Mass Transfer Coefficients I
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Module - 2
Mass Transfer Coefficients
Lecture - 2
Dimensionless groups and correlations for convective mass transfer coefficient

Welcome to the second lecture of module 2 which is on mass transfer coefficients. So, before going to this lecture, let us have recap on our previous lecture.

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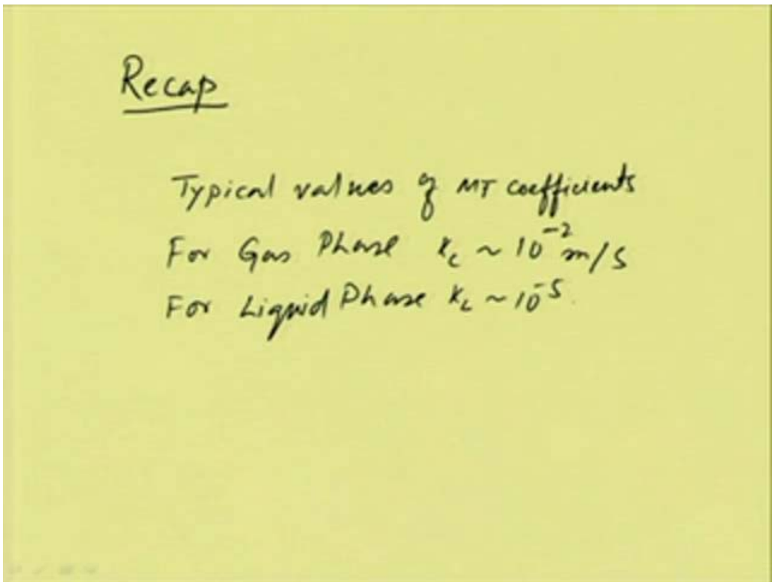

In the previous lecture, we have discussed the concept of mass transfer coefficients, where we have said that, the mass transfer coefficient is important for the convective mass transfer; and these mass transfer coefficients depends on the different systems - whether it is gas phase, or it is liquid phase, or whether the diffusion is occurring through non diffusing b, or the equimolar counter current diffusion. So, for each case, we have discussed the mass transfer coefficient and the relations among them. And finally, we have calculated the mass transfer coefficient for different systems and the typical values; typical values of mass transfer coefficients, coefficients for gas phase is about, K c is about 10 to the power minus 2 meter per second, and in case of liquid phase, K l is approximately 10 to the power minus 5 meter per second.

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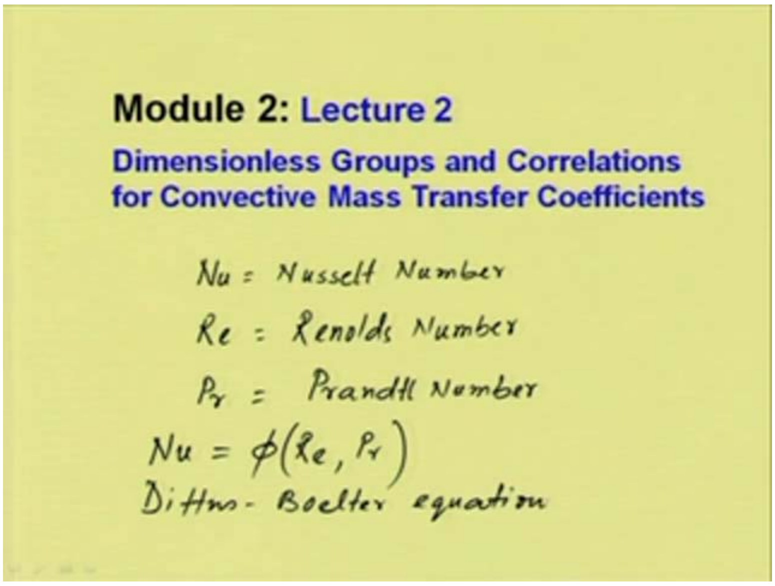

In this lecture, we will consider on two topics; one is dimensionless groups and the correlations for convective mass transfer coefficients. The dimensionless groups is generally important for the simplicity to represent the mass transfer coefficient and other variables, or physiochemical properties of the system. So, like in heat transfer, heat flux can be correlated with the heat transfer coefficient and the temperature gradient. This heat transfer coefficient, which is h, can be related with the Nusselt number Nu, which is Nusselt number. The other important dimensionless term in heat transfer is the Reynolds number and Prandtl number. For experimentally obtained data, under post convection in heat transfer, the Nusselt number can be related as a function of Renolds number and Prandtl number. The very important and useful correlations in case of heat transfer like this, is known as Dittus-Boelter equation.

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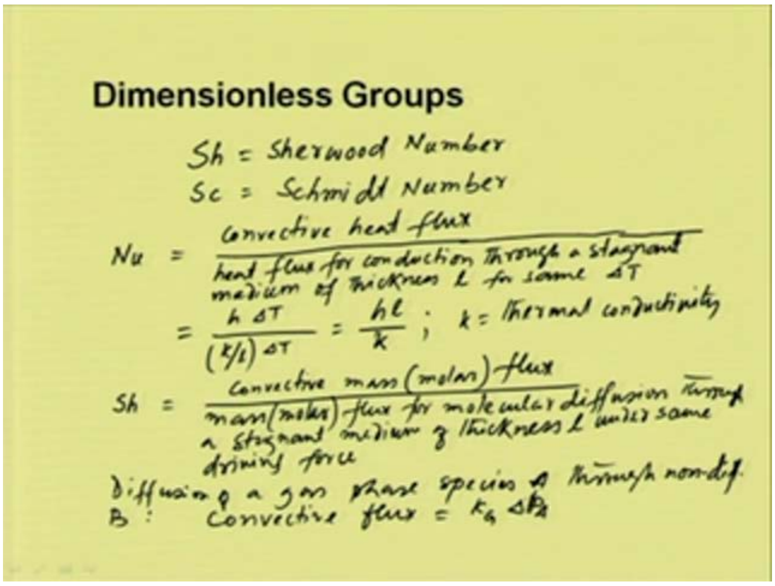

