Chemical Process Control
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Lecture - 19
Numerator dynamics

Hello students. The topic of this last, this portion this presentation, is numerator dynamics. So at the end of this lecture, you would be able to compute poles and zeros of any transfer function.

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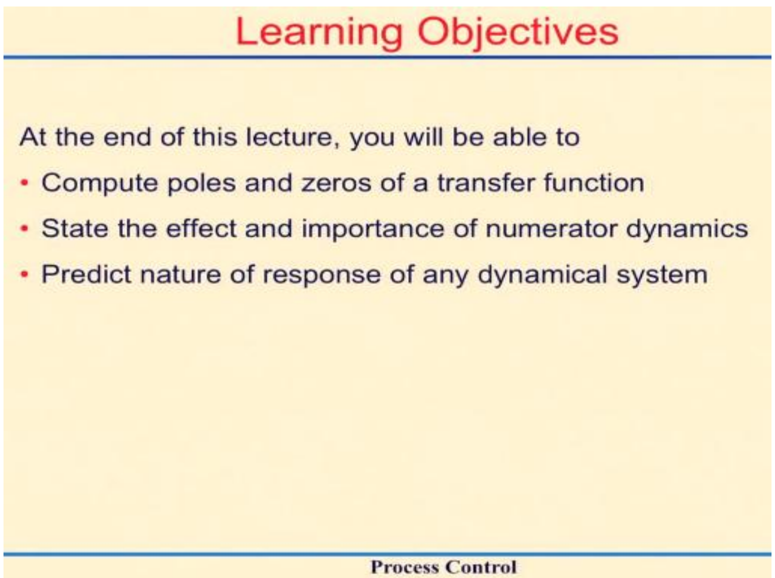

You would be able to state the importance of numerator dynamics which is the part of this lecture. And we will finish by giving you some guidelines about how do you predict step response of any transfer function by computing its poles and zeros. This will be very useful for us going forward when we move to the next module of this process when we talk about process control.

So we will finish the process dynamics portion by giving you the tools which are required to predict the response of any general transfer function. So let us get started. So far we have looked at different examples of dynamical systems. We started with the first order processes, second-order processes, and then we extended it up to higher order processes. In all those examples, what we considered was all the effect that was coming was from the denominator.

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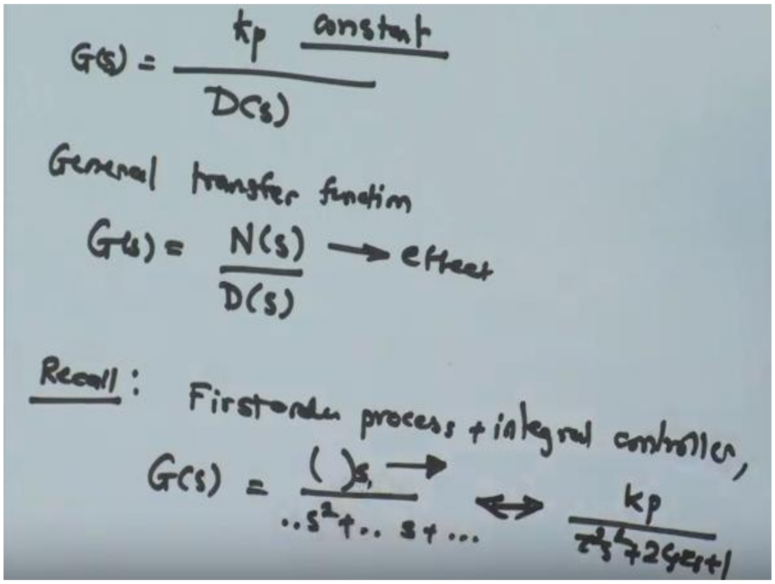

So whatever transfer function we had considered, it had some constant in the numerator and some polynomial in s in the denominator. But there are some examples of systems, and especially those are the, most of those cases come when we have a controller which is combined with the system. The numerator need not be independent of s. So a general transfer function will be of the form N(s) / D(s) where N(s) and D(s) are polynomials in s.

