Chemical Process Control
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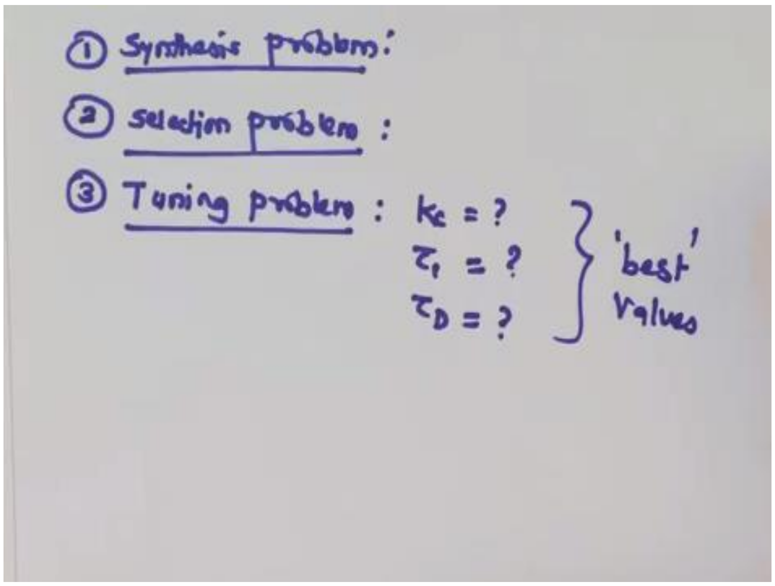
Lecture - 34
Criteria-Based Controller Tuning

Hello students. We are in the process of designing a feedback control system and so far we have seen how do you answer the first two questions. So let me recap.

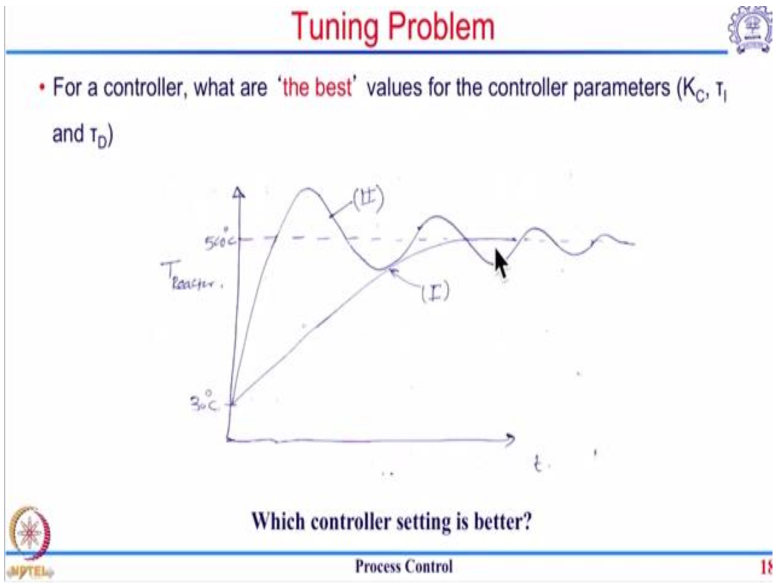


When you have to design a feedback control system, we start with the Synthesis problem where we select what are the control variables; what are the manipulated variables; how do you pair them and whether you pair them as a single input, single output controller or multi-input multi-output controller. So all those decisions about the architecture of the control system are taken care of in the synthesis problem.

Once the structure of the control system is finalized then we move onto the second problem which is known as a selection problem. In this problem, we look at the individual control loops and then try to identify within our current context of linear controllers whether that particular controller should be P controller, PI controller or a PID controller. Then the most important or more frequently encountered problem is known as a tuning problem.

For a given PID controller, how do you select good values of the controller called controller constant gain, the integral time constant and derivative constant depending on whether you are using a P controller, PI controller or a PID controller? So how do you select the best values for these would be would come under the context of tuning problem.

