Chemical Reaction Engineering - II

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Module - 11
Lecture - 54
Properties of RTD Function

After all these definitions that we have seen, that is the E-curve, F-curve and the I-curve and the mean residence time, variance and skewness, let us look at the residence time distribution in ideal reactors.

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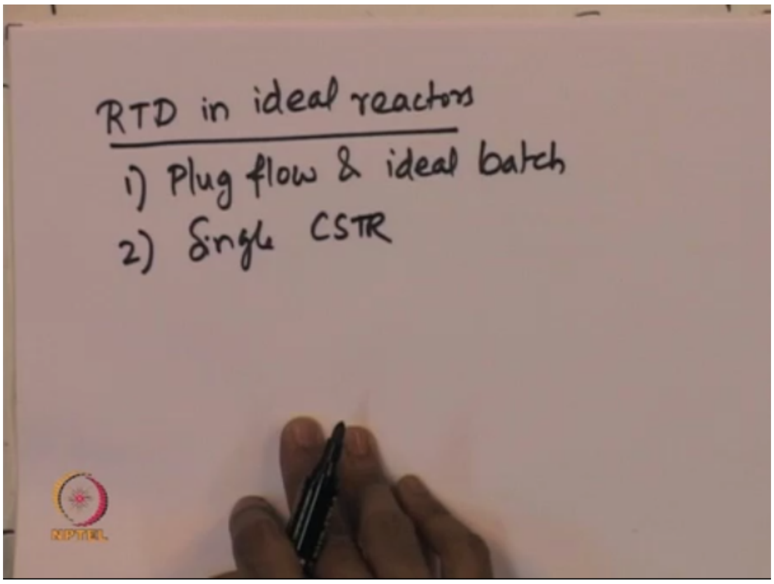

So, particularly we will consider 2 cases. One is a plug flow and ideal batch reactor. And second one is, we will look at the single CSTR case. So, these 2 we will look at. And we will attempt to find out how to get the RTD for, RTD curves for these 2 types of ideal reactors. So, let us first consider the plug flow reactor, let us consider the plug flow reactor.

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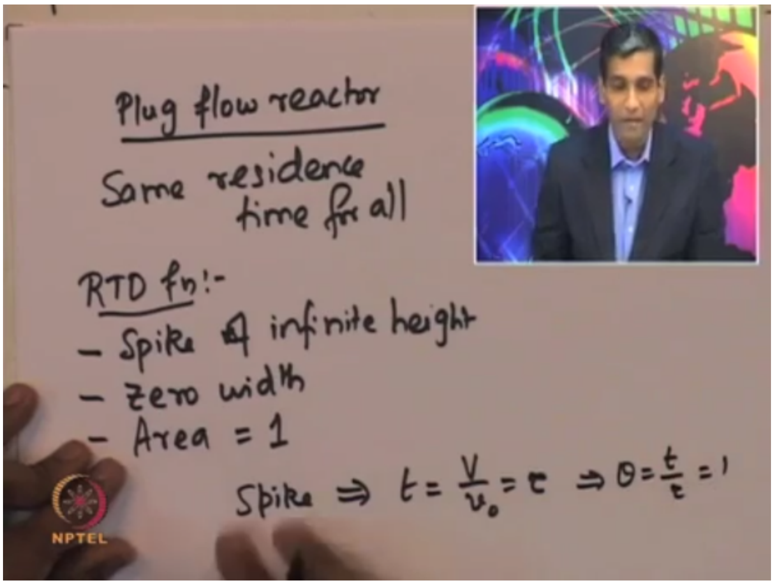

So, what is the property of the plug flow reactor? All atoms or all molecules of the material which is actually entering the reactor will spend exactly the same amount of time before they leave the reactor. Which means that, all elements or all molecules of the material will have exactly the same residence time. So, same residence time for all fluid elements that is actually entering and leaving the reactor.

So therefore, the RTD function must have the following properties. So, first thing is, it must have a spike of infinite height because all of them will have same residence time. Therefore, they will all leave like a plug. So, therefore the E-curve must have a spike of infinite height and also it must have 0 width. And not just that, the area under the curve should be = 1. The spike will be exactly at the mean residence time.

And that is very important because that is the property which actually captures the nature of the plug flow reactor. So, therefore the spike will be exactly at t = V by v nought. That is = tau which is the space time of the reactor. And because there is no dispersion the space time of the reactor will also be = the mean residence time of the reactor. Or in the non-dimensional terms, theta = t by tau, that is = 1. So, therefore the corresponding E-curve, because of these properties of the RTD function for the plug flow reactor.

