Chemical Process Control
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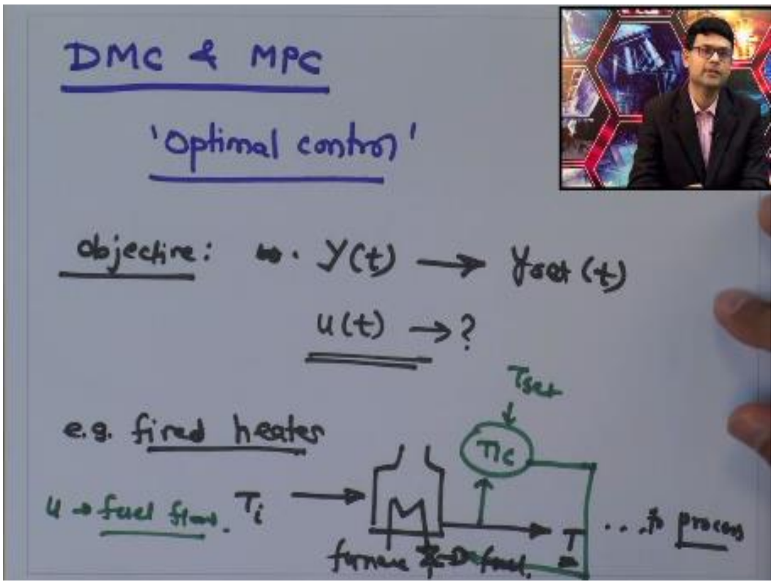
Lecture - 42
Dynamic Matrix and Model Predictive Control

Hello students. We have been discussing advance controllers and typically the relatively we are talking about now recent day's advance controllers. In the last lecture, we looked at IMC which was Internal Model Control. In this lecture, we will familiarize ourselves with a different type of controller. It falls under the domain of optimal control and we will be talking about a Dynamic Matrix Control and MPC which is Model Predictive Control.



Both these controllers fall under the domain of optimal control and let me first motivate what do we mean by optimal control. So far all the control strategies which we looked at right from a simple PID control, then cascade control or split range control or even for that matter IMC which is Internal Model Control, all the time our objective was to ensure that your output goes to the set value.

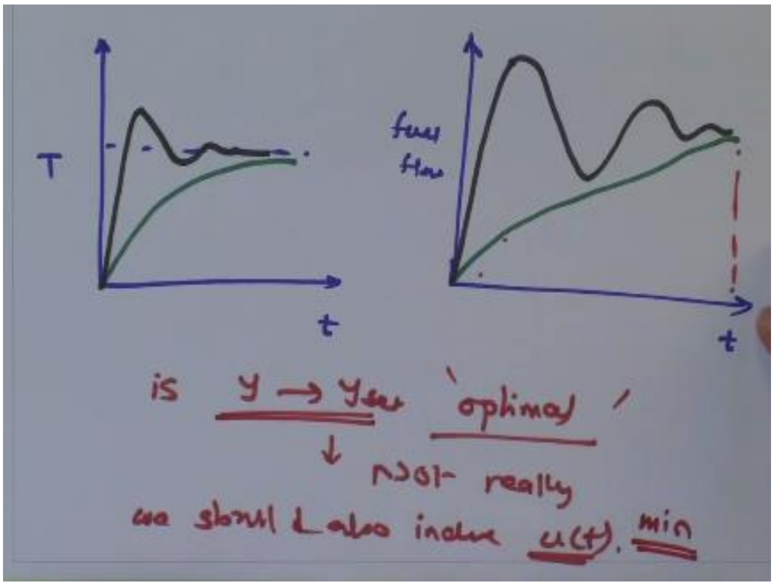
So all we want is our controlled variable should be maintained at the setpoint value which is very natural because that is why the control system has been implemented. But if you look at this objective from a practical point of view, you will realize that we have not said anything about the input. We want to achieve this control irrespective of what the input is required to achieve that.

Let me give you an example. Let us take an example of a fired heater. The fired heater is let say a process where you want to heat a certain stream by firing some amount of gas into a furnace. This is a furnace and at the end of it, you want to achieve a certain temperature because this particular stream will go to some other process which will require some high-temperature fluid.

There is some requirement in terms of what this temperature should be. In order to control this temperature, you would be firing a fuel gas or a fuel into this system. So what we have seen is in order to have a controller of control of this temperature or any control strategy would be like this you will measure the temperature, you will have a temperature controller, it will have a set value and accordingly, it will manipulate this particular valve.

