Chemical Process Control
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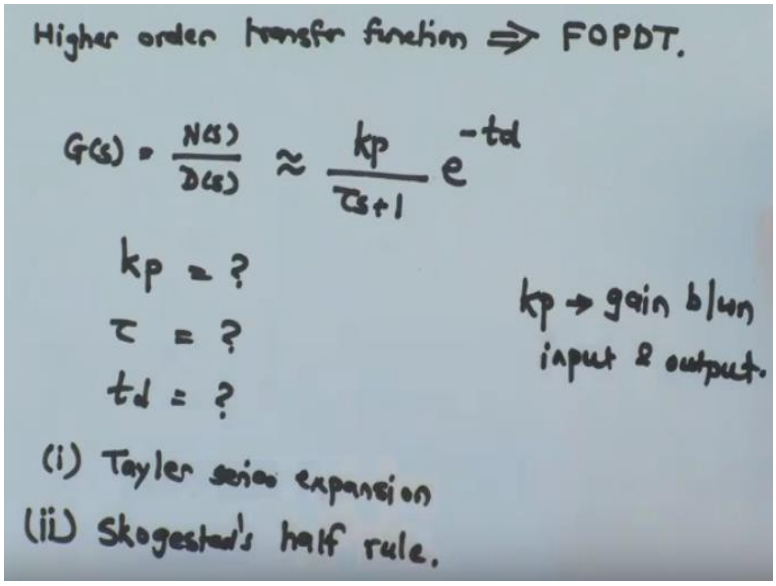
Lecture - 18
Approximation as FOPDT Model

Welcome back, students. We were looking at approximating any higher order system as a first order plus dead time. There are broadly speaking two ways of approximating a higher order system. Any higher order system into first order plus dead time depends on what information we have about the process. Either we may have the actual plant from which we can generate some data.

We may have raw data of the output variable as a function of time or we may also have the transfer function representation of the system which is of a higher order. We will see in both these cases how do we approximate this response as a first order plus dead time. So we will start with the case when we have the actual transfer function between the input and the output.

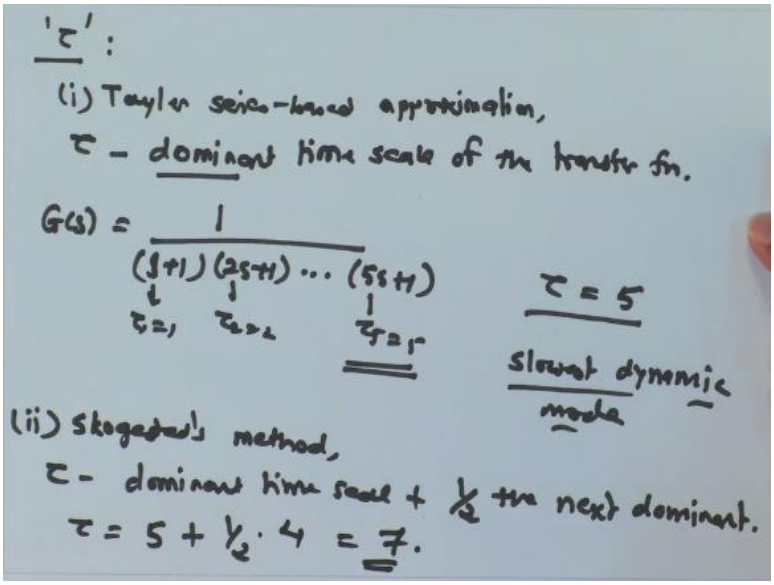


What we have is a higher order transfer function which we want to approximate into first order plus dead time model. So what we are essentially doing is we are converting this as,

$$G(s) = \frac{N(s)}{D(s)} = \frac{k_p}{\tau s + 1} e^{-t_d}$$

We need three unknowns. What is the ultimate gain, what is this time constant and what is the dead time given the fact that we have the transfer function for this process? So, there are two methods of doing this.

The first method is based on Taylor series expansion and the other method is Skogestad's half rule. In both cases, the notion of 'kp' is the same. 'kp' is the gain between input and output. For the given transfer function whatever is the gain between input and output, the same gain would get transferred to the final approximate form. The second term is how do we get tau?



