Chemical Process Control

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Lecture – 23
Proportional Control

Let us now look at the next type of control action which is known as a proportional control action or the controller will be known as a proportional controller.

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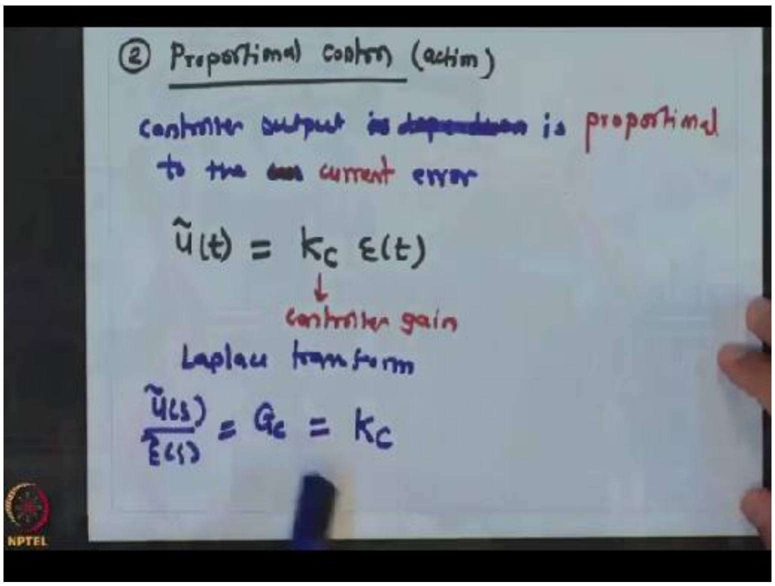

For the previous on-off controller, we saw that the control action or the control output was dependent on the size of the current error. So in this controller, the output is the current error. So there are 2 points which I have highlighted here for this control action. One is the controller output is proportional to the error, and that is why the controller is known as a proportional controller. All we are interested in is what is the current value of the error and proportional to that, will be the action taken by the controller. So in terms of the mathematical representation, what we will be writing as the deviation from the controller at any time, the signal generated by the controller will be proportional to the error at that time and that proportionality constant will be called as k_c, and is called as the controller gain. To find out the transfer function for this controller, it is very easy.

If you take the Laplace transform, we can write that u(s)/ε(s) which is same as the transfer function of the controller, is simply equal to kc. So this is an instantaneous controller which takes action as soon as there is some error which is detected. It gets multiplied by this gain which is known as a controller gain and accordingly the output of the controller changes. We will now see how implementation of proportional controller is going to help us in terms of disturbance rejection or set point tracking.

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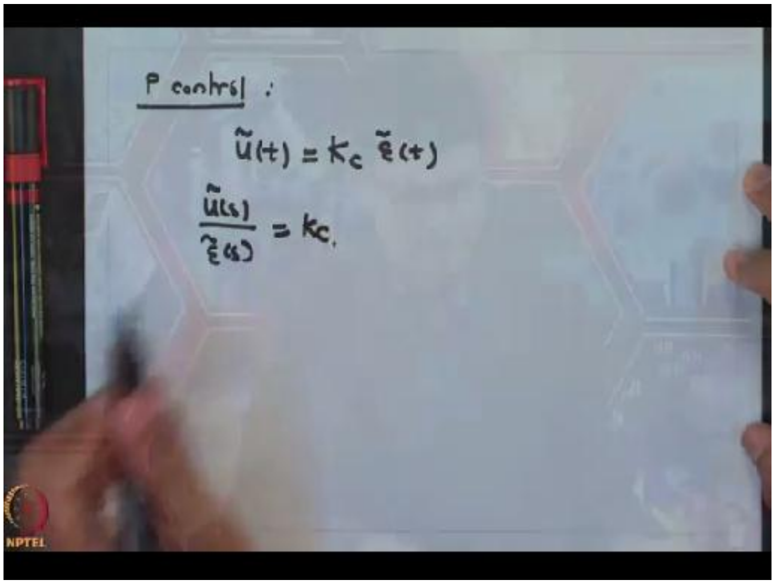

So just a while ago, we saw that for proportional controller, the controller output is proportional to the current error and the proportionality constant is the parameter of the controller which is known as controller gain. And in the Laplace domain, we saw that the transfer function for this controller is simply k_c. So let us look at the effect of what values are permissible for k_c depending on the action of the controller.