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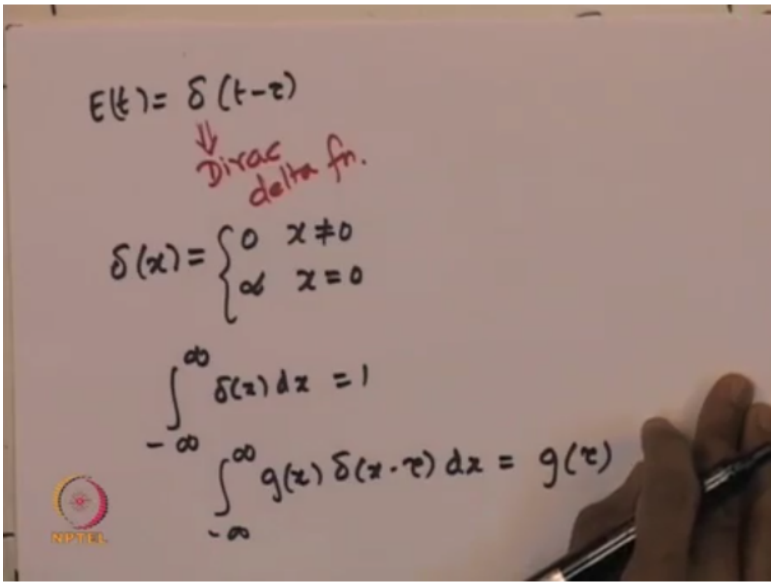

The E-curve should simply be represented by the Dirac delta of function, centred at the space time of the reactor. So, this is the Dirac delta function. That is the Dirac delta function and it is defined as follows. So, Dirac delta function delta x, that is = 0 if x is not = 0 and its = infinity when x is exactly = 0. And the property of this E-curve is actually given by – infinity to + infinity delta x d x should be = 1.

That is the property of the Dirac delta function. In addition to that, the another important property is, by, that the satisfies the convolution integral. That is = g of tau. So, integral of g x if g x is some function of x, multiplied by the delta function to d x. That is = g evaluated at that value of tau itself where x – tau is actually = 0. That is where the spike is actually present. So now, let us calculate the mean residence time for this RTD curve.

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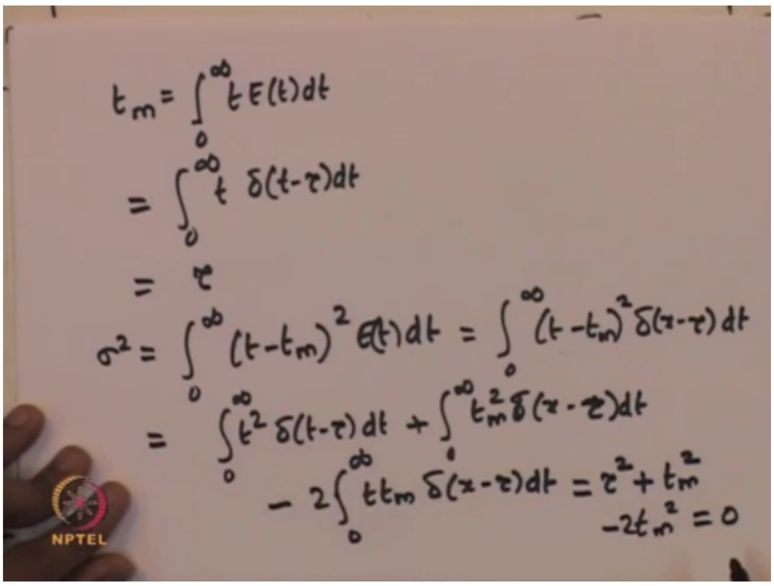

So, the mean residence time t m is actually given by integral 0 to infinity t into E of t d t. That is the definition for the mean residence time in terms of the RTD function. So, that is =, plugging in the E-curve for plug flow reactor, we will find that 0 to infinity t into delta of t – tau into d t. And that is nothing but tau itself. So, therefore the mean residence time is exactly = the space time.

And this actually, one would easily guess because, we said that the, an important property of the plug flow reactor is that, all material that is actually entering the reactor and leaving the reactor will actually have exactly the same residence time. And that the E-curve is actually going to be centred at a, at the space time. So, therefore the mean residence time must be exactly = the space time of the plug flow reactor itself which is, which one would actually guess and is also clearly shown by the RTD function also.

So now, let us look at the second moment that is the variance of the distribution. So, that is given by 0 to infinity t – t m the whole square into E of t into d t. So, that is = t m square into delta function into d t. And, so now, we open up this, the t – t m whole square and then one, if one integrates, we will find that this essentially reduces to t square into delta of t – tau into d t 0 to infinity + integral 0 to infinity t m square delta of x –, x – tau d t – 2 integral t into t m into delta of x – tau d t.

