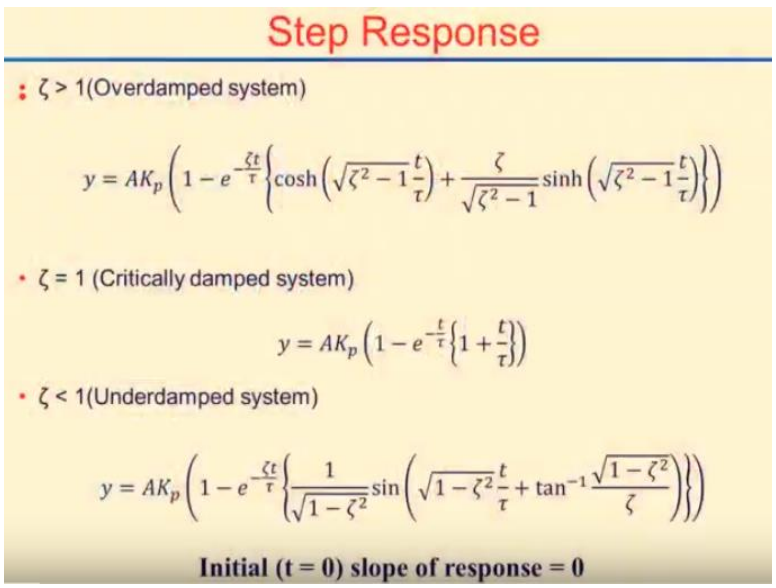
Chemical Process Control
Prof. Sujit S. Jogwar
Department of Chemical Engineering
Indian Institute of Technology-Bombay

Lecture - 16
Effect of Damping Coefficient

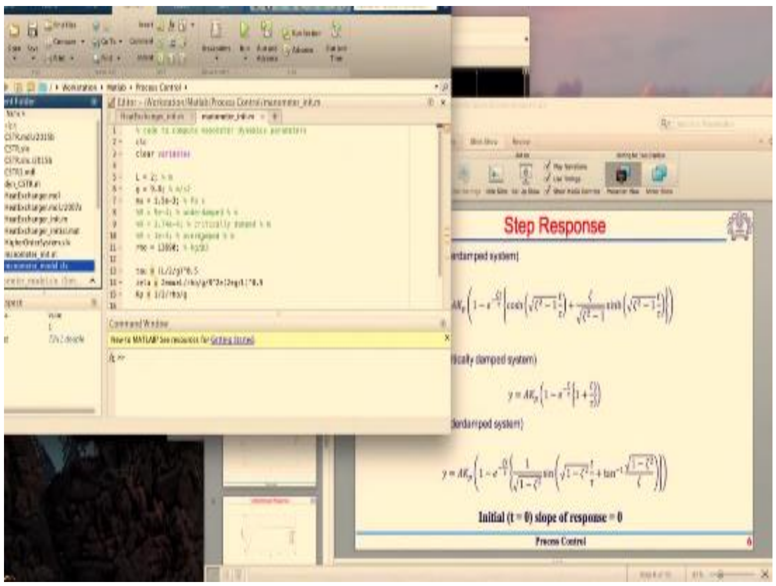
Okay, welcome back. Here is the summary of the expressions which we have derived for step response of a second order system and depending on whether we are in the regime of overdamped response, critically damped response or underdamped response, we will get these type of responses and you can see that the nature of these expressions is very similar. The first term 'A kp' gives you the ultimate value of the response.



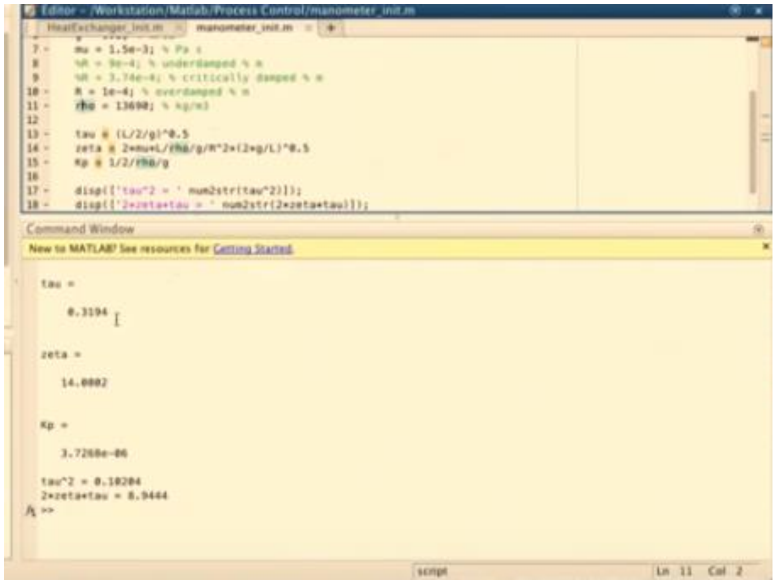
In all these cases, you should notice that if I take the slope of this response at time t = 0, that slope is going to be 0. So, the response does not directly start at t = 0.