So tau, as I had said, is a mathematical entity which just approximates or matches the given response with this approximated model. It does not have a direct physical significance. So, when we talk about Taylor series based method approximation, tau is taken as a dominant time scale of the process. And by dominant, we mean the slowest time scale. So if our process transfer function was something like this as

$$G(s) = \frac{1}{(s+1)(2s+1)\ldots(5s+1)}$$

Where,

$\tau_1 = 1, \tau_2 = 2, \tau_3 = 3, \tau_4 = 4 \; and \; \tau_5 = 5$

The slowest mode out of this is coming from this 1/(5s + 1). So, in that case, we would be taking tau as 5. It is the dominant or the slowest dynamic mode is taken as the time constant or tau. In the case of Skogestad's method, what we take is tau is taken as dominant time scale plus half of the next dominant one. So we are not relying only on the slowest time scale but we are looking at the top two slow time scales of the process.

And in that case for this particular example, what we would be getting is this tau will be,

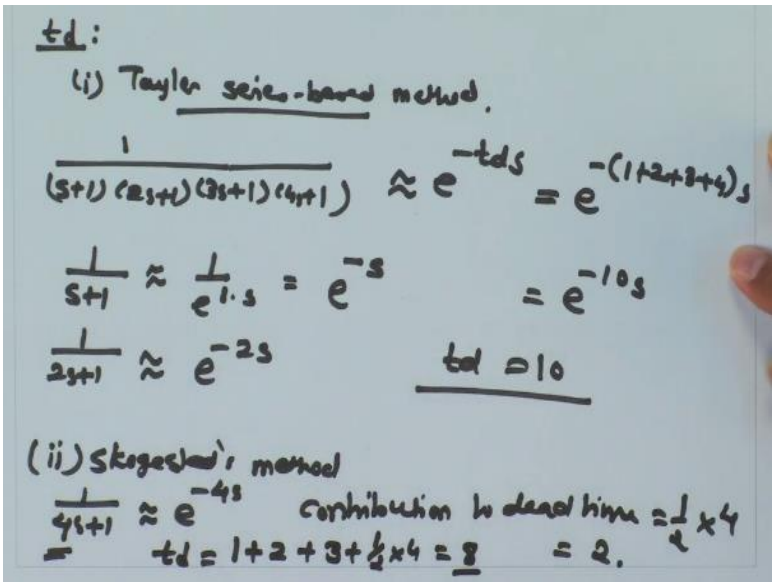
$$\tau = 5 + \frac{1}{2}\ of\ the\ second\ slow$$

So, if it is (s + 1), (2s + 1), (3s + 1), (4s + 1) times (5s + 1), it will be half of 4. In that case, we will get tau is,

$$\tau = 5 + \frac{1}{2}(4) = 7$$

So, the tau obtained for the same transfer function by using Taylor series expansion or Skogestad's method, they are going to be different. Skogestad's method will give you higher tau value.

And the last parameter which we want to get is 'td' which is the dead time.



The way we get the dead time is let us say if we look at the Taylor series expansion based method, so we look at the remaining transfer function after we have removed tau. So in this case the remaining transfer function was (s + 1), (2s + 1), (3s + 1), and (4s + 1). We approximate it as, $e^{-t_d s}$. The way we do it is, we try to write each of these. So let us say,

$$\frac{1}{s+1} \approx \frac{1}{e^{1.s}} = e^{-s}$$

Because if you take Taylor series expansion,

$$e^s = 1 + s + \frac{s^2}{2!}$$

So, this is approximately equal to,

$$e^s = 1 + s$$

Similarly, you can write that,

$$\frac{1}{2s+1} \approx \frac{1}{e^{2.s}} = e^{-2s}$$

So, this entire time for this particular example it will be equal to,

$$\frac{1}{(s+1)(2s+1)\dots(4s+1)} = e^{-(1+2+3+4)s} = e^{-10s}$$

So, in this case

$$t_d = 10$$

To get the dead time, you just approximate the remaining transfer function as the Taylor series approximation. In terms of Skogestad's method, we follow the same approximation procedure. The only difference is this,