So in this lecture, we will see what effect does this N(s) have on the dynamics of this particular system when it is a function of s. So if you recall one example, that was the example when we get a second order system by having an integral controller on a first order process. In that case, the transfer function which we had gotten for first-order process plus integral controller the transfer function, in that case, had s in the numerator.

And denominator was the second-order transfer function and the numerator there was s. So this s is going to have a specific effect on this response of this transfer function which will make it different from how a general $K_p/(\tau^2 s^2 + 2 \xi \tau s + 1)$. This response is much different than when you have s in the numerator. So that is exactly what we will be studying in this particular lecture that what happens when we have N(s) which is a function of s.

So to analyze these kind of systems we define two terms for a transfer function. So we will write the transfer function as a polynomial representation, the ratio of two polynomial N(s) / D(s).

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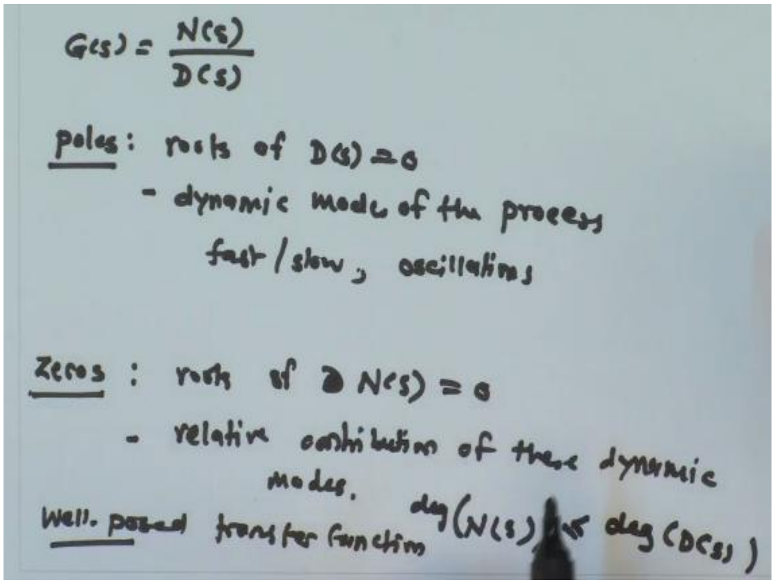

So we define poles as roots of the equation D(s) = 0. So you take the denominator polynomial, and you equate it to 0. The roots of that give you poles. So those are known as poles of the transfer function, and the poles give the dynamic modes of the process. So by dynamic modes what I mean is it tells you whether the system is fast or slow, would there be oscillations? So all those dynamic modes which are possible for a system those would be given by roots of this polynomial D(s).

And when we have the numerator polynomial as well then what we get is zeros. So those are the roots of the numerator polynomial. So when you take N(s) and equate it to 0, the roots of that are going to give you what is known as zeros of the transfer function. And poles give you dynamic modes, zeros give you relative contribution of these dynamic modes. So essentially it will just tell you which of these dynamic modes are dominant, are they moving, all the dynamic modes have the same contribution to output or some of them have an opposing contribution.

So all that effect will be compressed inside these zeros and for any typical transfer function for a well, what we call as well posed transfer function which is also like any real transfer function, practically implemented transfer function what you should have is the numerator degree, so the degree of the numerator polynomial should be less than equal to the degree of the denominator polynomial. So that will make it physically realizable.

If the numerator degree is greater than the denominator, we will see that those processes are not physically realizable. So let us try to see the effect of this by using a simple example.