There will always be 0 slope at time t = 0 and slowly the response will catch up. This is again a distinguishing factor between the first and the second order response. In the first order response, the response directly starts with a non-zero slope at time t = 0 which was (1/tau). In the second order system, the system takes a little bit time to get started and that is why the initial slope is going to be 0.

Now we will see how these responses look like with the help of the manometer example. That was the example we had considered as inherently second order system and depending on the values of parameters of the manometer we may have, we can make the manometer to behave as an underdamped system, critically damped system or an overdamped system. Let us look at it through simulation.

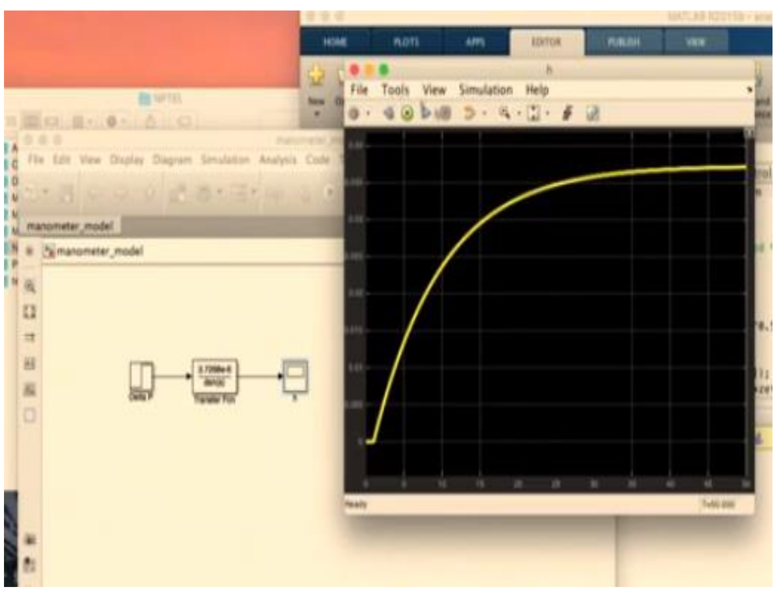


Let us start with how the response looks like when we have the overdamped response. For that, these are the parameters of the manometer and the only thing which we are going to change is the radius of that manometer. We will see that by just changing the radius of the manometer, we can move it from one type of second order dynamics to the other.

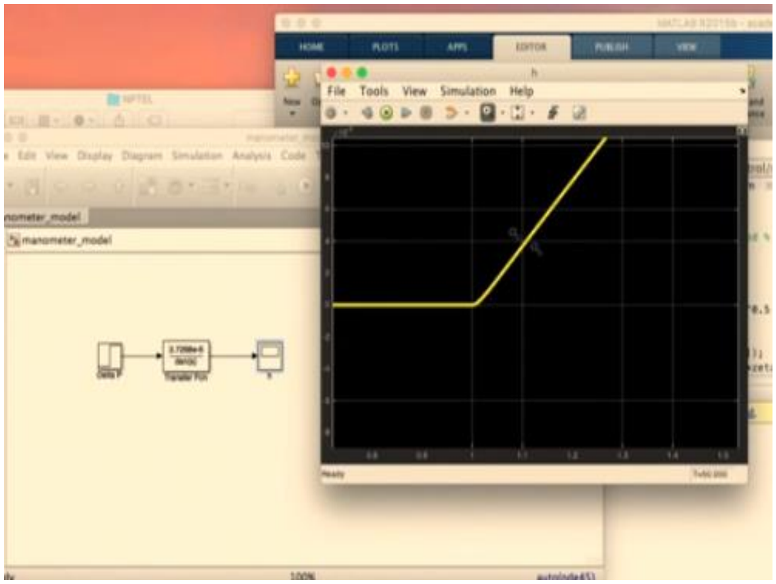


When we take the radius of this manometer to be 0.0001, we are going to get the overdamped response so let us see what are the parameters we get, we get tau = 0.3194. We have zeta which is greater than 1 which is, in fact, equal to 14 and the gain value is given here. In that case, this tau square will be 0.1 and this intermediate term will be 0.944. So, we will enter these values as a second order response system. The gain goes here.

