Chemical Process Control
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Lecture - 37
Frequency Response-Based Controller Tuning

Welcome back. We will know look at the final method of controller tuning and this time we will be talking about a method which falls in the domain of robust controller and this will be a method based on frequency response.

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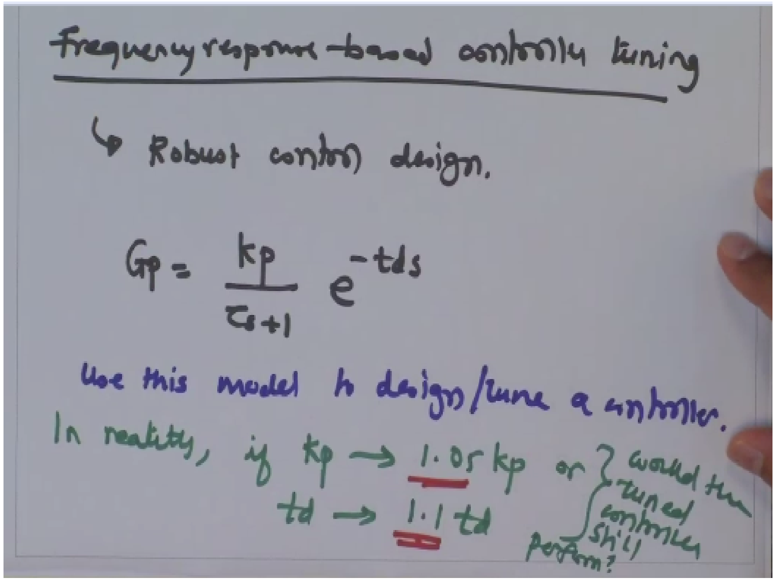

Frequency response-based controller tuning

↳ Robust control design.

$$G_p = \frac{k_p}{\tau s+1} e^{-t_d s}$$

Use this model to design/tune a controller.

In reality, if $k_p \rightarrow 1.05\, k_p$ or $t_d \rightarrow 1.1\, t_d$ } would the tuned controller still perform?

So we look at frequency response based controller tuning. And as I said this is under the domain of robust control design. So what we are interested in is let us say our process as a transfer function of this form and you can use any of the heuristic method or any direct synthesis method or criteria based method to design a controller. So you use this model to design or tune a controller.

In reality if Kp changes to let us say Kp increases by 5% or your dead time was wrongly calculated and actual date time is 10% extra of this under such scenarios would the controller still perform. So we are looking at the robustness of the controller that if some of the process parameters in the presence of certain errors or variations in the process parameter go to the controller still perform the way it is supposed to or would it still maintain the stability of the closed-loop system.

So if your controller is able to handle such a large variation in terms of process parameters then you can say that the corresponding control that is robustly design if not you will say it is not robust to modeling errors or errors to parameter values because these models are typically obtained from data and they may not capture the reality to the great extent so there is always a possibility that some errors might happen within these parameters, so your controller should be able to handle any variations in such kind of parameters.

So when you want to do a design such that it allows you a certain freedom in terms of variations in the process parameters that particular method will be known as the robust controller design and will be we will see how such a controller can be tuned by using frequency response. So for that we will revisit what is frequency response and what is its role in terms of stability.

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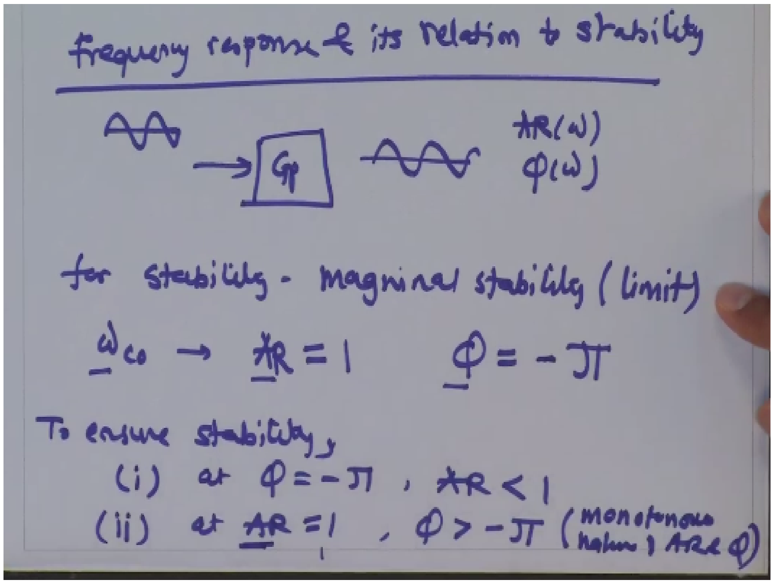