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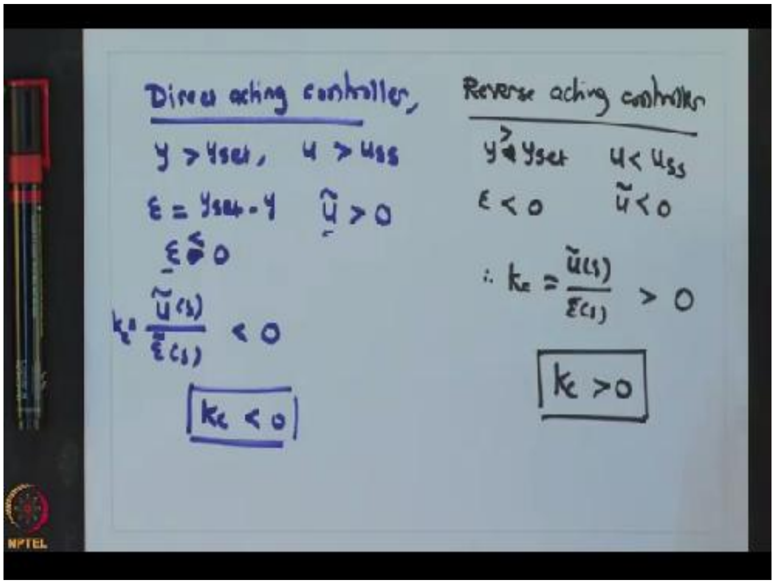

So let us say if the controller is direct acting, if $y > y_{set}$, then for a direct acting controller $u > u_{steady\text{-}state}$. Now error we have defined as y_{set}-y. So in this case, error < 0 and $u > u_{set}$, that means $\tilde{u} > 0$. So in terms of Laplace derivative, the Laplace transform, what we see is this has to be negative because a decrease in ε is going to cause an increase in u. So this was equal to k_c. So for a direct acting controller, k_c has to be negative.

Similarly, for the reverse acting controller, we can show that the exactly opposite holds. So when $y > y_{set}$, $u < u_{steady\text{-}state}$. So ε in this case is negative and $\tilde{u}$ is also negative. So therefore, k_c which is the gain which will be greater than 0. So for a reverse acting controller, the controller gain will be positive. For direct acting controller, the gain will be negative. Let us now look at the regulatory problem, or regulatory response of first order process using a simple P controller.

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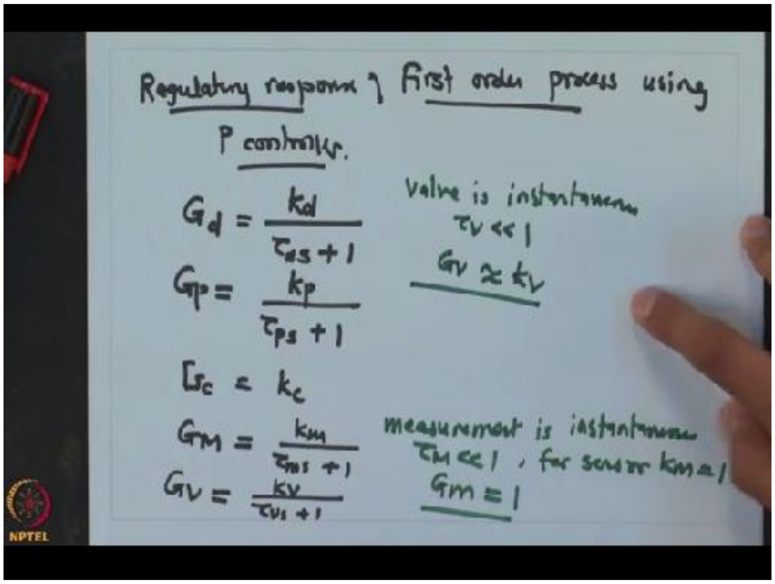

So in this what we are assuming that the process is represented as a first order process and we had seen that first order process are very common in chemical industry. And in this case, what we are going to have is the disturbance transfer function is going to be first order. The process transfer function we will also assume it to be a first order. And the controller is P controller. So the controller is this.

