Chemical Process Control
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Lecture - 17
Higher Order Dynamics

Hello students. So far we have looked at how the simplest which is the first order dynamic system response then we moved on to second order dynamics and in this lecture we will club anything which is of higher order than second order system as the higher order system and we would see why that is done in just next slide. So in this lecture our objective is to see where exactly or when do we get higher order dynamics.

And the second part is most of the times we would be approximating all these higher order dynamics in the form of a grey box model also known as first order plus dead time model. So we will be looking at all these higher order dynamics as an approximation and that is why we are not going to study each of them individually but we are just going to club all of them and we would see the rationale behind this and how that is done. So we will just define any higher order system or Nth order system.

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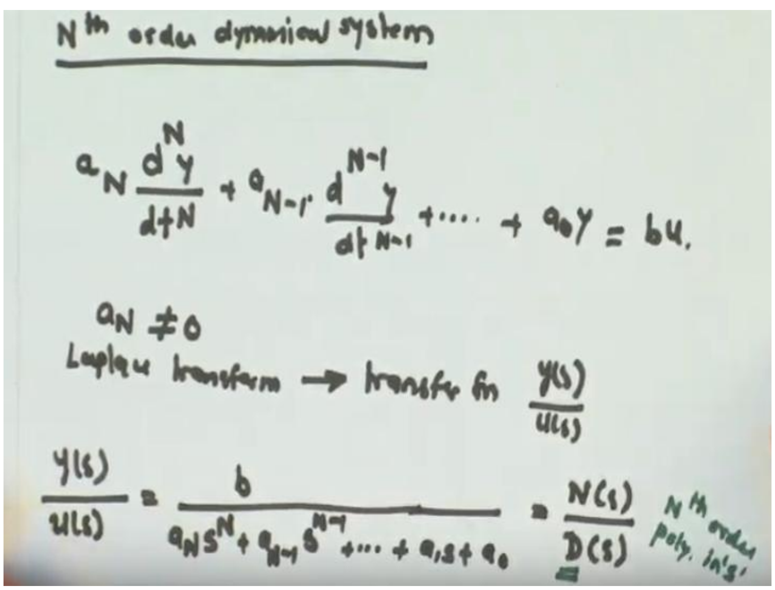

So Nth order dynamical system we will define as an extension of how we have been defining a first order system or a second order system. So it will be a system whose dynamics are given by

N^{th} order ordinary differential equation. So the general form of that will be this where *u* is the input and *y* is the output and as it is an N^{th} order system, a_N should not be equal to 0. Now most of the times we work in the Laplace domain.

So let us try to take a Laplace transform of this that will give us the transfer function which will be y(s)/u(s) that can be obtained by taking the Laplace transform of this equation. So we would get y(s)/u(s), for this particular case it will be $b/(a_N s^N + a_{N-1} s^{N-1} + ... + a_1 s + a_0)$ which in general form can be written as some numerator polynomial in s divided by a some denominator polynomial s.

So when we talk about an N^{th} order system so in that case this D(s) will be N^{th} order polynomial in s. So when we talked about a second order dynamical system that was s square. So second order in terms of polynomial in s for a first order system it was just (τs +1); so only s. Similarly, if it is power s^N, it will be an N^{th} order system. And then we will not try to find out how these systems respond to a step change.

Or what we are just going to say qualitatively is if this denominator, the roots of this denominator polynomial, so we will try to write that.

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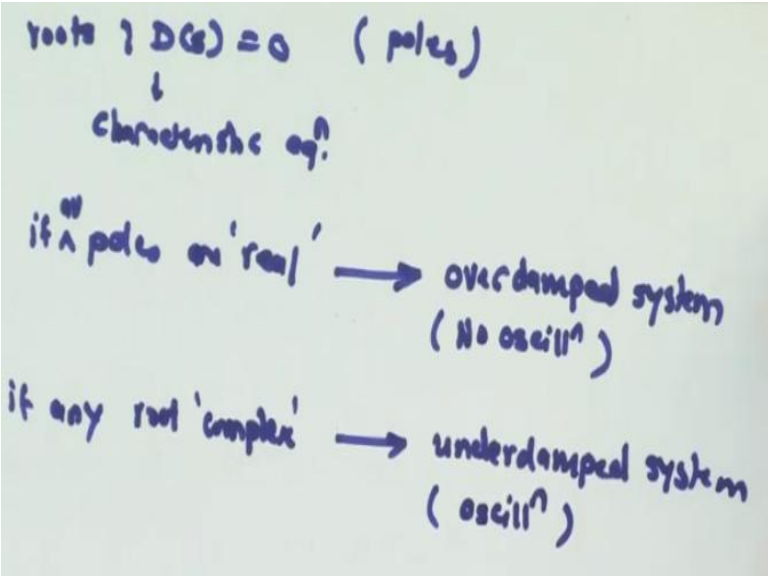

