Chemical Process Control
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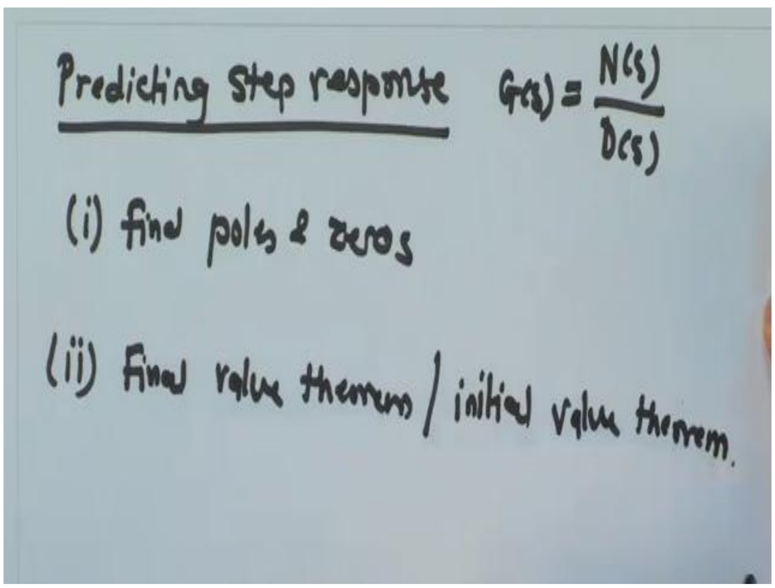
Lecture - 20
Prediction of Step Response

Okay, welcome back. We will try to finish this first module on process dynamics where we try to understand how process responds to changes in inputs by looking at how can we predict the response of any transfer function given there is some step in the input. We will try to see or I will try to give you some tools in terms of predicting the response of any transfer function and this we will be done by finding out poles and zeros of that transfer function.

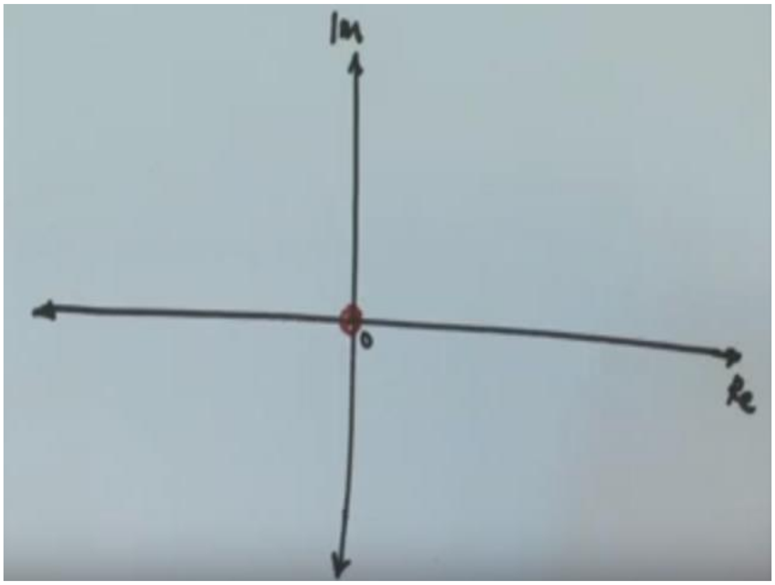
So what we need is the transfer function. Once we have the transfer function we should be able to predict its step response by using these steps.