Additionally, we will have the measurement dynamics as; and the valve dynamics like this. And here for simplification or for the results to be more easily visualized, we will try to make some simplifying assumptions. So we will assume that measurement is instantaneous. So because of that what we will try to assume is this is very small compared to 1.

And we can neglect τ and s in this case. So the transfer function becomes k_m and as it is the gain between actual value and the measured value, for sensor, k_m will be equal to 1 because we want the same change in the measured value as is the change in the actual variable. So in that case, G_m becomes a simple unity. Even for the valve, we will assume that valve is instantaneous, so that τ_v is very much smaller than 1. And in that case, G_v roughly becomes equal to k_v.

So now k_v need not be equal to 1 but what we can always do is, we can club the effect of the valve into the manipulated variable. What I mean by that is?

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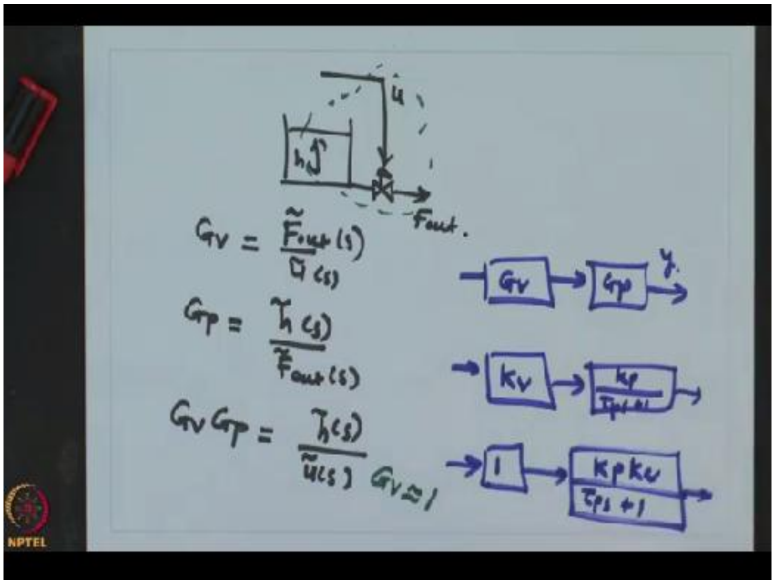

Let us say if we have our original system and whatever is the controller output, was u and the manipulated variable is F_{out}. So G_v was representing the transfer function between F_{out} and u and G_p was the transfer function between h and F_{out}. So what we are trying to say is what if we combine these 2. So what we have is a transfer function between the controlled variable and the controller output.

So this remains the same. This means we are looking at this entire part which is going to look like G_v, G_p and this is the y. So currently it is like this, so which is going to remain the same in terms of mathematics. If we say this is 1 and we include the effect of k_v as $k_p*k_v/(\tau_p s+1)$. So what I am trying to say is by incorporating the valve gain into the process gain, I can assume that the valve transfer function is also unity.

So these are just algebraic manipulations which we have done and then those are valid as long as I include that gain as a part of the process gain. With the help of this, I can assume that G_v is also equal to 1 because that simplifies our analysis for now. But having said that whatever be this sensor transfer function as well as valve's transfer function as first order TF, we can still carry out the analysis.

It will be little tedious but then the result which we are going to obtain, the effect of proportional controller, those are still valid. So you can try those as exercise if you want. So based on this if we look at regulatory transfer function was equal to $G_d/(1+G_pG_cG_vG_m)$ which is equal to how the output in the closed loop change as an effect to the disturbance.