So we will just try to find this root of the denominator polynomial in s. They are also known as poles of the system and this equation is known as a characteristic equation. So when we take roots from the characteristic equation to find out the poles, so if all the poles are real then the response of this higher order system will be similar to over damped response. So the response will be similar to an over damped response. That means no oscillations.

And if any root or actually pair of roots are complex, in that case the system will behave similar to an under damped response so that we will have oscillation. So for any higher order system what we are going to do is, we will try to find out whether the poles are real, if they are real, the system will behave more or less similar to an over damped system. It will be definitely slower than the second order over damped system.

But the response will look more or less similar to that. And if any of those roots are a pair of complex conjugate roots then the system is going to respond similar to an under damped system, again slower than second order under damped system. But based on the roots of these denominator polynomial we typically are able to at least gauge how the system is going to respond to a step change.

So let us now look at, are these higher order systems common in chemical engineering? Why do we need to study them? And when do we get a higher order response or a higher order dynamical system? So the answer again lies as an extension of a second order system. So we had seen that a second order system is obtained most of the times it is a series combination of two first order systems.

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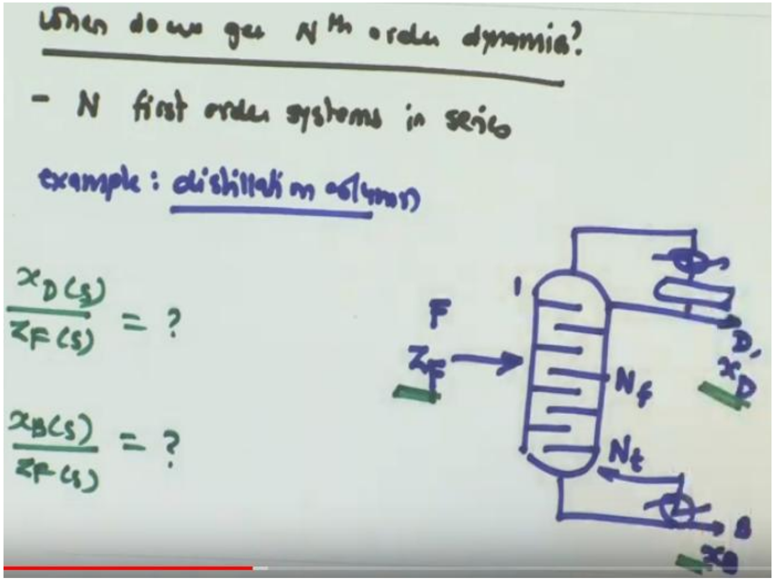

So when do we get, so if I want to answer the question when do we get Nth order dynamics, the most common place where we get it is when we have N first order systems in series. So it is just an extension of how we define or how we found the genesis of a second order system is that we have N first order systems in series that is going to give us Nth order dynamics. And as it turns out this is quite a common phenomena in chemical engineering.

So let us take an example of a distillation column. So we have this distillation column. The feed comes in at some stage we will call it as N_f. The stages are numbered from top. So we will say this is stage 1, we have a condenser, reflux drum. Product comes out here with a purity of x_D and then again in the stripping section we have few stages. The total number of stages let us say are N_t. Then we have this reboiler and then the product goes out at the flow rate of B and x_B.

And we typically have feed flow F and the feed composition of z_F. So this is a typical set of very conventional simple binary distillation column. It can be extended for any multi-component distillation as well. And what we are interested in is how does how do these product purities x_D and x_B change as a variation in the feed composition. So let us say this feed comes in from some reactor and we are trying to separate un-reacted reactant and the product.

So we are interested in how this separation happens so that we get the product with desired specification. And as this z_F or feed to the distillation column is coming from a reactor, there may be some upsets in the reactor which may cause this purity to fluctuate. So z_F would have fluctuations entering the distillation column. So in that case we want to see how these fluctuations affect the final product purity.

So we are interested in the response in terms of the Laplace domain. What we want is how do these transfer functions look like between x_D and z_F and between x_B and z_F. So in order to do that we will have to use material balance and let us try to write material balance for this simple system. So we will start with the feed tray.