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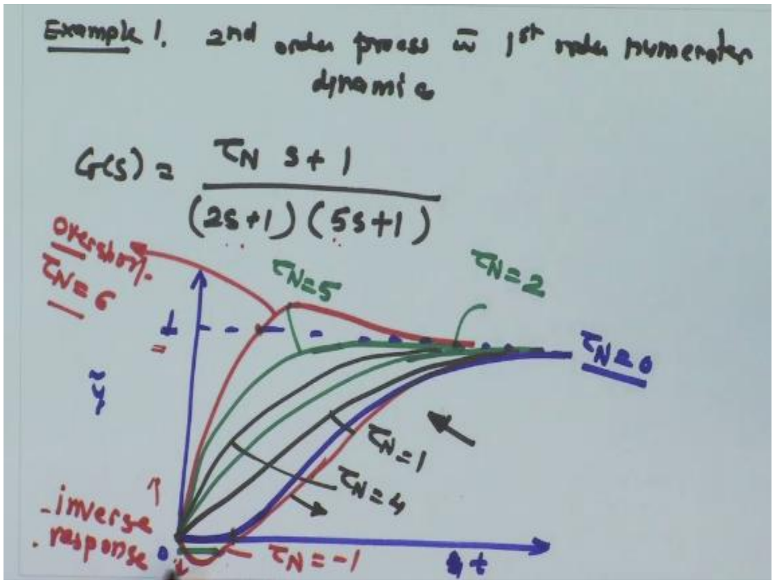

So we will take a second order process with first order numerator dynamics. So we will take a general form as $(\tau_N$ s + 1) as the numerator, and in the denominator, we will consider two first order series combination and then depending on the value of τ_N we will have different types of numerator dynamics for this particular process. So we will start with a case, and we will look at the step response of this particular system.

Let us say when we have $\tau_N = 0$. So when $\tau_N = 0$ the transfer function is 1/[(2s + 1) (5s+1)]. So there are no numerator dynamics to speak of, and the response is over damped. So the response will be something like this. The gain is 1. So the final value if the step response or the step change was unity what you would get is the final value is 1. So that is what will happen when there is no numerator dynamics.

Now let us consider that $\tau_N = 2$. So when $\tau_N = 2$ we are going to have 2N + 1 and 2N + 1.

So these two modes would get canceled and what you have is 1/(5s + 1). So, in fact, the system is going to behave like a first order system with a time constant of 5. So, in that case, the response, you can see that the response will be something like this. This is when $\tau_N = 2$. So you can see the stark difference between the original response and this response is that when there was no numerator dynamics, the system was idle for quite some time.

So the response was sigmoidal which is the characteristic of any higher order over damped system. But as soon as you have some numerator dynamics, the initial slope is nonzero, and the system reacts to the step change immediately. The same thing will be true when I take $\tau_N = 5$. In that case it will be (5s + 1)/[(2s + 1)(5s +1)]. So these two terms will get canceled and the response will be 1/(2s +1) which is significantly faster than this.

So in that case, the response will be like this. This is when $\tau_N = 5$. So these are very simpler cases to analyze. Now the interesting thing will happen when we keep on increasing τ_N beyond this 5. So let us say or before that let me just redraw some of the responses. When this τ_N was let us say greater than 0, but between 2 the response would be something like this which will be faster than $\tau_N = 0$ but slower than this. So this is when $\tau_N = 1$.

When $\tau_N = 3$ or 4 the response will be like this. So what you can see is as my τ_N is increasing from 0, the response is moving in this particular direction. So what happens if I increase τ_N beyond 5? So the response, it is going to push the response in this direction, and the response looks like this. So you can see that the response is exceeding the ultimate value. There are no oscillations, but the response is exceeding the ultimate value.

Because both the dynamic modes are contributing more than the ultimate value or more than the constant term inside the response and what you get is an overshoot. This is for τ_N let us say equal to 6 which is greater than 5. So what we are having is without oscillations or an over damped system is giving you overshoot. That is possible only when you have numerator dynamics. The other case would be what if we move in the other direction?