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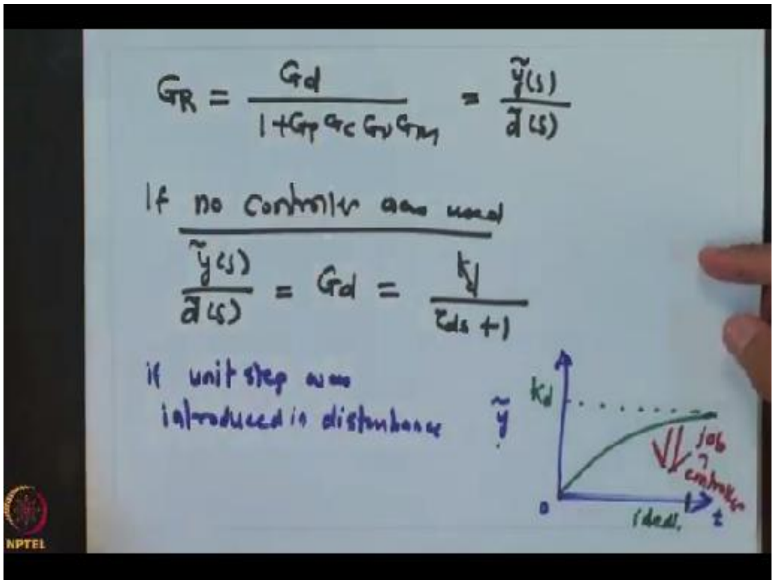

So before going forward, so if no controller was used, then we want to see how the output changes as a response to input that is same as G_d which is equal to k_d for this case, τ_d s+1. So if I had not used any controller, then if there was, if unit step was introduced in disturbance, then the output would had changed from its current value. And the ultimate value of the response would had been k_d.

So the response would had followed first order dynamics and we would had, the final value would had been k_d. So the output would had changed by an amount of k_d but in reality, we do not want the output to change because of changes in the disturbance. So the ideal value would had been this where $\tilde{y}$=0. So now the job of the controller is to bring this response below. So that is the job of controller. We will see whether the proportional controller is able to do this job or not.

So let us now substitute, let us now try to find out this y(s)/d(s) in the presence of a proportional controller. So we will substitute the transfer functions.

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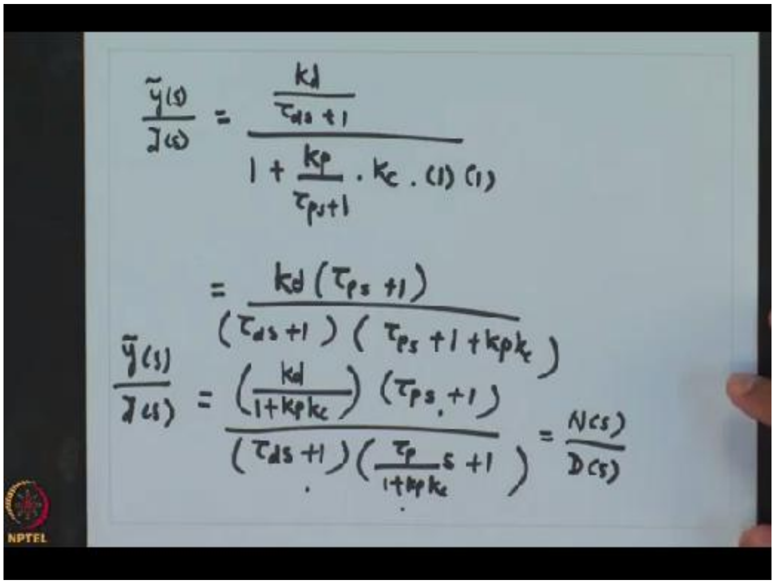

G_d was $k_d/(\tau_d$ s+1), denominator is 1+, G_p is $k_p/(\tau_p$ s+1), then G_c is k_c and then we have assumed that G_m and G_v are 1. As I said even though we do not assume them to be unity, you can still carry out the analysis and then try to derive the same results. So this will be equal to, k_d*(τ_p s+1)/[(τ_d s+1) and other term will be (τ_p s+1+k_p*k_c)] which I can simplify as [k_d/(1+k_p*k_c)]*(τ_p s+1)/{(τ_d s+1) and this will be [(τ_p*s/(1+k_p*k_c)]+1}.

