Chemical Process Control
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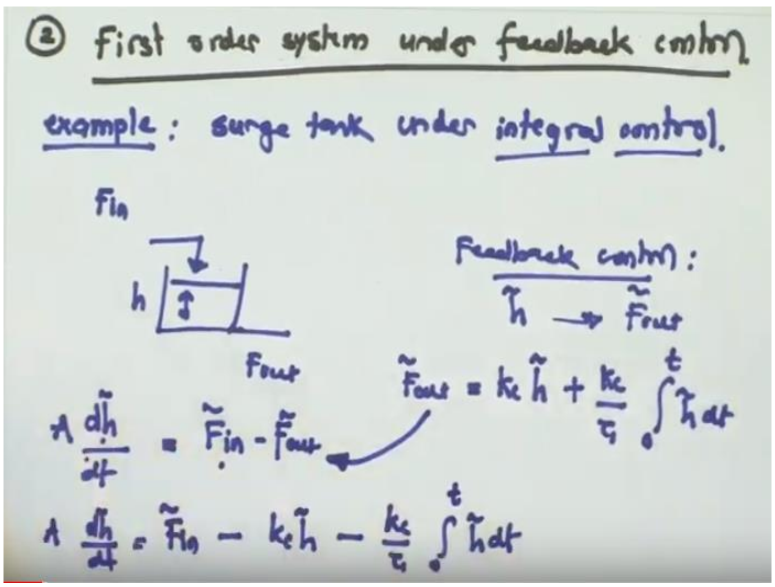
Lecture - 14
Examples of Second Order Dynamical Systems

Okay, welcome back. We are looking at the second way of getting a second order dynamical system. Again we will see that it is based on the primary dynamics which is first order system and then on top of that we have an integral controller which is going to make the entire system as well as control, this combined controlled system as the second order dynamical system. The example we are going to consider is a surge tank under integral control.



This is our surge tank where we have some feed flow coming in and have the outlet flow. We want to control the level ‘h’. Now in terms of the actual dynamics of the system, we had already seen that the height dynamics or the level dynamics of the system in deviation form can be written as

$$A\frac{d\tilde{h}}{dt} = \widetilde{F_{in}} - \widetilde{F_{out}}$$

i.e. the deviation in the input flow and the deviation in the outlet flow.

Now when we talk about the feedback control, that means we are going to measure the deviation in height and accordingly we are going to change the outlet flow rate. That is the philosophy of feedback control. The type of feedback control which we are going to look at, the integral control. At this point, I will just give you the form of this. In week 5, we will be looking at these type of controllers and that time it will be clearer about where and why this particular form of feedback control is used.

At this moment, let me just write down how this feedback control is going to operate. It is going to give you the value of the change in the outlet flow rate based on the measurement of height. So, it is some multiplication of deviation in height and another parameter and then you take the integral of height change as well. This is an integral control law and we will be substituting this into the dynamical system.

Then we will get the final dynamical system between 'Fin' and 'h' under feedback control law. So what we get is,

$$A\frac{d\tilde{h}}{dt} = \widetilde{F_{in}} - k_c\tilde{h} - \frac{k_c}{\tau_I}\int_0^t \tilde{h}\, dt$$

This is the final dynamical equation for this combined system plus controller. Now we are interested in getting a transfer function between this disturbance and the output level in the presence of the controller. In order to do that, we will have to take a Laplace transform of this equation.

Taking Laplace,

$$A\, s\, \tilde{h}(s) = \widetilde{F_{in}}(s) - k_c\tilde{h}\,(s) - \frac{k_c}{\tau_I}\frac{\tilde{h}\,(s)}{s}$$

Because the Laplace of f (t) dt is the Laplace of function divided by s.

Rearranging, we will get,

$$\left(As + k_c + \frac{k_c}{\tau_I\, s}\right)\tilde{h}(s) = \widetilde{F_{in}}(s)$$

$$\frac{\tilde{h}(s)}{\widetilde{F_{in}}(s)} = \frac{1}{\left(As + k_c + \frac{k_c}{\tau_I\, s}\right)}$$

$$\frac{\tilde{h}(s)}{\widetilde{F_{in}}(s)} = \frac{\tau_I\, s}{(A\tau_I)s^2 + k_c\tau_I s + k_c}$$

Again simplifying, transfer function G in the standard form by dividing 'kc' throughout

$$G(s) = \frac{\tilde{h}(s)}{\widetilde{F_{in}}(s)} = \frac{\left(\frac{\tau_I}{k_c}\right)s}{\left(\frac{A\tau_I}{k_c}\right)s^2 + \tau_I s + 1}$$

This is the final transfer function between the changes in the input disturbance 'Fin' and the output. You can see that under this control law, we have a quadratic function in terms of s. It is a second order system where,

$$\frac{A\tau_I}{k_c} = \tau^2$$

And

$$\tau_I = \ 2\,\tau\,\xi$$

It is actually, equivalent to $\tau^2 s^2 + 2\,\tau\,\xi s + 1$, so this is the second order dynamical system.