In order to give you a feel of what do I mean by best values let us look at this one example response wherein I am showing you a response of a temperature controller.



Let us say this is a reactor which has to operate at a temperature of 500 °C and we have put two controllers. Controller 1 which looks like a very overdamped response and there is a controller 2 which looks like an underdamped response. Both these are PID controllers and they are giving you a different response. However, in the end, both these controllers are able to achieve the response of the temperature setpoint is reaching the desired setpoint value.

So if I ask you a question which controller is better? So both these controllers have different Kc, τ_I and τ_D. And if I ask you a question which of these is better, can you answer that question? So I would ask you to pause the video and try to give a reason or select one of these two as the best option out of these two and try to reason out; I try to write down the reason why you have selected it to be the best one.

I hope you have thought about both these responses and try to compare them and then try to select which one is the best and let me make a case for each of those responses. So let us say that this reaction would not start unless the temperature reaches 500 ^{o}C or a very close vicinity of 500. Let us say 450 ^{o}C up to that the system will know the reaction would not start.

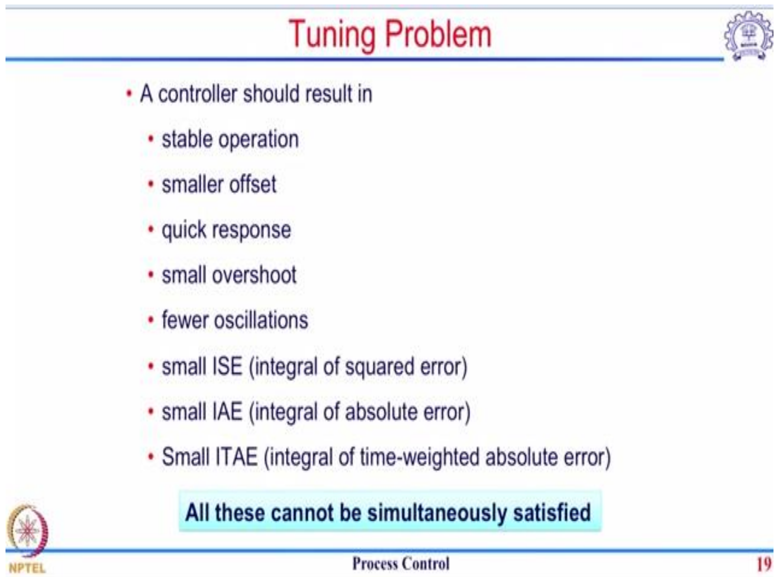
For the first controller, the 450 temperature would reach somewhere down here, so upto all this time, the reaction will not take place. So this is more like an idle time whereas in the second controller case the temperature reaches in the neighborhood of the final set point much quickly and therefore the reaction would get started much early, let us say somewhere here. Therefore you will have a very low idle time or non-productive time your profit your production will be high. So if that is my system then looks like the response 2, the PID controller 2 is better than 1.

Let me now give a different scenario for the same example. Let us say that for this system if the temperature crosses 510 ^{o}C, it is detrimental for the system. The way it can be detrimental is either the catalyst gets degraded or there is a reaction selectivity problem at 510 ^{o}C; some undesired product gets selectively produced so you are yield for the product goes down.

If that is the case, I would always want to ensure that my temperature inside the reactor never crosses that value or I would not want such a high offset or high overshoot above the setpoint value. So in that case, my controller 1 turns out to be a better option. So what I am trying to drive at is that depending on the scenario or what depending on what is your requirement, controller 1 or controller to either one can be a better option.

So there is no right or wrong in a very general sense. All you are interested in defining certain criteria. So for this particular example if my criterion is that I want a very fast response then and irrespective of any overshoot then my controller 2 becomes a better one. But if I say, I do not want any overshoot, in that case, the controller 1 becomes a better one.
So all I am trying to get at this tuning or the selection of the controller parameters heavily depends on what is my notion of being the best response. This is what we will use while developing these tuning methodologies. Typically when you design any controller, there are

different aspects which you would want the controller to have. So I have listed down a few of those aspects.