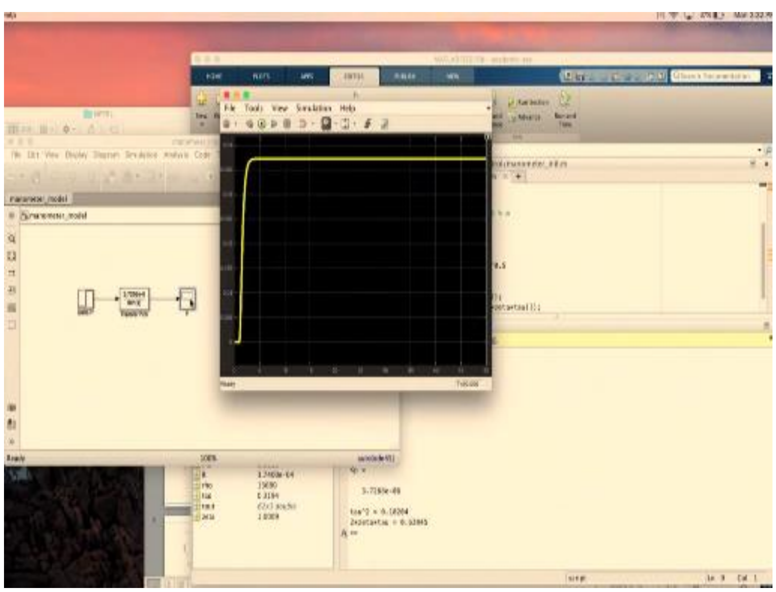
This is the tau square twice zeta tau and 1. And we see the response of this system to a step input in terms of the pressure drop we increase the pressure by this amount Pascal or 10 KPa. What we would get is the response of this system.



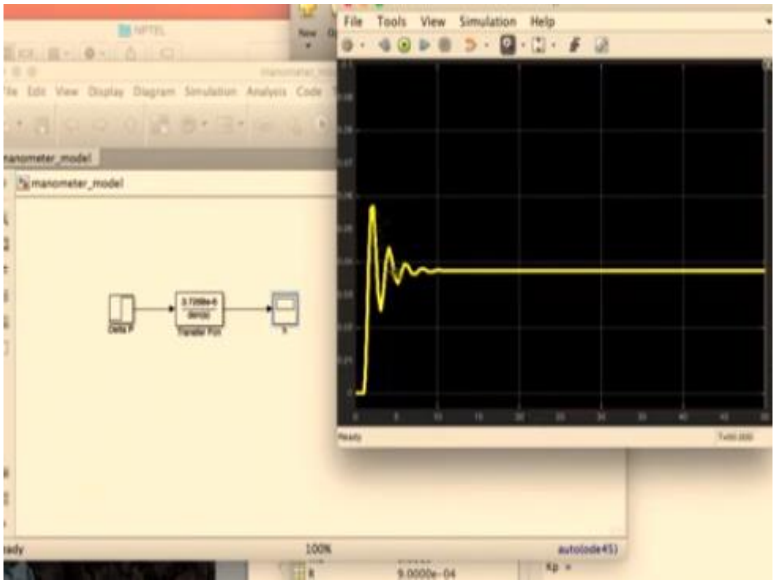
You can see that the response goes from the initial value to the final value. Here you may not be seeing what is happening right in the beginning.