Similarly, like heat transfer, two important dimensionless groups in case of mass transfer, is the Sherwood number, Sherwood number, and the other one is Schmidt number. So, like in heat transfer, in case of Nusselt number, we define convective heat flux divided by heat flux for conduction through a stagnant medium of thickness l for same delta t, which is equal to h delta t divided by K by l into delta t, which is equal to h l by K; K is the thermal conductivity. Similarly, for mass transfer, the Sherwood number can be defined as convective mass, or molar flux, divided by mass, or molar flux for molecular diffusion through a stagnant medium, medium of thickness l, under same driving force. So, in case of diffusion of, diffusion of a gas phase species through non diffusing B, convective flux, we can write K G into delta P A; P A is the partial pressure difference.

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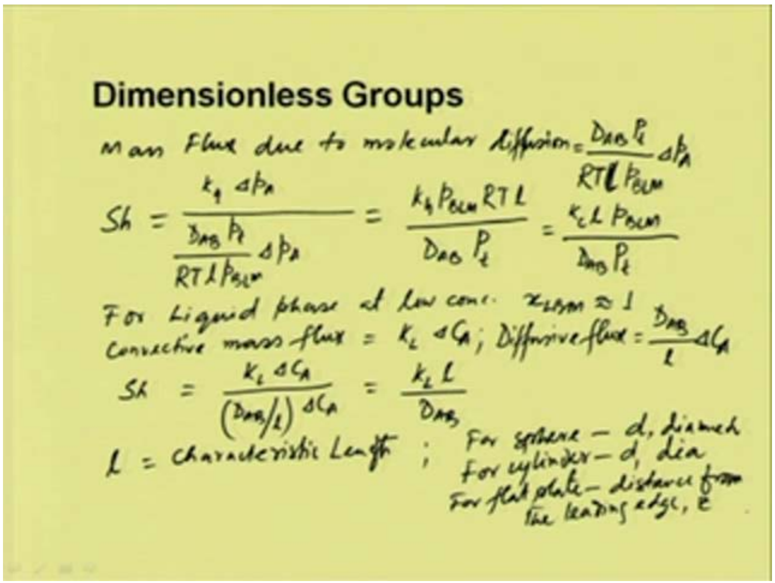

Now, the mass flux due to molecular diffusion, is equal to, we have learned in the last class, D AB P t divided by R T l P B L M delta P A. So, if we substitute in the Sherwood number definition, we will get, this K G into delta P A, which is convective flux, divided by D AB P t by R T l P B L M into delta P A, is equal to K G P B L M R T l divided by D AB P t. It is the total pressure. We can write K C l P B L M divided by D AB P t. If we consider, the transport occurs through a liquid phase, and at low concentration, for liquid phase, at low concentration, x L B M approximately equal to 1 and convective mass flux will be K L, concentration gradient, and the diffusive flux is equal to D AB by l delta C A. So, the Sherwood number, in this case, is equal to K L delta C A divided by D AB by l into delta C A, which is equal to K L, small l divided by D AB. The l is the characteristic length; for sphere, this d is the diameter, is the characteristic length; for cylinder, d dia is the characteristic length; for flat plate, distance from the leading edge, say z, is the characteristic length.

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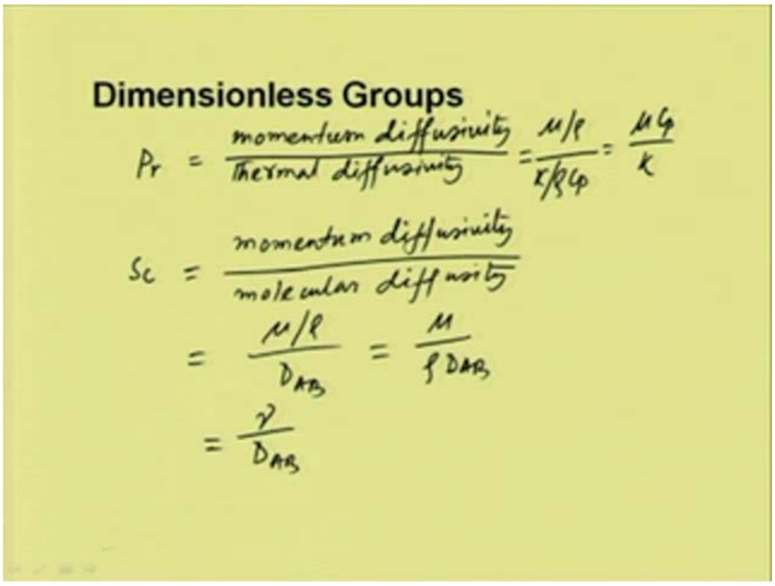

Let us consider another important dimensionless group in case of mass transfer, which is Schmidt number. We know the dimensionless group in heat transfer is Prandtl number Pr, which is defined as the momentum diffusivity, momentum diffusivity divided by thermal diffusivity, which we can write, mu by rho divided by K, thermal conductivity by rho C P, which is mu C P by K. The analogous number in case of mass transfer is Schmidt number, which is equal to the momentum diffusivity divided by the molecular diffusivity; so, which is equal to mu by rho by D AB, which is equal to mu by rho D AB, which we can write, nu by D AB. So, this dimensionless number, Sherwood number and Schmidt number, how the magnitude of these two dimensionless numbers varies for different systems?

