Chemical Process Control
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Lecture – 29
Laplace Domain Analysis - Part II

Welcome back, let us now consider another method of computing the maximum value of controller gain which can be used for that particular example or in general, in Laplace domain is known as direct substitution.

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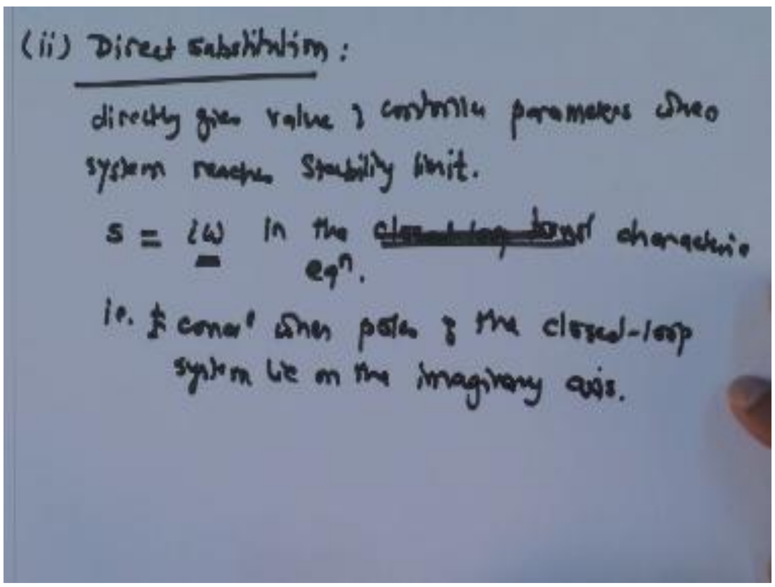

So, direct substitution method directly gives you the value of controller parameters when the system reaches stability limit. If you feel the effect of k_c or the controller gain on the stability of the feedback system with 3 CSTRs, we see that for a small value of k_c, the system was almost very much stable without any sustained oscillation. As you keep on increasing the value of controller gain, the oscillation will increase till a value of k_c = 64; the system just sustained oscillation which is marginal stability.

And then any value of k_c greater than that will result in growing oscillation or instability. So there is another way of calculating this limit when the system has sustained oscillation, and that is known as the direct substitution method. You substitute s = iω in the characteristic equation.

So, what it means is; it tries to find out the value of the controller parameters at which the poles will lie on the imaginary axis.

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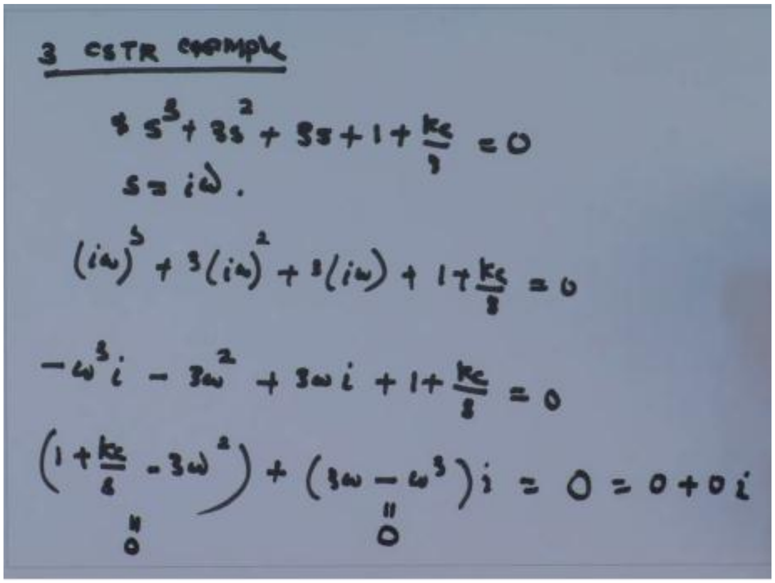