The first step, let us say this is for a transfer function,

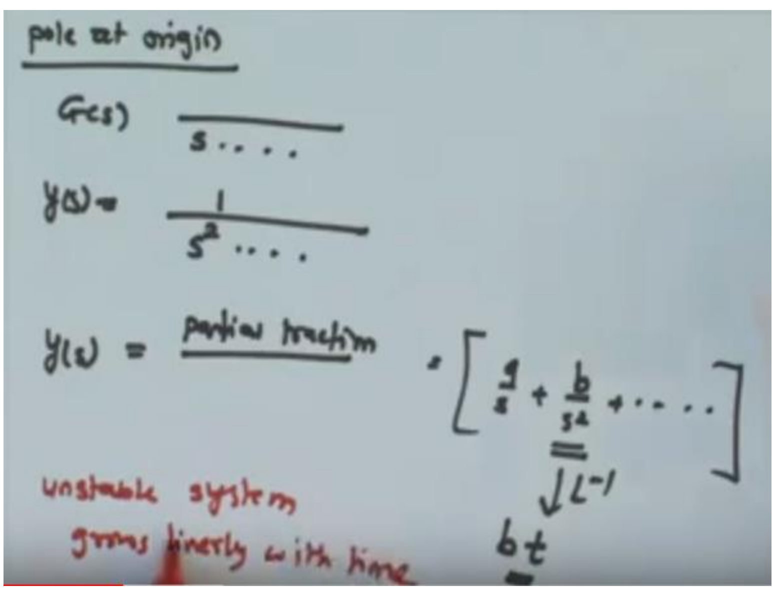
$$G(s) = \frac{N(s)}{D(s)}$$

The first step is to find all the poles and zeros. The next step is you use the final value theorem and in certain cases initial value theorem as well. With the help of these two, we would be able to predict response, step response of any transfer function. So for that what we do is we try to plot these poles and zeros into the complex plane.



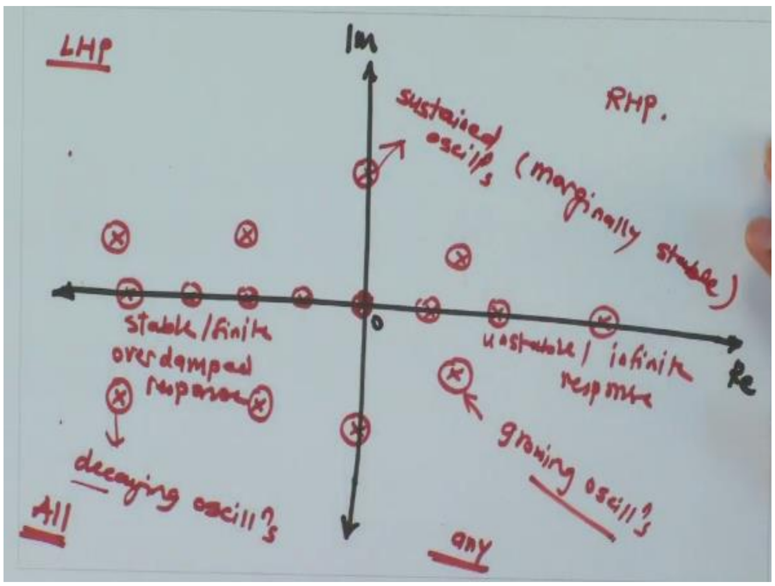
The real axis and the imaginary axis are shown in the above figure. In the figure for any transfer function, we will plot or we will put the poles as well as zeros. So we will start with the poles. Let us say the pole of the system falls at the origin. Let us say if the pole is at the origin. So when the pole of a system or transfer function is at the origin, the denominator of the transfer function will have a term '1/s' or the denominator will have a term 's'.

So when we have a step response, it will be get multiplied by '1/s'. So we will have terms like s^2 when we look at the Laplace of y(s).



In such a case when you have a pole at the origin the transfer function will have ‘s’ in the denominator. So when we talk about y(s), it will have s^2 multiplied by something. When we want to take y(s) in terms of partial fractions, in that case, we will be writing it as a/s + b/s^2 and the other terms. So what we are going to have is b/s^2. The inverse of that will give you ‘bt’.

Even though, this system will increase linearly with time or one of the dynamic modes of this system will increase linearly with time. So whenever you have a pole at the origin, it will be an unstable system or grows linearly with time. It will not have a stable final value. So that is the case when you have a pole at the origin.



When we have a pole on the real axis, any pole on this real axis in the left half plane with a negative value, what we are going to have is the response will have $e^{-\lambda t}$ terms which all will decay down to 0. So all these will give you stable response; so stable or finite response, overdamped response. There will be no oscillations in such a case.

