Chemical Process Control

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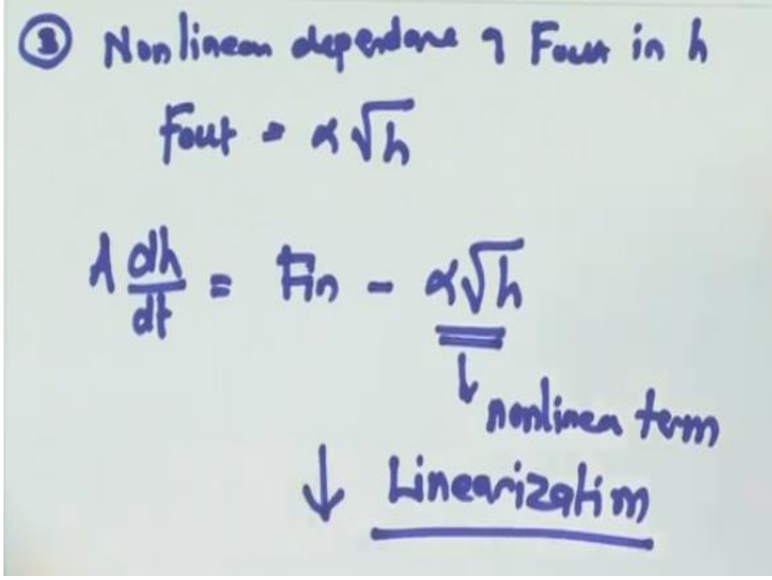
Lecture - 10
Linearization of Process Dynamics

Okay, welcome back. Before the break, we were working on liquid surge tank where we were considering that there is a valve at the outlet and we were considering a case when the flow through that valve is proportional to the square root of the height.



③ Nonlinear dependence of Fout in h

$F_{out} = \alpha\sqrt{h}$

$$A\frac{dh}{dt} = F_{in} - \underbrace{\alpha\sqrt{h}}_{\text{nonlinear term}}$$

↓ Linearization

The equation which we got for this particular system is,

$$A\frac{dh}{dt} = F_{in} - \alpha\sqrt{h}$$

We said that the last term in this equation, the $\sqrt{h}$ is a nonlinear term and we cannot proceed further with Laplace transform for this system because it is nonlinear. The first thing we have to do is to linearize this system. So, let us look at how we can linearize a nonlinear function.

The mathematical rule which we are going to use to do is Taylor series expansion. You might be aware of the Taylor series expansion. Let me revisit it again. If we have a function of variables x_1, $x_2,\ldots,x_n$, I can approximate this function around the function value at any point let us say $f(x_{10}, x_{20}, x_{n0})$. So, this is some point where we evaluate the function around which we are linearizing the function and this can be written as the multiple successive derivatives of the function.

$$f(x_1,x_2,\ldots,x_n) = f(x_{10},x_{20},\ldots,x_{n0}) + \left(\frac{\delta f}{\delta x_1}\right)_{(x_{10},x_{20},\ldots,x_{n0})}(x_1 - x_{10}) + \left(\frac{\delta f}{\delta x_2}\right)_{(x_{10},x_{20},\ldots,x_{n0})}(x_2 - x_{20}) + \cdots + \left(\frac{\delta f}{\delta x_n}\right)_{(x_{10},x_{20},\ldots,x_{n0})}(x_n - x_{n0}) + HOT$$

$(x_1 - x_{10})$ is the departure of any point away from the point around which we are linearizing the system. All the first order derivatives will be calculated for all the independent variables. All the first order derivatives are linear in terms of the variables of interest. The Taylor series expansion actually goes for the higher order terms (HOT), but for this approximation, we will make an approximation that the higher order terms are negligible. We will approximate the entire thing only up to first order point and say that this is approximately equal. If you consider only up to first order term, then we will have to replace equality by an approximate sign which gives,

$$f(x_1,x_2,\ldots,x_n) \approx f(x_{10},x_{20},\ldots,x_{n0}) + \left(\frac{\delta f}{\delta x_1}\right)_{(x_{10},x_{20},\ldots,x_{n0})}(x_1 - x_{10}) + \left(\frac{\delta f}{\delta x_2}\right)_{(x_{10},x_{20},\ldots,x_{n0})}(x_2 - x_{20}) + \cdots + \left(\frac{\delta f}{\delta x_n}\right)_{(x_{10},x_{20},\ldots,x_{n0})}(x_n - x_{n0})$$

So we will be approximating any nonlinear function as a function value at some point which would be typically a steady state in most of the cases. Also, we will see what is the advantage of doing that plus first order derivatives evaluated at the steady state times the deviations from the steady state. So, automatically it gives all the deviation variables when we linearize the system.

