Chemical Process Control
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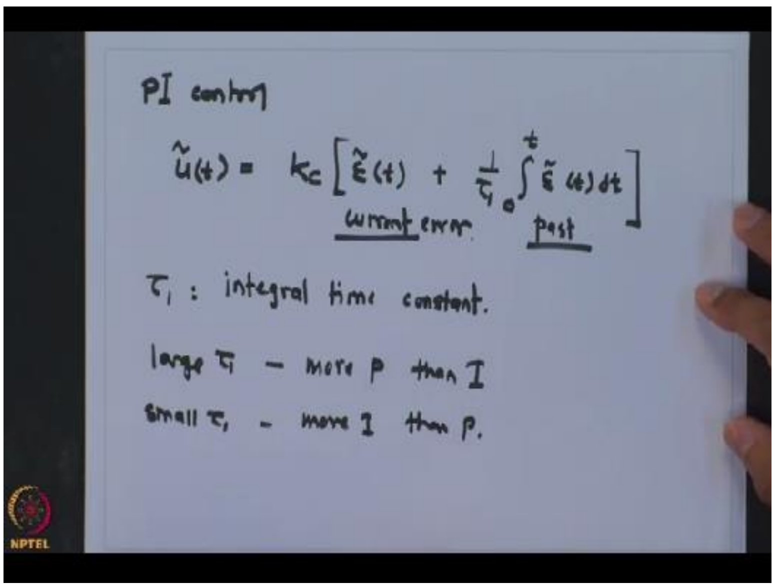
Lecture – 23
Proportional-Integral Control

Hello students. In the last lecture, we looked at the simplest of the linear controllers which was the proportional controller and we saw that one of the main limitations of a proportional controller is that it is not able to reach, it is not able to give you the response which reaches the desired value of the set point value and we always end up with offset which is non-zero. In order to get the offset free response, you have to increase the controller gain to mathematical infinity which in the reality is not possible.

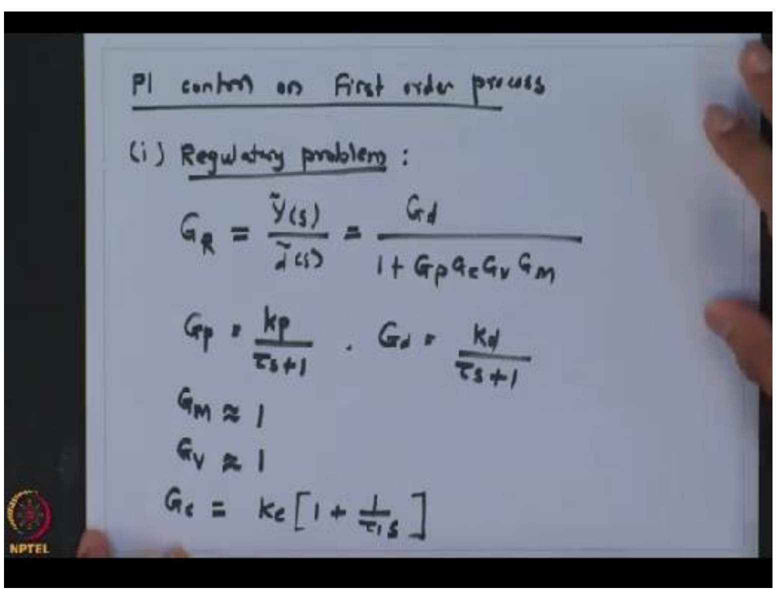


So what we saw was that when we have a proportional controller, the deviation in u was proportional to the error or the current error. Here, we are just looking at whatever is the instantaneous value of error and the controller is taking action based on the current instant. It is not looking at how the process has reached there. So there is no contribution coming from the past history of the process and that is resulting in to this offset.

In order to improve this performance or rather I would say in order to get offset free response, what we do is we along with the current value of error, we also get a term which is proportional to the integral of all the previous errors. So what we do is along with the current contribution, we also look at the past contributions or how the system has reached to that point and the weighing factor for that is 1/tau$_I$ will come to that significance of that in a bit.