This is the condition when poles of the closed-loop system lie on the imaginary axis. Let us apply this method to our example of 3 CSTR's for which the characteristic equation was,

$$S^3 + 3S^2 + 3S + 1 + Kc / 8 = 0$$

Now we substitute $S = i\omega$, so what we are going to get is;

$$(i\omega)^3 + 3 (i\omega)^2 + 3 i\omega + 1 + Kc / 8 = 0,$$

which on simplification what we get is,

$$-\omega^3 i - 3\omega^2 + 3 \omega i + 1 + Kc / 8 = 0.$$

And then we bring the real and imaginary terms together, so what we will have is $1 + Kc / 8 - 3\omega^2$ that is the real term + $(3\omega - \omega^3) i = 0$, so when we say this equation = 0 which is same as 0 + 0i, so we can equate the real term to 0 as well as the imaginary term to 0, so that will give us 2 equations.

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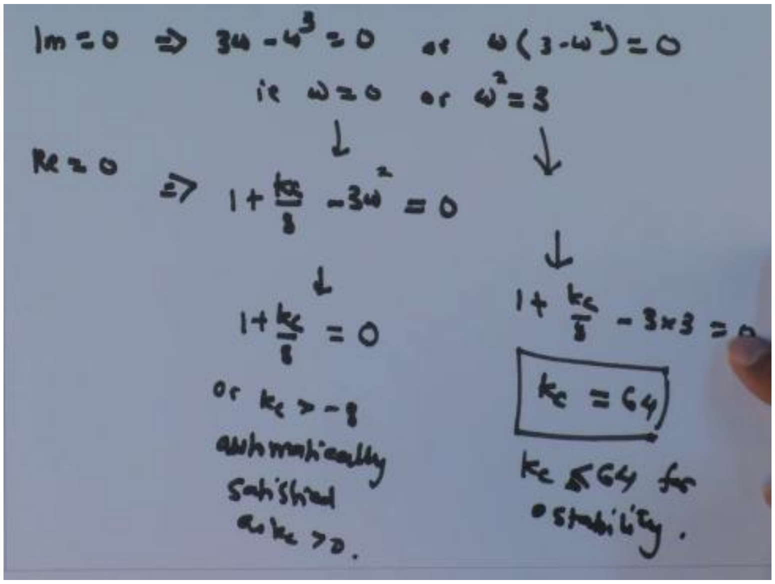

So, what we get is from the imaginary term = 0 gives us,

$$3\omega - \omega^3 = 0 \text{ or } \omega * (3 - \omega^2) = 0$$

That is $\omega = 0$ or $\omega^2 = 3$, when we say real term = 0 that gives us,

$1 + k_c / 8 - 3\omega^2 = 0$

When $\omega = 0$, we will get $1 + k_c / 8 = 0$ k_c should be > -8

When $\omega^2 = 3$, what we get is $1 + k_c / 8 - 3 * 3 = 0$, so $k_c = 64$.

As we have $k_c > 0$, we again get the maximum limit on the controller gain, which is the same as the one which we obtained by using Routh Hurwitz criteria. If you have a controller gain of 64, the closed loop will have sustained oscillation, if we increase the gain, it will lead to instability, and if we reduce the controller gain, here we will have decaying oscillations.

So, k_c should be < = 64 for stability. By using this method also we can compute the stability limit or what is the maximum value of controller parameter which we need to use to satisfy the stability criteria or to ensure that the feedback system will remain stable.

