Chemical Process Control
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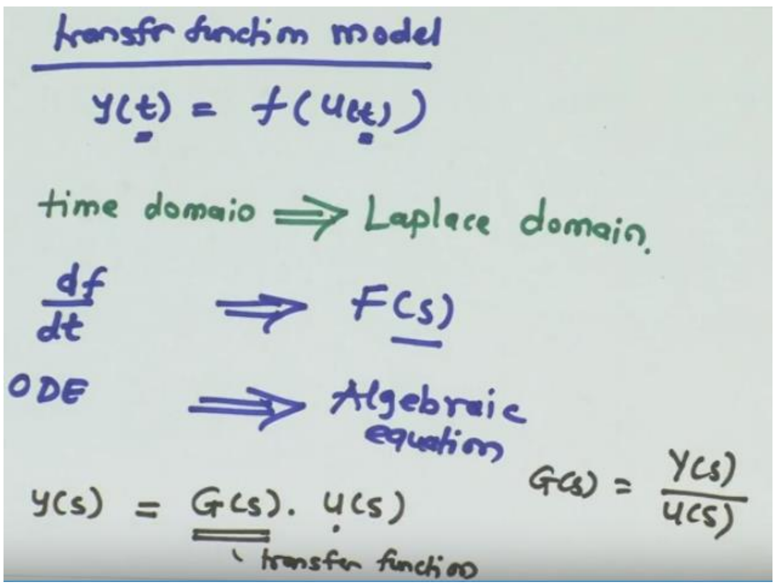
Lecture - 08
Degree of Freedom Analysis

We have looked at three different ways in which the dynamic model can be represented or can be formulated. Let us now look at how we would be representing it so that it can help to understand the dynamics or design a control system using that particular dynamic model. One of the most commonly used ways of analyzing process dynamics and also designing a control system is known as a transfer function model.



Any dynamic model tells is how does the output change as a function of input; as a response to input as a function of time. In the transfer function model, we actually move from the time domain to what is known as a Laplace domain. This is a very old way of analyzing process systems.

The whole philosophy behind that is back in let us say 1920, 30's or about 70, 80 years ago, the computing power of computers was very limited. In that case, we have seen that these dynamic models which we have generated, let us say for an example the first principle model, these are ordinary differential equations. So using those to predict the response of input on the output, you have to solve them numerically. As most of the time, the analytical solution might not be available and in that case, doing that repeated times becomes a very difficult job manually. The computers were also not that powerful like the ones which we have today. In that case, the researchers wanted to come up with an elegant way of analyzing these systems without directly solving them in time The nice breakthrough which they got was transferring the whole analysis from the time domain to a Laplace domain, a complex Laplace domain.

The major advantage or rationale behind this was if we have an ordinary differential equation in a time domain and convert it into a Laplace domain, it actually gives me an algebraic equation. So if it is a derivative of a function then what you get is an algebraic equation. So, the ordinary differential equation gets converted into an algebraic equation which is easy to analyze. We will see this as we go into this course that having analysis in the Laplace domain has very significant advantages compared to working with the time domain.

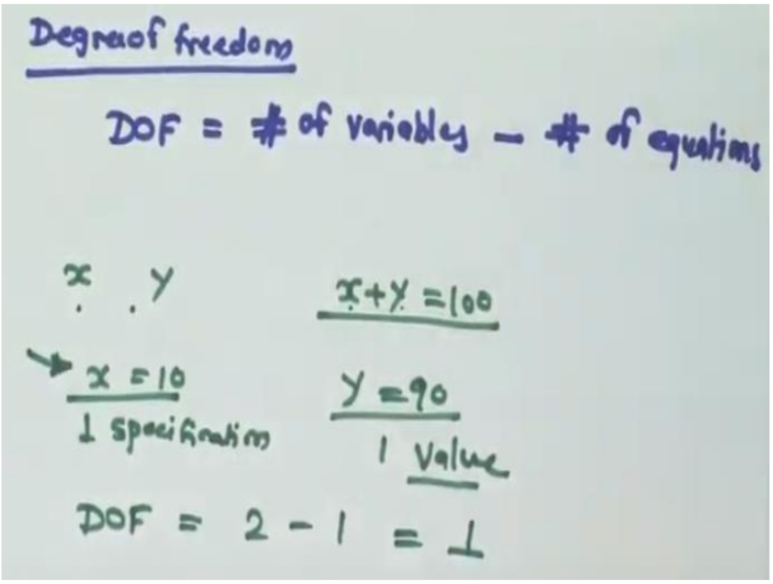
Even though nowadays our computing power is very advanced and we can simply solve these ordinary differential equations quickly, still the insights which we get by doing the analysis in Laplace domain are still more insightful than just doing a numerical analysis in a time domain. So