frequency response & its relation to stability

for stability - marginal stability (limit)

$\underline{\omega}_{co} \rightarrow \underline{AR} = 1 \quad \underline{\phi} = -\pi$

To ensure stability,

(i) at $\phi = -\pi$, $AR < 1$

(ii) at $\underline{AR} = 1$, $\phi > -\pi$ (monotonous nature of AR & ϕ)

And its relation to stability. So we have seen that frequency response is if you have a process you subject it to a sinusoid then your output will also be a sinusoid with a different amplitude ratio and a different phase than the input. So when you capture this as a function of Omega which is the frequency of sinusoidal oscillation you will get the frequency response. And how is it related to stability?

So for stability or I will say marginal stability or the limiting stability limit what we calculate is a crossover frequency such that when amplitude ratio is; when phase = –PI, amplitude ratio = 1 that will give you a marginal stability, so that point into this AR phase and Omega plane which are the based on these three parameters. If you select, if you find a point where phase is -pi and amplitude ratio = 1 then that system is at the limit of stability.

So in order to ensure stability there are two possibilities there are two options. Option 1 is at phase = -pi you can make AR to be <1 so automatically you can ensure that the system is stable. There is another way of ensuring stability that when your amplitude ratio is equal to 1 your Phi should be > -pi. So this is based on an assumption that monotonous nature of ARNC.

So if your amplitude ratio and phase are monotonous functions of Omega which is also a requirement of body stability criteria based on which this particular condition is designed then you can say that when your amplitude ratio is equal to 1 at that frequency if your phase is still away from –pi by the time it reaches –pi amplitude ratio would have fallen below 1. So both these conditions that sort of equivalent when you say both these monotonically decreasing functions of Omega.

And you can use any of these conditions to ensure stability. So these two conditions will give rise to two design parameters in terms of frequency response based design.

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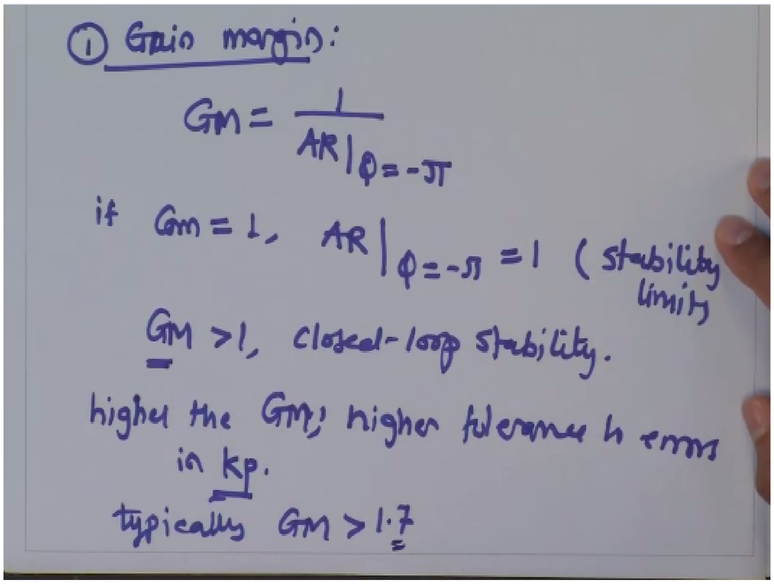