And that is essentially; so, the first time here, because of the property of the delta function, it will simply be = tau square. And the second property will simply be = + t m square. That will be the second one. And the third one will simply be 2 into t into t m t into delta function integral. t m is constant, so, that will come out of the integral. And t into the delta function will essentially be = the mean residence time. So, that will be = 2 t m square.

And that is = 0 because the mean residence time and the space time are exactly equal. So, therefore the variance is actually = 0 and that reflects the property of the RTD function that actually we intuitively guessed. That is, there has to be a spike at exactly t = tau with a with an area under the curve is = 1 and the height of the spike is = infinity which means that the variant should be = 0 for the distribution. So, let us look at the F-curve for the plug flow reactor.

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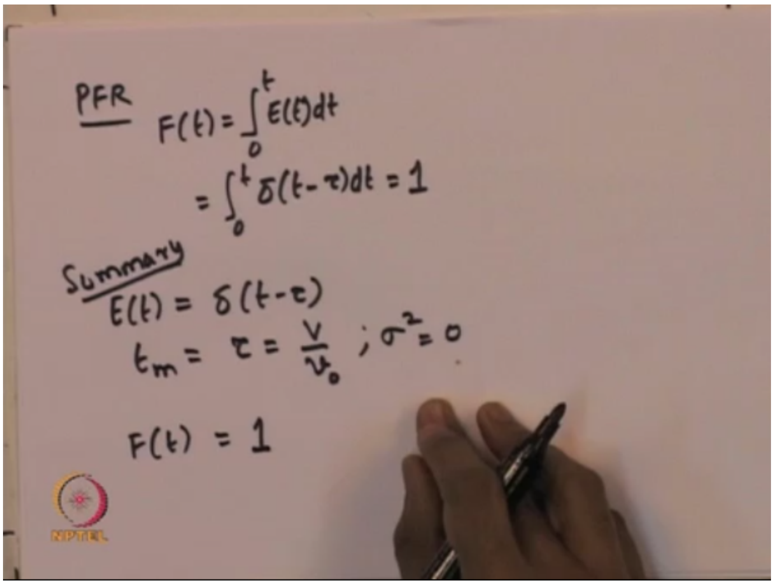

So, for a plug flow reactor, the F-curve F of t is essentially given by 0 to t E of t by d t. That is by definition. And so, that is = integral 0 to t delta of t – tau d t. That is = 1. By, we know that this integral is = 1. And therefore, the F of t curve is nothing but 1. And, so as a result the, as a result, so the properties or the RTD function for the plug flow reactor is essentially given by E of t. To summarise, is = delta function of t – tau.

So, that is the summary for plug flow reactor where the residence time distribution function is essentially given by delta t – tau. And the mean residence time is = the space time of the reactor which is the volume divided by the volumetric flow rate. And the sigma square is essentially 0. The variance is actually 0. And the F t is essentially = 1. So, therefore if we actually attempt to sketch the E-curve and the F-curve, we will find that;

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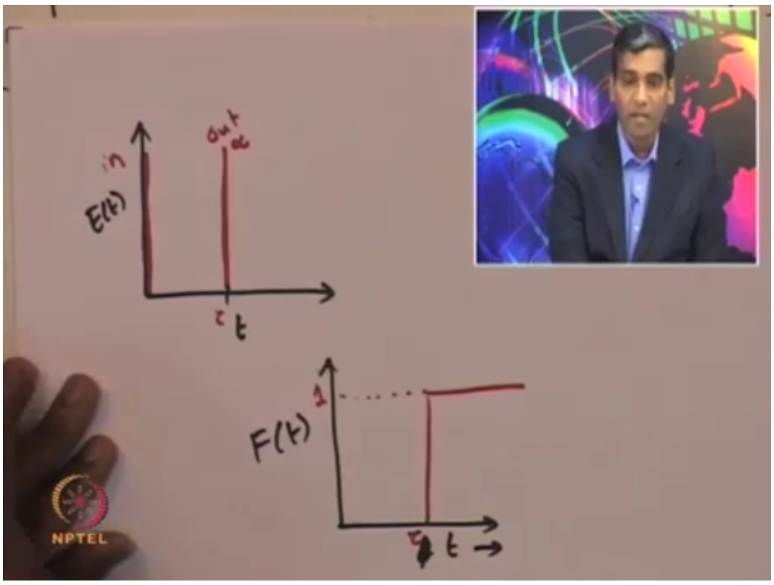

So, that is time. And suppose if this is tau here, at t = 0 if there is a spike tracer that is actually put into the plug flow reactor. So, that is the spike. Then, exactly after a delay of tau time which is the space time of the plug flow reactor, the tracer will actually come out and the same amount, same quantity of tracer will actually come out of the reactor. So, that is the out stream and the height will be infinity.