So what we have is a controller which takes action proportional to the error as well as integral of the error; that is why it is known as a PI controller. We will see how this addition of integral action is going to cause the offset to go to 0 even for finite values of proportional, even for the finite values of controller gain. So this tau$_I$ is known as integral time constant. In sense, it is used to kind of trade-off between how much contribution we want from the current error and how much contribution we want from the previous history of the process or past errors.

A large value of tau$_I$ means more P than I. So, there will be more penalty or the contribution of the output will largely depend on the current error and less on the history. When the small integral time constant is used, you will have more penalty on the integral term than the proportional term. This time constant tells you, so let us now look at how the addition of integral action, how it is going to respond in terms of a simple process which let us take a first order delay process, a first order lag process.



Let us start with a regulatory problem. We had seen that the regulatory transfer function is given between the output and the disturbance is as,

$$G_R = \frac{\tilde{y}(s)}{\tilde{d}(s)} = \frac{G_d}{1 + G_p G_c G_v G_m}$$

In this case we are looking at a first order process. So let us say,

$$G_p = \frac{k_p}{\tau s + 1}$$

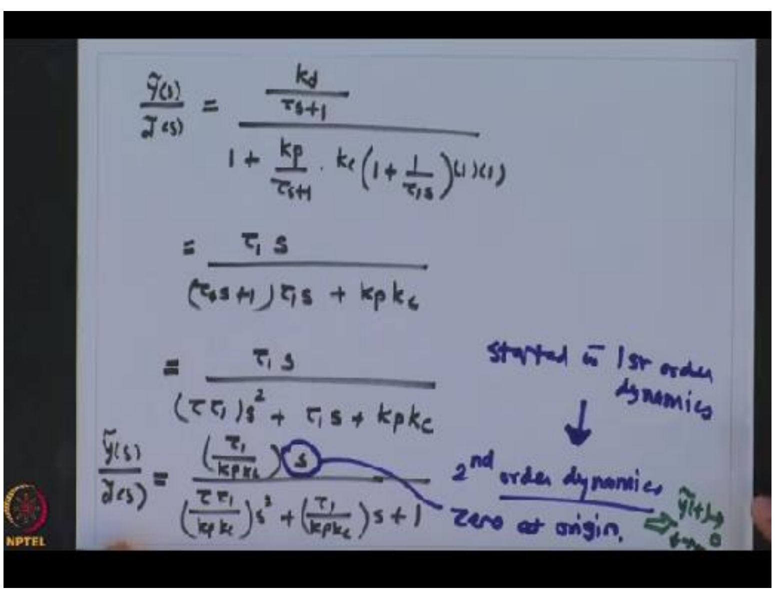
For simplicity, we will also assume, that G_d also has the same time constant. It is not necessary to have these two time constants to be the same but this simplifies the algebra.

$$G_d = \frac{k_d}{\tau s + 1}$$

We will also assume that the measurement is instantaneous and the valve's transfer function is also equal to 1. And then lastly, the controller we have used is the PI control. So the Laplace transform of that will be given by,

$$G_c = k_c \left[1 + \frac{1}{\tau_I s}\right]$$

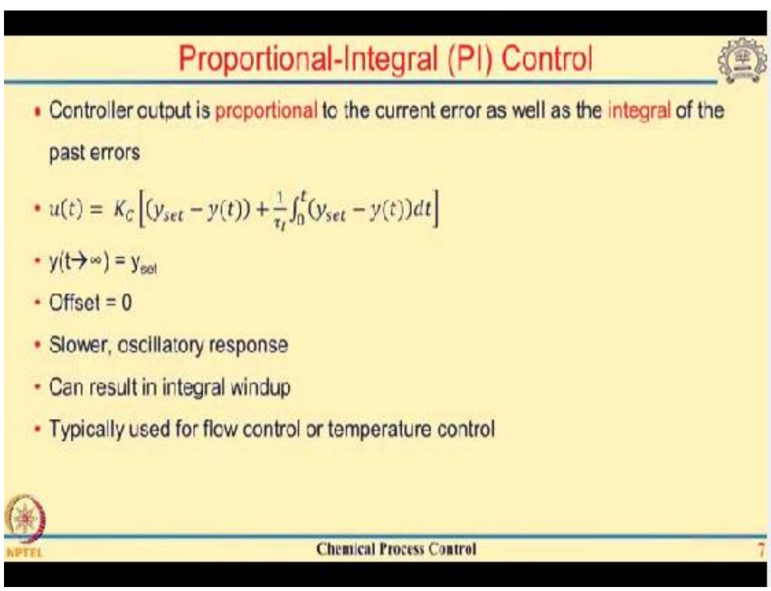
So let us see, how does this regulatory response looks like?