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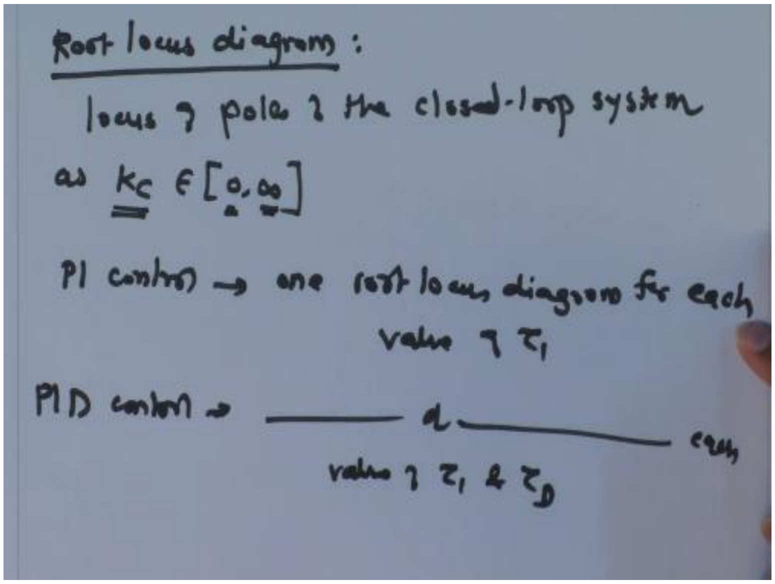

Now, all this information is typically presented in the form of a Root Locus diagram which will be used to design a feedback controller. Root Locus diagram is the locus of poles of the closed-loop system as the controller gain goes from 0 to infinity. It is the locus of all the poles of the closed-loop system as you move from an uncontrolled or open loop process to a closed loop process with infinite gain.

Now, this is always plotted as a trajectory of controller gain. If we are plotting this diagram for a PI control, we will have 1 Root Locus diagram for each value of τ_I and same way if it is the PID control, will have 1 Root Locus diagram for each value of τ_I and $\tau_{D.}$ You can see that if we have a PI controller or a PID controller, we will have large number of such Root Locus diagrams which we will have to draw depending on the values of τ_I and τ_D.

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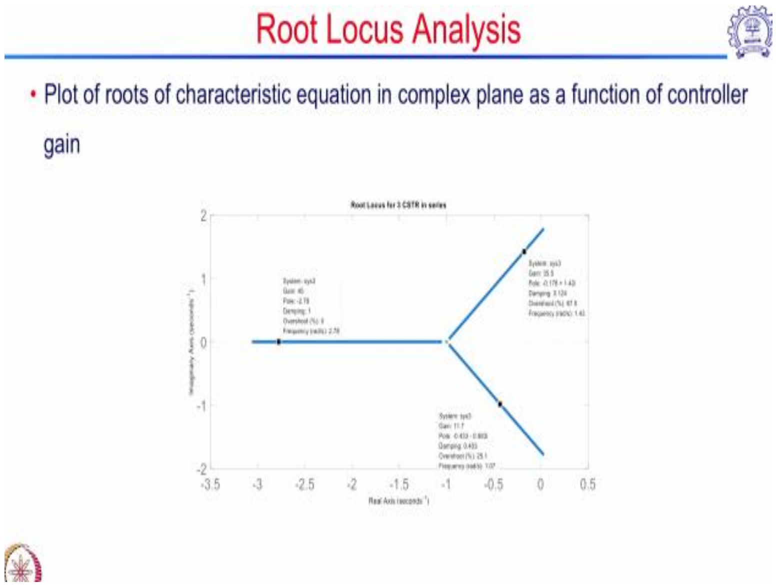

So, how does this Root Locus diagram look like; so if we see the 3 CSTR example, this is how the Root Locus diagram will look like. You will see that the Root Locus starts with the point which has a triple root of pole = -1.

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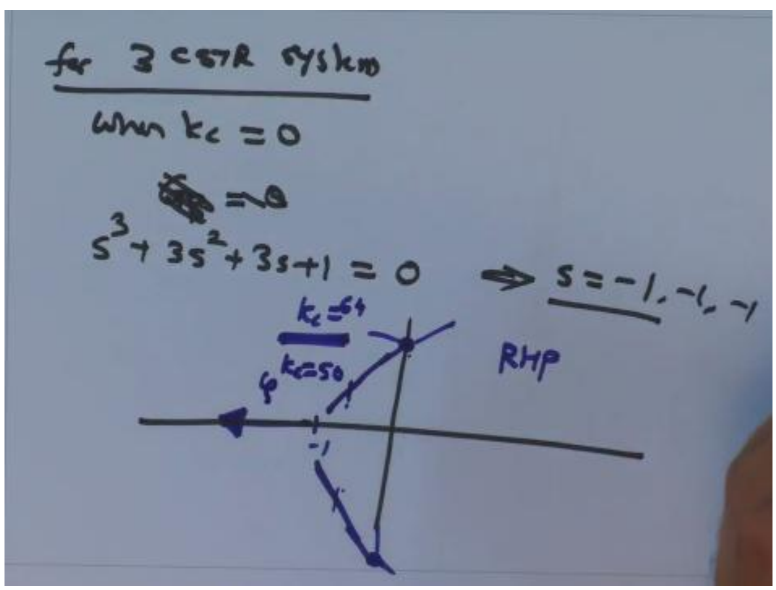