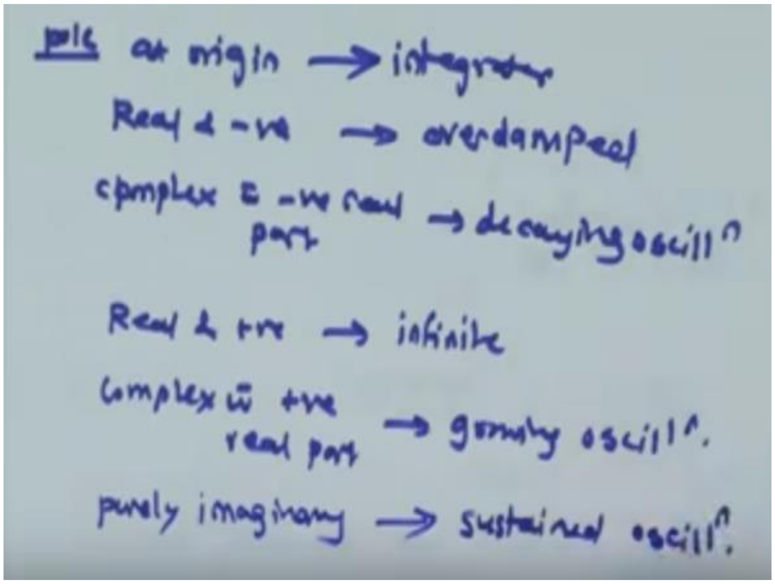
The oscillations will come if the poles are complex conjugate poles. So, if you have any pole in left quadrant or this quadrant again still in the left half plane, what you will get are decaying oscillations. These terms will give you e^{a+bi}. So there will e^{at} and e^{bit}. The first term will give you the magnitude of oscillation and the second term will give you sin and cosines. As the magnitude is on the negative side, the oscillations will decay but there will be oscillation when you have the poles on the left half plane but not on the axis or the real axis.

Exactly the opposite case would be when we talk about the right half plane. Previous this is the case when all the poles are on the left half plane. Now if any pole is on the right half plane, if it is on the axis, it will be e raised to power some positive number times t. So the response will just grow in time. The response will be infinite when time goes to infinity. So, all these would give you an unstable or infinite response.

This happens when any of the poles are on the right side, even though n-1 poles are on the left half plane and even a single pole on the right half plane will give you this kind of responses because all these will die down to 0 as time t goes to infinity. So the only thing which does not die down to 0 will be anything on the right half plane. Similarly, if we have some complex conjugate poles these will give you oscillations, but these will be growing oscillations. The oscillation magnitude will keep on growing as the function of time and again you will get an infinite response as time t goes to infinity. So if any pole or any pair of poles is on the right half plane you will have such infinite or growing responses.

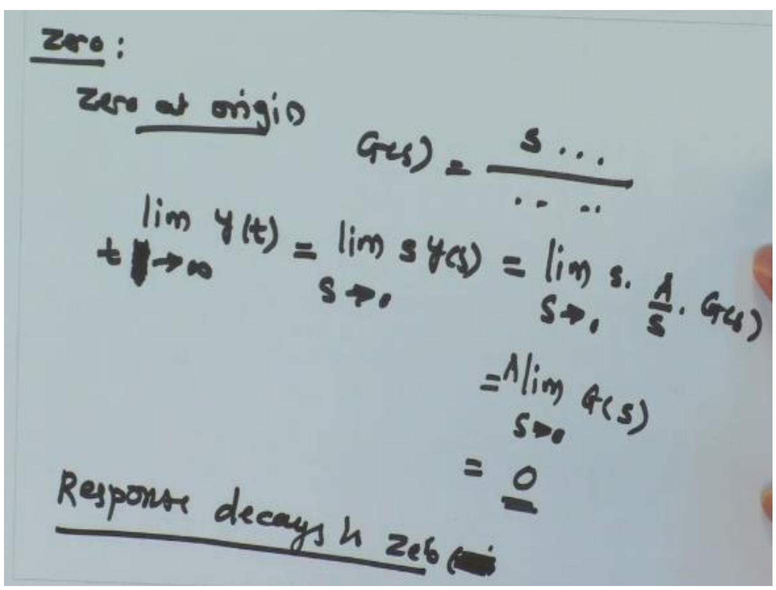
The last case which is remaining in this is if your poles are complex conjugate but or with the real part which is 0 or purely imaginary poles. In that case, what you will get are sustained oscillations which is also known as a marginally stable process. Depending on where your poles of the process lie, you may have an overdamped response, stable and finite. You may have decaying oscillations. You may have growing oscillations. You may have an unstable or infinite response. Or you may have sustained oscillations into your system.

Lastly, you may have an integrator which increases linearly as a function of time. So depending on the values of poles for a system, the overall dynamic modes of the process can be calculated by simply looking at the poles.
If I want to summarize this, what you would get is-



Pole at origin gives you integrator. Pole real and negative will give you overdamped response. Complex with a negative real part will give you decaying oscillation. Then the opposite side of that would be real and positive to infinite response. Complex with positive real part will give you growing oscillation. And then purely imaginary pole will give you sustained oscillation.