Now, suppose if I look at the F-curve. So, this is the E-curve. And suppose if I look at the F-curve of the reactor. Then, exactly at tau =, exactly at t = tau, that is the space time or the mean residence time of the plug flow reactor. The F of, F value will be exactly = 1. So, that is the E-curve and the F-curve for a plug flow reactor. Now, let us look at the CSTR case. What is the RTD function for a single CSTR.

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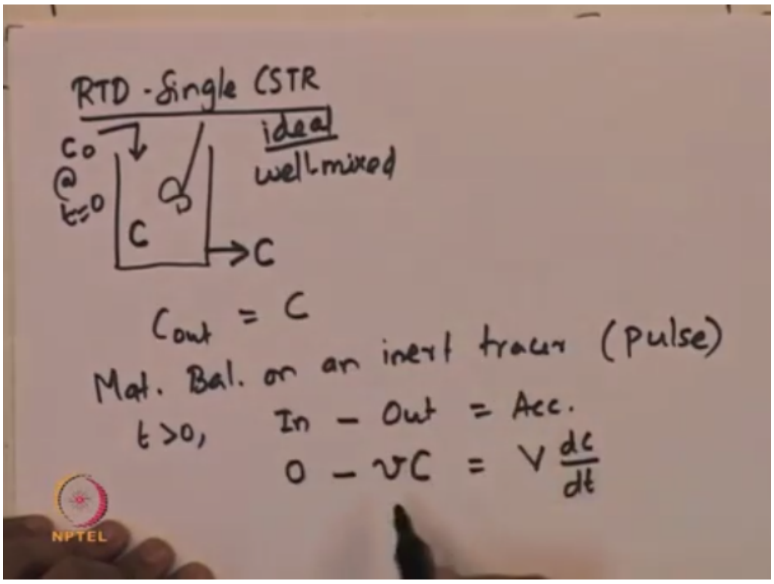

Now, suppose if here is a CSTR. And some, this is the inlet stream and this is the outlet stream of the CSTR. And it, the CSTR is well mixed and its assumed that it is a ideal CSTR. And therefore the, it is a completely well mixed system. And let us now, because it is completely well mixed system, the concentration of the species which inside the reactor should be = the concentration of the species in the effluent streams as well.

So, which means that the outlet concentration is = the concentration of the species in the reactor. And let us now write a material balance on an inert tracer. Suppose there is an inert tracer which is actually fed into the reactor. So, let us says that the inert tracer is fed into the reactor. And if the concentration of the inert tracer is actually C nought at t = 0. So, time t = 0, some t nought C nought quantity of tracer is actually fed into the reactor.

And now we can write a material balance in order to find out what is the RTD function. So, for any time > 0, whatever fluid is actually, whatever tracer is entering the reactor, that should; – whatever is actually leaving, that should be = the accumulation of the tracer inside the reactor. Now, if you assume that it is a pulse tracer, if it is actually a pulse tracer which means that the time at which the tracer is actually fed into the CSTR is exactly t = 0. And nothing before and nothing after t = 0. So, therefore at anytime > 0, no tracer is actually entering the reactor.

So, therefore the inlet is 0. – what leaves is the volumetric flow rate v of the effluent stream multiplied by the concentration of the tracer C. And that should be = v into d c by d t which is the accumulation of the tracer in the CSTR. Now, because the concentration of the species inside the reactor is = the concentration at which the species is actually leaving the reactor, the c here essentially represents the outlet concentration of the species from the reactor, reflects the concentration of the species with which it actually leaves the reactor in the effluent stream.

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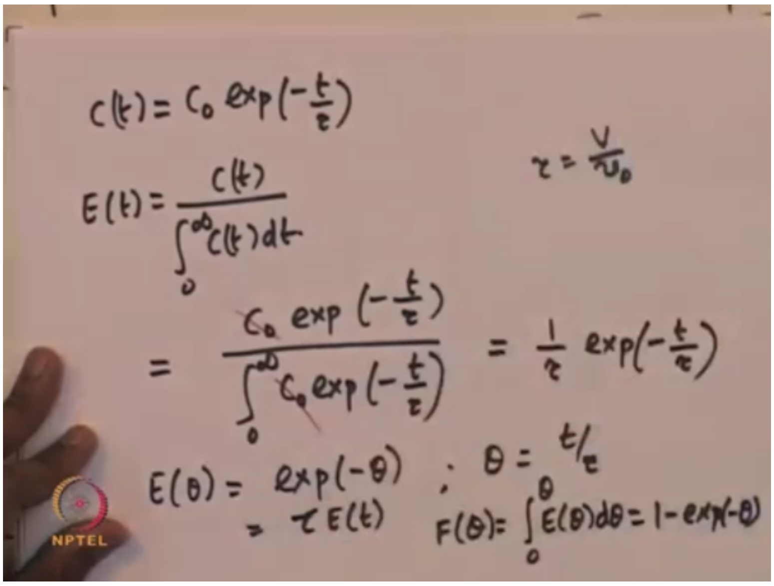