Let us now try to do this Taylor series expansion based linearization for our example. The nonlinear function was $\alpha\sqrt{h}$.

We will approximate it by a function value at steady state. This is a single variable function. So, we will have only one derivative and not even a partial derivative. It will be a pure derivative of $\sqrt{h}$.

We will have Taylor series expansion evaluated at the steady state as,

$$\alpha \frac{dh}{dt} \approx \alpha \left[\sqrt{h_{ss}} + \frac{1}{2\sqrt{h_{ss}}} \left(h - h_{ss}\right)\right]$$

Now, let us substitute this in the original dynamic equation which is,

$$A\frac{dh}{dt} = F_{in} - \left[\alpha\sqrt{h_{ss}} + \frac{\alpha}{2\sqrt{h_{ss}}} \left(h - h_{ss}\right)\right] \tag{1}$$

The steady state equation will be,

$$A\frac{dh_{ss}}{dt} = F_{in,ss} - \alpha\sqrt{h_{ss}} \tag{2}$$

Now we will subtract 2 from 1 and we will write the equation for deviation variable. This will give us,

$$A\frac{d\tilde{h}}{dt} = \widetilde{F_{in}} - \frac{\alpha}{2\sqrt{h_{ss}}}\tilde{h}$$

Here the term of $\alpha\sqrt{h_{ss}}$ will get cancelled and $\tilde{h} = \left(h - h_{ss}\right)$.

We can again simplify this equation or rearrange this equation and we will get,

$$\left(\frac{2A\sqrt{h_{ss}}}{\alpha}\right)\frac{d\tilde{h}}{dt} + \tilde{h} = \left(\frac{2\sqrt{h_{ss}}}{\alpha}\right)\widetilde{f_{in}}$$

We will again compare this with the original first order equation and that it gives us,

$$\tau\frac{dy}{dt} + y = K_p f(t)$$

So, for this system where we approximated a nonlinear system by a linear equivalent, we have obtained,

$$\tau = \frac{2A\sqrt{h_{ss}}}{\alpha}$$

and the gain as,

$$K_p = \frac{2\sqrt{h_{ss}}}{\alpha}$$

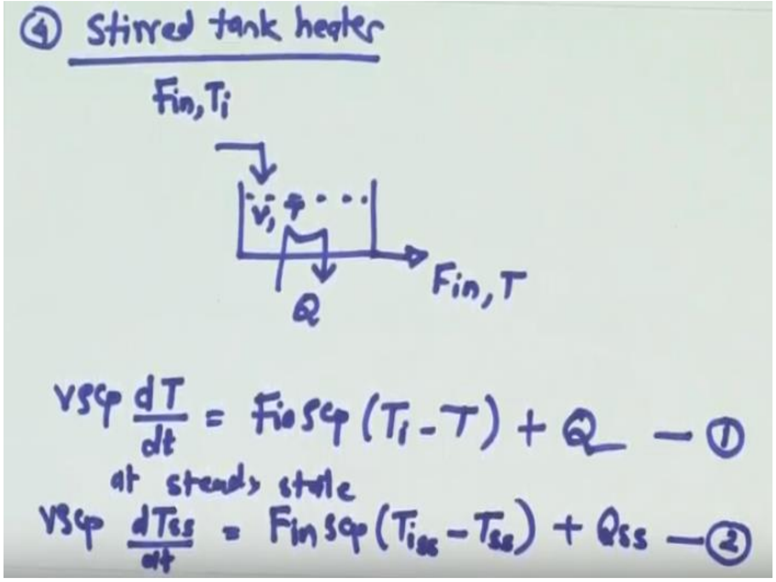
Here, I would like to point something that time constant (**τ**) and gain (Kp), both are the system constants and they are functions of the original steady state.

The interesting thing is, if I linearize the system at a different steady state, then the values of tau (**τ**) and Kp are to be different. For this example, if I linearize the system at 50% level, it will give me a value of the time constant and Kp; but if I linearize it around let us say 75% or 25% then these cases will give rise to different time constants. So even though I am saying it is the constant, it actually depends on the original steady state and the dynamics would heavily depend on the steady state around which that dynamics is linearized.

Whenever we linearize a nonlinear system, the parameters which we obtain for the system are going to be dependent on the steady state. As long as the deviation from that steady state is not significant, the approximation of linearization would work. We will see that through simulation also, sometime later in this lecture.

So here, we have seen that the tank where the outlet flow is a nonlinear function of height, we can approximate it as the linearized version. We can show that it is indeed a first order system and the resemblance to the first order system will always be there. The deviation from the first order dynamics will always be seen if the system is excited beyond a certain range, the changes in the height are more. In that case, the deviation from the approximation of the linearized version will also be more.