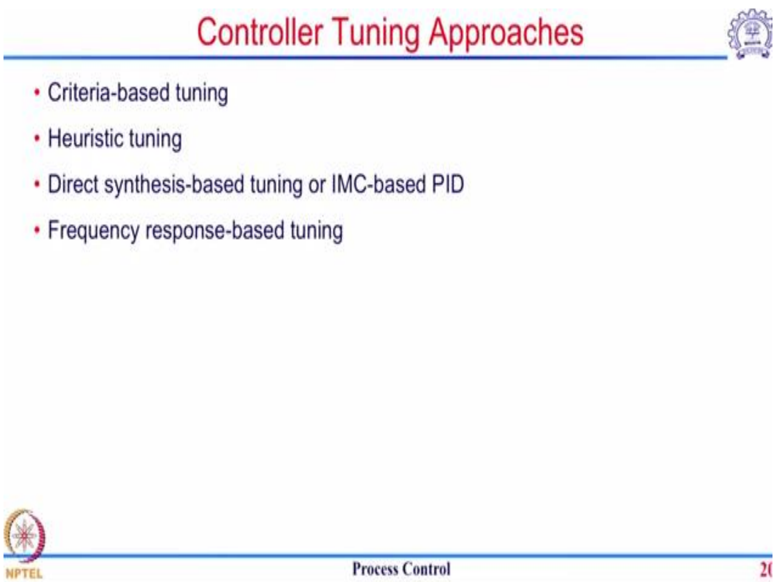
Let us see what are those. First and the foremost whenever you install a controller, the first thing you would want is the controller should not destabilize the process. We have looked at it when we talked about the stability analysis that there is always a limit on controller parameters up to which you can maintain stable operation. So irrespective of whatever is the type of a system, you would never want your controller to destabilize the process. So one of those criteria can be to ensure stable operation.

The other one would be in respect to the offset, we would definitely want to have a very minimal offset and most of the times if it is a temperature control or flow control or pressure control, in some cases, you would want 0 offset because you want to maintain that particular set point.

Then as we saw further yet the previous example, you may have a requirement that the response should be quick. In some cases, the requirement may be a small overshoot, the requirement may be about small oscillations and then there are some more quantitative terms like integral of square error; we will look at these last three criteria's in more detail when we talk about the methodology. So, all in all, there are a lot of different criteria which you can put on the closed-loop response of the system and you would realize not all of these requirements can be made by a single controller. A lot of times these are mutually exclusive.

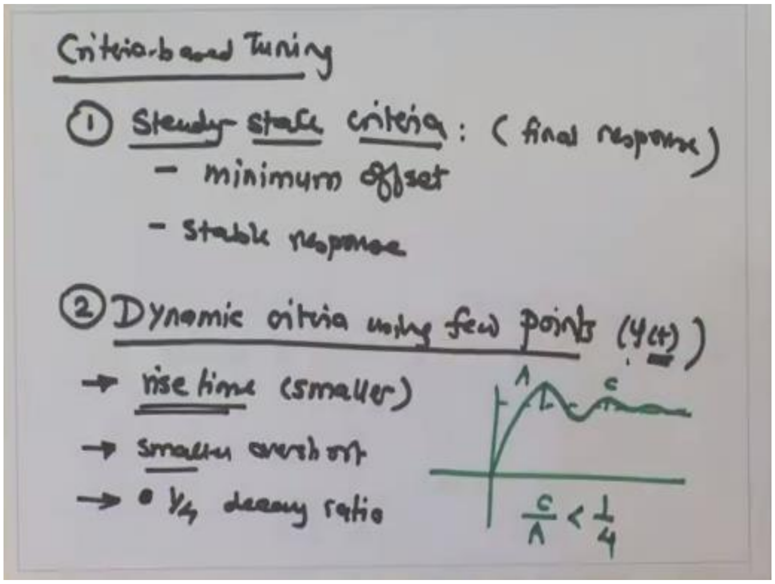
For example, if you want to improve the speed of response or if one of your objectives is a quick response, in that case, the oscillations in the system increase. We have seen that when we analyzed the response of a second-order underdamped system. So if I want to improve the speed of response then obviously I have to give away in terms of the oscillations. The bottom line is all these cannot be simultaneously satisfied. We have to pick the ones which are of interest or which are most important for the process and then we go forward in terms of designing the system.