So, now one can actually integrate this expression to find out that C of t is = C nought into exponential of – t by tau, where t C nought is the initial tracer concentration, initial pulse tracer, concentration of the initial tracer that is actually fed as a pulse tracer. And from this we can find out that E of t is given by C t divided by integral 0 to infinity C of t d t. So now, we know the expression for C t, the dependence of C on time and other properties.

So, we can plug that in here. We will see that is exponential of – t by tau divided by integral 0 to infinity – t by tau. So, performing the integration, we will find that, because C nought is constant, one can actually cancel out C nought from the numerator and denominator. And so, we will find that this will be = 1 by tau into exponential of – t by tau. So, that will be the residence time distribution function for a single CSTR.

Now, in terms of the dimensionless, in terms of the normalised RTD function, the E of theta is essentially given by exponential of – theta, where theta is actually t by tau. And E of theta is nothing but tau into E of t. So, that is the normalised residence time distribution function. And now we can actually find out what is the F-curve. So, F of theta is nothing but integral 0 to theta, E theta into d theta. That is actually 1 – exponential of – theta.

So, that is the F-curve. That is the expression for x, F-curve which is 1 – exponential of – theta, where theta is t by tau and tau is the space time of the reactor, where tau is nothing but V by v nought. That is the space time of the reactor. So, let us attempt to sketch the E-curve and the F-curve. So, the E-curve.

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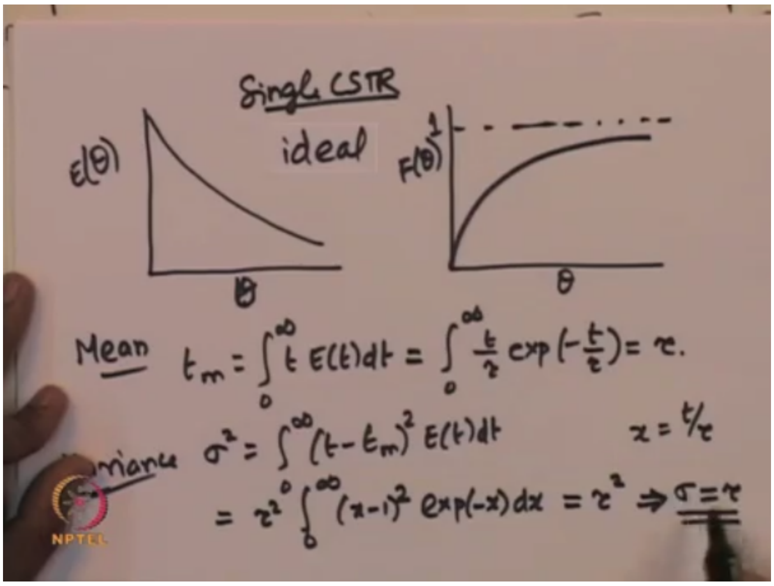

So, the normalised RTD function. So, the E-curve essentially looks like this. It is an exponential decay. And then, the corresponding F-curve, it actually looks like this. So, this is 1 and this is theta. So, it actually essentially looks like this. And so, the, mean can actually be estimated as t m. That is = integral 0 to infinity. These are different properties of the distribution t into E of t d t. That should be = integral 0 to infinity t by tau into exponential of – t by tau.

And that should be = tau. So, that is exactly what we observed before. If there is no dispersion, then irrespective of whatever is the RTD, then the mean distribution time should be = the space time of the reactor itself. And now the, next the variance sigma square is given by 0 to infinity t – t m square into E of t d t. And that should be = tau square integral 0 to infinity x – 1 the whole square into exponential of – x d x.

So, where the change of variable is done by setting x = t by alpha. And so, integrating this is a standard expression. So, while integrating this expression one can find the, that is = tau square which means that the standard deviation of the distribution is actually = the space time of the reactor itself. So, for a single CSTR the mean residence time is = the space time of the reactor and the standard deviation of the residence time function is also = the mean residence time of the reactor itself. So, now if we compare the various, compare the RTD function and the various properties of CSTR we can find that.