Here your ‘u’ represents fuel flow, right. Now for this particular system in order to maintain this temperature, I have to change the flow of fuel into this furnace and you can imagine that in a real process there is some cost associated with how much fuel you burn into the furnace. So your operational objective if you look at the end goal of making a profit out of this plant.

It is not necessary that always you should be ensuring this tight control of temperature because it can come at the expense of very high fuel flow. I want to show you two controllers which are giving you this temperature control.

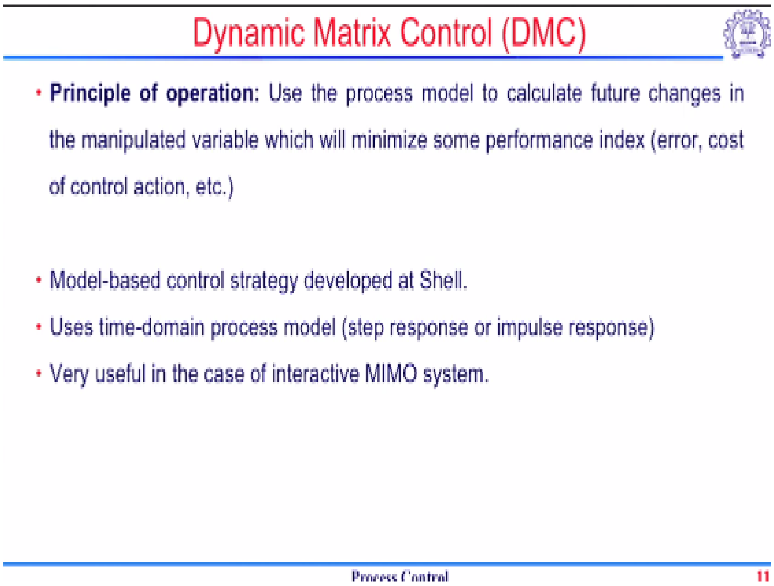


As shown in the figure, ‘T’ is the set value and here I will show you the fuel flow. One possibility is that you have a very fast controller and it reaches the setpoint value pretty quickly. This is, but the corresponding fuel flow what you require is very large and it varies like this. As against you can also have a different type of a controller which reaches that same temperature and in this case the fuel flow what you will realize is also relatively less compared to the other strategy.

Note that the final value in both the cases would be the same because that is dictated by the steady state energy balance of this system. But what you can realize is that if I see how much amount of fuel which was burnt till it reaches the steady state, in this case, it is considerably less amount of fuel flow is required compared to the previous case which was a very tight control.

So this brings in a question of is ‘y’ going to ‘yset’ that is tight control of the controlled variable, optimal in terms of making more profit from the process. The answer is not really. We should also look at how the input change as a function of time in order to reach to achieve that control and that is the main motivation of going for optimal control, that you want to somehow tie your control objectives with the final economics of the process.

One way is along with making sure that y goes to ‘y_{set}’ you also want to ensure that you want to use a minimum amount of control action, especially for this particular example. So that you can achieve this control at less amount of actual cost. So that is the whole motivation about any sort of optimal control and here we will be specifically focusing on what is known as the Dynamic Matrix Control and eventually gets developed into a Model Predictive Control and both these follow the same philosophy of optimal control.



So what does Dynamic Matrix Control do? Dynamic Matrix Control is a model-based control strategy and it uses process model to calculate or to predict how the output of the process would change and based on that you select how your manipulated input should vary in future with an objective to minimize a certain performance index. It is a very general definition in terms of performance index because the performance index also depends on the process which you are trying to control.