So instead of moving in this direction, we start moving in the other direction and try to reduce τ_N below 0. So let us say if τ_N is negative then the response is going to move in this direction. It has already reached a value where it is almost flat on the x-axis. If you decrease τ_N further, the response is going to go in the opposite direction and then reach the final value.

So what you are seeing is, initially the response is going in the opposite direction. So this type of response you can get when τ_N = - 1 and this response is known as an inverse response. The reason it is called as inverse response is that the final gain between input and output is positive. So when my input changes by 1 unit, the output is also going to change by 1 unit.

But initially, if you look, the change in the input or increase in the input is causing a decrease in the output. So initial response is exactly opposite to what the final response looks like, and there is a crossing of the response y = 0. So this kind of inverse response happens when also due to numerical dynamics when τ_N is negative in this case. So by just changing what is the numerator which is going to change the zero of the system, the poles of the system remain constant.

You can have a wide variety of dynamics simply based on the combination of efforts of these two poles. So by having the contribution change between these dynamic modes, you can have overshoot, and you can also have an inverse response. So all that is the product of this numerator transfer function which is τ_N s+ 1. So by just changing the numerator dynamics, you can have a wide variety of possibilities the system can exhibit.

We can take another example. This time the first order numerator and first-order denominator.

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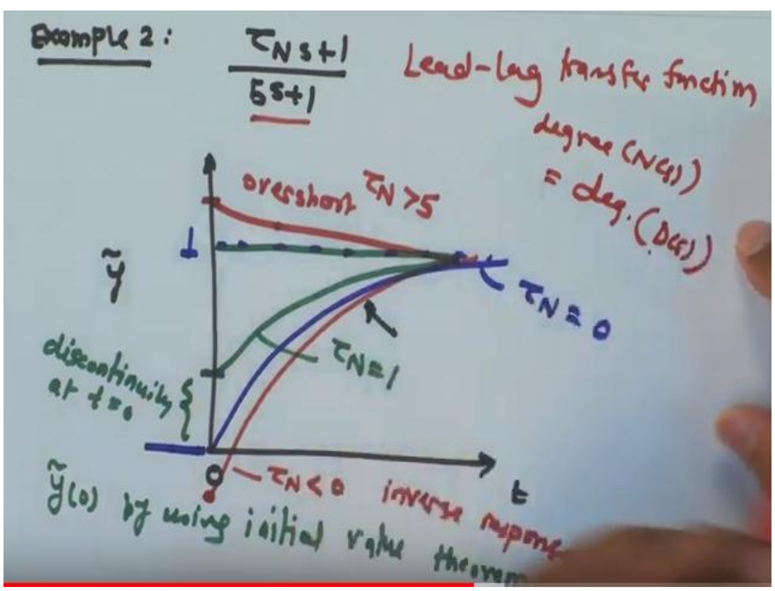

And again we can try to find out the response. When $\tau_N = 0$ we will have 1/(5s + 1). So the response stays at 0 and then goes to 1 as the first order response. So this is when $\tau_N = 0$. Now if we increase τ_N beyond this value what you would see is, in that case, the response does not start at 0. So up to time t = 0 response is 0, and suddenly the response starts at some value nonzero and then goes to this final value as the first order response. Let us say this is $\tau_N = 1$.

So there is a discontinuity at t = 0. You can find this initial value by using initial value theorem. So this discontinuity will keep on increasing as you increase τ_N. When $\tau_N = 5$ what you are going to get is 5s + 1/5s + 1. So the transfer function becomes unity. That means it is an instantaneous process with gain 1. So the input and output are identical. So input was the step so output will also be a step. So what you will get is a discontinuity of value 1.

So the output will also be a step unit step response or unit step. So we have been increasing τ_N, and the response is moving in this direction. As τ_N goes beyond 5 in that case what you would see is the response will start at a value higher than 1, and it will go down to 1. So again what you are going to have is an overshoot when τ_N is greater than 5. And similarly by the extension like the previous example if we move in the opposite direction and τ_N is negative the value will actually start below 0 and then it will go up to the value of 1.