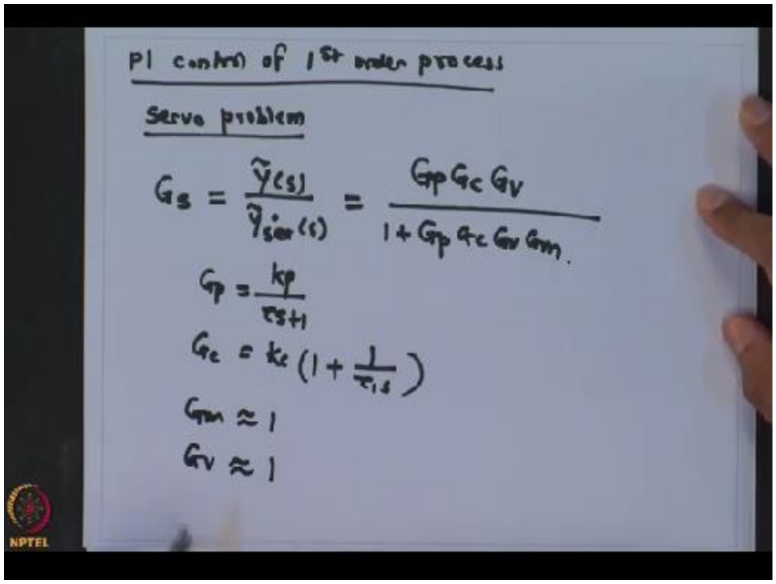


Simplifying, eventually we are getting the regulatory response is as a second order process. We started with a first order process and because of the integral action, we have increased the order of the system. If we try to connect to what we did in during the dynamics module, what it is going to cause is it is going to cause the system, the closed-loop system to response sluggishly compared to the original system as we are increasing the order of the system.

This disadvantage or limitation is coming at an advantage that we have a zero at origin. So that is the main contribution of the integral action. This ‘s’ is in the numerator is coming because of this additional integral action which we have done. And if we again recall to the numerator dynamics lecture, when we have a zero at the origin, the whole, this result in the response to go to 0.

So irrespective of the amount of step change in the, irrespective of the value of k_c and tau_I as long as they are not 0 or infinite, what you would see that the zero of this transfer function will always occur at origin and because of that the response, the y_t will always go to 0 as time goes to infinity. So what does that mean, irrespective of all these things even though a disturbance occurs, the response will always go to the desired value. We have seen that with the addition of this integral action, we have been able to cause this offset to go to 0. We will see that the same thing is true even for the servo problem.





PI control of first order process and if we look at the servo problem, in that case the transfer function Gs is between the output and the set point is,

$$G_s = \frac{\tilde{y}(s)}{\widetilde{y_{set}}(s)} = \frac{G_p G_c G_v}{1 + G_p G_c G_v G_m}$$