$$\frac{1}{4s+1} \approx \frac{1}{e^{4.s}} = e^{-4s}$$

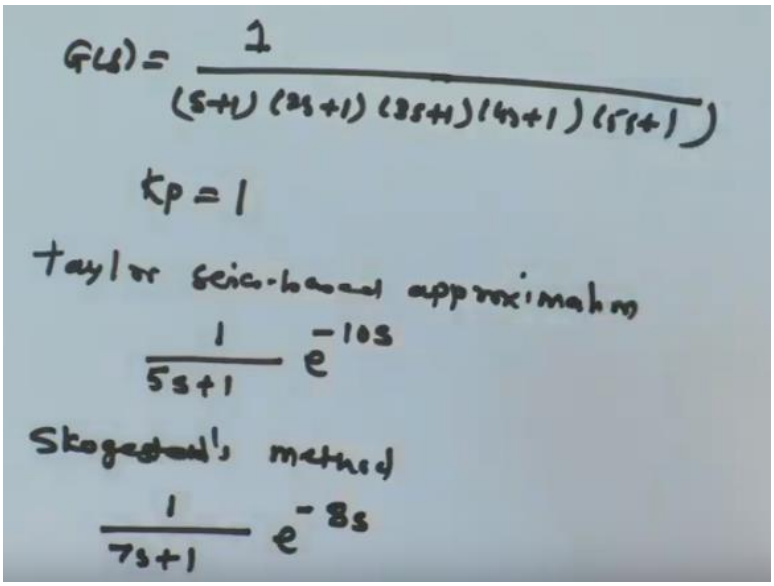
But in terms of calculating tau, we have already used half of this contribution. So that 2 out of that 4 was taken as tau. So in this case, the contribution to dead time is also taken as half.

So it will be, $\frac{1}{2}(4) = 2$

Based on this, the actual t_d here will be,

$$1 + 2 + 3 + \frac{1}{2}(4) = 8$$

So, the dead time in the Skogestad's method will be smaller than what you get from the Taylor series approximation.



For this particular example,

$$G(s) = \frac{1}{(s+1)(2s+1)\ldots(5s+1)}$$

If this was the transfer function, what we get is gain will be equal to 1 because the gain gets transferred directly.

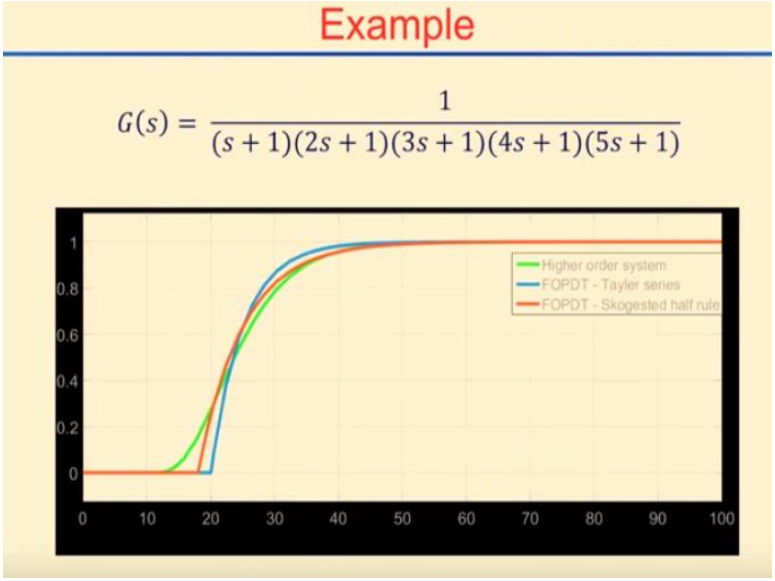
For the Taylor series based approximation, we get

$$G(s) = \frac{1}{(s+1)(2s+1)\ldots(5s+1)} = \frac{1}{(5s+1)}e^{-10s}$$

For Skogestad's method, we get

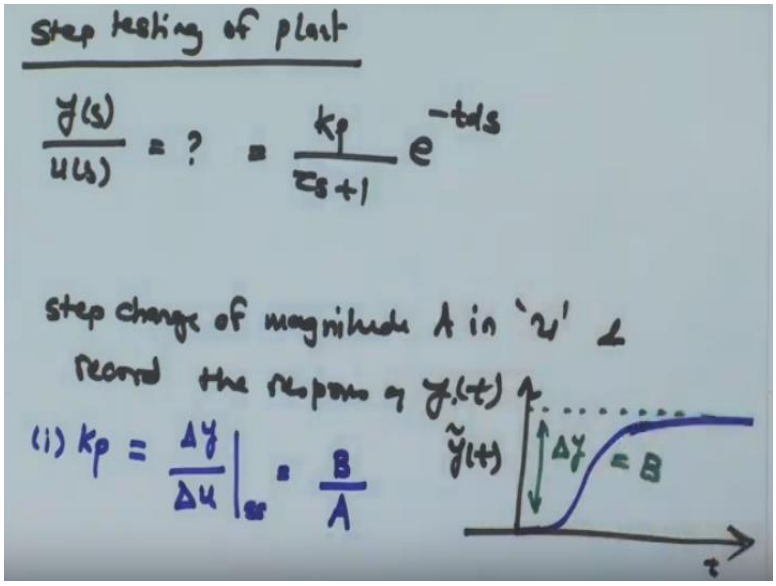
$$G(s) = \frac{1}{(s+1)(2s+1)\ldots(5s+1)} = \frac{1}{(7s+1)}\, e^{-8s}$$

Both these are approximations for this transfer function G(s) and if we plot a step response of this particular transfer function, you can see that the response is shown here.