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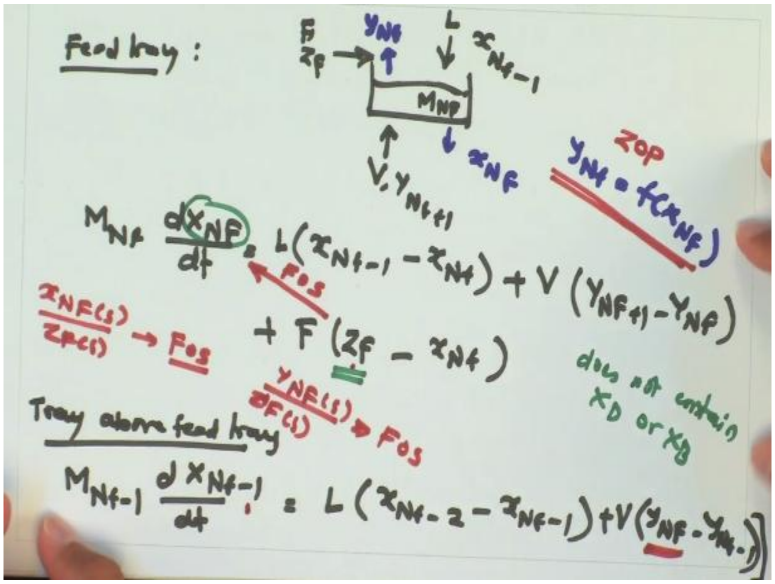

So if I write the dynamic material balance for the feed tray and if we make simplifying assumptions such as constant molar over flow so that the flow rate in the rectification section and flow rate in the stripping section remains constant from tray to tray. We will also assume that the feed is fully total liquid or fully saturated liquid. So these are just simplifying assumptions just to show this derivation in fewer steps.

It is not necessary that all these assumptions should hold to show that this is the higher order system. So let us take the feed tray and in the feed tray what is happening is you have some liquid coming in from N-1th tray. So the liquid flow rate we are assuming it to be constant as L and composition will be x_{Nf-1} and then the vapor is coming from the tray below at the flow rate of V and as a composition of y_{Nf+1}.

And the outlet streams are going to be liquid will go out at x_{Nf} and the vapor will leave as y_{Nf} and we typically assume that this x_{Nf} and y_{Nf} are in equilibrium. So y_{Nf} is a function of x_{Nf}. And then as this is a feed tray we have also one more input which is in terms of F_{zf}. And let us say the holdup on this particular tray is M_{Nf}. So then we can write the material balance for the more volatile component.

It will come out to be,

$$M_{Nf}\, d(x_{Nf})/dt = L(x_{Nf-1} - x_{Nf})$$

which is from the liquid side. From the vapor side we have and from the feed side we have this. So this is the equation which is going to govern the response of or how the feed composition is going to affect the distillation column. And we can see that this equation does not contain our desired outputs which are x_D or x_B. So definitely this system is not first order system in terms of x_D relationship between the purities, the final purities and the feed composition.

So what we are seeing is this z_F is going to first affect this x_{Nf} which is the purity of that feed tray and eventually this x_{Nf} is going to affect all the other trays, the trays above as well as below. So let us say how this change propagates upwards. So when we take the balance at tray above the feed tray, so in that case the equation would look like,

$$M_{Nf-1}\, d\, x_{Nf-1}/dt = L(x_{Nf-2} - x_{Nf-1}) + V$$

There is no separate feed, so the feed term will not come here, y_{Nf} - y_{Nf-1}. So we can now notice that the change in z_F is going to affect x_{Nf} as the first order system because this relationship between x_{Nf}, so if I want to write $x_{Nf}(s)/z_F(s)$ it is going to be a first order system based on this equation and we have said that this x_{Nf} and y_{Nf} are related by vapor liquid equilibrium which is a static equation. So this is like a 0^{th} order process or this is a algebraic equation.

So similarly, if I want to write $y_{NF}(s)/z_F(s)$ it is also going to be a first order system. And this y_{Nf} is entering here as an input. So when we talk about the transfer function between x_{Nf-1} and z_F it is going to have 2 capacities in series. The first is this that feed tray and the next one is the tray above the feed tray. So when we look at the transfer function between z_F and x_{Nf-1} it is going to be a second order dynamics.

So we will have to do this all the way up to the top tray and then the reflux Drum. So what we are going to end up with is, so we started with zF.