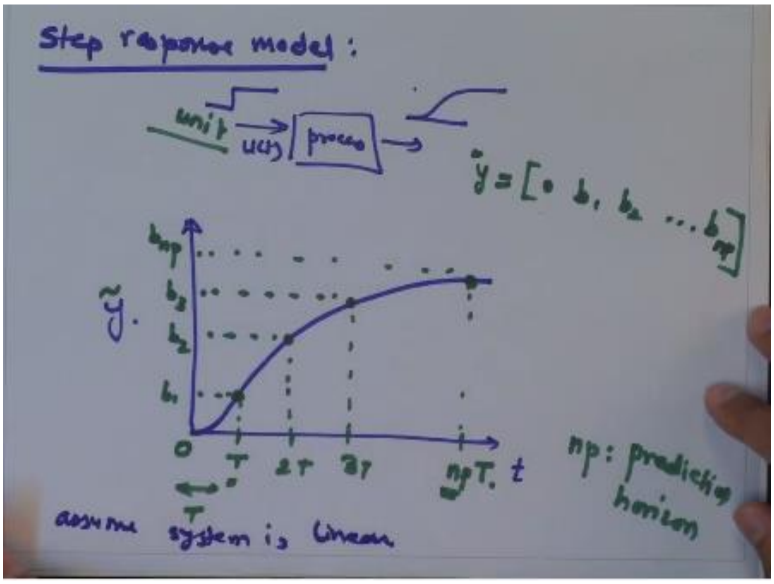
As I gave you an example of a fired heater, where the control action was important. So you want to penalize how much fuel goes into the process or how much manipulated input is used. In some other examples in some other cases like for an example if some process stream is getting cooled by using cooling water, then how much cooling water is used may not have that much impact in terms of the performance the final economic performance of the process.

So that is why the definition of DMC is given the very general sense that you have to first decide what is the performance objective of your process or performance index of your system and then accordingly you try to minimize it. As I said it is a model-based control strategy and what makes DMC very unique is that all the control strategy which we have studied so far, most of them were developed by research in academic research and eventually they got implemented into the industry.

However, DMC is in a unique position that it was developed inside industry because you will see that it has more to do with how things operate in real life and then based on that experience, the theory and everything for DMC were developed later on by academia. It was developed in a company called as Shell and again another distinguishing factor about DMC is that rather than using Laplace Domain Model as it was not developed inside academia which was mostly focusing on Laplace Domain Analysis.

This uses a time domain representation of the process, it can be a step response model or an impulse response model. Even though we have not yet looked at MIMO type of control which stands for multiple inputs multiple output control, we will be discussing that in the next week. The main utility of this DMC or MPC comes when you have multiple inputs into your process and you want to use those inputs to control multiple outputs and these are all interactive.

We will talk more about that in the next lecture, but what you can note that the main utility or the main effectiveness of DMC lies when you have multiple inputs and multiple outputs. Though in this lecture we will focus on the simplified version of DMC where you will have only one output and one input, okay. Let me tell you what this model looks like.



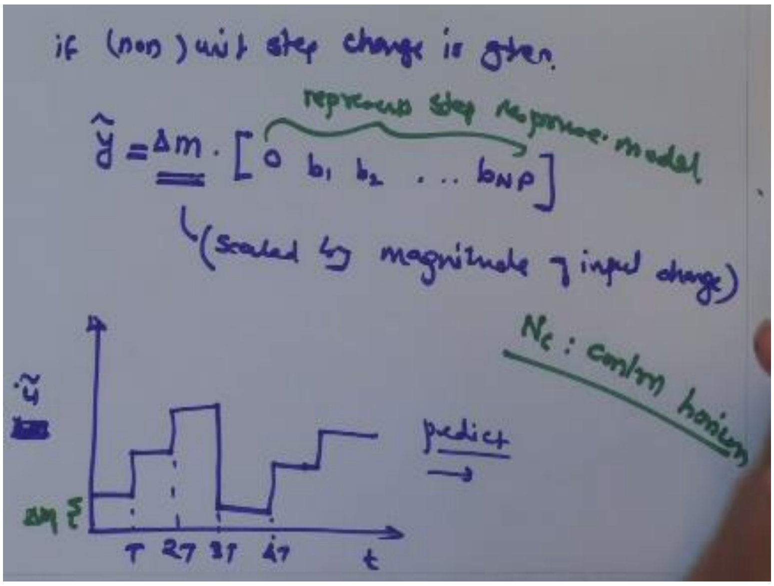
DMC uses what is known as a step response model. In the first few lectures first few weeks of this course, we saw first order dynamics, second order dynamics, and even higher dynamics and how they respond to a step input. This is exactly the same thing. So you give a step input into your process. So you give a step input and you record how your output changes.

Let us say your output changes like this and rather than representing it as first order plus dead time sort of a model; what you really do here is- you take some sample points along the trajectory. Let us say this is time ‘t’ and then you take a certain amount of time interval where ‘t’ is the interval gap so it can be minute or it can be 5 minutes or it can be hour depending on how slow or fast the process is.