So when τ_N is less than 0 you will have an inverse response. So again having the same dynamic mode which is 1/(5s + 1) by changing the numerator transfer function you get overshoot, some discontinuity in the input at time t = 0 and then the inverse response as well. This type of transfer function is also known as lead lag type of transfer function because the degree of the numerator is equal to the degree of the denominator.

So denominator is a first order lag and numerator is a first order lead. So this type of transfer function is known as a lead-lag type of a system. When we analyze control systems, we will see that some of the transfer functions which we get which will be of this form. So depending on the value of τ_N, we may have some kind of overshoot or inverse response. So we have seen that by having something, some polynomial in the numerator, we could get different types of dynamics which were not possible when we had a constant in the numerator.

Specifically, we can get overshoot even for non-oscillatory behavior, and we can get something which is known as an inverse response which is one of the difficult or the most difficult type of dynamics to control in any chemical processes. So we will try to analyze this inverse response a little bit further about why it happens or how it happens.

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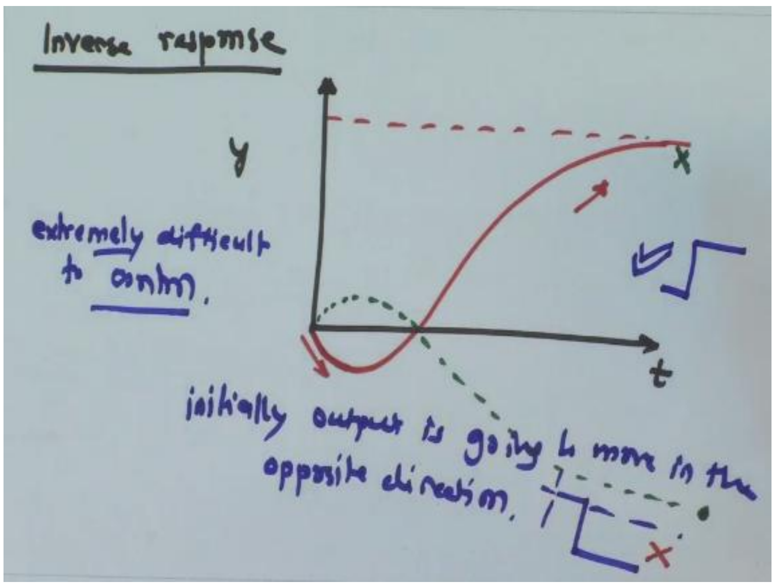

So the inverse response is a type of response when the initial direction of output is different than the final direction of output change. So initially the output goes in the negative direction, but the final gain or the final direction is positive. So having such kind of a response is known as the inverse response when the system initially shows exactly the opposite direction of movement as compared to the final direction of movement which is an increase in the output.

But initially, you can see that there was a decrease in the output. Now, this type of process is extremely difficult to control. Because what is going to happen is as soon as the controller makes a move, so let us say in this case the controller has to move the output from this point to this point and this will happen if he makes a positive move or positive step change in the input.

So now what is going to happen is whenever the controller makes a positive step in the input the initially the output is going to deviate or show, so initially output is going to move in the opposite direction. So the controller is going to think that it has made an exactly wrong move or exactly opposite move. So the controller is going to make a reverse move in that case which is wrong because for this particular system the correct input is an increase in the input.

But if the controller makes an opposite move, it is not going to be able to get this particular type of response. The response is going to lead the system to is the mirror image of this, and the system will never be able to reach this particular point. It will go in the opposite direction. So every time the controller will try to make an exactly opposite move and the system will either end up at this point, or it will keep on oscillating between the operating points.