So broadly, if I want to, the type of approaches which we are going to see for controller tuning are these.



One is a Criteria-based tuning, the second one will be Heuristic tuning, the third one would be more of a model-based control strategy where we talk about direct synthesis-based PID controller tuning or IMC-based controller tuning and lastly, we will make use of frequency response to get a very robust PID controller tuning.

Let us start with the criteria-based tuning as it is the most the simplest one to understand. When I say criteria base controller tuning, what we are going to do is we are going to select few of the criteria from the list which I had given you earlier and then have a closed-loop transfer function for the system in terms of the PID controller whichever is your controller of choice and then you solve those equations to maintain that criteria. These various type of criteria can be written down. So let us; which can be broadly classified into three types of criteria.



The first type is known as the Steady-state criteria. So which would include you would want or the Steady-state criteria means you are only going to look at the final response of the system. Based on the final response requirement from the controller, these criteria would be set so, one of those will be minimum offset. So offset you would typically get when the response steady. So one of the criteria can be to have a minimum offset and 0 offsets would definitely mean that you have to either use a PI or a PID controller.

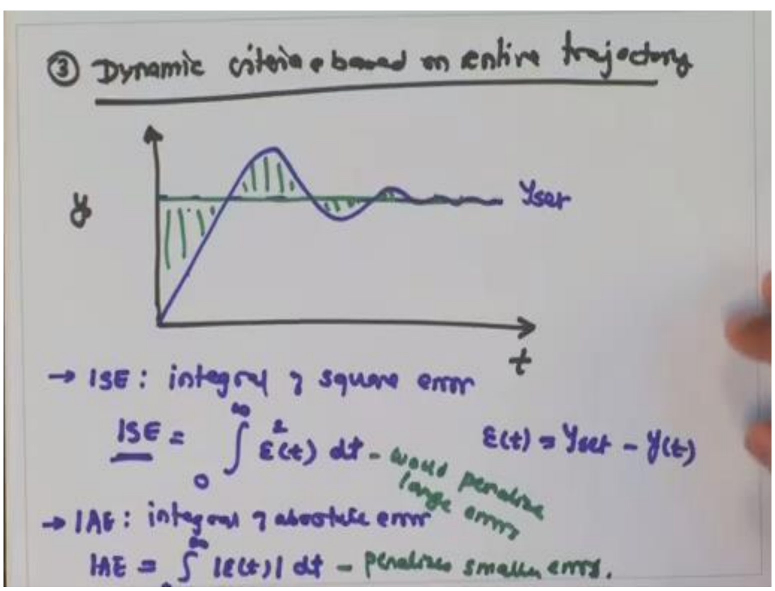
The other one can be a stable response. So we have seen that stability again eventually is governed by the steady-state, finally, whether the system response is bounded or unbounded. So again this becomes the steady-state criteria that may controller tuning should give you a stable response.

The other type of criterion is dynamic criteria using a few points. So here we will look at the actual trajectory of the output as a function of time and then have certain criteria on the type of response or the value of the response at certain important points.

So to give you some example, it may be about a rise time and a smaller rise time. So one of the criteria may be very fast reached; the steady-state has to be reached very quickly. We have seen the difference between a rise time and response time, so when I say rise time being smaller it means the reactor or the output reaches the setpoint value or in the neighborhood of the setpoint value very quickly; it may not stay there but the first time it reaches that setpoint value that will kind of govern that the process has been moved to the in the neighborhood of the set point.