But what you really care for is you take such N_p points where N_p is known as the prediction horizon. It typically when you say N_p, the process typically should have reached a steady state by these many time points and then for every time point you notice what is the value of your output and let me also specify this was the unit step change. So what we had given was a unit step input change in the input and accordingly, you get these different values.

So you can say this is b_1 this is b_2 this is b_3 and the last one is b_{Np}. The whole step response of this system can be represented by a vector. So I can say that this change in y can be given by 0, b_1, b_2,…, b_{Np}. By using only these coefficients, I can predict the response of this system to any sets of inputs.

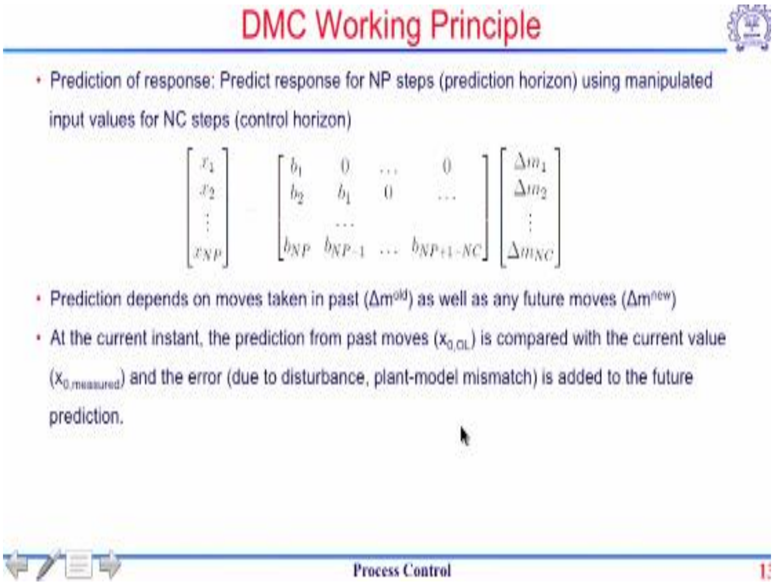
So what we are going to assume here is that the system is linear. If that is the case, for a unit step change, these are the coefficients by which the system response is going to be like this. Then if I want to calculate the response of this system to any other input.



So if non-unit step change is given, then the corresponding y tilde will be Δm times this vector 0, b_1, b_2 up to b_{Np}. It simply says that the response will get scaled by the magnitude of input change. You can verify this for all the sorts of systems which we have already studied first order dynamics, second order dynamics. We have all studied the linear systems and in all those cases this was true that if you double the input change then the corresponding output change will also get doubled.

By using just these coefficients, so these coefficients have to be obtained one time. This sort of represents the model. By using this, you can predict the output of any process when there is some step input is given and let us say if you give successively different steps in the input. Let us say the input has a certain change at every time interval, then accordingly you can predict the output at different time points by using this formula.

So what you can see here is that at the first instant, the amount of change in the input was Δm_1.



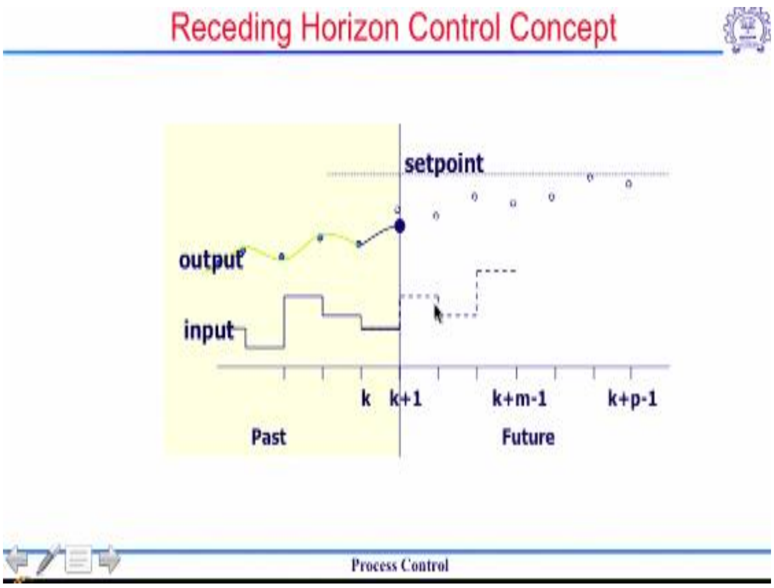
It will affect the output for the first time and eventually all the way up to x_{Np}, it will have an effect based on these b's. Then in the next interval, you gave an input of Δm_2, so it is not going to affect the output at the previous time, but it is going to start affecting any future outputs. And accordingly, let us say variable at t_{Np} at that time what you will see is that the corresponding output would be dependent on all the previous moves which were made.