Let us now take the fourth example of a stirred tank heater where the fluid comes in at flow rate ‘Fin’ and the inlet temperature of ‘Ti’.



④ Stirred tank heater

$$V\rho c_p \frac{dT}{dt} = F_{in}\rho c_p (T_i - T) + Q \quad — ①$$

at steady state

$$V\rho c_p \frac{dT_{ss}}{dt} = F_{in}\rho c_p (T_{i,ss} - T_{ss}) + Q_{ss} \quad — ②$$

It gets heated by some source. The amount inside the tank is ‘V’, the temperature inside the tank is ‘T’ and the outlet flow rate is the same as inlet but with the updated temperature of ‘T’. For this system, we had derived a dynamic equation for the system in the last lecture which looked like

$$V\rho C_p \frac{dT}{dt} = F_{in}\rho C_p (T_i - T) + Q \tag{1}$$

We can write a similar equation at steady state as,

$$V\rho C_p \frac{dT_{ss}}{dt} = F_{in}\rho C_p (T_{i,ss} - T_{ss}) + Q_{ss} \tag{2}$$

We will assume for now that ‘Fin’ is not changing as a function of time and the only disturbance here is ‘Ti’. ‘Qss’ is the steady state value of the heat input. In order to write this dynamic equation in deviation form, we will take the difference between equation 1 and 2.

We will get as,

$$V\rho C_p \frac{d\tilde{T}}{dt} = F_{in}\rho C_p (\tilde{T}_i - \tilde{T}) + \tilde{Q}$$

Now we will take the Laplace transform as,

$$V\rho C_p[s\tilde{T}(s) + \tilde{T}(0)] = F_{in}\rho C_p\left[\tilde{T}_i(s) - \tilde{T}(s)\right] + \tilde{Q}(s)$$

Here, $\tilde{T}(0) = 0$, because we are assuming the steady state as the initial point.

Simplifying this equation, we will have-

$$V\rho C_p s\tilde{T}(s) + F_{in}\rho C_p\tilde{T}(s) = F_{in}\rho C_p\tilde{T}_i(s) + \tilde{Q}(s)$$

From this, we can come up with an expression for T(s). We will try to write it in a standard form of $\frac{K_p}{\tau s+1}$. It can be written as,

$$\tilde{T}(s) = \frac{F_{in}\rho C_p}{V\rho C_p s + F_{in}\rho C_p}\tilde{T}_i(s) + \frac{\tilde{Q}(s)}{V\rho C_p s + F_{in}\rho C_p}$$

We can see from the above equation that the outlet temperature transfer function, the Laplace domain variation of $\tilde{T}$ is composed of the effects of the changes in inlet temperature and changes in the heater duty. Both of them follow the same form of the first order, so the first transfer function is,

$$\tilde{T}(s) = \frac{1}{\left(\frac{V}{F_{in}}\right)s + 1}\tilde{T}_i(s)$$

Similarly, the second transfer function is,

$$\tilde{T}(s) = \frac{\left(\frac{1}{F_{in}\rho C_p}\right)}{\left(\frac{V}{F_{in}}\right)s + 1}\tilde{Q}(s)$$

Summarizing,

$$\tilde{T}(s) = \frac{K_{p1}}{\tau_1 s + 1}\tilde{T}_i(s) + \frac{K_{p2}}{\tau_2 s + 1}\tilde{Q}(s)$$

Where,

$$\tau_1 = \tau_2 = \frac{V}{F_{in}}$$

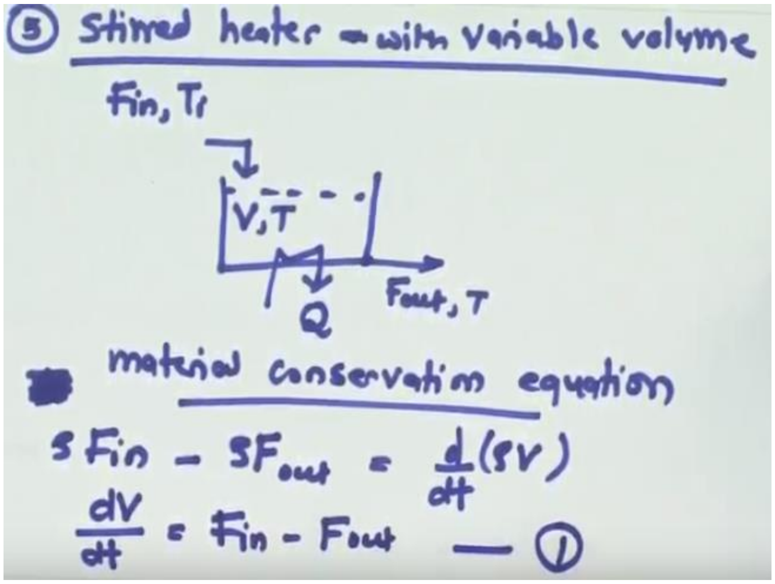
And

$$K_{p1} = 1$$

$$K_{p2} = \frac{1}{F_{in}\rho C_p}$$

The system is composed of the parallel effect of two first order dynamics. Typically, V/F represents the residence time in a CSTR and from here, we can recollect that ‘V/Fin’ is similar to the residence time of the system. So, that is exactly the same time is appearing as the time constant of the system. The system has two capacities and both are of first order nature.