And the system will never be able to reach this particular final output point. Because of that, the inverse responses are very difficult to control, and we should know when we get inverse responses. It is always better to know that the system exhibits inverse response then some corrective action can be taken or has to be taken to control that process. So let us look more into when do we get an inverse response, that is what is the physics of the process.

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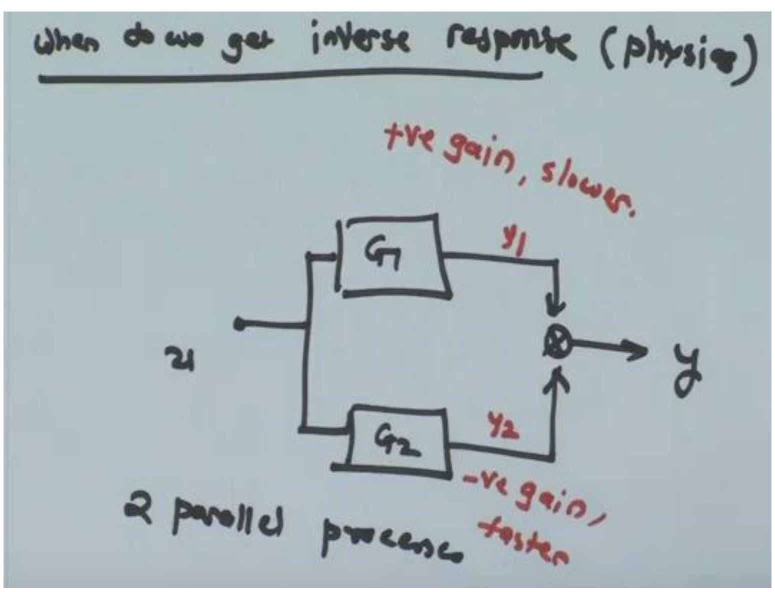

What is the physics of the process which will give you an inverse response? So as it turns out inverse responses happen because there are two competing effects which are triggered by the input. So if this is my input, when I change my input it is going to change the output in two ways. So there are two parallel processes which take place, and the net output is the combination of the two. So there are two parallel processes which are taking place.

And as it turns out, they have to follow certain rules. Then only you will get an inverse response. So let us say G_1 is which has a positive gain. So this y_1 increases when u increases. But it is slower. As against G_2 which has negative gain, so that means when u increases y_2 in fact decreases, but it is a faster response. So you need to have two transfer functions between the input and output which are in parallel with each other.

The one effect has a positive gain and slower, and the other has negative gain and faster or vice versa. So the idea is there are two parallel processes which are operating at different time scales and the gains are opposite to each other. The gain being opposite is essential, and they also have different time scales.

So when you have that under certain conditions or values of these different gains as well as time constants, you will get the inverse response, and mathematically you can find that out. So any physical system which is going to give you inverse response is going to have this kind of parallel processes which are happening. So if I want to give you some literature examples, the very common system which gives an inverse response is,

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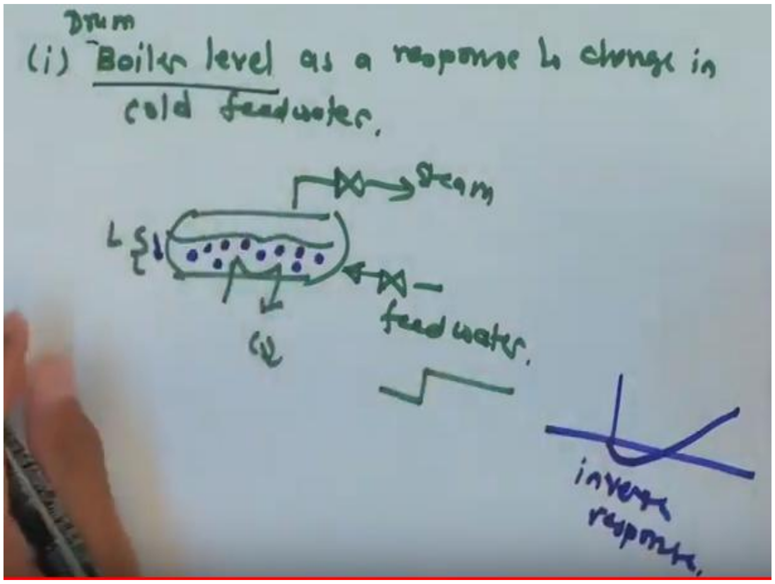