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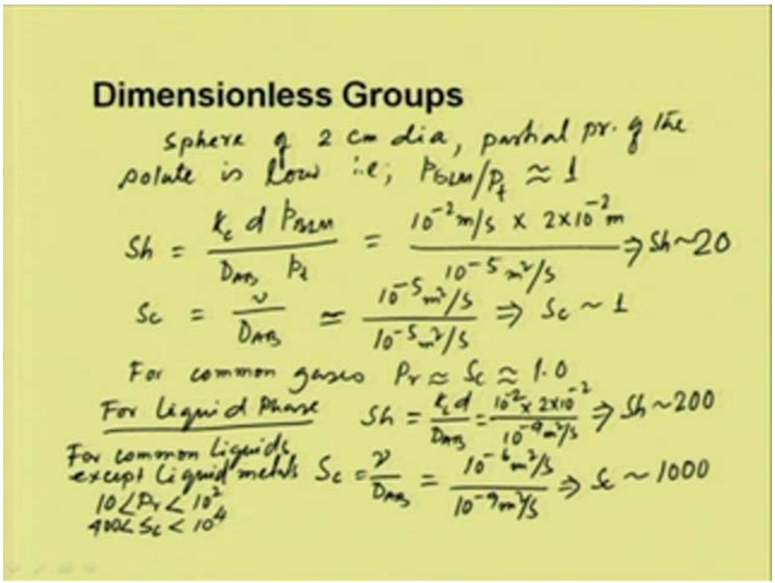

Let us consider a sphere of 2 centimeter dia, where the gas phase mass transfer is occurring, which is flowing first through this sphere; and the partial pressure of the solute is low; that is, P B L M, log means special gradient, by total pressure, will be approximately 1. The Sherwood number in this case, will be equal to K C d P B L M by D A B P B. As we know, the mass transfer coefficient is in the order of 10 to the power of minus 2 meter per second and the diameter is given 2 centimeter, which is 2 into 10 to the power minus 2 meter, and the typical values for the diffusivity is 10 to the power minus 5 meter square per second in case of gas, so, from which we can get the typical Sherwood number is approximately 20.

But Schmidt number, for this case, is equal to nu by D A B and nu, which is mu by rho will be in the order of 10 to the power minus 5 meter square per second and this value the diffusivity 10 to the power minus 5 meter square per second. From this we can obtain Schmidt number is approximately equal to 1. For common gases, gases, these values, the Prandtl number is equal to Schmidt number and this is equal to 1.0. For the liquid phase, and for similar geometry, Sherwood number is equal to K L d by D A B, which is equal to 10 to the power minus 2, into 2 into 10 to the power minus 2 divided by 10 to the power minus 9 meter square per second, which is the diffusion coefficient in case of the liquid phase. So, from here, the Sherwood number is approximately 200. And Schmidt number, in this case, is nu by D A B, and in this case, the nu is to 10 to the power minus 6 meter square per second, divided by the diffusivity value is 10 to the power minus 9 meter square per second, which leads to Schmidt number is approximately equal to 1000. So, for common liquids, except liquid metals, the Prandtl number is in the range of greater than 10, and less than 100. For the same case, the Schmidt number is greater than 400 and less than 10 to the power 4.

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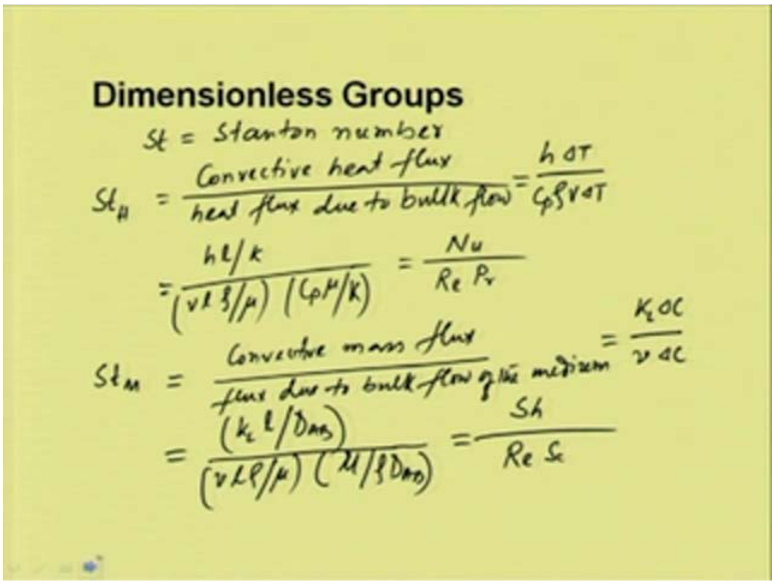

Now, let us consider another dimensionless number, which is important in case of mass transfer. A similar number in heat transfer is also exists. For heat transfer, the Stanton number S t, Stanton number. The Stanton number for heat transfer, we define the convective heat flux divided by heat flux due to bulk flow, which is equal to h delta T divided by C P rho v into delta T; which we can write, h l by K divided by v l rho by mu into C P mu by K, which is equal to Nusselt number divided by Reynolds into Prandtl. The analogous number for mass transfer is Stanton number for mass transfer, we can define the convective mass transfer, mass flux, divided by flux due to bulk flow of the material; so, which we can write K L delta C divided by v into delta C, which we can write K L l by D A B divided by v l rho by mu into mu by rho D A B; which is, we can write in terms of Sherwood number divided by Reynolds into Schmidt.