So the closed loop response of this P controller for this disturbance is having a second order response. So you can see that the denominator transfer function has a square term or 2 first order capacities and numerator also has a first order capacity. So this is of the form N(s)/D(s) which has 1 zero and 2 poles. So this is a system which we had considered in our example of higher order numerical dynamics as well.

And this is going to behave like a first order process because the numerator has, s power as 1 and the denominator has power of 2. So we will now see how this looks like. So for that we will have to find out what are the poles and zeroes of this system.

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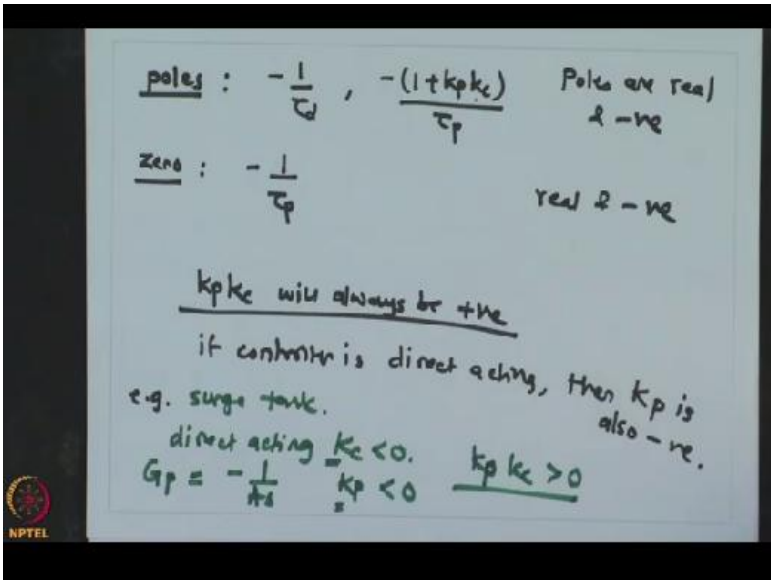

So there are 2 poles of the system. So the first pole is $-1/\tau_d$ and the second pole is $-(1+k_p*k_c)/\tau_p$. And zero of the process is $-1/\tau_p$. So for this particular system, you can note that all the poles are real and negative and zero is real and negative. Now here let me make a comment that earlier we had seen that this k_c can be positive or negative. But let me tell you that k_p*k_c will always be positive in these cases.

That is because if the controller is direct acting, then manipulated variable gain is also negative. So you can verify that if you take the example of the surge tank, there it was direct acting. So k_c was negative and if you see what was the process transfer function? It was -1/(As). So this k_p is also negative. In this case, k_p is -1/A. So you can see that this, whenever the controller is direct acting, the corresponding manipulated variable gain will also be negative.

And in effect k_c*k_p will always be greater than 0 which means this number will always be positive and that tells me that all the poles are negative and real and the zero is also negative. So as the zero is negative and real, it is not going to give me any inverse response. The only case which is of interest is if this zero is closer to the origin than the poles, in that case, you may have overshoot. So if we look at the zeros and poles, so you can see that this pole, k_p*k_c being positive will always be away from origin compared to the zero. So this is not of interest. The only interesting case is; can this pole be farther closer to origin than zero?