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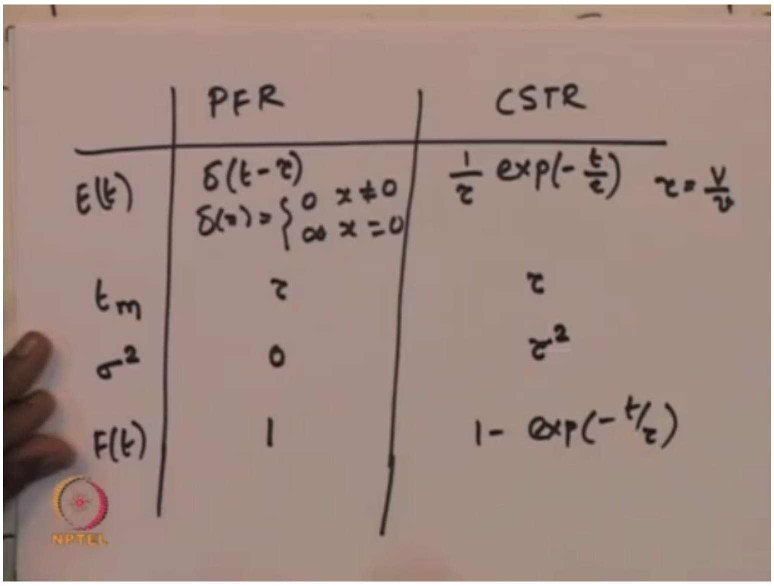

So, suppose if we make a comparison, we can summarise the function and the properties that we have found so far, for a plug flow reactor and a CSTR. So, the residence time distribution function E of t is essentially the delta function for a plug flow reactor. Which means that there is just a delay and whatever is fed into the reactor is going to come out of the reactor exactly after a certain delay. And the delay is given by the space time of the reactor.

And here, delta x is actually defined as 0 for x nought = 0 and infinity for x = 0. And the corresponding RTD function for CSTR is 1 by tau exponential of – t by tau, where tau is given by the V by v. Tau is the space time which is given by volume of the reactor divided by the corresponding volumetric flow rate. And then, the mean residence time for a plug flow reactor is given by tau. And it is the same for the CSTR because there is no dispersion.

And so, the mean residence time should be = the space time of the reactor itself. And the variance for a plug flow reactor is 0, while for a CSTR it is actually = the square of, space time of the reactor itself. And then the F-curve, actually is 1 for a plug flow reactor and it is 1 – exponential of – t by tau for a CSTR. So, that summarises the various properties of the RTD, that summarises the RTD function and the various properties of the function for the plug flow reactor and a CSTR.

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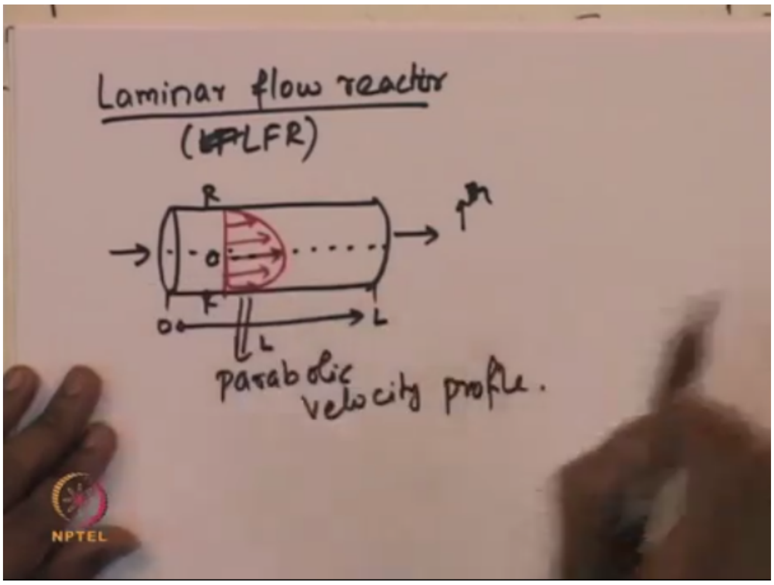

So next, let us look at the, another reactor, laminar flow reactor. Let us try to estimate the RTD function for the laminar flow reactor, LFR. Will be referred to as LFR hereafter. So, suppose if there is a tank and there is the fluid stream which is actually entering at 0 and leaving at L. So, that is the length of the reactor. That is the, that is L. And the fluid is actually entering under laminar conditions.