I would like you to pause here for just a moment and try to compare this particular transfer function with the standard transfer function which we had written. Then will try to see if there is any difference between this transfer function and the general transfer function. I hope you could figure out the difference.

The standard transfer function is,

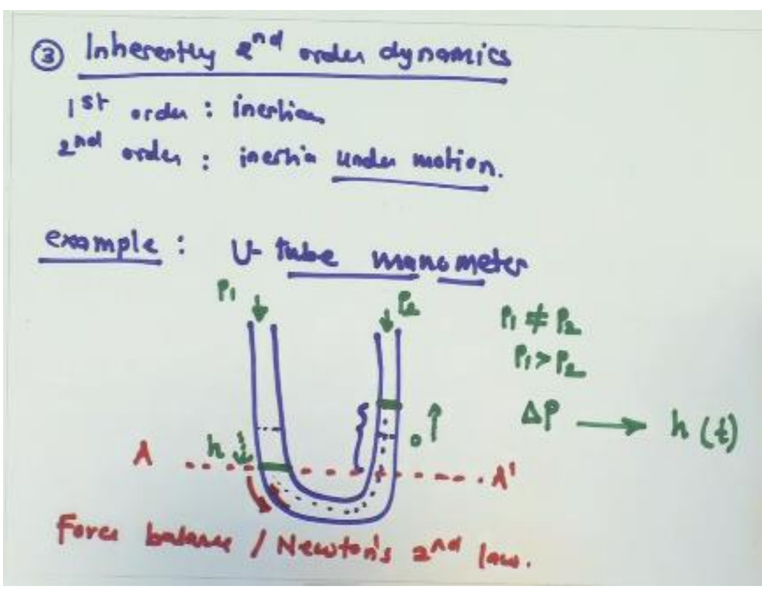
$$G(s) = \frac{K_p}{\tau^2 s^2 + 2\,\tau\,\xi s + 1}$$

The difference which you could have noticed is the numerator was constant when I looked at the standard form. In this case, there is some function of s in the numerator. There is no s in standard form. That is the difference between the standard form and this particular transfer function. However, the good point is, most of the dynamics depends on denominator of the transfer function. So, this is same.

The dynamics of this particular system are going to follow whatever the dynamics we are going to study as a general second order dynamical system. But interesting to know that as we have something in the numerator which is also a function of s, it gives rise to known as numerator dynamics. This is additional dynamics to whatever we are going to get based on this denominator dynamics.

This will be covered in later part of this particular week where we will be looking at what effect this s in the numerator is going to have. But more or less the major dynamics is still governed by the denominator, so whatever results we are going to derive are going to be applicable to this system as well. The actual significance of this s from a more fundamental point of view in terms of this control system we will see when we talk about this integral control and why we need integral control.

As it turns out that the s is going to play a very key role when we are trying to control this particular system. This is the second way of getting a second order dynamical system. Again we had started with the first order system and we had added an integral controller on top of that. Now lastly, the third way of getting a second order system and as it turns out it does not require a first order dynamical system to get it.



These we will call as inherently second order system dynamics. These are the systems which by themselves are going to give rise to second order dynamics. If I want to distinguish them from first order systems; first order system was a system which has a capacity to store mass or energy or it has some inertia associated with it. First order process, it has inertia and when we talk about inherent second order system, this inertia is going to be under motion. Those are the type of systems which are going to give rise to inherently second order dynamics. These are very uncommon in chemical engineering systems because most of the times are mass 2 a systems or energy storage systems are stationary. They are not moving from one point to the other. So very rarely we would have a system which is inherently second order.

Most of the times our systems will be the second order systems which we are going to get are going to be either a series combination of first order systems or a first order system under an integral control. Then we will still try to cover this third way of getting a second order system because there are still some systems which are going to demonstrate this behavior.

These are quite common in other types of engineering systems like mechanical systems are many times inherently second order. The reason being most of the times they work with forces and displacement and we can see that force is related to acceleration and acceleration is a double derivative of the displacement. So that is the natural way of getting a second order dynamical system.

These type of systems is common in other engineering domain. But in chemical systems, these will not be that common. The example we are going to consider is something which you might have seen in your chemical engineering laboratory or even in a physics laboratory. That is U-tube manometer. U-tube manometer is shown in following figure. Under normal conditions, it will be at rest. We will call it as the 0 position, when the pressure on left hand side which is P1 and on the right hand side P2, when these two pressures are equal we will have a constant height in both the limbs.

