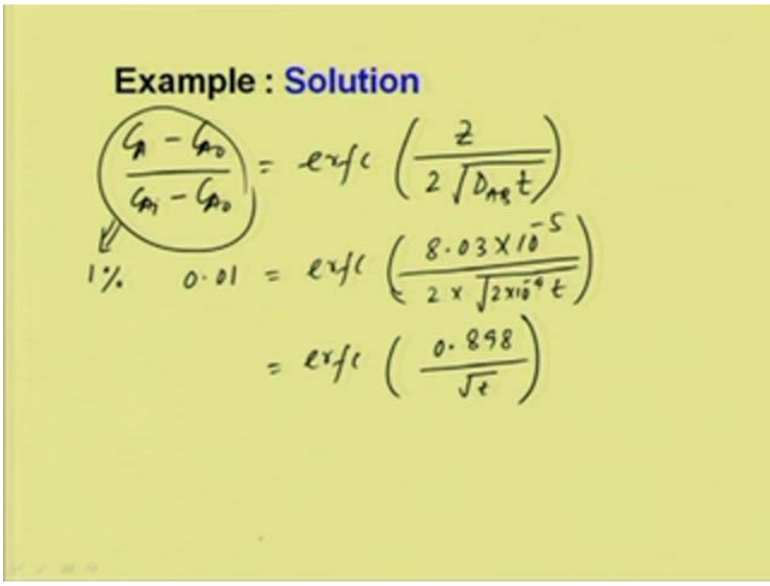

Now, the concentration profile C A minus C A naught divided by C A i minus C A naught is equal to complimentary error function into z divided by 2 into D AB t. It is given that, it should not reach more than 1 percent of the saturation value. So, this value is given, 1 percent, which is 0.01. So, 0.01 is equal to complimentary error function into z; this thickness is 8.03 into 10 to the power minus 5 divided by 2 into root over, diffusivity value is given, 2 into 10 to the power minus 9 into t; so, which is equal to complimentary error function of 0.898 divided by root over t.

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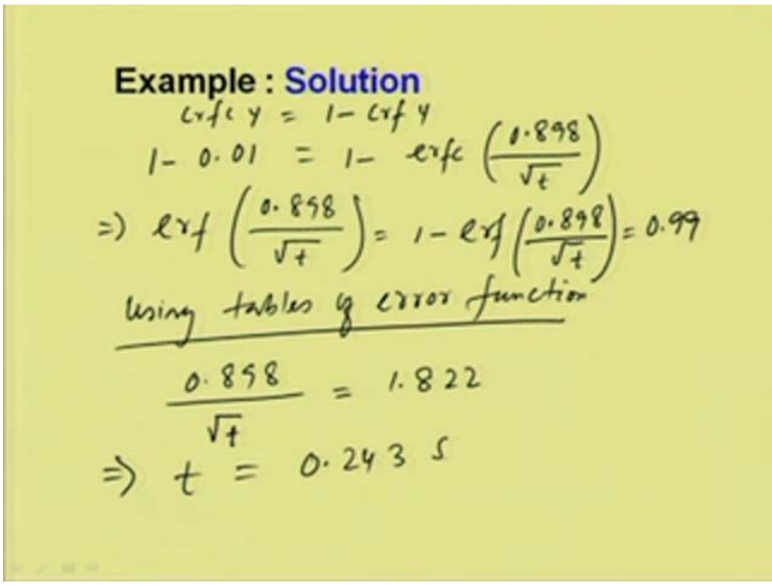

So, we can write the definition, 1 minus 0.01 is equal to, since we know c r f c y is equal to 1 minus c, complimentary error function of y. So, we can write, 1 minus 0.01 is equal to 1 minus error function, complimentary error function of 0.898 divided by root over t. Now, we can write from this, e r f 0.898 divided by root over t, is equal to 1 minus e r f 0.898 root over t, is equal to 0.99. So, if we use the table, error function, we can obtain, 0.898 divided by root over t is equal to 1.822. So, t is equal to 0.243 second.

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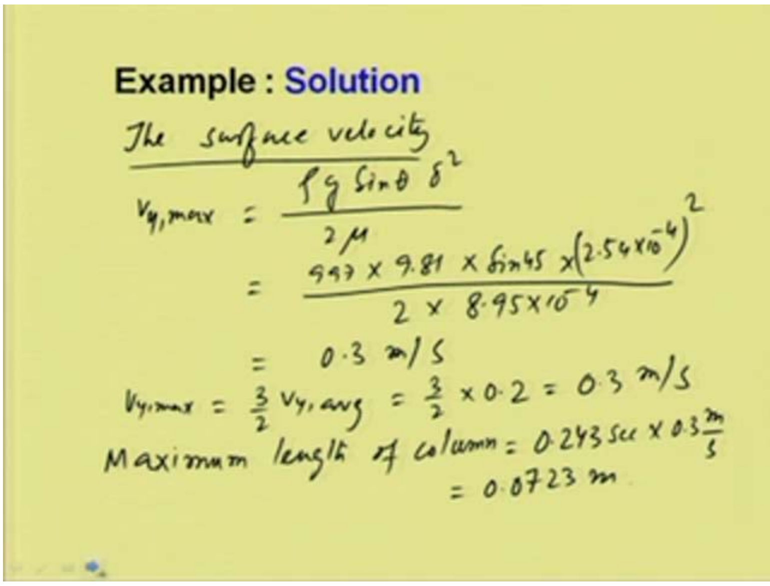

So, the surface velocity, that is V y max, is equal to rho g sin theta delta square by twice mu; so, which is equal to 997, into 9.81, into sin 45 into 2.54 into 10 to the power minus 4 square, divided by 2 into 8.95 into 10 to the power minus 4, which is equal to 0.3 meter per second; or, in other words, we can calculate V y max is equal to 3 by 2 V y average, which will be the same value, 3 by 2 into 0.2, which is equal to 0.3meter per second. So, the maximum length of the column would be equal to time, 0.243 second into 0.3 meter per second. So, it will be 0.0723 meter. So, this way, we can calculate the maximum length for the column to be required.

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