what we do is we have this relationship in the time domain. We just convert it into a Laplace domain and what we get is Laplace transforms of output, Laplace transforms of input and a factor in ‘s’ which kind of multiplying or transfers the effect of input into the output. This is known as a transfer function.

We can write a transfer function for the process as Y(s)/u(s). It transfers the effect of input on the output in the Laplace domain. We would see that the form of this transfer function will give us a lot of insights about the process dynamics as well as we will be using this particular transfer function model for designing a control system.

The advantage of using a transfer function model is that it simplifies the analysis by converting the differential equation into an algebraic equation. The major limitation of transfer function based analysis is that we can use Laplace transforms only for linear functions. So as long as the relationship between input and output is linear, we can use the Laplace analysis. Especially when the processes are nonlinear which is quite common for chemical engineering systems, we would try to see how we can get over this requirement of linearity by linearizing a nonlinear function.

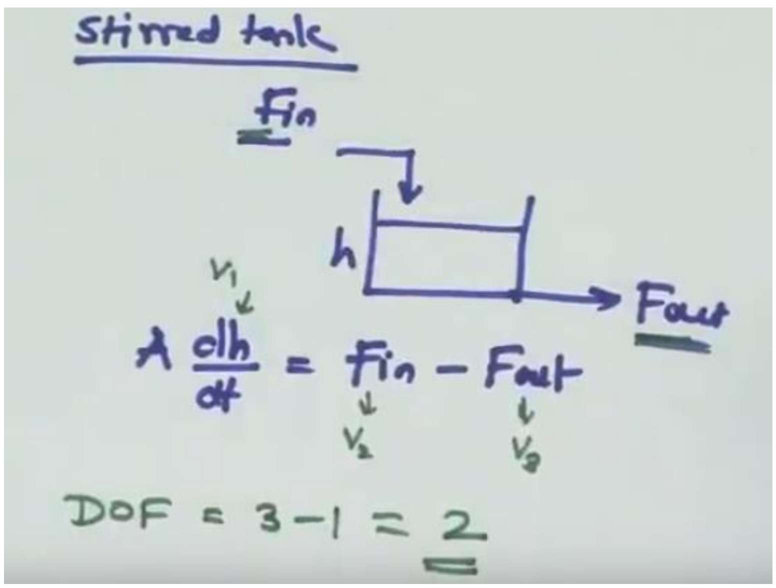
When we will talk about a nonlinear, we will touch upon the aspect of how do we still use Laplace based analysis when the process is nonlinear. So we will close out this first lecture with the final topic on the analysis of degrees of freedom of a process. We saw that the control system is implemented to satisfy a certain objective on a process. So the natural question is how many objectives can I satisfy in a given system? Is there any maximum limit on that? And we can easily guess that we cannot indefinitely keep on satisfying multiple objectives. There will always be some maximum limit on the number of objectives which can be satisfied and that will be given by what is known as an operational degree of freedom analysis. We know what is the degree of freedom for any system.



The degree of freedom is how many handles do we have in that system or how many independent specifications we can make in that system without compromising the final outcome of the process. The degree of freedom represented as DOF is generally given by a difference between the number of variables and the number of equations.

To drive this point, let me take a very simple example. Let's say that I have two variables x and y, and the only information available is the addition of them gives me 100. If I ask you, in how many different ways I can choose x and y; then there will be infinite ways in which I can choose x and y. But if I say that x is 10 or I make one specification, then automatically y can take only a single value which is 90. So this particular system has 1 degree of freedom and if I specify one variable then the system is completely defined. If we use the same formula of the degree of freedom, we have a number of variables, in this case, there are 2 variables and the equation is only 1. So, I have 1 degree of freedom.