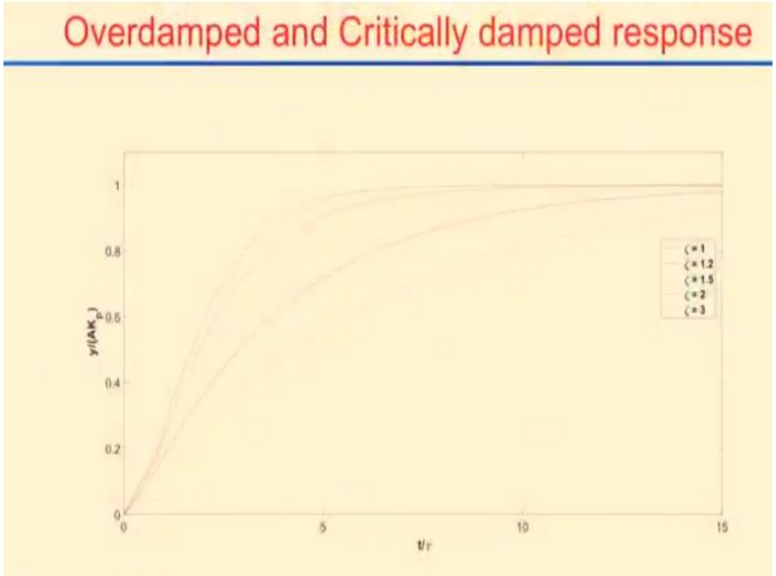
But if we zoom in, we can see that the response has a slight 0 slope in the beginning and then slowly it kicks off. Now when we look at a critically damped response, in that case, we will be increasing the radius of this manometer and if the radius goes to 0.00037, you will notice that this damping coefficient becomes almost equal to 1, tau and Kp remain the same.



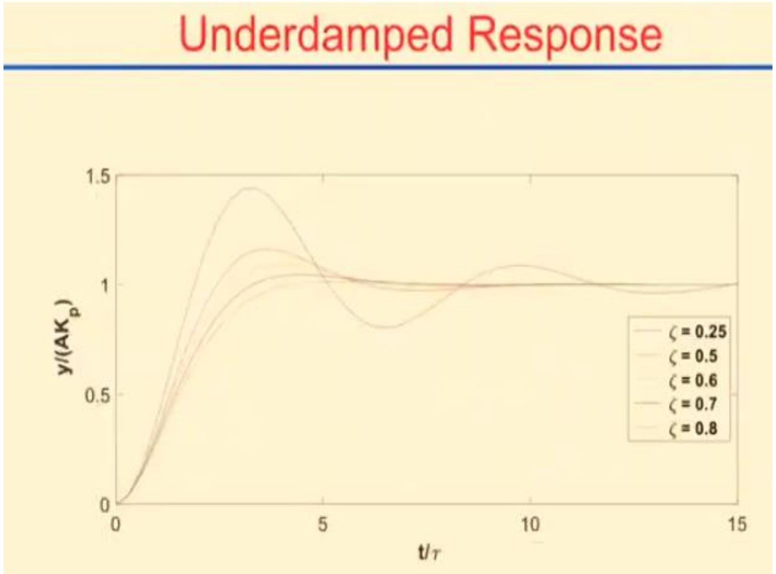
If we put that value, you will see that the response has spread up significantly. It looks similar to the previous response but the only difference is it becomes a little faster. When we look at the response when zeta is less than 1, in that case, let us say, radius goes to 0.0009. We will see that the damping coefficient, in that case, becomes 0.1728.



You can see that the response of the system has oscillations which decay over time and that is the typical response of a second order system. So, this is how even a single system just by changing a parameter, we can move from overdamped to critically damped to underdamped. Let us now look at these responses in more detail and what is the effect of these parameters. So, here are the responses when we are either looking at a critically damped or overdamped response.