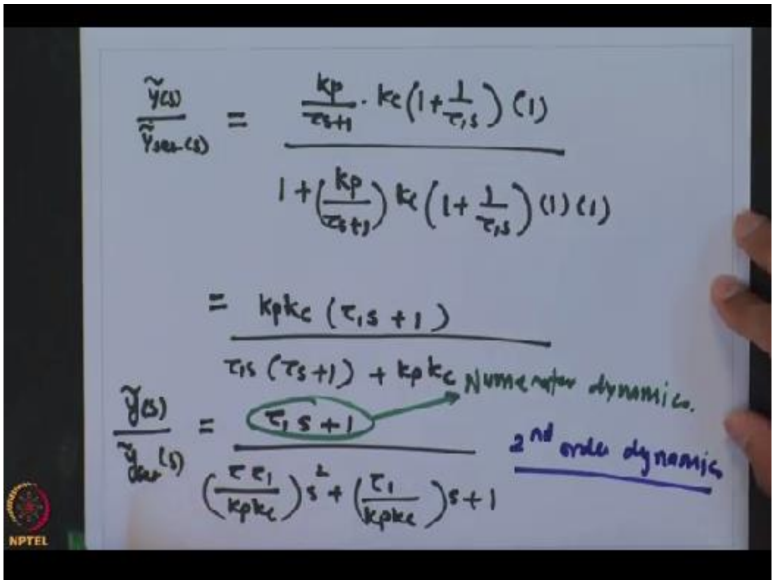
Again for this process, we have,

$$G_p = \frac{k_p}{\tau s + 1}$$

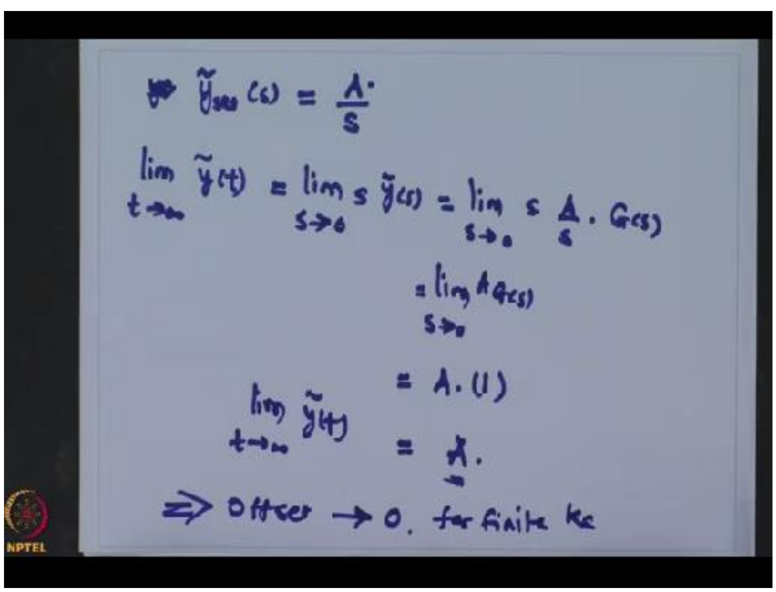
$$G_c = k_c \left[1 + \frac{1}{\tau_I s}\right]$$

$$G_m \approx 1 \; and \; G_v \approx 1$$

In that case what we are going to get is as follows-



So, ultimately the response or the transfer function between the set point and the final output is again a second order. So again the integral action has increased the order of system by 1. The system is going to be slower but that statement is not entirely correct. The reason being we also have numerator dynamics which is of order one. So the net effective order of the system still is 2-1=1. The system is still going to behave more like a first order system with initial non-zero slope.