And it is expected that there will be a parabolic velocity profile with maximum at the centre and 0 near the walls. So, suppose if the centre of the reactor is; so, that is the centre of the reactor. And that is r = 0. So, if I label this coordinates as r. And at r = 0, it will be maximum velocity. And at R = R which is the periphery, the velocity will be 0. So, that is a parabolic velocity profile, that is the parabolic velocity with which the, velocity profile with which the fluid is actually flowing through the reactor.

Now, clearly this suggest that the fluid particles which are actually, fluid elements which are actually at the centre, they will actually have the shortest residence time because they have the maximum velocity. And so, they will leave the reactor much faster than the, they will leave the reactor faster than the other fluid elements which are actually present in other radial locations other than 0.

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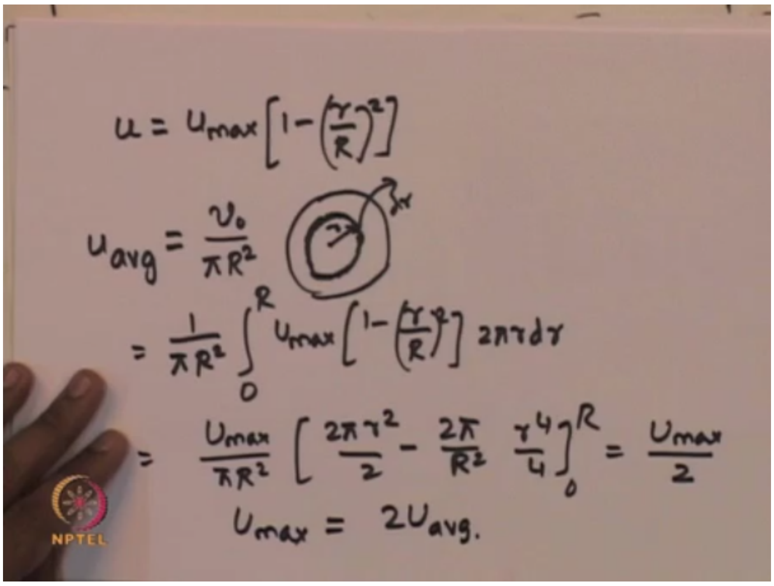

So now, so therefore the velocity profile u is actually given by u max which is the maximum velocity at the centre multiplied by 1 – r by R the whole square. Now, often this maximum velocity may not be known. So, instead what may be known is the average velocity. That is the velocity of the fluid stream averaged across the whole cross section. And that can actually be estimated from the velocity profile, from the local velocity expression.

So, u average which is the average velocity at a given cross section is given by volumetric flow rate divided by the area of the reactor at that cross section. And that is given by 1 by pi R square into integral 0 to R U max into 1 – r by R the whole square into 2 pi r d r. So, here I have assumed that if this is the cross section of the reactor, if that is the cross section of the reactor, then let us assume that there is a small element which is present here from the centre.

And that is located at a distance r. And the thickness of this is strictly d r. So, therefore the volumetric flow rate of the fluid and in, at any cross section is given by the local velocity multiplied by to pi r d r into integrated over the, integrated between 0 and R. So, that gives the volumetric flow rate at that cross section. And pi R square is the corresponding area at that cross section. So, from this, integrating this expression, we will find that it will be = U max by pi R square multiplied by 2 pi r square by 2 – 2 pi by R square into r power 4 by 4.

And the limits are 0 to R. And that is = U max by 2. So, the maximum velocity is simply twice the average velocity. That is the averaged over the cross section of a, of the reactor. In fact, the average velocity is also called as the cup mixing average. And so U max is = 2 times U average. So, substituting this in the expression for the velocity, we can actually rewrite the velocity expression as;

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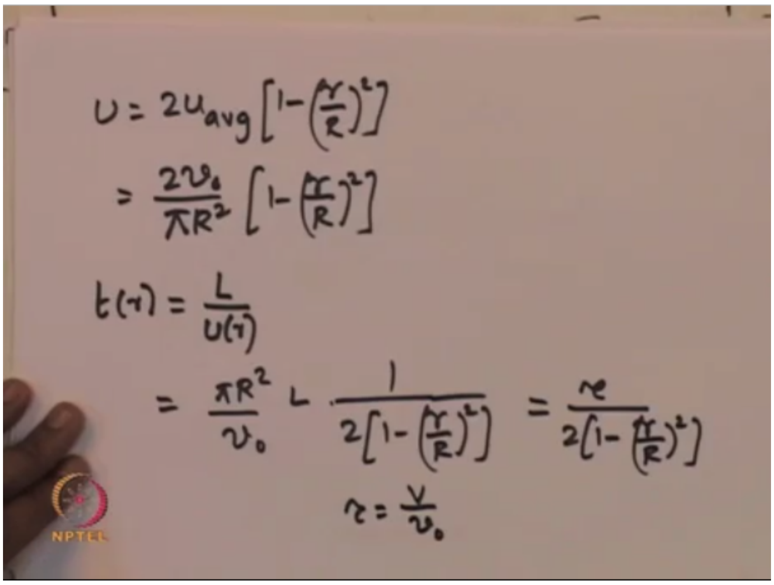