So, for 3 CSTR system, when $k_c = 0$, at that time, what we have is, $S^3 + 3S^2 + 3S + 1 = 0$, which gives me S = -1. It is triple root, so all the Root Locus diagram will start with the point of -1, and you can see that as the k_c value of the controller gain increases, one of the root moves along the real axis towards negative infinity.

It is never going to cross the imaginary axis and will never go into the right of the plane. The other roots bifurcate into complex conjugate roots, and they have the real part which is decreasing as you increase the controller gain and when the controller gain reaches 64, you have a pair of complex conjugate roots, if you increase the controller gain, it will go into the right of the plane.

By representing this closed loop behavior using Root Locus diagram, you can again represent what is the maximum value of controller gain which you can have. The additional information which Root Locus diagram also provides is the nature of instability, so here it shows that a pair of complex conjugate roots are moving into the right of plane.

So that means the closed-loop system will have growing oscillations. Root Locus diagram additionally gives you an information about how does the response of the system look like, even before the controller gain value < 64, let us say some value around 50, it will say that the system will have decaying oscillations, it will also tell you what is the corresponding damping coefficient, so you can also calculate the period of oscillation.

So, Root Locus diagram kind of condenses all this closed loop information along with the limit of the stability. This is a very commonly used method to represent the stability analysis data for a closed loop system or a closed loop feedback system. You draw this Root Locus diagram, and then that will tell us what is the maximum value of controller gain which can be used for that particular system, may be it is the P controller, PI or a PID controller.

So far we have seen these 2 methods in the Laplace domain; one was the Routh Hurwitz criteria, and the other one is the direct substitution, both these methods give you a limit up to which the controller gain can be o increased or controller parameter can be changed, so that you maintain the stability of the closed-loop system.

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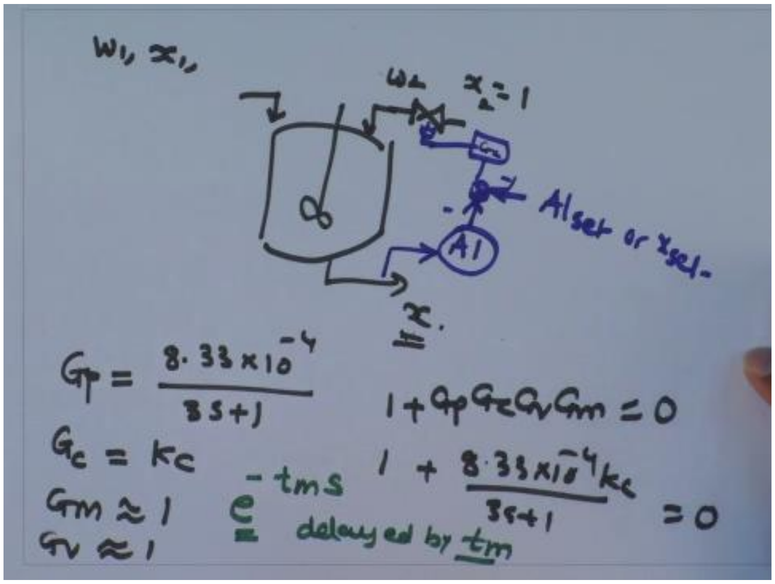

Now, let us say we have a blender. Blender is a very commonly used chemical engineering piece of equipment. Here a stream of flow rate W_1 or the mass flow rate W_1 and mass fraction of x_1 is coming into the system and what you desire is some particular fixed purity x. To achieve that you generally add a blending agent which is a pure component, so that you can achieve the desired value of the product purity.