We will be using a similar concept to find out what is the degree of freedom of a dynamic model of a process because that is what is eventually going to give rise to process control.

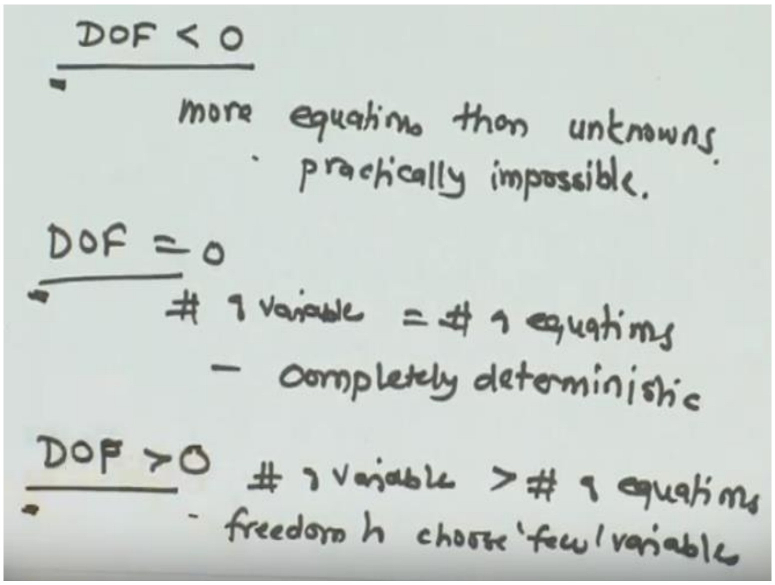


Let us look at an example of the stirred tank. Here we are saying that there are some input and some output in this process. You have some level in the tank and then we have written the dynamic equation for this as,

$$A\frac{dh}{dt} = F_{in} - F_{out}$$

For this particular system, we have 3 variables. Height ‘h’ is a variable number 1, ‘Fin’ is a variable number 2 and variable number 3 is a ‘Fout’. Here, we have only 1 equation. So, the degree of freedom is 3 - 1 = 2. There are 2 independent specifications we can make which are either Fin and Fout. As long as we specify Fin and Fout, the rest of the system will get specified.

Now let us look at what are the different values a degree of freedom can generally take. The degree of freedom can take various values for any system.

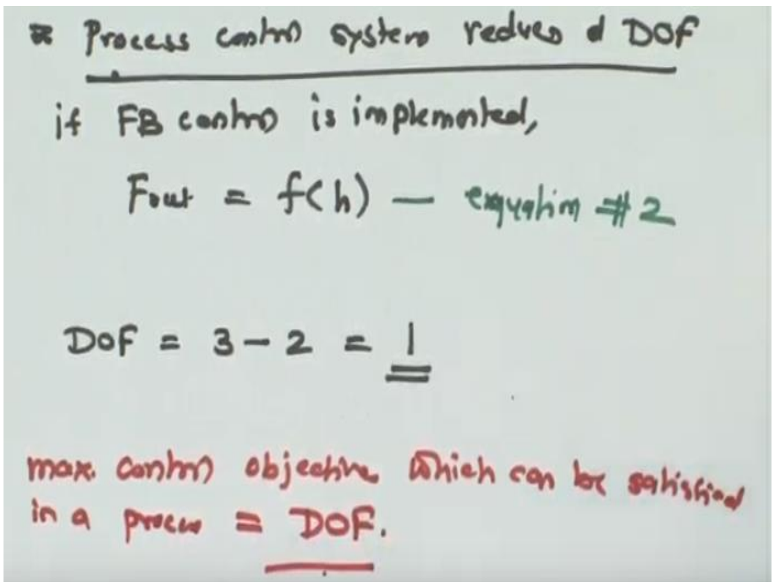


Let us say if the degree of freedom is less than 0, that means I have more equations than unknowns. So these systems will be mostly practically impossible. The degree of freedom may take a value of 0, which means a number of variables are equal to the number of equations and this is a completely deterministic case. We can just solve this system of equations and get a solution. There is no degree of freedom or there is no way to manipulate this particular process.