Boiler or drum boiler level as a response to change in the cold feed water. So if you have a boiler, this is the outlet for steam, and typically you want to control this level by changing the feed water. So as it turns out when you increase the feed water flow rate ideally what you expect is that the level should increase because you are adding more material in the process.

However, what happens is whenever you have such kind of a system when you are boiling this liquid there will also be some bubbles of this steam which are inside the liquid and the height is actually the addition of actual liquid height plus the rise in the height because of these bubbles. When you add cold feed water in, then the volume of these droplets go down and as the volume of this droplets goes down the actual level goes down.

So the response of the level looks something like this as a response to a step change in the input, and you get an inverse response. It is a very commonly studied process where you get an inverse response.

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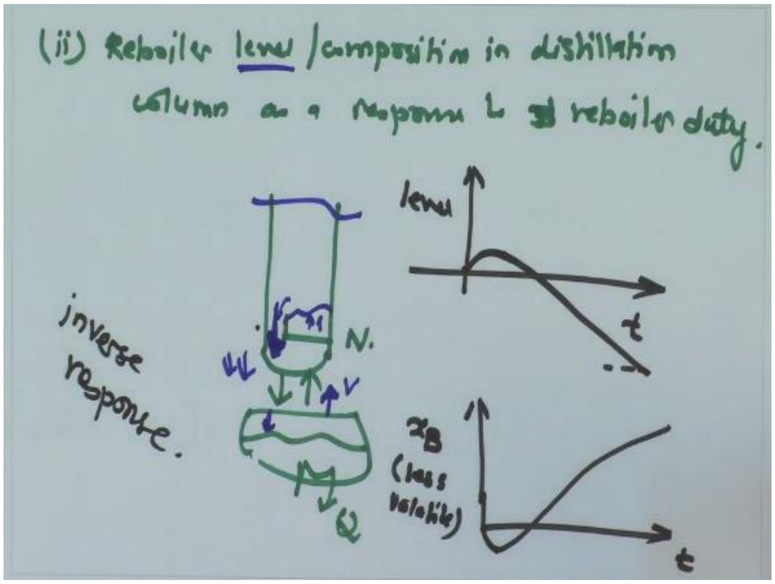

The other system which can give you inverse response is the reboiler part level or even composition in distillation column as a response to reboiler duty. So what happens is let us say if you take the bottom of the distillation column. Let us say this is the Nth tray and then this is your reboiler where you provide heat, what is going to happen is when you increase the reboiler duty, the amount of vapor which goes up into the column increases and as this vapor increases, it puts a force on this liquid which is on N^{th} tray and it pushes more liquid down. So the amount of vapor increase, in this case, is countered by more liquid which falls down and even though we are boiling more liquid inside this reboiler the net holdup of the reboiler momentarily increases. So even though you increase the reboiler duty the level inside the reboiler keeps on increases momentarily before it starts to reduce.

And same way the composition as you are mixing some low composition liquid into the reboiler because of this effect the purity also goes down. So ideally what you expect is when the reboiler duty increases, the reboiler level you want to go down but initially you see that the level increases and then the level goes down, and the opposite effect is on the composition. So if you look at the composition of less volatile, as you increase the reboiler duty you want it to be purer but what you see is initially the purity goes down.

That is because of the mixing of this low quality liquid into the reboiler. And then eventually it goes to a higher value. So in both these cases, you get an inverse response. So we will stop here for this lecture. Thank you.