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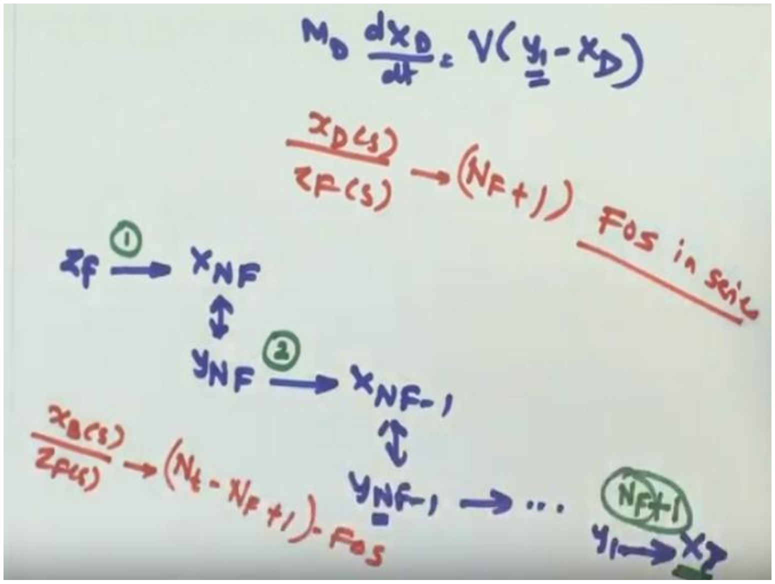

So z_F to x_{NF} this is the first and then x_{NF} and y_{NF} are related through equilibrium and then this y_{Nf} again has a first order response with x_{Nf-1} which will be in equilibrium with y_{NF-1} and then this will continue further till we get x_D as the top purity and the equation for x_D will be this will be M_D d x_D/dt = V(y_1 – x_D). So we have to eventually go all the way up to y_1 when we go on reducing N_{F-1}.

So after N such steps we would have reached y_1 which will give me this first order response with x_D. So what we are going to have is this is the first order process. This is the second order. So when we go all the way up to N_F, after N_F such steps we are going to get the top purity. In fact it will be N_F+1. So the response between, so when we want to write the response between x_D(s) and z_F(s) we have N_F+1 first order system in series.

So this response is, the order of this response is definitely higher order and the order is given by how many trays are there in the rectification section. So we have N_F trays in the rectification section and this one comes from the reflux Drum. So that is also first order process. So total there are N_F +1, this is an N_F+1 order system. So depending on how many stages you have in the distillation column we would have such higher order systems or higher order dynamics in a single distillation column.

And you are aware that distillation columns are one of the most commonly used separation processes in our chemical industry. So whenever we have a distillation column, we are going to have a higher order system because our distillation column is never going to have one or two trays. It is definitely going to have multiple such trays. So we will have such responses and the same thing which we can also show for the stripping section.

And then if we want to write the bottom purity response to the feed composition change again it will be of higher order system. In that case it will be the stages in the stripping section which will be $N_t - N_F + (1)$ the reboiler. So these will be the these many first order systems will be in series. So that will be the order of this response between x_D and z_F. So the whole purpose of this little example was to show that such higher order systems are quite common in chemical industry.

And if I want to show you the response of this what we are going to see is the response of a distillation column. Here is an example of a binary distillation column and it had around 95 stages. The feed was around the middle of the column and you can see that the response looked something like this. So for quite some time, almost up to first 5 minutes, there is the output does not change.

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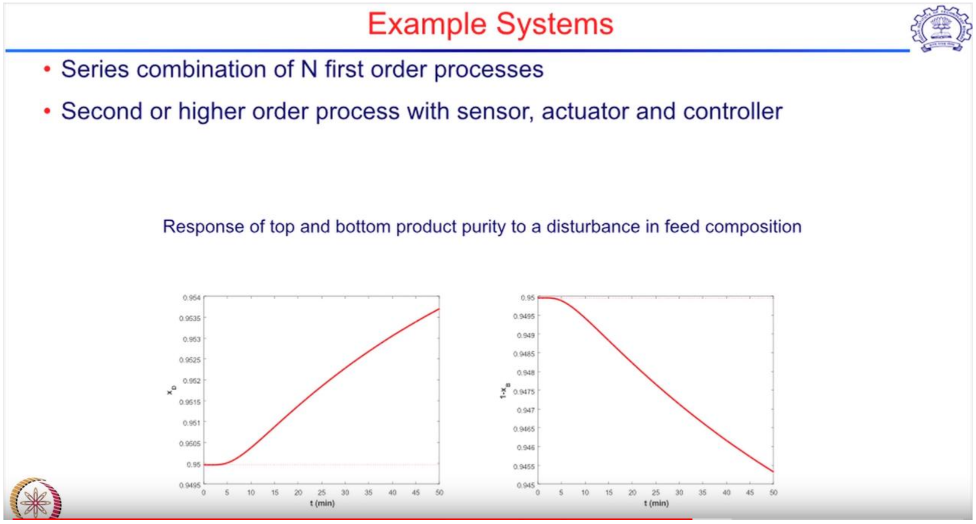