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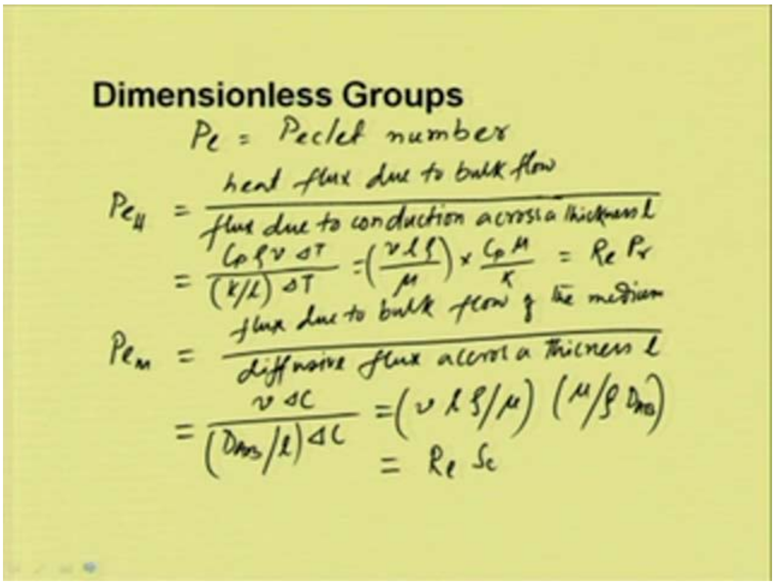

The another important dimensionless group in mass transfer, similar to heat transfer, is the Peclet number. This Peclet number, in case of heat transfer, we can define, is the heat flux due to bulk flow divided by flux due to conduction, due to conduction across the thickness, thickness l; so, which is C P rho v delta T divided by K by l into delta T, which we can divide into two groups, two dimensionless group, v l rho by mu into C P mu by K, which is Reynolds number into Prandtl number. The analogous number for mass transfer is Peclet number, is flux due to bulk flow of the medium divided by diffusive flux across the thickness, thickness l, which we can write velocity into delta C divided by D A B by l into delta C; which we can write also in two different dimensionless term, v l rho by mu into mu by rho D A B, which is equal to the Reynolds number into Schmidt number. So, these are the 4 important dimensionless number, or important, in case of mass transfer, there are two more dimensionless terms are available in case of mass transfer.

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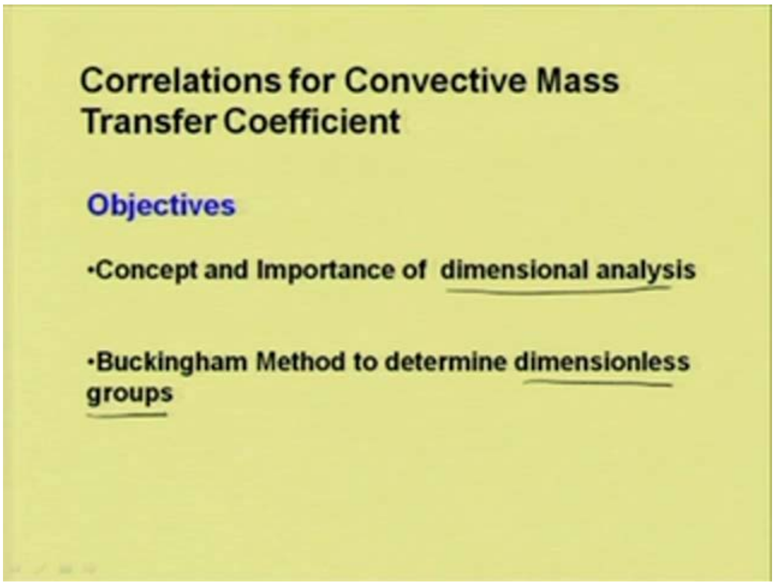

Now, the correlations for convective mass transfer coefficients; so, what are the objectives for this? It is to explain the concept and the importance of dimensional analysis for the experimental data and to obtain a useful correlation. And, how to use the Buckingham method to determine dimensionless groups involved for a particular systems. Correlations are required for the mass transfer in turbulent flow, where the mass transfer coefficient calculations is not easy from the theoretical considerations.

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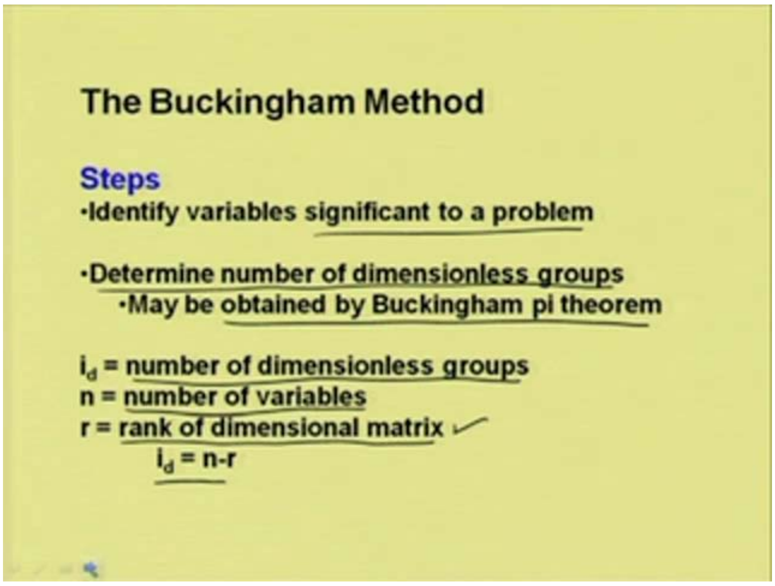

So, we will discuss the Buckingham method to obtain the correlations. So, in this method, first, we have to consider the certain fundamental dimensions. What are those? Length, which we represent by l. Length, is one fundamental dimensions and which is symbolized as l; and like area, we can write l square; volume, we can write l cube. Another dimension is time. It is symbolized as t and velocity, we can represent as l length per time, and acceleration, we can write, length per time square.