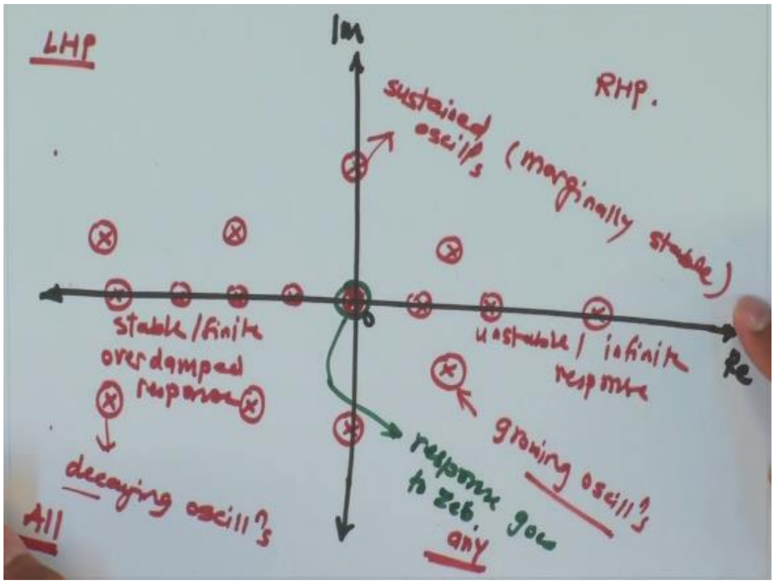
So far we have not looked at where the zeros of the system are. So we will now see how this analysis changes when we have zeros into the system as well.



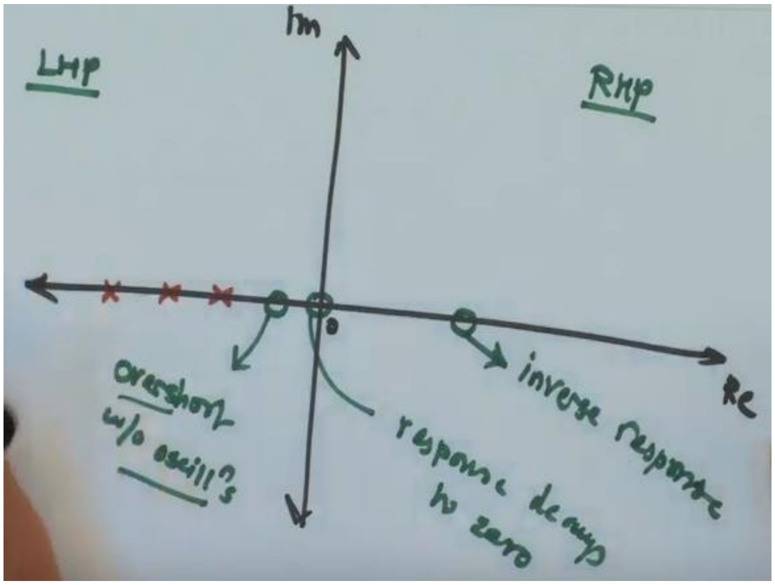
So when you have zero into the system or zero at the origin, the transfer function will be of the form s in the numerator. So whenever you try to find out the final value by the final value theorem,

$$\lim_{s \to 0} s\, y(s) = \lim_{s \to 0} s \frac{A}{s} G(s)$$

As zero is at the origin, G(0) is 0. So the final value of the response will be 0. So whenever there is a zero at the origin, the response decays to 0. The final value of the output will always be equal to 0 whenever there is a zero at the origin. So if we go back to the previous figure.



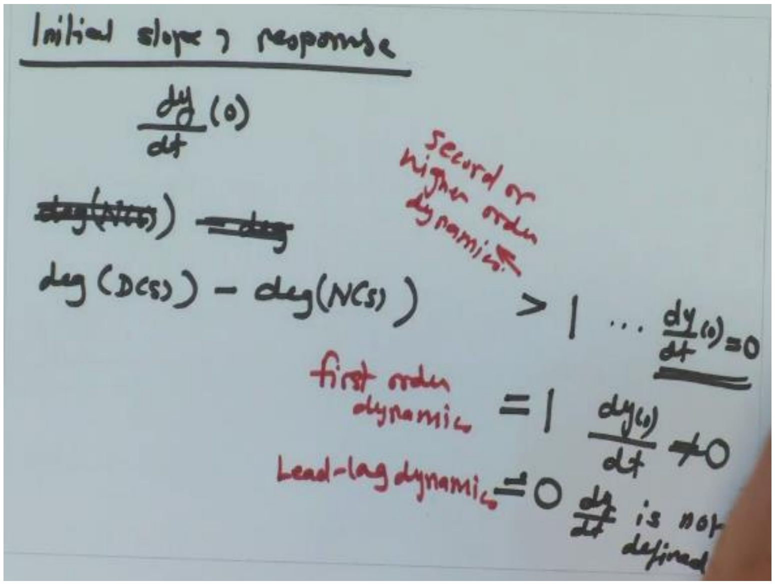
If I say there is a zero at the origin. Then it will give me a response goes to zero. Now for most of the parts, zero would just change the relative contribution which would not be directly predictable. The only case when you can predict the response of zero, so let us try to use a different figure for zero.