By using this compact notation about how the output changes as a function of time and how the input change and this particular matrix can be represented as a b matrix. You can predict the output of this particular process or this system for any changes in the input and another parameter which I want to define here is N_c which is called as a control horizon. Earlier we saw prediction horizon, it was the number of time steps in the future up to which you want to make the prediction and control horizon up to how many intervals you want to change your input. There is no necessity that if you want to predict it up to 4 hours into the future, your input should also change for 4 hours. You may select that the input change only up to 2 hours and later on inputs are held constant and you want to see that based on these inputs, the final output reaches its desired value or not.

These are the two important parameters and when it comes to DMC that what is your prediction horizon which is typically decided based on the time constant of the process and the control horizon is typically taken as up to 50% of the prediction horizon. You can see that the changes in the input are done up to the control horizon and the prediction of the output is done up to the prediction horizon.

The prediction not only depends on any new moves which you are going to make, but also it is going to depend on what were the previous or past moves and therefore you want to compute the final prediction by using these 2 quantities that any future inputs which you are going to calculate now and any past inputs which you have already implemented. At any time, instant whenever you are arriving out of this at that instant you can get a prediction of what my output should have been and then you also incorporate feedback from the system.

Because again these models you are assuming that the system is linear, in reality, the system may not always be linear and this super-position may not always work. In such a case it is a very good idea, we have seen that feedback control is very robust. You would get feedback from the system and you will try to compute what is the error between the current value which the process is giving and what is the predicted value. So all the difference between that gets added to the new prediction so that you have a better sense of prediction into the system. Let me simply explain to you how this works from a figure.



You can see here that this is your process. You had given some sort of inputs and this is how the process is responding and you have a setpoint request for this particular process and here ‘k’ stands for any time instance. Now what happens in this sort of a controller, you first predict what should be your new future input, so those are shown here. And by using this prediction model you calculate how your output profile looks like.

We will see how this input profile is calculated, but for that particular input profile, you see that eventually, the variable which you want to control reaches its setpoint value and then what you do is, you just implement the first input. We will see how that is done. Even though you calculated 4 or 5 moves in the future, you actually implement only the first move and then this is where the process reach.

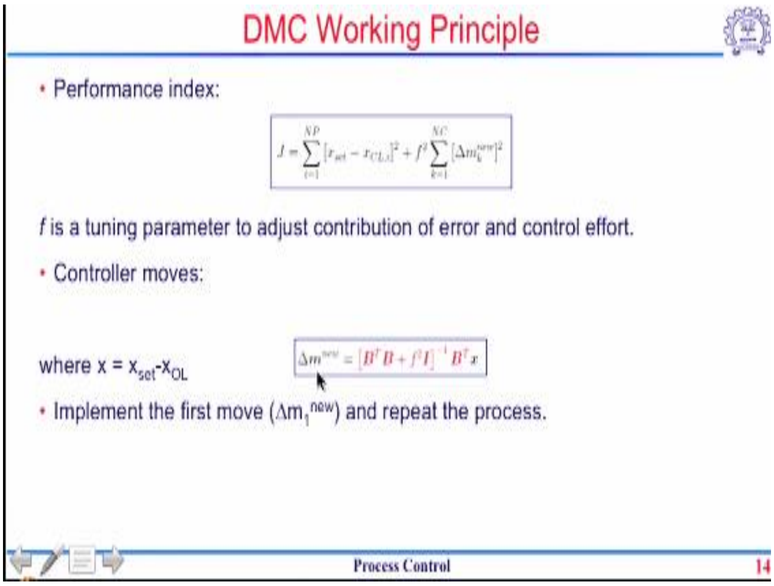
Your prediction was this hollow circle, but your plant actually reaches to a value of this solid circle that is what I was referring to in terms of the model-plant mismatch that even though you may predict it in a certain way your process may not actually follow exactly the same value. So what you do because of that is one of the main reasons why you do not incorporate the entire trajectory.