Lastly, let us take an example which combines both the effects, i.e. the thermal effect and the material effects.



$$\rho F_{in} - \rho F_{out} = \frac{d}{dt}(\rho V)$$

$$\frac{dV}{dt} = F_{in} - F_{out} \quad \text{— ①}$$

The fifth example we are going to take is the stirred heater with variable volume. We will remove one of the assumptions of the previous system that the volume of the tank remains constant because there is always the possibility that the inlet and outlet flow rate are not kept at the same value. So we may have inlet ‘Fin’, inlet temperature ‘Ti’, volume at any time inside the vessel is V, the temperature is T and what goes out let us call it as ‘Fout’ at T where ‘Fin’ may not be equal to ‘Fout’ all the time. We have the same heat duty ‘Q’.

In this case, the original equation still remains the same. Now here instead of just the energy balance, we also have to write the material balance. We will start with the material conservation equation as,

$$\rho\, F_{in} - \rho F_{out} = \frac{d(\rho V)}{dt}$$

There is no generation or consumption of material which is then eventually equal to the rate of change of material content which is rho times V. We have assumed that density is independent of temperature or time and in that case,

$$\frac{dV}{dt} = F_{in} - F_{out} \qquad (1)$$

Here, equation (1) captures the dynamics of material balance.

Let us now write energy balance for this system which will get slightly modified because 'V' is no longer constant.

$$F_{in}\rho C_p\left(T_i - T_{ref}\right) + Q - F_{out}\rho C_p\left(T - T_{ref}\right) = \frac{d\left[V\rho C_p\left(T - R_{ref}\right)\right]}{dt} \qquad (2)$$

As there is no generation or consumption of energy in the system. We can see that here there is a multiplication of variable volume (V) as well as temperature (T) on the right hand side. So, we have to use the product rule for the derivative.

The right hand side will be,

$$\frac{d\left[V\rho C_p\left(T - R_{ref}\right)\right]}{dt} = \rho C_p\left(T - T_{ref}\right)\frac{dV}{dt} + V\rho C_p\frac{dT}{dt}$$

Substituting Eq. (1) in above,

$$\frac{d\left[V\rho C_p\left(T - R_{ref}\right)\right]}{dt} = \rho C_p\left(T - T_{ref}\right)\left(F_{in} - F_{out}\right) + V\rho C_p\frac{dT}{dt}$$

Again substituting the above in Eq. (2), we will get-

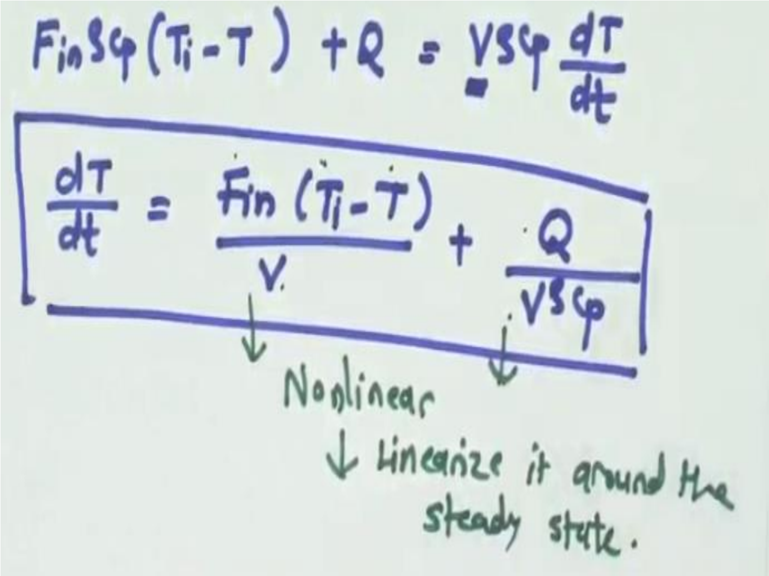
$$F_{in}\rho C_p\left(T_i - T_{ref}\right) + Q - F_{out}\rho C_p\left(T - T_{ref}\right) = \rho C_p\left(T - T_{ref}\right)\left(F_{in} - F_{out}\right) + V\rho C_p\frac{dT}{dt}$$

we will simplify this equation to write it in terms of $(\frac{dT}{dt})$.