Typically if I am using a feedback control on the system, you will first analyze what is the corresponding purity, it will be given to the controller, the desired set value or x_{set}, will be given to a controller and then that controller will change the flow rate of this blending agent.

So, for one such example, the process transfer function which is the transfer function between this product purity and the flow rate of this blending agent that can be given as $8.33*10^{-4}/(3S+1)$ and we can again see, what is the effect of controller gain. If we are using a proportional controller, we can assume that measurement is instantaneous, valve dynamics is also fast. So, in that case, if we are trying to analyze the Laplace domain. We will find out that the closed loop transfer function is $1 + G_p\, G_c\, G_v\, G_m = 0$.

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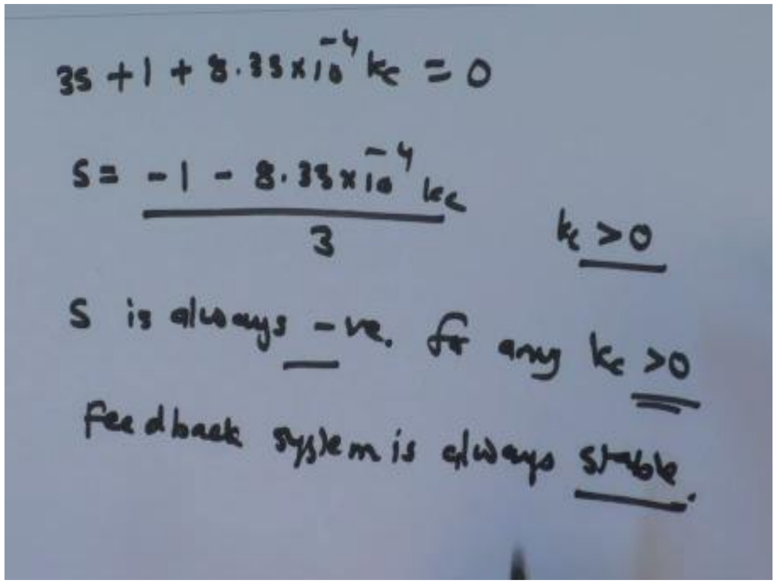

So, we get $1 + 8.33*10^{-4} *k_c /(3S + 1) = 0$, which on simplification will give us $3S + 1 + 8.33*10^{-4} *k_c = 0$ or $S = (- 1- 8.33*10^{-4} *k_c)/ 3$. Again this is the reverse acting controller, so k_c is always > 0, so you can see that S is always negative, for any $k_c > 0$. What we will realize is that the feedback system is always stable. Now, let us slightly tweak this system and try to look at a more realistic picture.

What we are doing here is that we measure the composition and accordingly, the system is taking action. Composition measurement is very slow, so typically, temperature, pressure, level and flow, all these can be measured almost instantaneously, and there is almost no lag in terms of change in the variable and what you get as a measured value.

However, the same cannot be said about composition. Composition measurements are very tricky and even though, we have sophisticated instruments like GC, it will still have some amount of lag by the time, you have the measurement in a digital form. So even though I am saying that the measurement is almost reliable, what typically ends up happening is a transfer function where you have some delay.

So, measurement is delayed by an amount of time = t_m, which is known as measurement delay. It is a very common feature whenever we are working with composition measurements, so let us see what happens to this stability analysis if you have some measurement delay.