Another fundamental dimensions is mass, which is symbolized as m. Like density, if we consider density, then, it is mass per volume; this is m by l cube. So, we have to have these fundamental dimensions initially, and then, in the Buckingham method we have to identify the variables significant to a particular problem. And then, we have to determine the number of dimensionless groups. And this number of dimensionless groups may be obtained by Buckingham pi theorem. What is that? If i d is the number of dimensionless group for a particular system and there are n number of variables for that particular problem, and r is the rank of dimensional matrix, then we can write, id is equal to n minus r, the number of dimensionless groups is equal to number of variables minus rank of the dimensional matrix. So, now, we will talk about what is dimensional matrix and how to determine the number of dimensionless groups.

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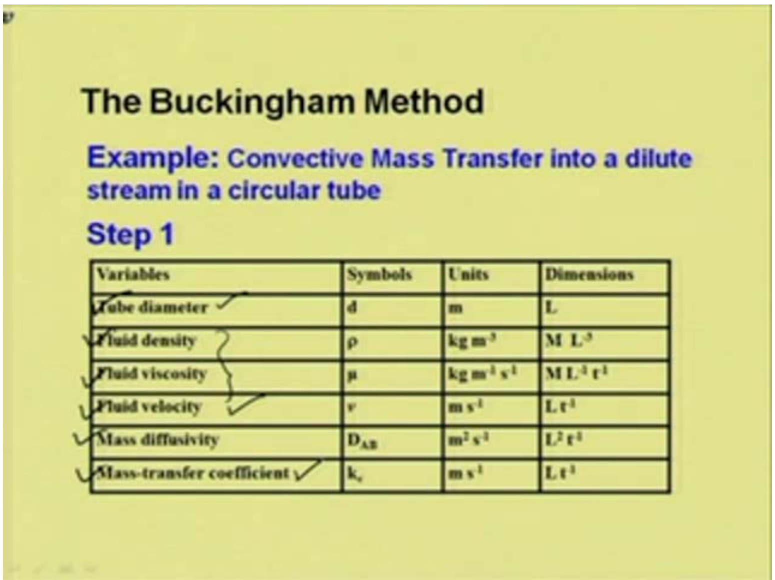

So, let us consider a simple example for convective mass transfer into a dilute stream, in a circular tube. So, the first step is to identify the variables pertaining to that particular problem. For this problem, these are the variables - tube diameter, which is symbolized as d and the unit is m and dimension is L; similarly, fluid density, fluid viscosity, fluid velocity, mass diffusivity, mass transfer coefficients. So, these are the variables which incorporate the system geometry, which is diameter. And then, fluid properties – density, viscosity and (( )) properties and the other primary quantities, that is, mass transfer coefficients.

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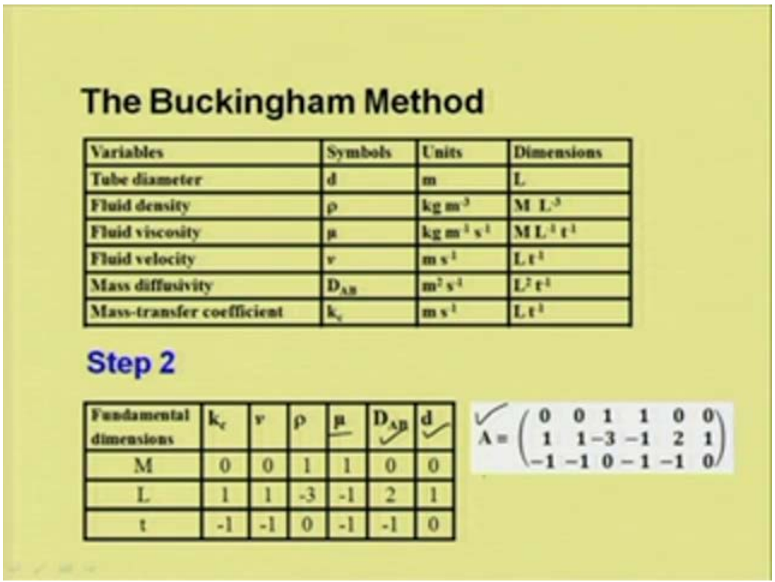

Now, if we take the fundamental dimensions M, L and t, and make another table, where the M will represent for all the exponent of the fundamental dimensions, which is in case of k C, we have l and t, l 1 and t minus 1. So, l is 1 and t is minus 1 and M is 0, in case of k C. Similarly, we will obtain this table for velocity, density, viscosity, diffusivity and diameter. So, M, we can represent all these variables in terms of fundamental dimensions. Then, the exponent of these dimensions will form a matrix which is known as the dimensional matrix. So, this is our dimensional matrix and we have to find the rank of this matrix. The rank of this matrix, we can find out using Matlab or some other program; for this case, the r is equal to rank of A, which is 3.