$$F_{in}\rho C_p(T_i - T) + Q = V\rho C_p \frac{dT}{dt}$$

Rearranging, equation appears as,

$$\frac{dT}{dt} = \frac{F_{in}(T_i - T)}{V} + \frac{Q}{V\rho C_p} \qquad (3)$$



$$F_{in}\rho C_p(T_i - T) + Q = \underline{V}\rho C_p \frac{dT}{dt}$$

$$\boxed{\frac{dT}{dt} = \underbrace{\frac{\dot{F}_{in}(\dot{T}_i - \dot{T})}{V}}_{\downarrow} + \underbrace{\frac{\dot{Q}}{V\rho C_p}}_{\downarrow}}$$

Nonlinear

↓ Linearize it around the steady state.

Eq. (3) is the final form of the dynamic equation for temperature. Here note that this is again nonlinear equation because our volume is a variable which is appearing in the denominator. Another variable, ‘Fin’ which is multiplying the third variable like ‘Ti’ or T. Similarly, Q is a variable and volume is a variable. So both these terms are nonlinear. Our next step is to linearize it around the steady state.

Let us take each of these terms separately and try to linearize it.

The first nonlinear term we will linearize is,

$$Nonlinear\ term\ (F1) = \frac{F_{in}(T_i - T)}{V}$$

Here, we have four variables. Let us assume,

$F_{in} = x_1, T_i = x_2, T = x_3 \text{ and } V = x_4$

We will approximate it around the steady state value. The first term will be the function value at steady state followed by the four first order partial derivatives. First, we will partially differentiate with ‘Fin’, the second term is ‘Ti’. The third term will be with respect to ‘T’ and the last term will be with respect to the volume. The linearized approximation will be as,

$$\frac{F_{in}(T_i - T)}{V} \approx \frac{F_{in,ss}\left(T_{i,ss} - T_{ss}\right)}{V_{ss}} + \left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right)\left(F_{in} - F_{in,ss}\right) + \frac{F_{in,ss}}{V_{ss}}\left(T_i - T_{i,ss}\right) - \frac{F_{in,ss}}{V_{ss}}\left(T - T_{ss}\right) + \frac{F_{in,ss}\left(T_{i,ss} - T_{ss}\right)}{{V_{ss}}^2}\left(V - V_{ss}\right)$$

You can see from the above equation that there are a bunch of steady state values which are multiplied by the deviation forms of the variables around the steady state.

Now, we will take the second term which is,

$$\text{Nonlinear Term } (f2) = \frac{Q}{V\rho C_p}$$

In this term, there are two variables. Let us assume,

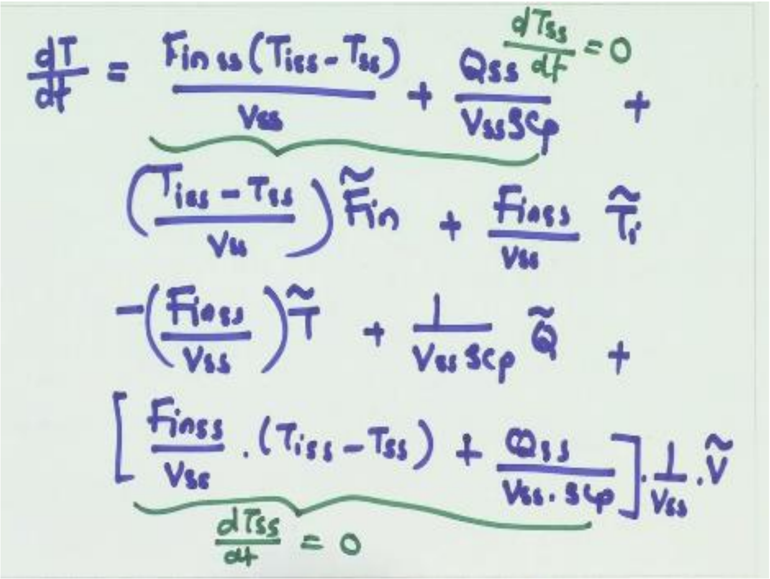
$$Q = x_1 \text{ and } V = x_2$$

So, the approximation for the second term will be,

$$\frac{Q}{V\rho C_p} \approx \frac{Q_{ss}}{V_{ss}\rho C_p} + \frac{1}{V_{ss}\rho C_p}\left(Q - Q_{ss}\right) + \frac{Q_{ss}}{{V_{ss}}^2\rho C_p}\left(V - V_{ss}\right)$$