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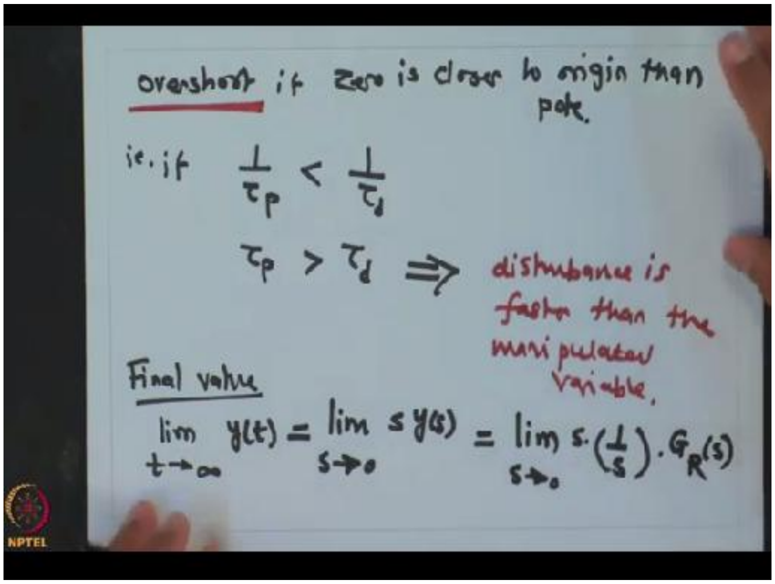

So we will get overshoot if zero is closer to origin than pole which is possible. That is if $1/\tau_p < 1/\tau_d$ which means $\tau_p > \tau_d$. So the overshoot is there if disturbance is faster than the manipulated variable. So the response which we are going to get, we will have overshoot if that is the case. Otherwise, the response will look more or less like a first order response and before completing the response, let us find out the final value which will be limit $t \to \infty$, y (t) which by final value theorem we can write as limit $s \to$ to 0, s*y(s) which is again equal to limit s tending to 0, s*y(s) is d(s)*the regulatory transfer function.

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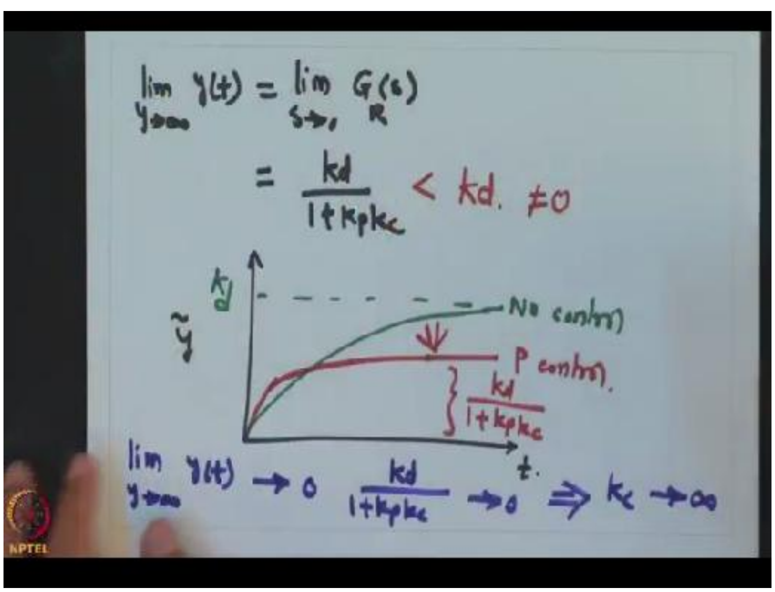

So using this, the final value which we are going to get is limit $s \to 0$ G(s) which in this case is = $k_d/(1+k_p*k_c)$ which you can easily show that this is going to be less than k_d. So the response is not exactly equal to 0 but it is definitely less than k_d. So if I plot this response and I try to compare what was the case when there was no controller. In that case, the response was a simple first order reaching the final value of k_d.

Now in the presence of controller, in the presence of this P controller, the response is still first order. And we can even see that it is faster than this. And which will reach a value which is smaller than k_d by this factor $1+k_p*k_c$. So you can see that the proportional controller has started doing what it was suppose to do. It was supposed to bring this response close to the x axis. It has done that but only partially and in order to make this response.