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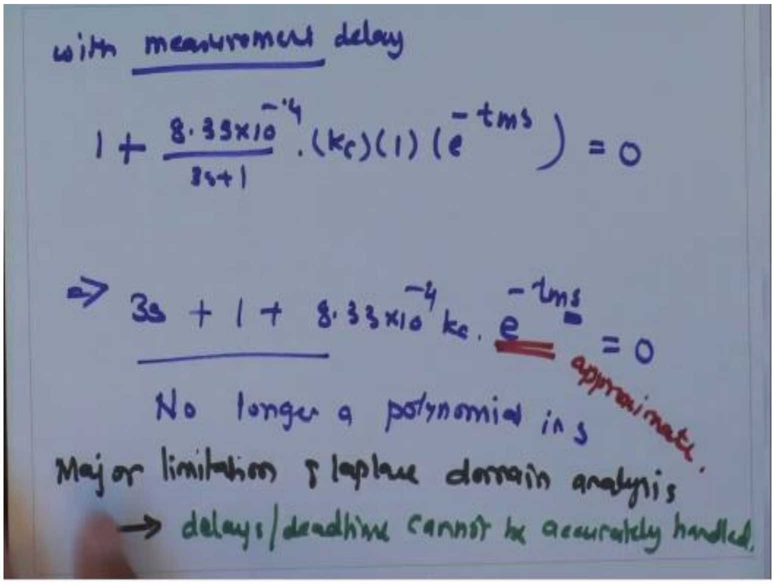

So, with measurement delay, the closed-loop transfer function on simplification will give us $3S + 1 + 8.33\ 10^{-4} *k_c * e^{-tm*s}$. Now you can realize that this is no longer a polynomial because of this exponential term, because of that we cannot directly use the method of Routh Hurwitz to assess the stability or even the direct substitution method will not be directly applicable.

This is a major limitation of Laplace domain analysis. Whenever we have any dead time in the system, or there is measurement delay or the original process transfer function has a delay, they cannot be accurately handled, so delays or dead time cannot be accurately handled. So, does that mean if we have a delay, we cannot use Laplace domain analysis?

So that is not entirely true, we can do sort of an approximate analysis, and that can be done by approximating this dead time term. So will have to approximate e^{-tm*s} and that approximation is done by what is known as Pade's approximation.

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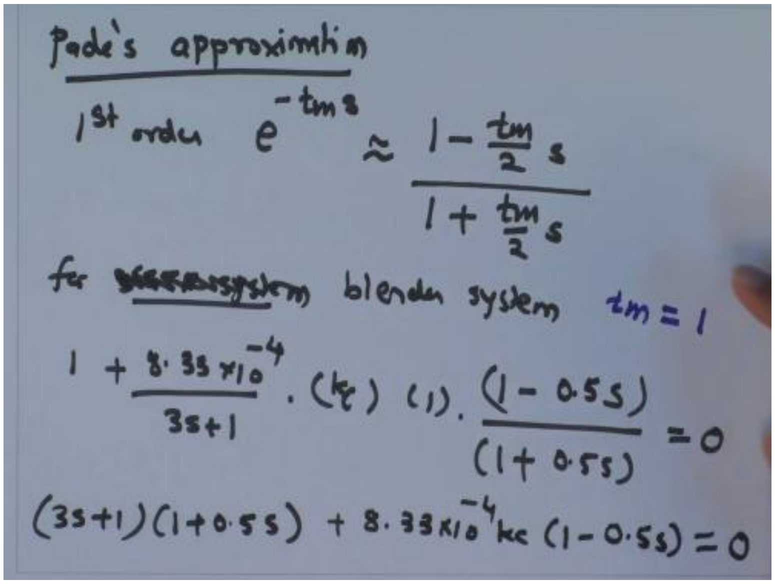