The real part and an imaginary part is shown in the figure. if there was a zero at the origin, the response decays to 0. Now if the zero is on the real negative real axis and it is closer than any of the poles. This zero is on the negative real axis and it is closer than any of the poles to the origin, in that case, you will get overshoot without oscillations obviously. So this was the case when we had overshoot into that first order or second order system.

This is the type of zero which is closer to the origin than any of the poles those cases will give you overshoot. And the other case which is going to give you interesting results is if the zero is on the right half plane. So if the zero is real zero on the right half plane it is going to give you inverse response. So by putting the zero also into the complex plane, you can predict whether there will be an overshoot without oscillation or whether there will be an inverse response.

This will be on top of the way we predicted the response for the poles and the combined response will be the combination of these two factors. And then lastly, something which you should also keep in mind while predicting the response is the initial slope of response.

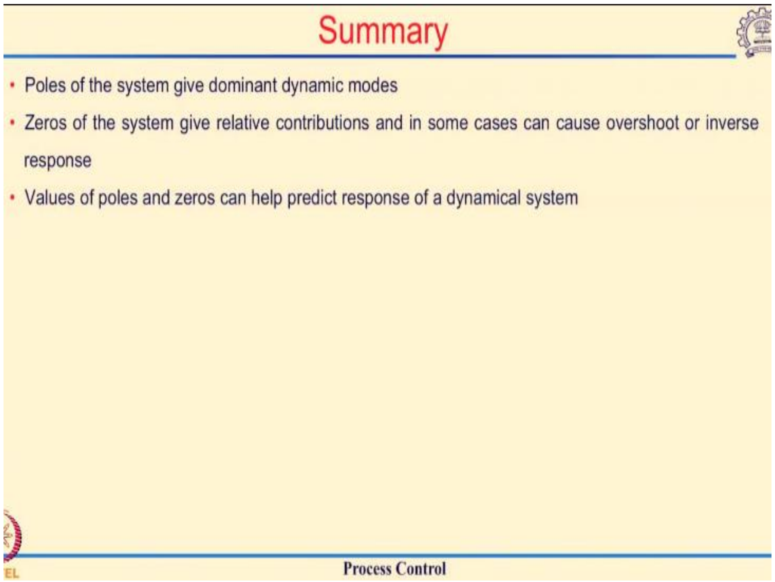


So that is dy/dt at t = 0. So that depends on the difference, I did the other way round. The degree of the denominator polynomial – the degree of the numerator polynomial, if this difference is greater than 1 then the initial slope will be = 0. That was the case when we had a pure second order, pure higher-order system. In that case, the denominator polynomial degree was greater than the numerator by 1, more than 1. So, in that case, the response does not start immediately.

The response has a zero slope at time t = 0. When the difference is = 1, then the slope is finite. The slope is not equal to 0, the response immediately starts and if this difference is = 0, in that case, dy/dt is not defined and there is a discontinuity in y. So this was the case when we have the lead-lag type of dynamics. This was the case of the first order dynamics. And this is the case when you have second or higher order dynamics.

By knowing the initial slope or how the response starts then looking at whether you have overshoot or inverse response and then also combining it with whether you have a stable response, overdamped response, decaying oscillations, growing oscillations all that in combination you would be able to predict the response of any transfer function for a step change.

To summarize what we have seen is the poles of the system or transfer function will give you what are the dominant modes of that particular dynamic transfer function. The zeros will give you the relative contribution and will also give you some conditions when you can get overshoot or inverse response.



And then these values of poles and zeros will help you to predict the response of any dynamical system. So at this point, we have finished analyzing how dynamical systems behave. So in terms of, in the context of this course on process control what we have seen is given a change in the input how the system responds. So now we have a better understanding of the process. Once we, we are now at a position that we know how the system is going to behave.

So now we can have a way to control the process. Now we are at the stage where we know how the system behaves. So we know exactly how in order to move the process from one point to the other how we should change the input of the process. So it is more like solving an inverse problem. So as we know the way process behave we should be able to now tame its behavior or make the process behave the way we want. That will be the part we will be discussing under the second module of this course which will be on process control. Thank you.