① Gain margin:

$$GM = \frac{1}{AR|_{\phi=-\pi}}$$

if $Gm = 1$, $AR|_{\phi=-\pi} = 1$ (stability limit

$GM > 1$, closed-loop stability.

higher the GM; higher tolerance to errors in k_p.

typically $GM > 1.7$

So one is known as Gain margin, so gain margin is defined as an inverse of amplitude ratio when phase is equal to –pi. So it tells me of that; so if gain margin = 1 then I have AR at phase = -pi. So that means it is a stability limit. If the gain margin is >1 you have closed loop stability. And we can show that higher the gain margin more is the tolerance in terms of controller process gain errors in the process gain.

So typically gain margin is selected beyond 1.7 so which will ensure that if only process gain has certain uncertainty then up to 70% uncertainty can be accommodated by ensuring by keeping gain margin of 1.7, so even if the gain increases by 70% the controller would still remain stable, so that is the primary notion of what is a gain margin. So it tells you how much additional safety you are putting, so whatever beyond 1, is the safety which we are putting into in order to counter any uncertainty in the process gain. Similarly, we can also define a criteria based on the phase.

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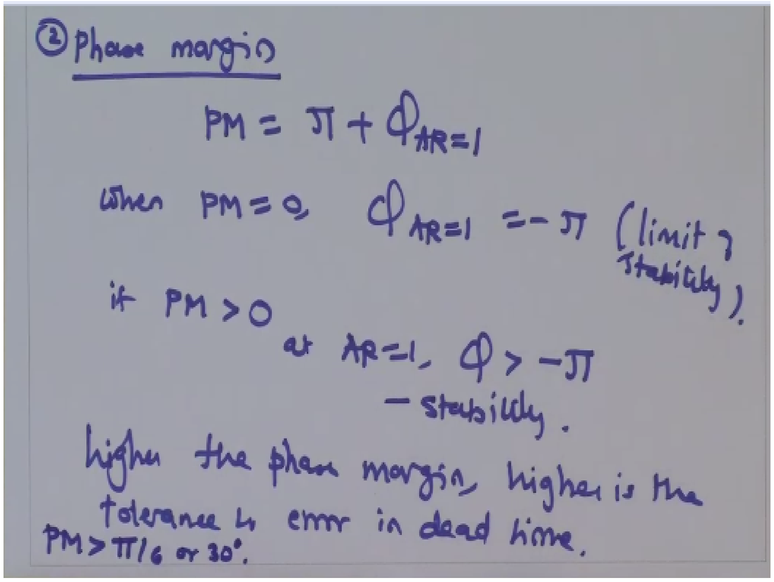

② Phase margin

$$PM = \pi + \phi_{AR=1}$$

When $PM = 0$, $\phi_{AR=1} = -\pi$ (limit of stability).

if $PM > 0$

at $AR=1$, $\phi > -\pi$

− stability.

higher the phase margin, higher is the tolerance to error in dead time.

$PM > \pi/6$ or 30°.

So that is known as the Phase margin. So phase margin is defined as pi + phase when amplitude ratio = 11. So when phase margin is equal to 0 we have phase at amplitude ratio -1, amplitude ratio = 1 is –pi, so again that is the limit of stability. If the phase margin is positive what we have is at AR= 1 your phase will be > -pi and therefore you will ensure stability. And higher the phase margin, higher is the tolerance to error in dead time.

So this deals with any uncertainty in terms of dead time calculation so if the process has a lot of variability in terms of dead time then we can go for a higher value of phase margin. And you can note that this gain margin as well as phase margin as they are dependent on amplitude ratio and phase calculations they are also functions of controller parameters. So by selecting a particular gain margin and phase margin we will get equations based on the controller parameters which will be Kc Tau and Tau D.

And then we can accordingly select the values of controller parameters which will ensure a certain minimum gain margin and minimum phase margin typically phase margin of greater than pi/6 or 30 degrees is quite common. So let me show you how these are related to uncertainties.