U = 2 times u average multiplied by 1 – r by R the whole square. And that is = 2 v nought by pi R square. v nought is the volumetric flow rate with which the fluid is actually flowing at that cross section into 1 – r by R the whole square. So, that is expression for the velocity with which the fluid is actually flowing as a function of the radial position. Now, we can now estimate what is the time that is actually spent by the fluid particles at a, that is entering at a given location r.

So, that is actually given by the length of the reactor L divided by the velocity with which the fluid is actually flowing in that radial location r, which is actually U r. And that is given by pi R squire by v nought into L into 1 by 2 times 1 – r by R the whole square. So, that is the time that is taken by different fluid elements that is actually entering the reactor at any r location. Okay. So, that is actually = tau divided by 2 into 1 – r by R the whole square.

Where tau is given, tau is the space time of the reactor which is given by V divided v nought. So now, we need to relate the, we need to now relate what is the, we need to find out what is the RTD function E t. So, in order to find that, we need to know what is the fraction of this fluid that is leaving and what is the age of that particular fluid.

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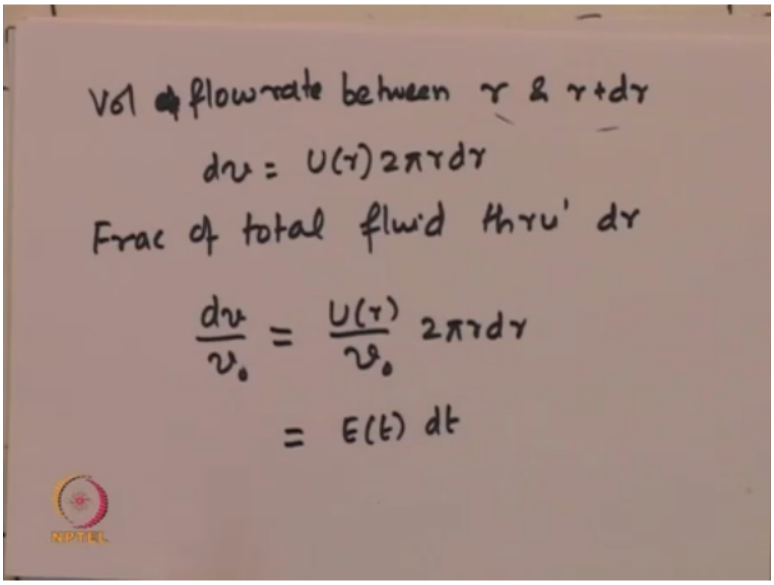

So now, the volume of the fluid, the volumetric flow rate between r and r + d r. So, that is the volumetric flow rate of the fluid which is actually flowing between r and r + d r. And that is given by d v, that is = U r into 2 pi r d r. So, that is the volumetric flow rate of the fluid which is actually flowing in this element d r. That is, between r and r + d r. So now, the fraction of the total, that actually, that is actually flowing through this small element d r is actually given by d v divided by v nought where v nought is the total volumetric flow rate.

d v by v nought gives the fraction of the fluid that is actually flowing through this element d r. So, that is given by U r divided by v nought into 2 pi r d r. So, that is the fraction of the fluid that is actually flowing through this element d r. And in fact, that is nothing but the E of t into d t. Because, the fraction of the fluid that is actually flowing through the, this small element d r and also the fluid which is actually between v and d v which is spending the time t and t + delta t is what is given by this RTD function E of t d t.

And that should be = d v by v nought which is actually the fluid which is flowing between v and v + d v whose residence time is actually between t and t + delta t. So, what we have seen so far in this lecture is essentially different properties of the residence time distribution which is the mean, you have looked at the variance and you have looked at the skewness. And then, we went on, moved no to the residence time distribution of the ideal reactors, particularly, we considered the plug flow reactor.

And then, we found out what is the residence time distribution for this particular reactor and what are the properties of the residence time distribution. And in, specifically we found out what is the E-curve and the F-curve as related to the time, as a function of time. And next, we looked at the residence time distribution function for a single CSTR. We found the E-curve and the F-curve and the corresponding properties and then initiated discussion on the laminar flow reactor. Thank you.