We actually input only the first value of the next manipulated variable or the next value of the manipulated variable. And then you simply move to the next time instance and now you can see that this step or this situation very similar to what we had done at time t=k. Again you have a previous input profile and you have output profile and whatever was the error here that gets added to the new predictions so that was the error I was talking about.

Then this same process is repeated at this instant also. Again you will predict the new manipulated variable profile, you will predict the output and then implement only the first move of the manipulated input. So in some sense, this horizon for which you are predicting and taking control action keeps on moving or keeps on receding that is why it is also known as a Receding Horizon Control Concept.

It is very commonly used in DMC as well as MPC that your horizon for which you are making predictions as well as taking any control moves keeps on receding from you. So if my horizon is for 3 hours, I can operate the same controller for a few months because every time I will be looking at 3-hour window in the future, the window keeps on moving from you. So now we will come back to how do we compute.

Now we saw how the controller works. Now we need to find out how do we compute the value or how do we compute these future values of the manipulated variables. Based on the definition what we had said was this controller works based on minimization of certain performance criteria.



Here is one of such performance criteria which is shown. 'J' is known as the objective or performance index. The first part of this index deals with the actual controlled variable. So it tells me that if x_{set} is the value of my setpoint and x_{CLi} are the predictions about the output, what you want is the prediction it should be very close to the setpoint. Because ideally if you want to control this process you want to ensure that your controlled variable reaches the setpoint value.

That is why it sort of captures the error between the current value or between the value of the controlled variable and the setpoint value and this is done up to the prediction horizon because that is the time up to which you are predicting the output. At the same time, you also have another term which penalizes how much is the change in the manipulated variables, this is known as the move penalty.

So whatever you are manipulated input moves, accordingly you would also penalize how much control action is taken in order to reach this controlled value and that is done up to the number of steps you take are the control horizon. And this ‘f’ is a tuning parameter so it tells me how much penalty how much is the relative weight between the control performance and the control action or the control cost.

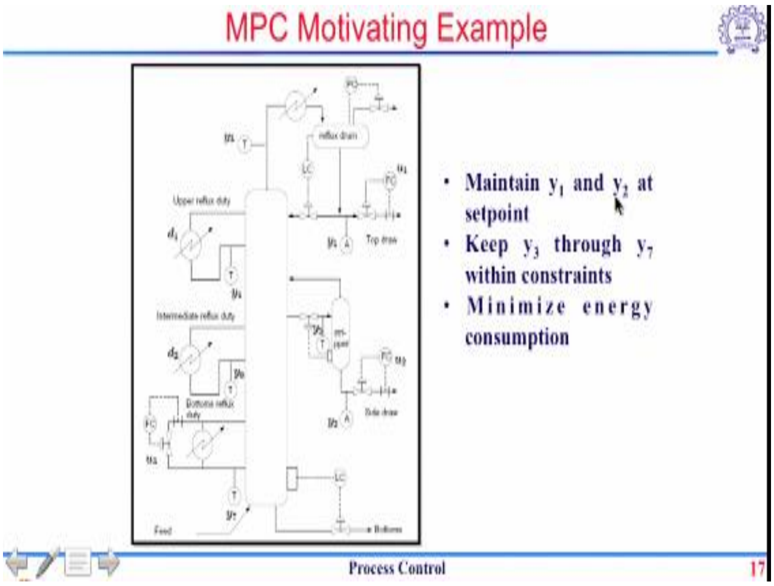
Depending on, if ‘f’ is very small, then it will typically work as a normal traditional control where you want to achieve very tight control. If your ‘f’ is large, what it tells me is the manipulated input is very costly and I should not make very big moves in terms of the manipulated variable. And then it is a simple optimization problem, you can solve it by using if it is a linear system by normal optimization least square optimization, then it will get it will give you how your future input should move.

As I said you just take the first move and repeat the process, so that is how a DMC controller works. Now let me tell you these DMC controllers started in the late 1980s and since then these are very commercial sort of controllers. You will see that most of the industries today if they are using an advanced controller, you will see an advanced version of DMC there and that advanced version of DMC is known as a Model Predictive Control.

In philosophy, it is almost the same as DMC with some added benefits. Because as I said DMC originated in the industry and then eventually the academia adopted those ideas and started making improvements on those. And the main improvement what is made is that in a physical process along with this performance objective minimization, you also have some sort of a constraint.