So having a smaller rise time generally gives you a quick response so that can also be a criterion. Here you are looking at only one point out of the response. Rise time is calculated based on the time instant when the output first reaches the setpoint value, so it is based on a single point of the dynamic response.

The other one can be a smaller overshoot. It can also be based on decay ratio and very common criteria which are used is quarter decay ratio. So you want oscillations inside the system to die down very quickly. So one of the very popularly used criteria is a quadratic ratio, so if this is the response of a controller, then you would want that this C/A should be less than 1/4. So you can see that if this is my criteria and we have already seen that.

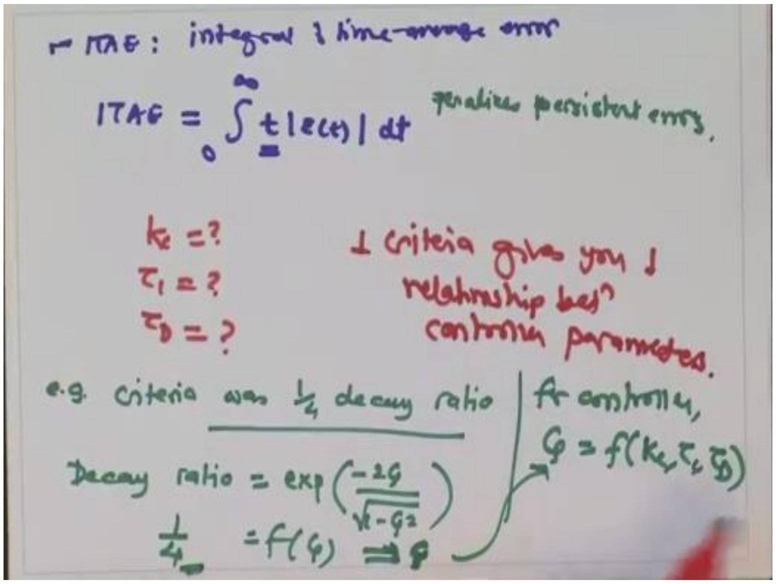


Then the last type of criterion is again dynamic criteria but they are based on the entire trajectory. It looks at overall how does the controller perform in terms of tracking the set point or maintaining the set point. The setpoint and the response are as shown in the above slide, so it is going to capture; these are going to capture how much is the deviation from the set point. All these represent a deviation from the setpoint, so ideally the controller should have maintained the output along this yset line which is not practically possible.

Any deviation from this value is going to give you an error. So these criteria are based on minimizing this error and there are different ways this error can be quantified. One of those is ISE or Integral of square error. ISE you would define as an integral over the entire trajectory of squared errors where $\varepsilon(t)$ will be $(y_{set}-y)$. So you would try to calculate how much is the overall deviation from this set point value as a function of trajectory.

The more amount of the more these trajectory is closer to the set point, smaller will be the integral of square errors. Similarly, IAE is also defined as Integral of Absolute Error. IAE will be defined as integral over the entire trajectory of absolute error. Now you may want to say that both these are equivalent; the difference between the two is these ISE your integral of square errors would penalize large errors. Because if the error is small then the square of the error becomes even smaller, so ISE tends to penalize large errors only.

In order to penalize the smaller errors, you would define an integral of absolute error. However, both these methods are not going to account for how long the error was there in the system. Indirectly it is accounted because you are taking an entire integral over the entire trajectory. But if you want the error to go or decay very quickly, then the third type of criteria is defined which is known as integral of Time-Averaged Error.



ITAE will be defined as t times ε(t) dt. If the error remains for a longer time then the penalty will be more, it will tend to penalize persistent errors. These are the different criteria which somebody might enforce.