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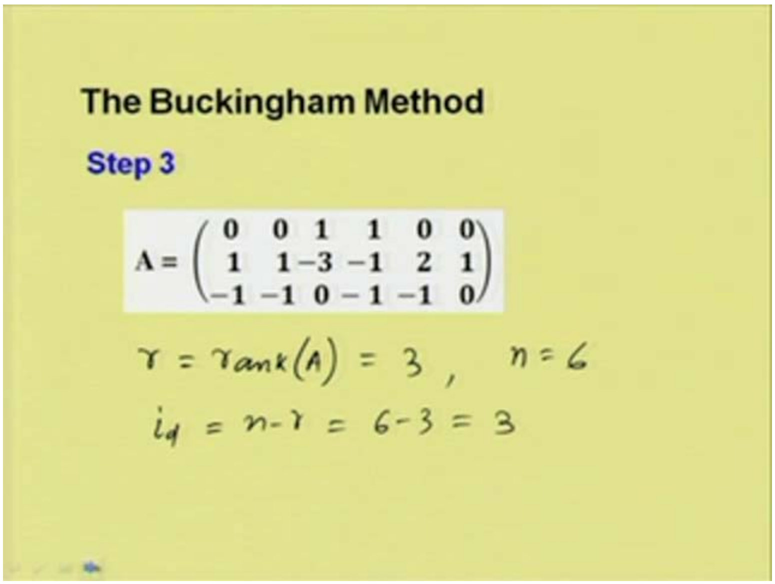

So, we can obtain id, the number of dimensionless group for a particular system will be, the number of variables n is, here is, if we can go back, we can see, there are 6 number of variables, we have considered for a particular system. So, n is 6 and rank of the matrix r is 3. So, id is n minus r. So, it will be 6 minus 3, is equal to 3. So, there are 3 dimensional groups for this particular system.

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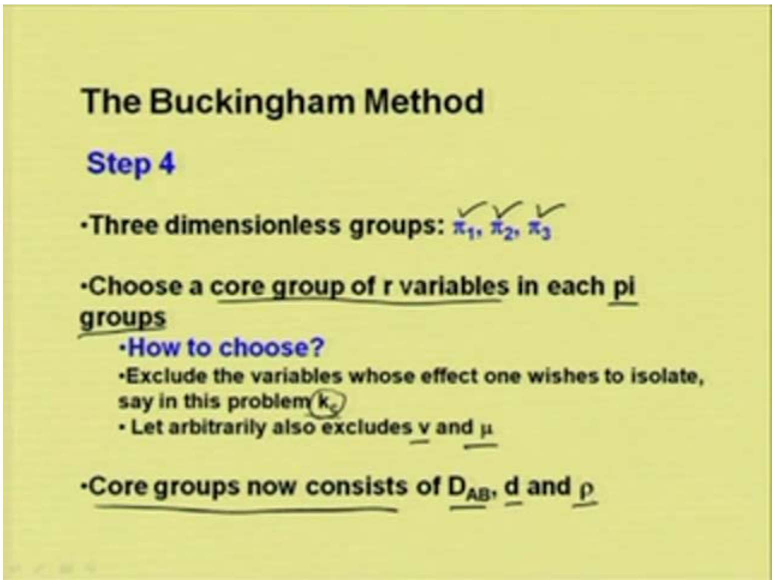

Let us symbolize this dimensionless group as pi 1, pi 2 and pi 3. So, step four, the three dimensionless group we represent pi 1, pi 2 and pi 3. Now, from the system, we have to choose a core group of r variables in each pi groups. So, how to choose these core groups? The one way to choose these core groups, is to exclude the effect of a particular variables which we want to isolate. Say, in this problem, we want to isolate the mass transfer coefficient k C. And also, let us arbitrarily exclude other variables velocity and mu for the particular system; velocity and viscosity of the fluid, we can exclude, as the variables which will not be included in the core group. So, the core group now consists of diffusivity, diameter and the density of the fluid.

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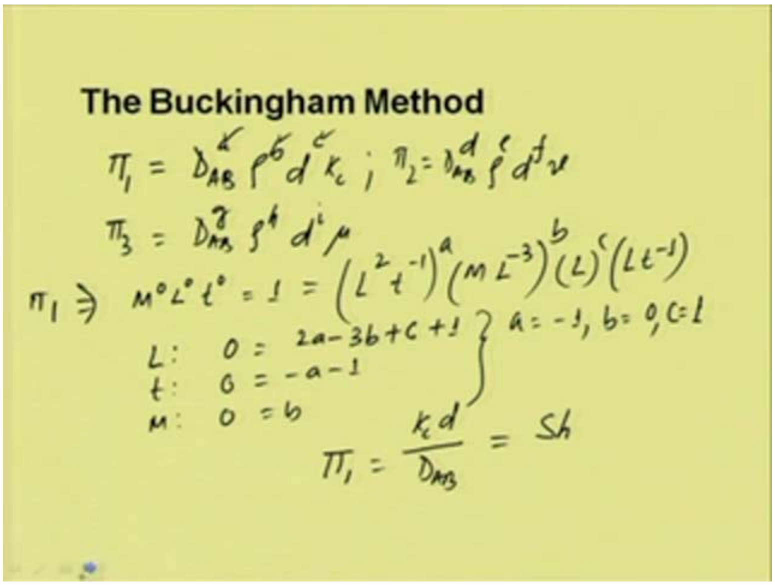

Now, we can write pi 1 is equal to D A B to the power a, rho to the power b and d to the power c, that is core group and we will include the other excluded variables, that is, K c; and, pi 2 is equal to D A B to the power d, rho to the power e and d to the power f and v ; and, pi 3 is D A B to the power g, rho to the power h and d to the power i and mu. These are the 3 groups we have identified. Now, from the dimensional form, we can write, M 0, L 0, t 0 is equal to 1, is equal to L square; for pi 1, we can write L square t minus 1 to the power a, M L to the power minus 3 to the power b and L to the power c L t minus 1. So, from this, if we solve these equations, we will have for L, we will have 0 is equal to twice a minus 3 b plus c plus 1; and 0 for t, 0 is equal to minus a minus 1 and for M 0 is equal to b. So, if we solve these three, we will have a is equal to minus 1; b is equal to 0, and c is equal to 1. So, with this, we can write, pi 1, if we have obtained power of these variables, and if we put this, pi 1 will be K c d divided by D A B, and which is known as

Sherwood number; and it is analogous to heat transfer of the dimensionless group which is Nusselt number.