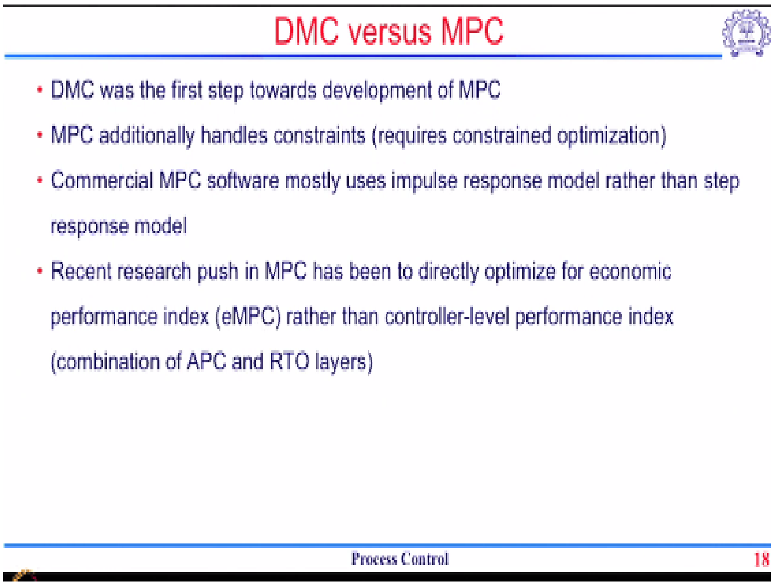
Your manipulated input may not show move beyond a certain value, you may also have to maintain certain constraints. For example, if it is a distillation column, then you would want to maintain that the column does not flood based on the control actions you are taking. So this kind of having additional constraints incorporated into your process adds a more the feel of the real operation into your control algorithm.

So far we never saw that the controller was worrying about how the process operates internally. All we cared about was the input-output relationships, but having these additional constraints into your process this may also be some safety constraint you want to incorporate into your control algorithm. All these additional operational constraints can also be incorporated and when you have those, then you have to solve is known as a constrained optimization rather than a simple optimization which we solved for DMC and that is the beauty of Model Predictive Control that it allows you to specify these additional constraints into your process as well.



Here is one motivating example of MPC. It is a refinery column and you will see that for this system there are about 7 controlled variables, but only a few manipulated inputs. I think there are about 3 manipulated inputs here u_1, u_2, and u_3. So what you do here is that you specify your control problem such that you want to tightly control y_1 and y_2 which are around final products. Other outputs 3 to 7 you want to maintain within a certain constraint, you may have the maximum limit and minimum limit about those and you want to ensure that it need not be at that value, but it can float between the maximum and minimum value and along with that you also want to minimize any energy consumption into this process. Because energy consumption will be based on how much is the coolant you are using here or how much the reboiler duty you want to specify.

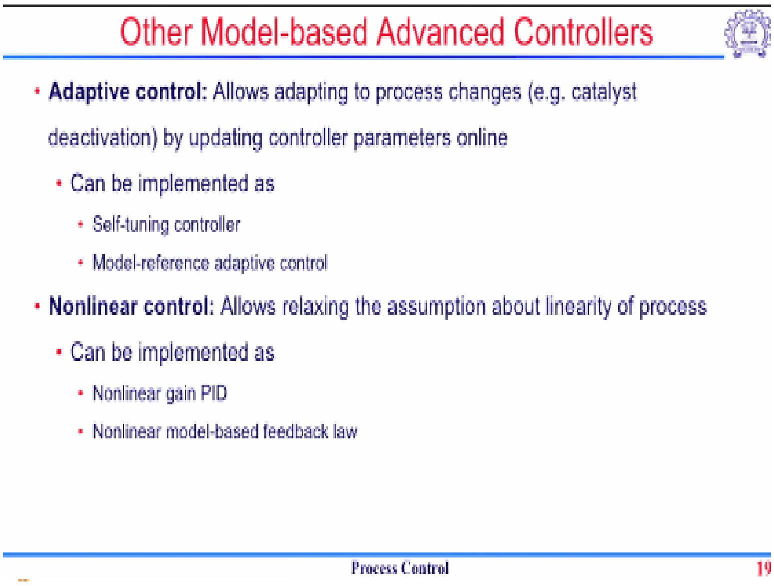
So all those things can be incorporated. This sort of makes it as a when you want to control a certain process it takes into account how the operationally what are the objectives in terms of operating that process and those all can be incorporated within the framework of Model Predictive Control.