Lastly, the degree of freedom can take values greater than 0, which means the number of variables is more than the number of equations. In this case, some of these variables can be selected by us and then we can finally arrive at the case where the number of equations is equal to the number of variables. So there is a freedom to choose few variables which we also call as there are few ways to manipulate the process. These will be the processes which are manipulable. This will be the process which will be relevant from the control point of view.

The first case is definitely practically impossible. The second type of case tells me that there is no way to manipulate the process. So only third type of cases are the ones where the process can be manipulated and we know from the control that we have to be able to manipulate the process. So the degree of freedom greater than 0 is a requirement when we talk about any process control.

If we come back to the example of a stirred tank, we have a degree of freedom 2. That means this process can be controlled and the next thing which we want to look at is what would happen when we have a process control?



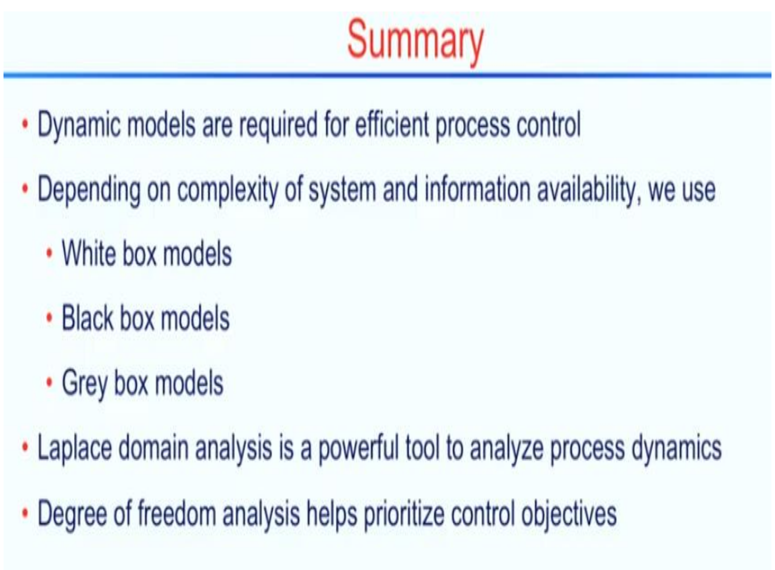
Process control system reduces the degree of freedom. Let us look at the previous example of the stirred tank and in this case let us talk about a feedback control strategy. In a feedback control strategy, we would measure the height and accordingly change the output flow rate. If we have implemented a feedback control system, then your output will be a function of height. So the equation 2 is,

$$F_{out} = f(h)$$

If I write the degree of freedom again, now I have still the same number of variables 3 but then I have 2 equations, so the degree of freedom is 1. By implementing one control loop, I have reduced the degree of freedom by 1. Every process control loop will reduce the degree of freedom by 1. So it automatically tells me how many maximum numbers of control objective that can satisfy in a process. The maximum control objective which can be satisfied in a process will be equal to the degree of freedom.

If we look at the previous example, we had 2 degrees of freedom which means any two of Fin, Fout, and h can be specified. Here, even though disturbance 'Fin' is considered as something which we can manipulate; the typical analysis of this kind of systems shows that in terms of 'Fin', there is a very little capability we have that we can manipulate it. We would typically not consider it to be something which we can vary. So, the degree of freedom automatically gets reduced by 1 because there is no freedom in choosing Fin. The practical degree of freedom is 1 and therefore only 1 control loop which is the controlling height will just make the system completely deterministic and then that is the only control objective or only a single control loop can be implemented on this particular system.

To summarize this second part of the first week's lecture, we have seen that dynamic models are required for efficient process control because they will give us the way of how the system behaves which can be later on refurbished to design a control strategy. Depending on the type of system for which we want to write the dynamic model, any one of first principle or white box model, empirical or black box model and grey box model can be used.



These are the three different ways in which the dynamic model can be formulated and depending on the system complexity, one model may be better than the other. We typically represent all these dynamic model into the Laplace domain because it simplifies the analysis as well as controller design. Then lastly, the degree of freedom analysis is typically carried out before starting any control work because that gives you an upper limit on how many control objectives which can be satisfied for that particular process. Thank you.