So if I want that this limit y →∞, y(t) should go to 0, that means my $k_d/(1+k_p*k_c)$ should go to 0. This requires that this kc goes to infinity. Only in that case what we will have is that this intercept or this final value will go to 0. As I keep on increasing kc, this number will keep on increasing and then the value of this response, the final value will keep on reducing and ultimately only when kc tends to infinity, this number will be equal to 0.

So for a proportional controller to give me response which does not, where the complete disturbance rejection is possible, will require the controller gain to be infinity. Mathematically, the way we represent this is through the definition of a quantity called as offset.

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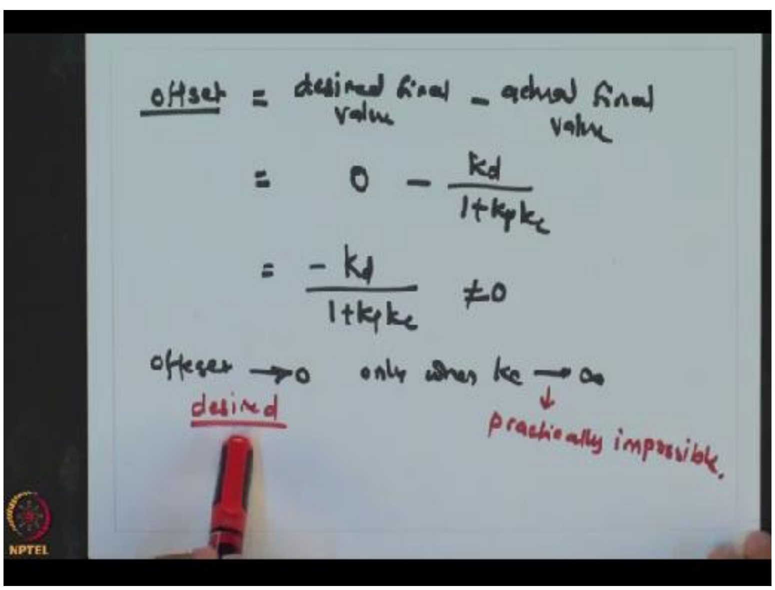

So offset is defined as the (desired final value - the actual final value). So in this case for disturbance rejection, the desired final value is 0 and the actual final value we got was $k_d/(1+k_p*k_c)$. So the offset is equal to - $k_d/(1+k_p*k_c)$. And this offset is not equal to 0 for any finite values of controller gain, offset tends to 0 which is desired only when kc tends to infinity.

Now this is desired and this is practically impossible as you cannot infinitely increase the gain, only mathematically it is possible because this will eventually lead to the manipulated variable going to its upper or lower bound. So the manipulated variable will get saturated for some high finite value of kc. And you will never be able to achieve 0 offset in the presence of a only proportional controller. The same thing is valid even when we look at the regulatory control or the servo control.

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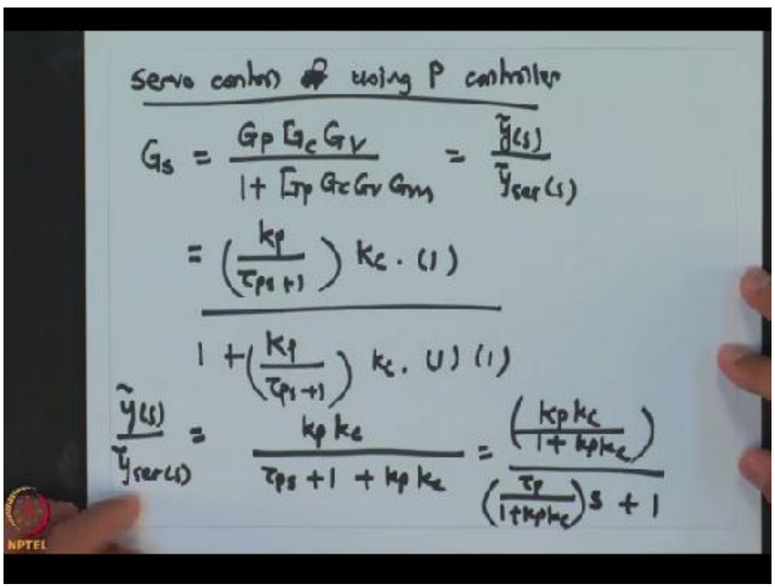

<u>Servo control using P controller</u>.