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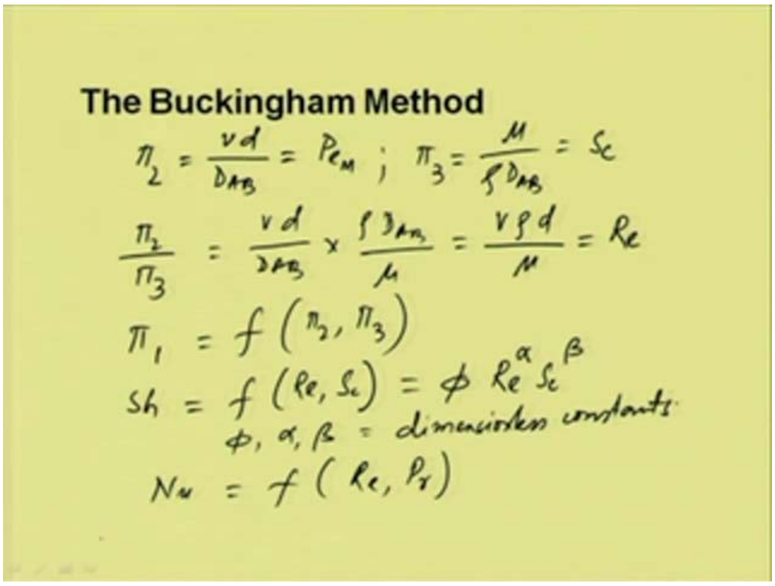

For pi 2, we will do the similar analysis, and we will obtain pi 2 is v d divided by D A B, which is equal to Peclet number for mass transfer we have discussed earlier. And, for pi 3 also, similar analysis will give you, mu by rho D A B, so, which is Schmidt number. Now, if we divide pi 2 by pi 3, so, it will give you, v d by D A B into rho D A B by mu, so, which is v rho d by mu, so, which is known as Reynolds number. So, the dimensional analysis will give us pi 1 is a function of pi 2 and pi 3; that means, Sherwood number, we can write as function of Reynolds and Schmidt number, which is in the form of phi function of, is phi into R e to the power alpha, and Schmidt number to the power beta; alpha, phi, alpha, beta are the dimensionless constants. So, this is analogous equations in heat transfer; Nusselt number is equal to function of Reynolds number and Prandtl number.

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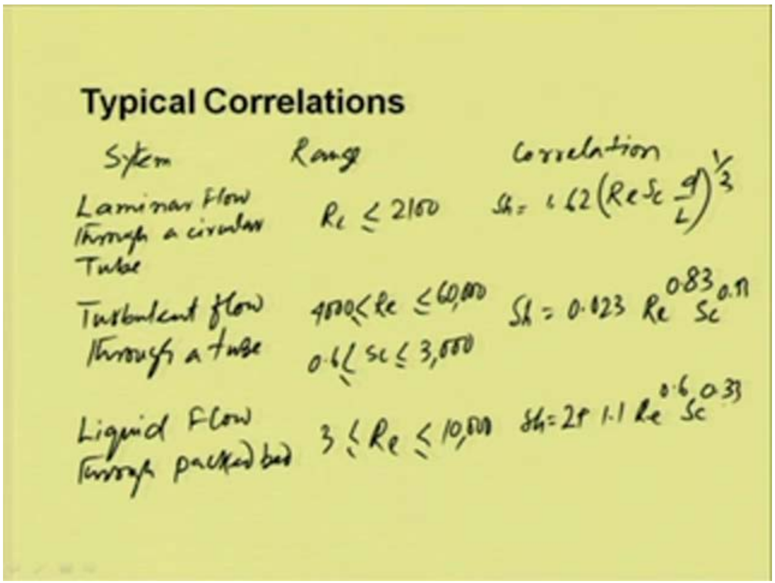

Similar correlations, we can get for different systems. Let us consider few examples of correlations like this system, Range and correlation. Laminar flow through a circular tube - if Reynolds number less than 2100, then, Sherwood number is 1.62; Reynolds Schmidt d by l to the power 1 by three. Turbulent flow through a tube, turbulent flow through a tube - if Reynolds number greater than equal to 4000 and less than equal to 60000, and Schmidt number greater than 0.6 and less than 3000. So, we have Sherwood number is equal to 0.023; Reynolds number to the power 0.83 and Schmidt number to the power 0.33; similarly, liquid flow through a, liquid flow through packed bed. So, here, Reynolds greater than equal to 3 and less than 10000; the Sherwood number is equal to 2 plus 1.1, Reynolds to the power 0.6 and Schmidt to the power 0.33.