Let us now include both these linearized terms into the original equation (3) and we will get is,

$$\frac{dT}{dt} = \frac{F_{in,ss}\left(T_{i,ss} - T_{ss}\right)}{V_{ss}} + \frac{Q_{ss}}{V_{ss}\rho C_p} + \left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right)\widetilde{F_{in}} + \frac{F_{in,ss}}{V_{ss}}\tilde{T}_i - \frac{F_{in,ss}}{V_{ss}}\tilde{T} + \frac{1}{V_{ss}\rho C_p}\tilde{Q} + \left[\frac{F_{in,ss}}{V_{ss}}\left(T_{i,ss} - T_{ss}\right) + \frac{Q_{ss}}{V_{ss}\rho C_p}\right]\frac{1}{V_{ss}}\tilde{V}$$



$$\frac{dT}{dt} = \overset{\frac{dT_{ss}}{dt}=0}{\underbrace{\frac{F_{in\,ss}(T_{iss}-T_{ss})}{V_{ss}} + \frac{Q_{ss}}{V_{ss}\rho c_p}}} + \left(\frac{T_{iss}-T_{ss}}{V_{ss}}\right)\tilde{F}_{in} + \frac{F_{inss}}{V_{ss}}\tilde{T}_i - \left(\frac{F_{inss}}{V_{ss}}\right)\tilde{T} + \frac{1}{V_{ss}\rho c_p}\tilde{Q} + \underbrace{\left[\frac{F_{inss}}{V_{ss}}.(T_{iss}-T_{ss}) + \frac{Q_{ss}}{V_{ss}.\rho c_p}\right]}_{\frac{dT_{ss}}{dt}=0}.\frac{1}{V_{ss}}.\tilde{V}$$

This is the final equation which we get and you can notice that there are some terms which are coming directly from the steady state. So, if I write the original equation (3) at steady state, we get as,

$$\frac{dT_{ss}}{dt} = \frac{F_{in,ss}\left(T_{i,ss} - T_{ss}\right)}{V_{ss}} + \frac{Q_{ss}}{V_{ss}\rho C_p} = 0$$

This simplifies our analysis and the final form which we get is,

$$\frac{d\tilde{T}}{dt} = \left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right)\widetilde{F_{in}} + \frac{F_{in,ss}}{V_{ss}}\tilde{T}_i - \frac{F_{in,ss}}{V_{ss}}\tilde{T} + \frac{1}{V_{ss}\rho C_p}\tilde{Q}$$

So this is the final dynamic equation in the deviation form. Taking a Laplace transform will get-

$$s\tilde{T}(s) - \tilde{T}(0) = \left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right)\widetilde{F_{in}}(s) + \frac{F_{in,ss}}{V_{ss}}\tilde{T}_i(s) - \frac{F_{in,ss}}{V_{ss}}\tilde{T}(s) + \frac{1}{V_{ss}\rho C_p}\tilde{Q}(s)$$

As at time zero, the system is at steady state, it gives

$$\tilde{T}(0) = 0,$$

$$s\tilde{T}(s) = \left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right) \widetilde{F_{in}}(s) + \frac{F_{in,ss}}{V_{ss}}\, \tilde{T_i}(s) - \frac{F_{in,ss}}{V_{ss}}\, \tilde{T}(s) + \frac{1}{V_{ss}\rho C_p}\, \tilde{Q}(s)$$

$$\left(s + \frac{F_{in,ss}}{V_{ss}}\right) \tilde{T}(s) = \left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right) \widetilde{F_{in}}(s) + \frac{F_{in,ss}}{V_{ss}}\, \tilde{T_i}(s) + \frac{1}{V_{ss}\rho C_p}\, \tilde{Q}(s)$$

Rearranging the equation, the final form of Laplace transform for the output will be,

$$\tilde{T}(s) = \frac{\left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right)}{\left(s + \frac{F_{in,ss}}{V_{ss}}\right)} \widetilde{F_{in}}(s) + \frac{\frac{F_{in,ss}}{V_{ss}}}{\left(s + \frac{F_{in,ss}}{V_{ss}}\right)} \tilde{T_i}(s) + \frac{\frac{1}{V_{ss}\rho C_p}}{\left(s + \frac{F_{in,ss}}{V_{ss}}\right)} \tilde{Q}(s)$$

Lastly, we want to compare it with the transfer function of the form of $\frac{K_p}{\tau s+1}$. So we will have multiply everywhere by (Vss/Fin,ss) and end up getting the final form as-

$$\tilde{T}(s) = \frac{\left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right)}{\left(\left(\frac{V_{ss}}{F_{in,ss}}\right) s + 1\right)} \widetilde{F_{in}}(s) + \frac{\frac{F_{in,ss}}{V_{ss}}}{\left(\left(\frac{V_{ss}}{F_{in,ss}}\right) s + 1\right)} \tilde{T_i}(s) + \frac{\frac{1}{V_{ss}\rho C_p}}{\left(\left(\frac{V_{ss}}{F_{in,ss}}\right) s + 1\right)} \tilde{Q}(s)$$

So this is the final form of the transfer function. The relationship of output temperature is a summation of three effects. It is a summation of three first order capacities and they are corresponding to the three input variables ‘Fin’, inlet temperature ‘Ti’, and heat duty ‘Q’.