And it is quite intuitive that when I change z_F, the portion of the column is not going to see any change happening. So unless this change makes its way all the way through all the trays into the rectification section, these purities will slowly keep on changing right from X_{NF} then X_{NF-1} and slowly when the final X_1 changes then it is going to start showing some effect on the top purity. So the response is always going to start very slowly and then it is going to pick up.

So that is a very trademark feature of any higher order system that they are always characterized by some dead phase when nothing happens. So that is also known as a dead time and then the response starts to move. So let us say if this was the example and the controller is placed on x_D. So when the feedback type controller is on x_D it is going to reject any changes in x_D.

So when a disturbance in z_F happens it has not up to that time period let us say for this example it was 5 minutes; up to 5 minutes the controller is going to think that nothing has happened. But by that time the controller starts seeing that something has happened all the entire column profile has changed. So all the compositions on each of the trays has changed. So in that case it takes quite some time for the controller to reject all these disturbances.

So reject all these effects and bring the column profile to the original steady state. So that is why having a dead time in the system can have very detrimental effect in terms of seeing that the disturbance has happened. And also in terms of the control action that whatever action we take if the response does not show any change then the controller will feel that the effect has not happened and it will keep on increasing the manipulated input.

And it may happen that the change in the manipulated input is so high that the system just goes out of bounds. So again dead time systems are very challenging in terms of control and when we look at control we will be spending more time on that.

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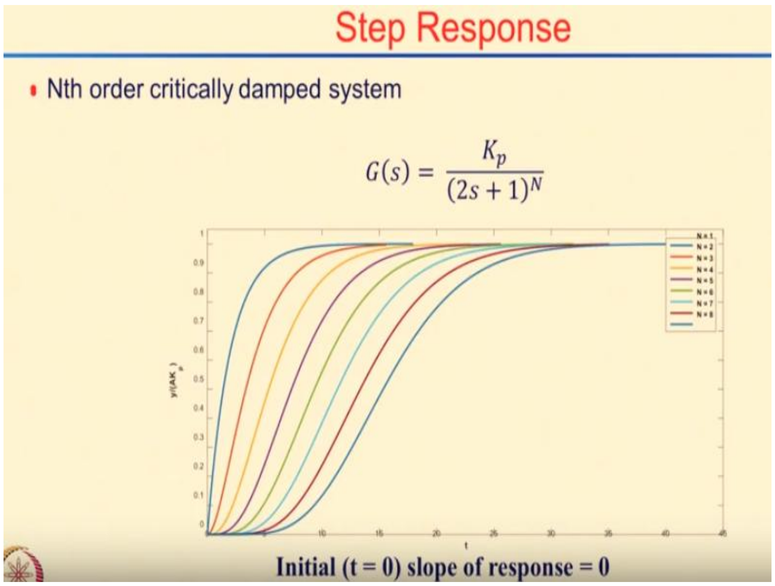

Similarly, this was one example and if we take just a series combination of these first order systems which has a transfer function of 1/(2s +1) and we are going to see how the response of this transfer function changes as we keep on increasing N. So when N = 1 it is a very simple first order system and the response is the leftmost response shown in the blue color, light blue color.

You can see that it is a very fast response and it reaches the ultimate value of 1 in this case because K_p is considered to be 1. And we keep on increasing N. So when N = 2 we have a critically damped response which is slightly slower than the first order response and as we keep on increasing this N we are going to see that the response becomes slower and slower. In control term we call it as a sluggish response.

And you can see that when that N was equal to 8 the response almost did not take off for quite some time. I am talking about the rightmost curve here and you can see that there is a significant amount of dead time and then the response shoots up. So the typical feature of this higher order response is that they are characterized by some dead time and then there is a sigmoid response which follows it. And then ultimately all of those reach the final value of AK_p.

So this phenomena we will use to approximate any higher order system. So we are seeing that any higher order system has a dead time and then followed by a very sharp response. So we will be approximating this higher order systems as first order plus dead time systems. So we will say that the response looks like this.