But whenever this P1 is not equal to P2 or let me just show P1 > P2 there will be some change in the level. One of the limbs will go down and liquid in the other limb will go up. We will be monitoring. So the height which we are going to be measuring. So this particular system what we are interested in is whenever there is a change in pressure applied across this particular manometer, it is going to change the height from the steady state i.e. from the basic base point. We are going to see how that height changes as the function of time. In some time, I will be able to show you that this particular dynamics is going to be a second order system. In order to do that, we will be writing is a force balance or in the more common term, it is Newton's second law of motion. We will be writing force balance occurs this particular plane. Let us assume as AA' plane.

We will be writing the entire force across this plane and then whatever is the net force that is going to cause acceleration into this particular inertia which is the total material of mercury or the manometer fluid present inside this system. Let us write this particular force balance. The net force acting on this particular manometer would be,

$$P_1 A - P_2 A - \rho(A\,2h).g - \Delta P_{friction} \cdot A = \rho(AL)\frac{d^2h}{dt^2} \quad \text{— } (1)$$

Laminar flow (H.P. eqn)

$$\Delta P_{friction} = \frac{8\mu L Q}{\pi R^4}$$

Q: flow rate

$$Q = \pi R^2 . v = \pi R^2 \frac{dh}{dt}$$

$$\Delta P_{friction} = \frac{8\mu L \pi R^2}{\pi R^4}\frac{dh}{dt} = \frac{8\mu L}{R^2}\frac{dh}{dt}.$$

The last force which is going to act on the system is the viscous force, can write as a ΔP frictional pressure drop time area and this net force is equal to mass times acceleration.

To get this frictional pressure drop, we have assumed a laminar flow. Because, most of the times this flow will be very slow. The manometer fluid will not move that fast and it is very common or reasonable to assume that this flow is going to be laminar. If it is laminar, we can use Hagen Poiseuille's equation to get this ΔP which would be,

$$\Delta P_{friction} = \frac{8\mu L Q}{\pi R^4}$$

Where Q is the flow rate which is an area times velocity and velocity is going to be,

$$v = \frac{dh}{dt}$$

$$P_1 A - P_2 A - 2\rho g h A - \frac{8\mu L}{R^2}\frac{dh}{dt} A = \rho L A \frac{d^2h}{dt^2}$$

$$\rho L \frac{d^2h}{dt^2} + \frac{8\mu L}{R^2}\frac{dh}{dt} + 2\rho g h = \Delta P \quad —(2)$$

@ steady state,

$$\Delta P_{ss} = 2\rho g h_{ss} \quad —(3)$$

(2) − (3)

$$\rho L \frac{d^2\tilde{h}}{dt^2} + \frac{8\mu L}{R^2}\frac{d\tilde{h}}{dt} + 2\rho g \tilde{h} = \widetilde{\Delta P}$$

$$\left(\frac{L}{2g}\right)\frac{d^2\tilde{h}}{dt^2} + \left(\frac{4\mu L}{\rho g R^2}\right)\frac{d\tilde{h}}{dt} + \tilde{h} = \left(\frac{1}{2\rho g}\right)\widetilde{\Delta P}$$

The above is the dynamical equation which is going to dictate how the height in the manometer is going to change when you apply some differential pressure drop, differential pressure across this manometer as a function of time.

Before going into the Laplace transform, we also have to write this in a deviation form. At steady state, what we have is,

$$\Delta P_{ss} = 2\rho g h_{ss}$$

It gives us the final form as,

$$\left(\frac{L}{2g}\right)\frac{d^2\tilde{h}}{dt^2} + \left(\frac{4\mu L}{\rho g R^2}\right)\frac{d\tilde{h}}{dt} + \tilde{h} = \left(\frac{1}{2\rho g}\right)\widetilde{\Delta P}$$

As you can see, this particular manometer is going to show a second order dynamical response.

Where,

$$\left(\frac{L}{2g}\right) = \tau^2$$

$$\left(\frac{4\mu L}{\rho g R^2}\right) = 2\,\tau\,\xi$$

And

$$\frac{1}{2\rho g} = K_p$$

If you go through the derivation, we never encountered any first order dynamical system here. This particular dynamics cannot be decomposed into two first order capacities in series or a first order capacity with the control. This is by default a second order dynamical system. So, whenever you make a change in ΔP, it is going to make a change in h which is going to look like a second order dynamical system. So, these are the 3 ways in which you can get a second order dynamical response in the chemical engineering system. Thank you.