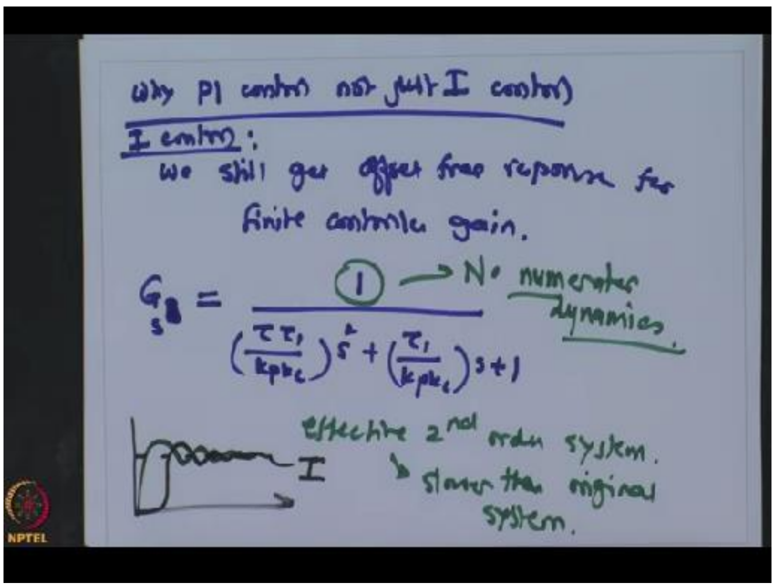
And if you look at the gain of this system, if I say the set point had a step change of magnitude ‘A’ and I am looking at where does the output reach as time goes to infinity with limit using the final value theorem,

$$\lim_{s \to 0} s\tilde{y}(s) = \lim_{s \to 0} \frac{A}{s} s\ G(s) = \lim_{s \to 0} A\ \ G(s)$$

So,

$$\lim_{t \to \infty} \tilde{y}(t) = A$$

So, what you get is the final value of the output, is also same as whatever was the set point change which was given. This implies offset goes to 0 for finite value of controller gain. Using both this regulatory mode as well as servo mode, what we have been able to see is that because of this addition of integral action, we have been able to move this offset to 0 even for finite values of controller gains.



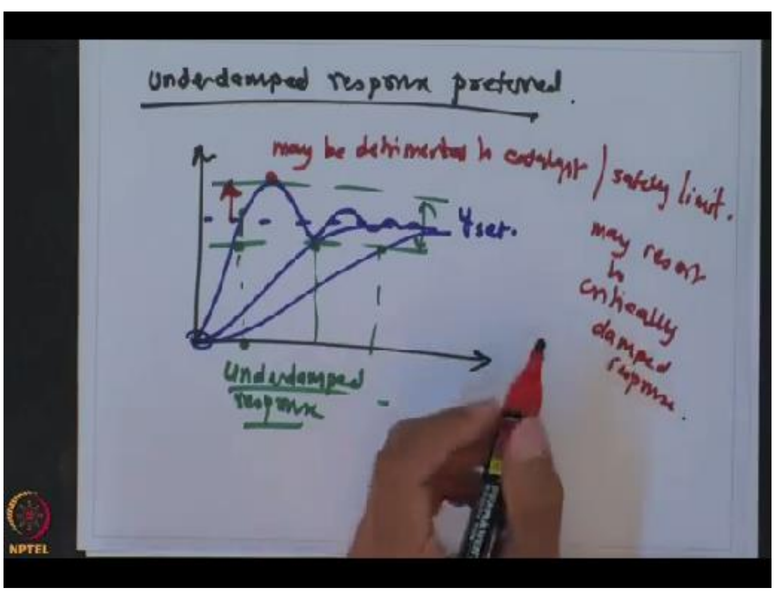
Now before proceeding further, let me also make a comment on why do we always go with the PI control and not just I-control. What if I had used only the integral action and no proportional action? In that case, we still get offset free response. If I say I-control, so it does solve the problem of not getting any offset even for finite values of gain, controller gains. But if you look at what you would get as the servo transfer function, it will give you,

$$G_s = \frac{1}{\left(\frac{\tau\tau_I}{k_p k_c}\right)s^2 + \left(\frac{\tau_I}{k_p k_c}\right)s + 1}$$

The difference is that it has no numerator dynamics. The implication of that is this is effective second order system so which is slower than original system. So if we use only the integral action or the penalty only on the past values, what we are going to end up is the system, the closed-loop system becomes slower than the original process and the system does not respond that fast.

If we compare these two responses; in one case, so if I use only I-controller, the response is underdamped which is shown in above figure. The addition of this P, the proportional action is going to make this response as shown in same figure. So this is still reaches the final set point much quicker compared to just the I-control for the same values of integral time constant. That is the main reason why only integral action is never used. It is always used in conjunction with the P controller.