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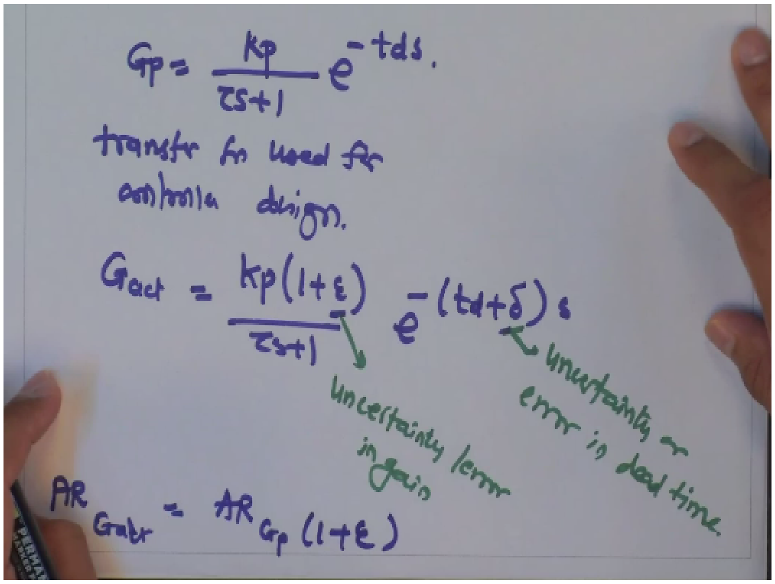

$$G_p = \frac{k_p}{\tau s+1} e^{-t_d s}.$$

transfer fn used for controller design.

$$G_{act} = \frac{k_p(1+\varepsilon)}{\tau s+1} e^{-(t_d+\delta)s}$$

uncertainty/error in gain

uncertainty or error in dead time

$$AR_{G_{actr}} = AR_{G_p}(1+\varepsilon)$$

So let us say our process transfer function for which we have designed a controller is this, so this is the transfer function used for design. And let us say the actual transfer function is, has certain error in terms of gain Tau remains the same and certain in uncertain in terms of dead time. So this can be uncertainty or error in gain. This is uncertainty or error in dead time. Now if we see what is the amplitude ratio of the G actual = amplitude ratio of your original Gp because this is not going to cause any contribution towards the amplitude ratio 1+ epsilon.

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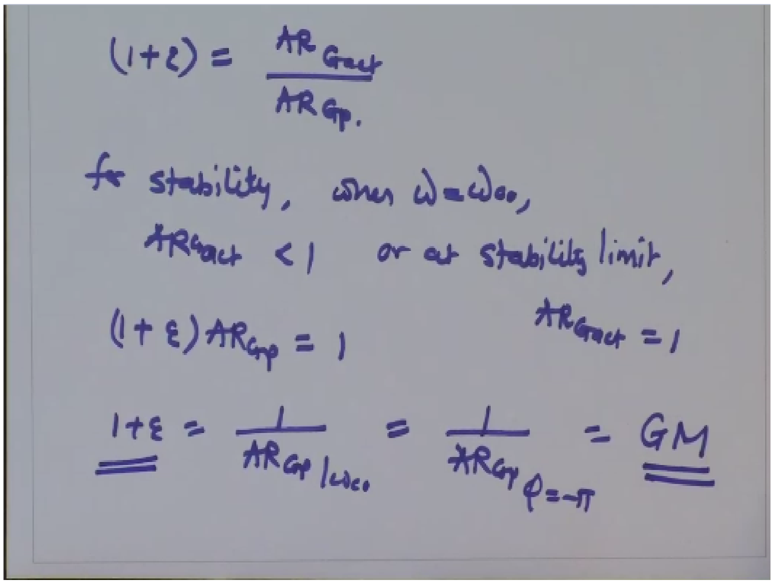

$$(1+\varepsilon) = \frac{AR_{G_{act}}}{AR_{G_p}}.$$

for stability, when $\omega = \omega_{co}$,

$AR_{G_{act}} < 1$ or at stability limit,

$AR_{G_{act}} = 1$

$$(1+\varepsilon) AR_{G_p} = 1$$

$$\underline{\underline{1+\varepsilon}} = \frac{1}{AR_{G_p}|_{\omega_{co}}} = \frac{1}{AR_{G_p}|_{\phi=-\pi}} = \underline{\underline{GM}}$$