Quickly let me summarize what is the distinction between DMC and MPC. DMC was the first step and then MPC is sort of an advanced version of DMC. MPC additionally handles constraints, so that is why it requires a constrained optimization. Most of the commercial MPC software so all these commercial companies like Aspen, Honeywell, when they sell you MPC software, those who rather than having a step response model which is slightly difficult to get, they use what is known as an Impulse Response Model. The philosophy is similar. Just that instead of maintaining coefficient from a step input, you maintain the coefficient from an impulse input.

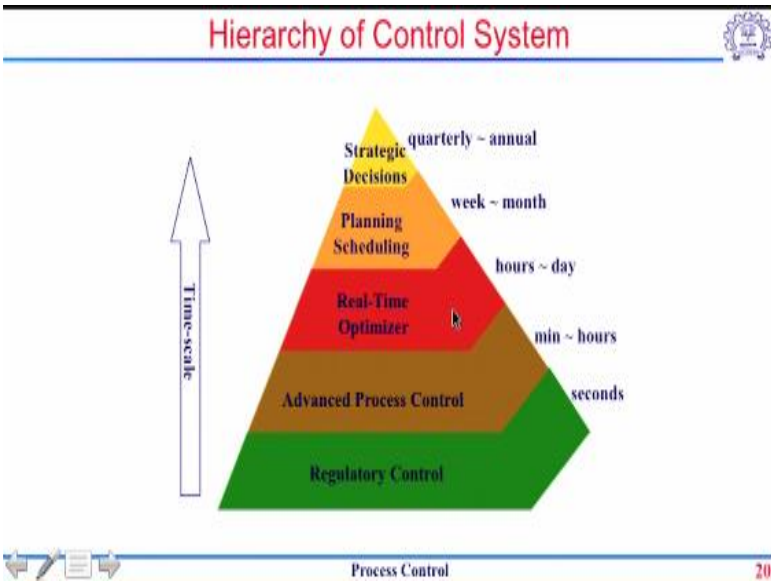
Nowadays if you look at what is the state of the art in terms of MPC or what researchers are doing in terms of MPC, they are kind of coupling 2 stages in terms of the advanced control. They want to say that this performance index rather than writing it in terms of the controlled variable and manipulated input, eventually what I am interested in is making a profit. So can I put this performance objective as the actual profit of the process and then use the same philosophy of Model Predictive Control, so that is known as economic MPC and that is one of the very hot topics in terms of research of Model Predictive Control.

Let me also tell you that we consider these IMC as well as DMC as to representative advance controller. These are by no means the only advance controller which are out there.



You can also have what is known as Adaptive control which sort of adapts its parameters as the process changes over time. One classic example is that if you have a catalytic reactor, in that case, your catalyst activity keeps on degrading over time. If you have a simple process model, then the process itself is changing, so the model can not represent, the same model cannot represent the process over time. So what this Adaptive controller does? It kind of adapts the model parameters as you move from one year to the next year and accordingly changes the controller parameters.

Another set of the control strategy is known as nonlinear control. All the while even though we looked at IMC and even MPC, most of the time we made an assumption about the linearity of the process and we know that the real process may not always be a linear process. So in such a case, if the linearity is a very severe, a lot of times people also go with what is known as nonlinear control where the control law itself is nonlinear. So that it represents the model is also nonlinear and the control law can also be nonlinear. So that it captures the predictions are much accurate compared to a linear model.



Then we will try to close out by showing you this pyramid which I had shown you in almost a first lecture and we had seen that there are different levels of control hierarchy when it comes to the real plants and most of the lectures in this particular course, we focused on regulatory control. When we talked about simple PID control or even up to this traditional advance controller, all those forms what is known as base regulatory control.

Then IMC and DMC, these are sort of the advance control layer. These would give you setpoints for the regulatory control and on top of that is known as real-time optimization which optimizes a certain steady state performance of the process and accordingly gives this performance objective for the Advanced Process Control. Then on top of that is planning and scheduling which sort of tells you how much of a product should be made within a month or within a quarter.

Those are different areas in which process control is moving these days. When I said this economic MPC, this sort of ties up with the combination of this real-time optimization and advanced control. It tries to bridge these 2 layers and make it into a single layer. We will stop here in terms of advanced control and in the next week, we will talk about multivariable control and batch process control. Thank you.