The green line represents the actual response where the step change was given at time t = 10. You can see that both these models are approximating the behavior fairly with a good match with you can see that the red line is closer to the real response and that corresponds to the Skogestad's rule. So it is a more empirical method which was developed by Professor Skogestad and by just taking the second dominant scale and increasing the tau and while reducing the dead time, it is able to match the response in a much better way.

Next, we look at if we do not have a transfer function representation of the process but we have the actual process and how do we come up with this first order plus dead time approximation. So in that case what we do is known as step testing of the plant.



Let us say we are interested in the relationship between, so we are interested in output is y and input is u which we are going to represent as first order plus the dead time model. So what we do is in the given plant, we make a step change in u let us say of magnitude 'A' and we record the response of y as a function of time. So what we are going to get is a response of this type and when I say y(t), it is always a deviation variable.

We give a step change when the plant is at steady state and we record the response as time goes on increasing away from this particular steady state and the typical response what we will get will be of a sigmoidal nature which will start at 0, take some time and then reach the final value. From this response, you can record and then from this response we would be able to find the values of 'kp', 'tau', and 'td'.

In order to get 'kp', this is the gain of the process. It is the total change in y divided by the total change in the input and this is at the final steady state. So for this particular example,

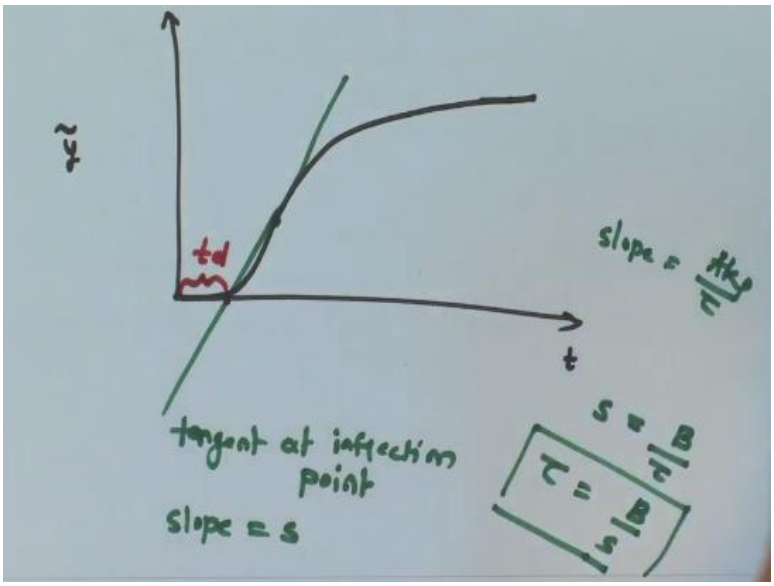
$Total\ change\ in\ y = \Delta y = B$

So the gain, in this case,

$$Gain = k_p = \frac{Total\ change\ in\ y}{Total\ change\ in\ u} = \frac{\Delta y}{\Delta u} = \frac{B}{A}$$

So, we will be able to get the gain of this process.

Let us now look at how do we get ‘tau’ and ‘t_d.’



Let me redraw the same figure again. In order to get ‘tau’, what we do is we take the slope of this sigmoidal response at the inflection point. We draw a tangent at the inflection point so which is somewhere here. Let us say that tangent looks like as shown in the figure. The tangent at an inflection point and let us say its slope is equal to s. So, from the first order dynamics lecture, you should recall that the slope of the first order response at time t = 0 is given by,

$$slope = \frac{A\,k_p}{\tau}$$

We will use the same relationship. In this case, the slope is ‘s’ and ‘A kp’ is nothing but B; then