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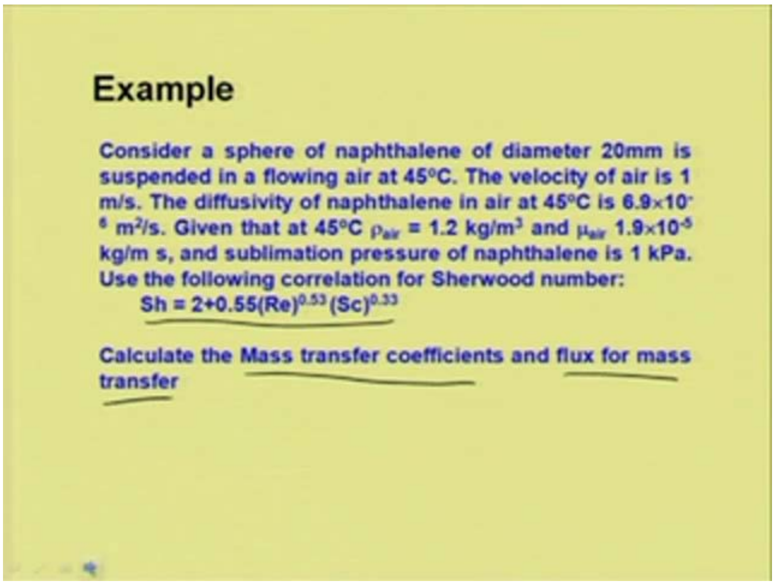

Now, let us consider one example. A sphere of naphthalene of diameter 20 millimeter is suspended in a flowing air at 45 degree Centigrade. The velocity of air is 1 meter per second. The diffusivity of naphthalene in air at 45 degree Centigrade is given. Given that the density and viscosity of the air and sublimation pressure of naphthalene as 1 kilopascal and using this correlation, calculate the mass transfer coefficient and flux of mass transfer.

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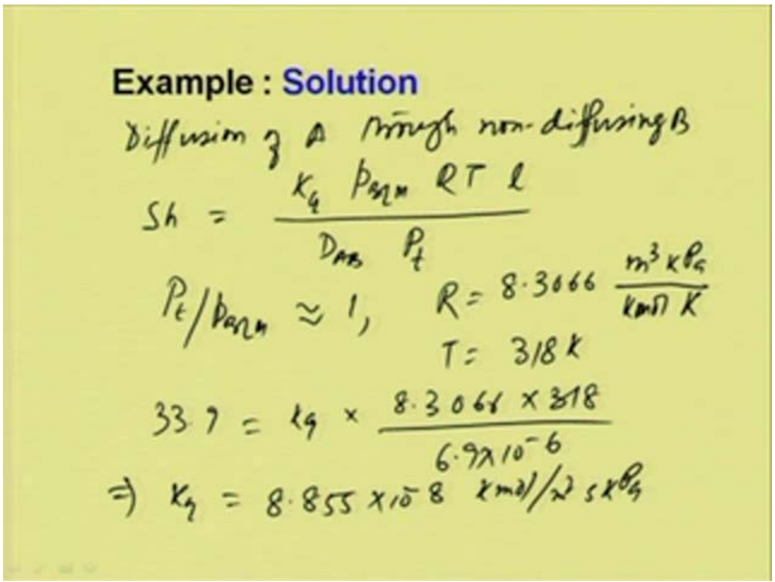

So, given the data, we can calculate, Reynolds number will be d v rho by mu, which is equal to 0.02 into 1 into 1.2, divided by 1.9 into 10 to the power minus 5, which is 1260. And, Schmidt number also we can calculate, mu by rho D A B, which is equal to 1.9 into 10 to the power minus 5, divided by 1.2 into 6.9 into 10 to the power minus 6; so, it will be 2.29. And, Sherwood number, as the equations given, we can calculate 2 plus 0.55, Reynolds to the power 0.53 and Schmidt to the power 0.33. Substituting the data, it will be 33.9. Now, we can write the diffusion, diffusion of A through non diffusing B. So, we can calculate Sherwood number is equal to K G P B L M R T l divided by D A B P t and at 45 degree Centigrade, the vapour pressure is small. So, P t by P B L M is approximately 1; R is equal to 8.3066 meter cube kilopascal per K mol Kelvin and t is 318 Kelvin. So, we can write, Sherwood number is given, 33.9 is equal to K G into 8.3066 into 318 divided by 6.9 into 10 to the power minus 6. So, from this we can calculate K G, which is 8.855 into 10 to the power minus 8 K mol per meter square second kilopascal.

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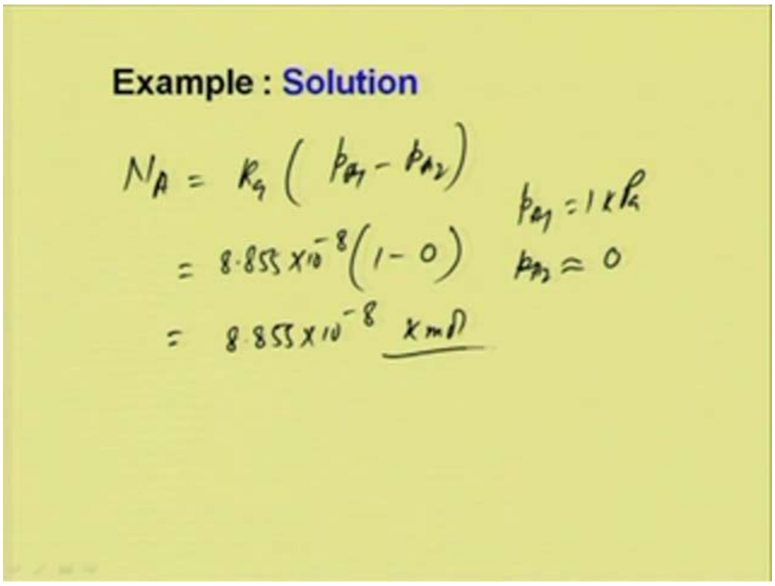

Flux, we can calculate, N A is equal to K G P A1 minus P A2. So, P A1 is given; is 1kilopascal and P A2 can be considered approximately equal to 0. So, it will be 8.855 into 10 to the power minus 8, 1 minus 0. So, it will be 8.855 into 10 to the power minus 8 K mol per meter square second. So, this is end of this lecture.