So the Pade's approximation can be of different orders. Very commonly we use the first order Pade's approximation. It says that if you have the transfer function of this form(e^{-tm*s}) which is a pure delay, we can approximate it as [1 - (t_m /2) s] /[1 + (t_m /2) s]. So we split the contribution of this transfer function as numerator and denominator dynamics. So, if we use this for our example, for the blender system, we get,

$$1 + \{8.33\ 10 - 4 * kc\ /\ (3S + 1)\} * (1 - 0.5\ S)/(1 + 0.5S) = 0.$$

So, now if you try to simplify this system, what we will get is,

$$(3S + 1)(1 + 0.5S) + 8.33\ 10^{-4} * k_c * (1 - 0.5S) = 0.$$

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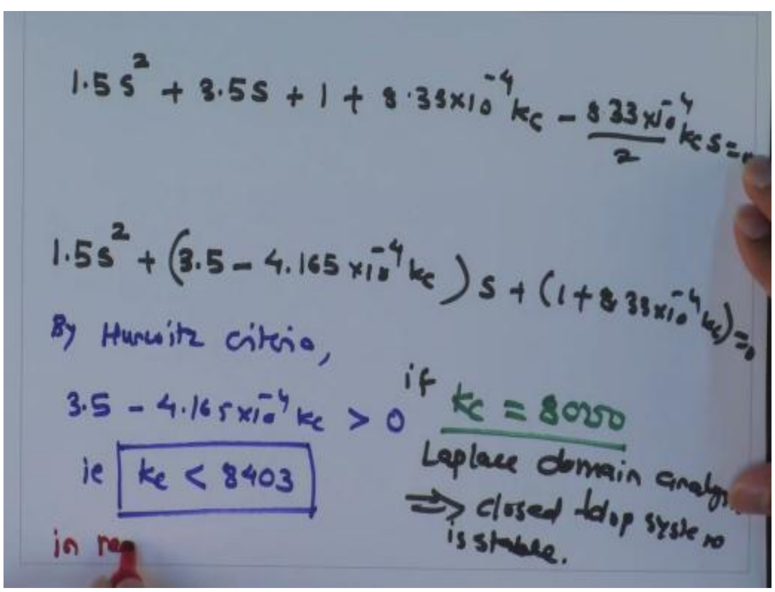

So, if we simplify this further what we will get is,

$$1.5\ S^2 + 3.5\ S + 1 + 8.33\ 10^{-4} *k_c - (8.33\ 10^{-4} *k_c /2)* S = 0$$

In a condensed form what we will get is;

$1.5\ S^2 + (3.5 - 4.165\ 10^{-4}\ k_c\ )s+ 1 + 8.33*10^{-4}\ k_c$.

So, now if we use the Routh Hurwitz criteria, we will have $3.5 - 4.1652*10^{-4} *k_c$ should be > 0 that is k_c should be < 8403.

So, you can note that earlier when there was no delay in terms of measurement, I could use infinitely large controller gain and the system would remain stable. However, the moment I have a measurement delay of let us say one unit, the controller gain reduces to 8403 and you can easily test that if the delay in the measurement keeps on increasing, the stability limit will also keep on reducing.

So, if there is a larger measurement delay, then the maximum controller gain which you can use without affecting the stability or without destabilizing the system keeps on reducing. By using this Pade's approximation, we can approximately do the stability analysis, and the reason why I say approximately is; let us say for this particular system, if I use k_c = 8000, then Laplace domain analysis tells me that closed-loop system will be stable.

However, if I actually implement/stimulate this particular system in k_c = 8000, what I will realize is or in reality, k_c = 8000 is unstable. So why is this happening? My Laplace domain analysis is telling me that the maximum controller gain I can use is 8400 however, even if I use k_c = 8000 for this system, the oscillations are growing, and I get instability. The reason for that is the way we are doing the stability analysis is that we are approximating the delay as by Pade's approximation as a proper transfer function of numerator and denominator which is not correct.

Whenever we do approximation, we are going to get an approximate stability, and the limit is not going to be true, and you can see that it is quite a drastic difference. Even if I use k_c = 8000 which is quite low compared to the stability limit. Especially for that reason, we need to have a method which would not require such an approximation of dead time.

That will be possible if we do the stability analysis in frequency domain. We will take a break here and when we comeback, we will see how stability analysis for systems with dead time can be accurately handled. If we do the analysis in frequency domain which is inverse time domain as against the Laplace domain which is the complex time domain. So, we will stop here for this lecture and when we come back, we will look at the frequency domain stability analysis, thank you.