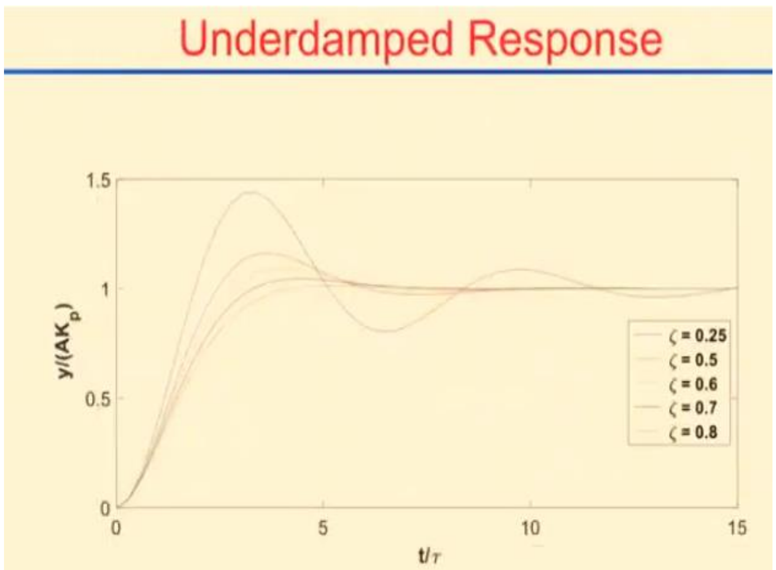


You can see that as the damping coefficient is very large, the response becomes sluggish or slower compared to a critically damped response. The critically damped response would actually represent the fastest way to go from the original point to the final point and staying there. This is how the response would change when the damping coefficient changes. You can see that as we are reducing the damping coefficient from the value of 3, the response is getting pushed towards the y-axis.



As we move fast zeta = 1 and go into zeta < 1 territory, we will start seeing that the pushing towards the origin actually moves the response above 1. So the response is going to overshoot the value, the ultimate value and what we are going to see is as the damping coefficient goes beyond the value of 1, we will have oscillations and the damping oscillations. If the zeta value is closer to 1, the oscillations are not that significant. As the damping coefficient value goes much below 1, we will see that the oscillations are quite significant. And we had seen last time that if the damping coefficient becomes 0, then the oscillations do not die and you will be getting a phase of sustained oscillations. That is how these responses of a second order system change as we change the damping coefficient right from a very high value to a very low value close to 0.

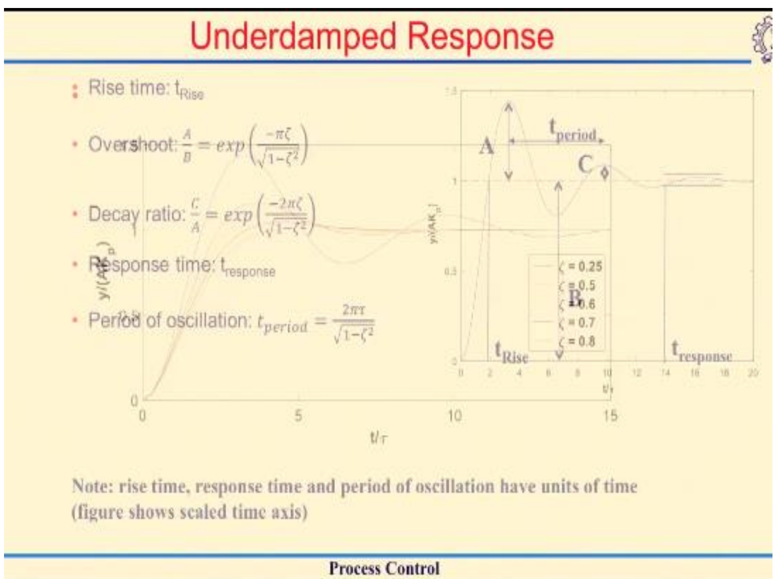
Now the second order response is the underdamped response or critically damped response, they will play a very important role when we are designing a control system.



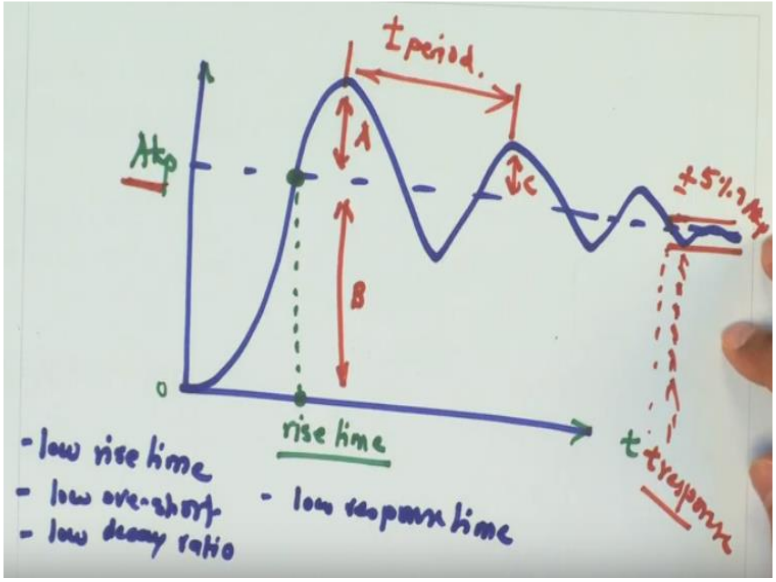
Let us say for an example, we want the response which is shown in Figure, is the underdamped response which is a response of a controller which is trying to move the parameter let us say, a scaled temperature from the original steady state value to the final steady state value which is given by the scaled value of 1. In that case, what we would want is the response or the controller should move the response from the original value to the final value as soon as possible.