We can rewrite this equation as,

$$\tilde{T}(s) = \frac{K_{p1}}{\tau_1 s + 1}\, \widetilde{F_{in}}(s) + \frac{K_{p2}}{\tau_2 s + 1}\, \tilde{T_i}(s) + \frac{K_{p3}}{\tau_3 s + 1}\, \tilde{Q}(s)$$

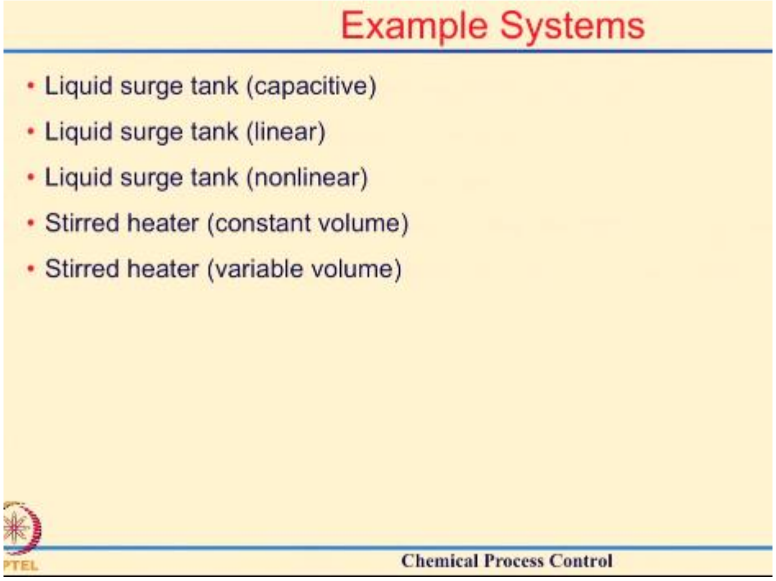
Where,

$$K_{p1} = \left(\frac{T_{i,ss} - T_{ss}}{V_{ss}}\right) \; and \;\; \tau_1 = \left(\frac{V_{ss}}{F_{in,ss}}\right)$$

$$K_{p2} = \frac{F_{in,ss}}{V_{ss}} \;\; and \; \tau_2 = \left(\frac{V_{ss}}{F_{in,ss}}\right)$$

$$K_{p3} = \frac{1}{V_{ss}\rho C_p} \; and \; \tau_3 = \left(\frac{V_{ss}}{F_{in,ss}}\right)$$

We have seen the 5 examples where we could show that the relationship between output and different inputs follows first order dynamics. So out of these examples, let us try to delve further and see what is so special about these 5 systems.



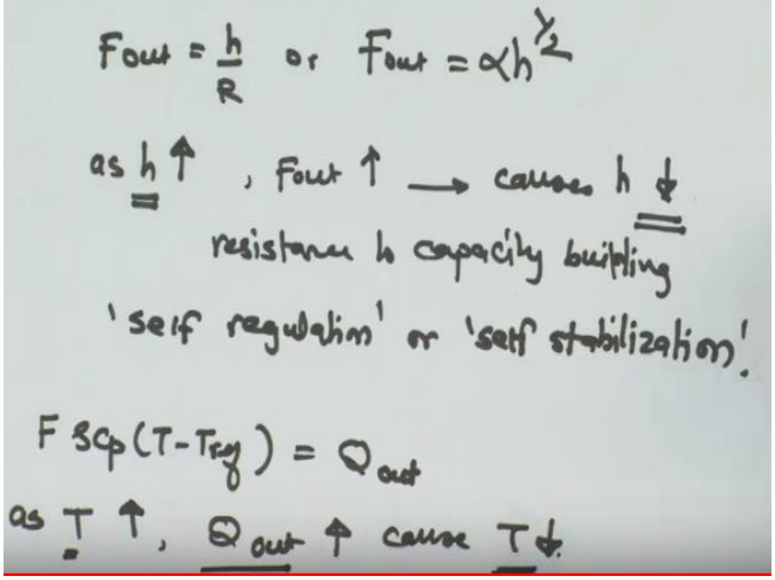
Example Systems

- Liquid surge tank (capacitive)
- Liquid surge tank (linear)
- Liquid surge tank (nonlinear)
- Stirred heater (constant volume)
- Stirred heater (variable volume)

Chemical Process Control

All these 5 examples if you see, the first 3 examples, those systems have a capacity to store mass or material. Whereas the last 2 systems have a capacity to store energy and especially the last one has the capacity to store mass as well as energy. So, all of these systems have a way to store material or energy. That is a very defining characteristic of a first order system that they have the capacity to store material.