Again for first order processes, we are interested in G_s which is $G_pG_cG_v/(1+G_pG_cG_vG_m)$ with the same sort of assumptions which we have made. This is equal to y(s)/yset (s). It will be $k_p/(\tau_p\ s+1)$, G_c is k_c, G_v we have assumed to be 1. In the denominator, we will have $k_p/(\tau_p\ s+1)$ and then we have G_c as k_c, 1 and 1. So the servo, what we get is equal to $k_p*k_c/(\tau_p\ s+1+k_p*k_c)$ which we can simplify as $[k_p*k_c/(1+k_p*k_c)]/[\{(\tau_p/(1+k_p*k_c)\}s+1]$.

So what we are seeing is as a response to a change in the set point to the output, the transfer function looks is a first order process with this as gain and this as a time constant. So let us now see what happens if there is a step change in the set point.

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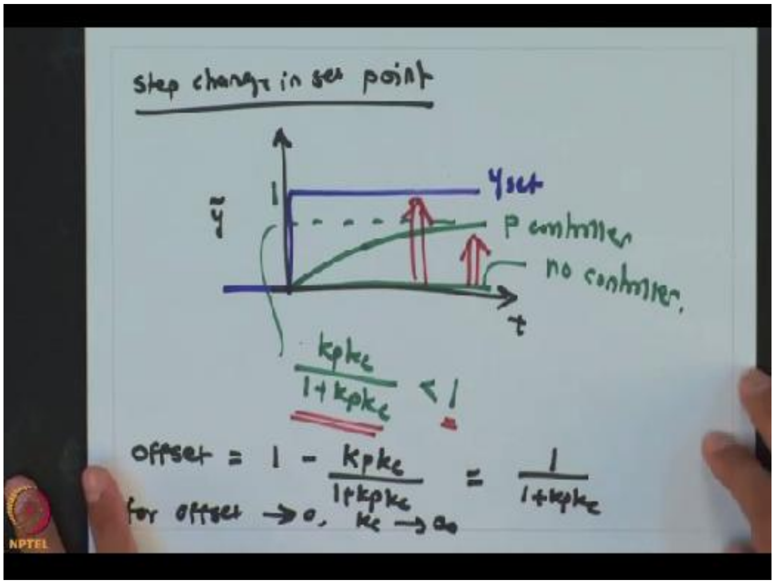

So we have given a step change in the set point. So this is y_{set} and what we are seeing is if there was no controller in place, then the output would not had changed. So this would have been the output when no controller was in place. But as we have put in a controller, the final value it is going to look like a first order response with a final value which is equal to $k_p*k_c/(1+k_p*k_c)$. So this is for a unit step change.

So this was the value 1 and we are actually reaching a value which is less than 1. So what we are seeing is the proportional controller, it was suppose to move this original response up to here. The proportional controller is coming short of its requirement again even in the case of servo problem. So it is moving the process towards the set point but not exactly and what you see that it fails to reach the final value of 1 because the step change was unity and what you are getting is the final value of $k_p*k_c/(1+k_p*k_c)$.

And you can again define the offset as earlier. The desired final value in this case is 1 and the actual final value is this which again is equal to $1/(1+k_p*k_c)$. So you can see that it has the same form and for offset to go to 0, you require k_c to go to infinity. So again in the case of servo problem, what we see that the proportional controller cannot give you the desired final value and for that it will require infinite controller gain which is again a mathematical entity.

The only thing which is of interest is that it is moving the process towards the desired direction. So we will now see that how do we improve this in order to get this offset to be 0. With the finite controller gain, what we need is an additional controller action which will be the integral action. So we will take a break here and we will see how addition of integral action will help us achieve this offset to go to 0 even for finite values of controller gain. Thank you.