So 1+epsilon = amplitude ratio of actual over amplitude ratio of the controller transfer function the transfer function which is used for controller design. So for stability when Omega = Omega cross over we want AR G actual to be <1, so we want or at marginal stability they actual = 1, so you can show that 1+ epsilon = 1 over AR of Gp at Omega cross over which = 1/AR of Gp when G = -pi which is equal to the gain margin. So gain margin is related to any uncertainty which we can tolerate in terms of the process gain value. Similarly, we can make a case for a phase margin.

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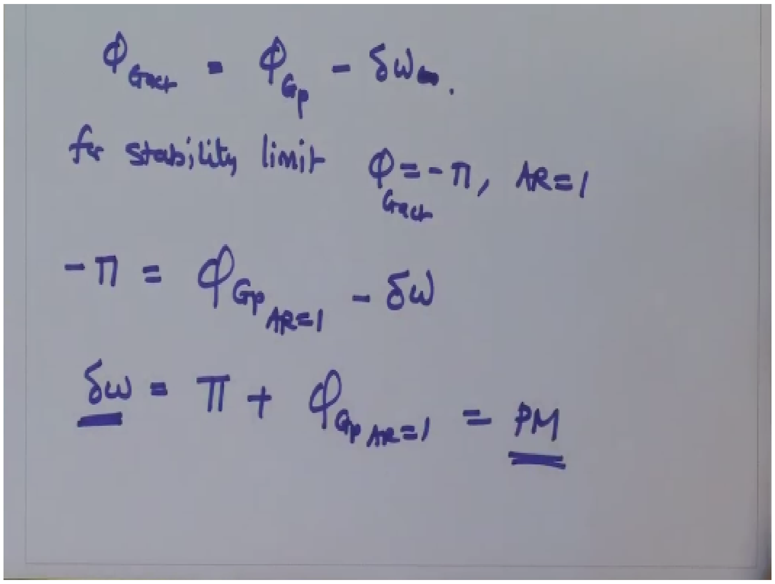

$$\phi_{G_{act}} = \phi_{G_p} - \delta\omega_{co}.$$

for stability limit $\phi_{G_{act}} = -\pi$, $AR = 1$

$$-\pi = \phi_{G_{p_{AR=1}}} - \delta\omega$$

$$\underline{\delta\omega} = \pi + \phi_{G_{p_{AR=1}}} = \underline{PM}$$

So if we see phase of G actual = phase of Gp - delta Omega cross over. And now we want to say that or at Omega, so you want to say that at for stability limit when Q = -pi AR=1 so we can say that -pi = phase of Gp when AR=1- delta Omega, so delta Omega = pi+phase of Gp when AR=1 which is equal to the phase margin. So you can say that whatever phase margin we choose higher the phase margin higher will be the tolerance in terms of the dead time of the process.

So by using all this by this method we can select we can specify a certain gain margin or a phase margin and accordingly we can and find out the controller parameters. **(From 16:36 To 25:35 - Media Offline)** So to summarize we have seen this entire feedback control design system; what we are seeing is consists of three sub-problems for the first problem is about the synthesis where we want to identify what are the control variables, manipulated variables and how do you pair them.

The second part of the problem is a selection problem where we depending on what needs to be controlled, what type of a simple controller has to be used whether it should be a P controller, PI controller or a PID controller and then lastly we looked at four different methods in which we can select the parameters for the feedback controller those are based on performance based tuning; it can be a heuristic based tuning; it can be your direct synthesis based tuning or just now we looked is at is a frequency response based controller tuning.

And that is how you would end up selecting the values of controller parameters. So we will stop here. Thank you.