Now if you look further, except the first example all the other examples there is also a mechanism or a resistance for this capacity building. If we look at the surge tank with a linear or nonlinear outlet flow rate, as the flow is proportional to the height inside the tank.



$F_{out} = \frac{h}{R}$ or $F_{out} = \alpha h^{1/2}$

as $\underline{\underline{h}}$ ↑ , F_{out} ↑ → causes $\underline{\underline{h \downarrow}}$

resistance to capacity building

'self regulation' or 'self stabilization'.

$F \rho c_p (T - T_{ref}) = Q_{out}$

as $\underline{T}$ ↑, $\underline{Q_{out}}$ ↑ cause $\underline{T}$↓

So if you see,

$$F_{out} = \frac{h}{R} \; or \; F_{out} = \alpha h^{1/2}$$

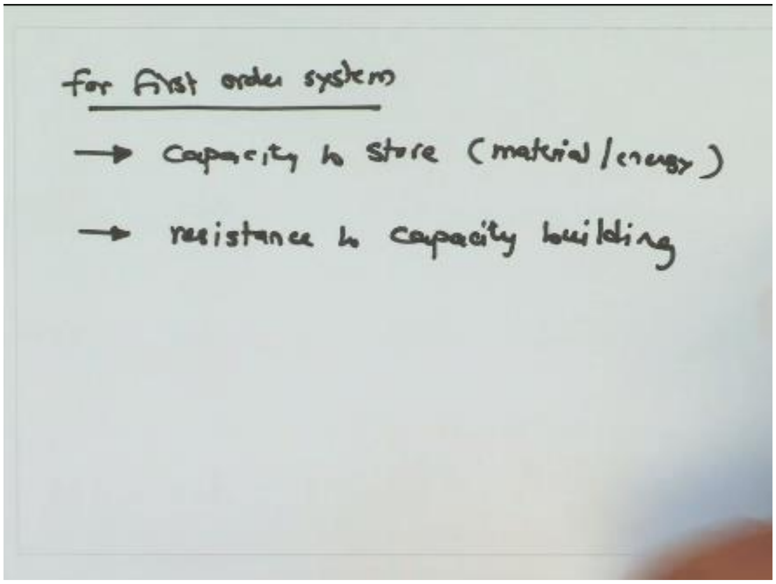
What does this do is, as 'h' tends to increase which means the material inside the tank is increasing, the outlet flow rate also increases. This causes 'h' to decrease. So, an increase in 'h' is eventually triggering an action which is causing a reduction in 'h'. This is the resistance to capacity building. This will allow the system to do what is known as self-regulation or self-stabilization. This feature will allow keeping a check on how the height changes inside this tank and it will not under most of the cases allow an infinitely large change in height. So as the height increases the 'Fout' also takes care of reducing the height. Same phenomena you can see is also present in the last two examples where there is a storage of energy.

So what happens in those cases,

$$Q_{out} = F\rho C_p\left(T - T_{ref}\right)$$

This is the mechanism by which energy can exit the system. Now if you can see that as the temperature inside the tank increases, 'Qout' is also going to increase and as 'Qout' increases, there is more exit of energy than the inlet and eventually, it will cause the temperature to go down.

You can see the similarity between the second and third example and the fourth and fifth example. Whenever there is an increase in the state variable, there is a mechanism which eventually leads to a decrease in the total inventory. The total inventory of energy in this example would again have resistance. So there is a resistance to capacity or inventory building. So all these examples 2 to 5, they have two features.



for first order system

→ Capacity to store (material / energy)

→ resistance to capacity building

We have seen that first order systems are characterized by the capacity to store which may be material, energy. Mostly for Chemical Engineering systems, these will be the two types of storages. And sometimes they are also accompanied by resistance to capacity building. These are the two important features or characteristics of first order systems.

Now for the second point, I excluded the first example. So, when we have the purely capacitive process which is also known as an integrator, the outlet flow rate we had considered as an independent of the height. There was no such resistance to capacity building. There was only a capacity to store and there was no resistance and that is why these systems are also known as purely capacitive systems or integrators. Thank you.