Now let us look at what is the relationship between having the criteria and the end goal of finding what are the values of controller parameters; we have not at all seen how is this related to tuning the controller. So what you end up doing is you select criteria and then accordingly let us say if your criterion was, so for example criteria was quarter decay ratio. Then we know that decay ratio is for a second-order system given by,

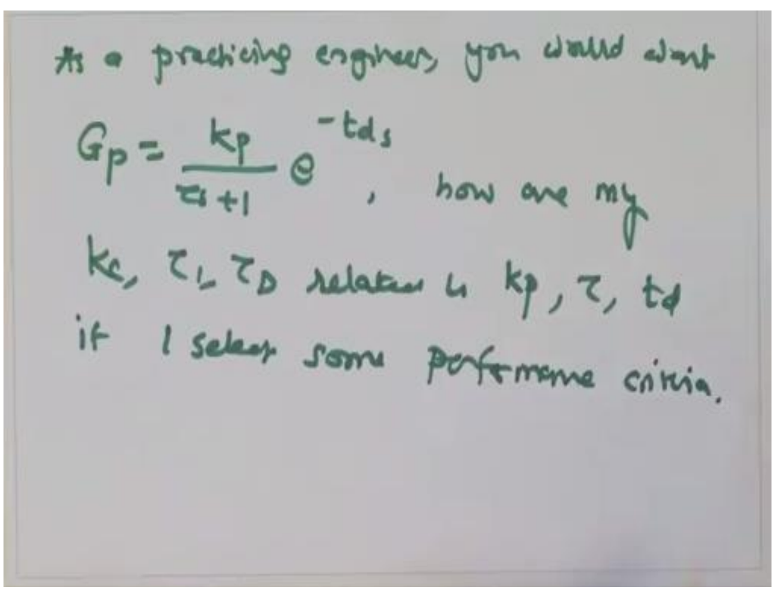
$$Decay\ Ratio = \ exp\left(\frac{-2\zeta}{\sqrt{1-\zeta^2}}\right)$$

It is a function of the damping coefficient, smaller is the damping coefficient, larger will be the decay ratio. So what you would end up doing is you would specify the value of damping coefficient into your closed-loop transfer function equation which will give you what is the required value of the damping coefficient. And for the controller, this damping coefficient will be a function of K_c, τ_I, and τ_D. So once you get this damping coefficient you can put it here and then you will get one relationship between the controller parameters.

So every criterion will give you one relationship. One criterion gives you one relationship between controller parameters. So by having multiple such criteria depending on how many parameters you have, you may get a number of equations in these controller variables with these controller parameters as variables and we can solve them to find out the best values of the controller parameters which are going to satisfy these performance specifications.

That is how one would end up tuning a controller using the criteria-based method. However, what you would realize is that this is turns out to be a very time consuming exercise because this; let us say if I have to use the quadratic decay ratio then what you are going to have is that you would require a relationship between damping coefficient and decay ratio and you would also require a relationship between controller parameters and a damping coefficient.

A lot of process transfer function information is required. You have to do a lot of mathematical rearrangement and equation solving to eventually come up with the values of the controller parameters. It turns out to be very time consuming and when you are operating a plant you may not want to be spending your time going through all this rigorous algebra and you would really want to have some easy way to get these controller parameters given the information about the process. So what you would ideally want is;



So as an as a practicing engineer, you would want if my process is let us say first order + dead time, how are my Kc, τ_I, and τ_D related to the process parameters if I select some performance criteria? So when you want to narrow down your problem something like this, then you rely on someone else to carry out a lot of experiments or mathematical analysis for you for this kind of a system and then give you some quick formulas where you can plug in the values of Kp, $\boldsymbol{\tau}$ and t_d and then get the values of corresponding PID controller.

This is exactly done by using what is known as a heuristic controller tuning where there are a set of heuristic rules which are available in the literature which also tells what are the performance criteria which they have used and they will help you in terms of deciding the value best values of Kc, $\boldsymbol{\tau}_I$, and $\boldsymbol{\tau}_D$. So we will take a short break here and when we come back we will look at how do you go about tuning using this heuristic controller tuning rules. Thank you.