We would also want this value should stay there. We would also want that it should not overshoot by a large amount. In order to do all those characterizations, many times we prefer to have an underdamped response of the final system plus controller because in that case what we see that the response reaches quickly to the neighborhood of the ultimate value. Even though it does not stay at the ultimate value, it reaches in the neighborhood of that value much quickly compared to either critically damped or overdamped system.

When we make a change in the setpoint, the controller reacts very fast. It takes you very close to the final value even though it oscillates around it. we are still very close to that final value. Because of that, what we are going to get is the system will move to the new, stay close to the new steady state and even though we have some oscillations, we will try to minimize those oscillations. Typically we would want to have an underdamped response and in order to boost this response for controller design, we need to define some of the parameters of a second order system.



Let us look at what are the different parameters which characterize the second order underdamped response. We will start with the first one which is rise time.



If we draw the second order response, ‘A Kp’ is the ultimate value and we are going to get a response the as shown in the figure. This is the typical response of a second order system. The rise time is defined as the time corresponding to the point shown in the figure. It is the time when the response value reaches the ultimate value. The ultimate value was ‘A Kp’ and the response reaches the ultimate value for the first time. It may not stay there but it is the time in which it reaches there the first time. So, if it is a controller, then we would want this rise time to be as small as possible because we want to move from original point to the final point as quickly as possible so that our operating point has shifted. It may not have stabilized but it is still shifted to the new point.

The next term we define is known as an overshoot. So as an underdamped response overshoots the final value, we want to quantify how much is the overshoot. We quantify that as if we measure A and the ultimate value to be B, then the overshoot will be defined as A/B. It tells you how many fractions of overshoot is happening inside the system. If this is a controlled system, again we would want a low value of overshoot because we do not want the response to go much beyond the designed final value. So our designed operating point is let us say ‘A Kp’ we do not want this A to be much different from that.

Because there may be some physical limitations whether the system cannot go there or it may be that let us say if there is a reactor temperature, there may be some safety limit in terms of the catalyst which is used or explosive condition. So you do not want to move too much away from the desired operating point. Typically in a controller, you would want this overshoot to be as low as possible.

Then the third term which we define is known as a decay ratio. It tells you how these oscillations are going to decay. If I take C as it is shown in the figure, the decay ratio would be defined as C/A. It tells you how quickly these oscillations are decaying. Again if this is a controlled system response, then we would want these oscillations to decay very fast, so that the response ultimately reaches that value and stays there. Again from a controller point of view, we would want a low decay ratio.

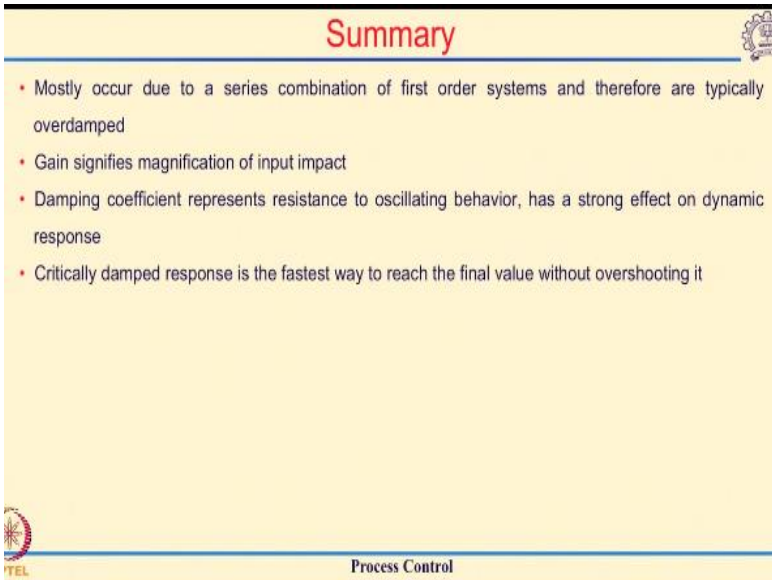
The other term which we are going to define is known as the response time. So response time typically is characterized as the time by which the response has reached within the plus minus 5% of the ultimate value. As the response is oscillating, we will have to wait for infinite time till it reaches exactly equal to 'A Kp'. But all practical purposes we can say that if the response is between 95% and 105% of the ultimate 'A kp' as soon as this sinusoid enters this particular tunnel, we will call this as the response time. Again if this is a controlled system response, we would want this response time to be as low as possible. Because in that case, we would have final stabilization of the system to the new operating point in a quicker time.