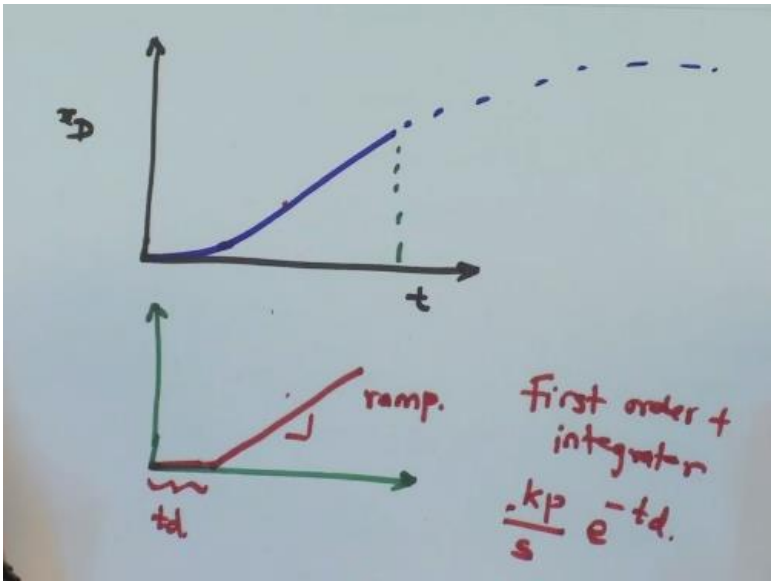
$$\tau = \frac{B}{s}$$

This tau value which we are interested in is equal to the ultimate value of output y divided by the slope of this tangent at the inflection point.

Then lastly, now the only thing remaining is the dead time. We take whatever is the intersection of this tangent to the time axis. So this intersection we take as ‘t_d’.

This will be our dead time. If we have the actual process and do not have a transfer function then using this method we would be able to approximate this process as first order plus dead time. So this is a very common way any real plant data would be analyzed and converted into first order plus dead time model.

Now just I would also like to make a small note that sometimes the order of the system is very high. In that case, running a step response and trying to get the final output, it takes a very long time. So, in that case, the typical example of this would be let us say if there is some, we are interested in finding the transfer function model between the product purities of the distillation column and the feed composition. We had seen this as a motivating example and the order of the system is equal or is of the same order as that of the number of stages.



If the column has a large number of stages, this becomes order 20, 30 dynamic models and it takes a very long time to settle. So the response would typically in such a case if I say x_D versus time, the response would be so slow that it will take a long time of the step testing to reach the final steady state. So, in that case, most of the time these models would be used to make the initial predictions about how the system responds.

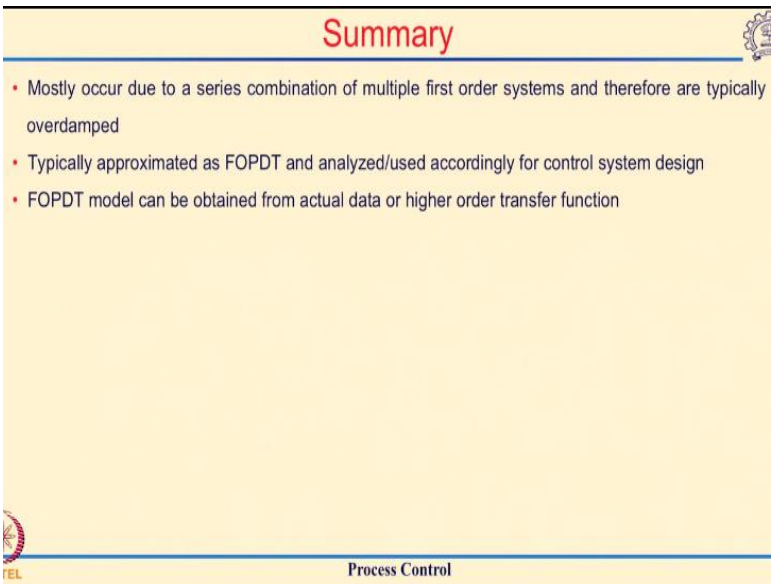
So what we consider is we just consider the response only up to this point and what we say is it is similar or we approximate it not as a first order plus dead time but we consider that the response is like some dead time followed by a ramp. We approximate this as a ramp rather than a response of a first order process. So, in that case, this is known as a first order plus integrator.

So it is not a first order lag but it is a purely capacitive process for which the transfer function will be,

$$G(s) = \frac{k_p}{s} e^{-t_d}$$

In this case, the only two things which we are interested in is what is the dead time and what is the slope here which would give the value of gain 'kp'. So especially this type of approximation is made when the system is really of higher order and it does not reach the final value in a perceivable or whatever is your prediction horizon.

To summarize we have seen that in order to get a higher order process, it is typically a series combination of multiple first order systems and therefore they are typically overdamped and rather than analyzing each of those order separately, we typically convert them or approximate them as first order plus dead time model and that is the model which we will be using to make prediction or design a controller for that particular higher-order system.



And then that FOPDT model can be obtained either as an approximation of the original transfer function which we have generated or it can be obtained from the experimental data or the real plant data. We will stop this lecture here and in the next part of this lecture, we will look at the numerator dynamics. Thank you.