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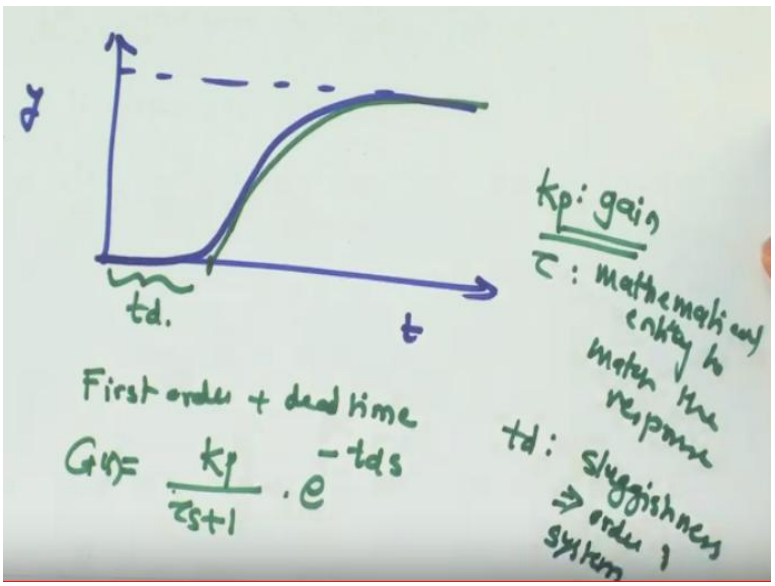

So what we are seeing is this y versus t response is something like this. This is the ultimate value. So we are going to approximate it as up to a certain time nothing happens and then it follows a first order response. So that is how we are going to approximate any higher order system as first order + dead time. So if you want to take the transfer function of this, so this dead time is t_d then the transfer function of this is going to look like first order process which is $K_p/(\tau s + 1)$ times the dead time transfer function is $e^{-td\, s}$.

So we are going to approximate any higher order transfer function as a first order plus dead time process. So in terms of the parameters , K_p is gain and it has the same significance as the original transfer function. So if the original transfer function had the gain of K_p even this first order plus dead time system has the same gain. So it has the same significance as the original gain.

τ is just a mathematical entity to match the response. Most of the time it does not have any physical significance. It is justly simply going to give us the best match between the approximation and the original response and then this td is the dead time. So it characterizes or gives us how much time does it take before the system starts to move. It kind of gives you qualitatively how much is the sluggishness in the system. And it also sort of gives you what is the order of the system qualitatively.

Because it is not going to give you the number value of the order but as the td increases we will see that the order of the system goes on increasing. So let us now look at how do we approximate any higher order system as a first order plus dead time model. And there are depending on what information we have, we can approximate a higher order system as a FOPDT or a first order plus dead time system if we have a transfer function of the system.

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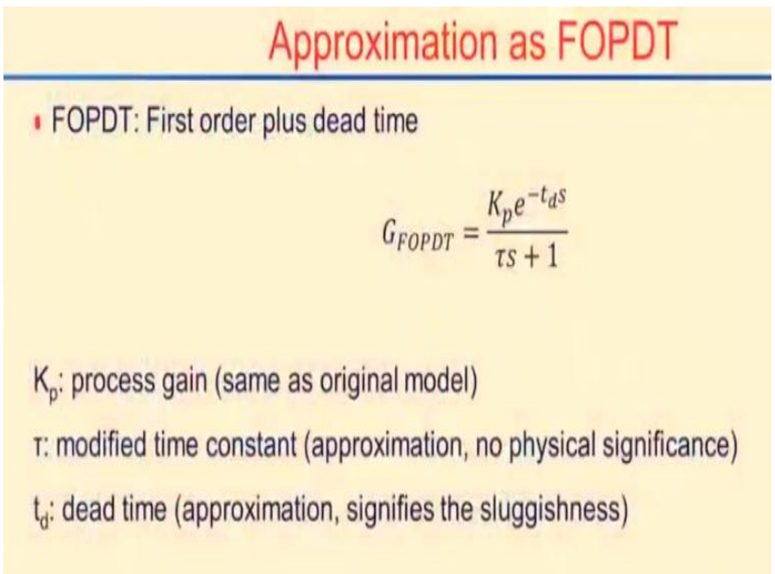

So we will take a short break here and after the break we will look at how do we approximate higher order system as a first order plus dead time system. Thank you.

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