Lastly, the parameter which we define is the period of oscillation which is a very fundamental characteristic of any sinusoid. If we have this as a sinusoid, then it will be the difference between any two successive peaks. So the period of oscillation is as shown in the figure. So I have made some comments here in terms of if this is a controlled system response then what do we require?

What we would want is a low rise time. We would typically like low overshoot. We would also want a low decay ratio. We would also want .a low response time. If you recall the responses I had shown you for underdamped system as we change the damping coefficient if I want a low rise time, if I try to push the response towards origin then these oscillations are going to get bigger, so the overshoot is going to increase as well as the response time is going to increase.

There is always a tradeoff when we are trying to design a control system. If we want a low rise time or a quick response to the new steady state, then obviously I have to live up with larger overshoot or decay ratio. We will be looking at more of these when we look at a control system design, but here I just wanted to give you some glimpses of what kind of tradeoff is going to exist when we are trying to design response for a second order system or a controlled system which behaves like an underdamped response.

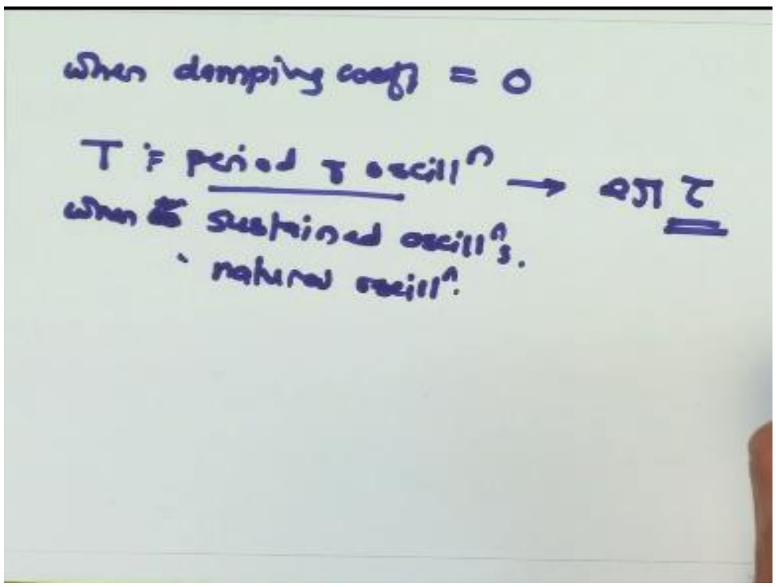
To summarize, mostly what we have seen is a second order system. Most of the times we would be getting it as a series combination of two first order capacity of the first order system.



Very rarely we would have something which is inherently second order. Out of these parameters, the gain has the same significance as that of a first order system where it signifies the magnification of the input. The damping coefficient represents it represents the resistance to the oscillating behavior of the system and we have seen that it has a very strong effect on the dynamic response. The response may show oscillations or may not show any oscillation.

The critically damped response is the fastest way to reach the ultimate value without overstepping it or overshooting it. If we are okay with overshooting, then the underdamped response will be a faster way to go there but may not, the ultimate value may not stabilize there before the critically damped response. And lastly, I have not given you the significance of ‘tau’ yet.

If it is a ‘tau’ you had called it as a natural period of oscillation and when the damping coefficient becomes 0 we have shown that the system oscillates with the radian frequency of ‘1/tau’ or the overall frequency of ‘2 $pi\ tau$’.



When damping coefficient = 0, then we had shown that the response oscillates and the final period of oscillation is ‘2 pi tau’. So, ‘tau’ has the significance in terms of the period of oscillation. When there are sustained oscillations, also known as natural oscillation, this ‘tau’ is known as the natural period of oscillation. So we will stop for the second order system here and in the next lecture, we will look at the higher order systems. Thank you.