Now you can see that as we have a second order dynamics, this opens up lot of possibilities in terms of what kind of dynamics we might expect from closed-loop system. This closed loop system either this or the PI control, what we can see that depending on the values of k_c, tau, tau_I; so tau and k_p are sets based on the process. But by choosing the values of kc and tau_I, we can always make this response either become overdamped, critically damped, or underdamped and most of the times what we end up doing especially for servo type of problems, is that we try to make it as an underdamped response.



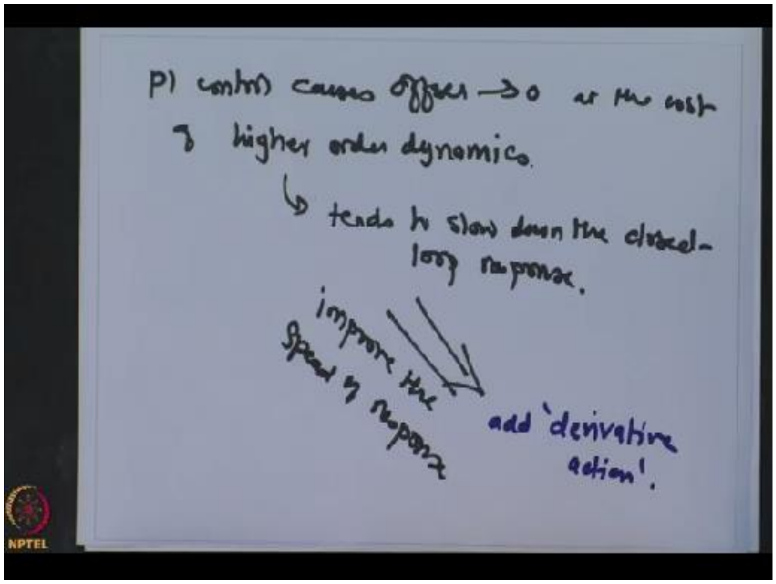
Let us say if we have, the set point which we are tracking. The responses of overdamped, critically damped and underdamped system are shown in the above figure. In all these cases, what we will have is the initial slope will be non-zero as we have numerator dynamics and then if we have underdamped response, this is what we get.

Now the advantage of underdamped response is that the system reaches the neighbourhood of the set point much quicker compared to any of these responses. So this, let us say if this is the point at which it is reaching within this particular neighbourhood of the final response, what we can see that the underdamped response is much faster to reach within that particular value.

Let us take an example that it is some sort of production rate which we want to attain. So initially we were operating at certain production rate and now we want to increase the production to this new value, then you would want to reach to within the neighbourhood of that final value as quickly as possible so that we start making lot of profit rather than having a very slow trajectory towards that final value. So most of the times, underdamped response will be desired.

The only cases where we would not want to go with the underdamped response is when we have something like, the set point is also close to the stability or the safety limit. If it is for example a reactor temperature which is getting controlled and this final set point is also close to where the catalyst activity is maximum. And the catalyst may get degraded if we increase the temperature beyond that value. So obviously as we are going for some portion, we are already above the safety limit of this good temperature at which the catalyst can work. So this temperature may be detrimental to catalyst, for example it may cause some explosion or the safety limit. So in that case, such kind of oscillations or the overshoot may not be desired. In such cases, you may resort to critically damped response.

But having a PI control gives you that flexibility that you can move from one type of dynamics to the other type of dynamics. So what we have now seen is that by the addition of integral action, we have now a system which can reach its desired set point. Then what you should then see that PI controller should be present everywhere in the world and there is no need for any additional thing. So in fact that is not the end of the world. There is always something which can improve the performance even from PI control.



The PI controller even though it causes the offset to go to 0, but at the cost of some penalty and that penalty is higher order dynamics. Because we are increasing the order of the system by 1, so it tends to slow down the closed-loop response. So, in order to improve the speed of response, what we do is, we add derivative action. We will talk about the effect of PID control after the break. Thank you.