Heat Transfer
Prof. Ganesh Viswanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture - 09
Extended surfaces - 1 General formulation

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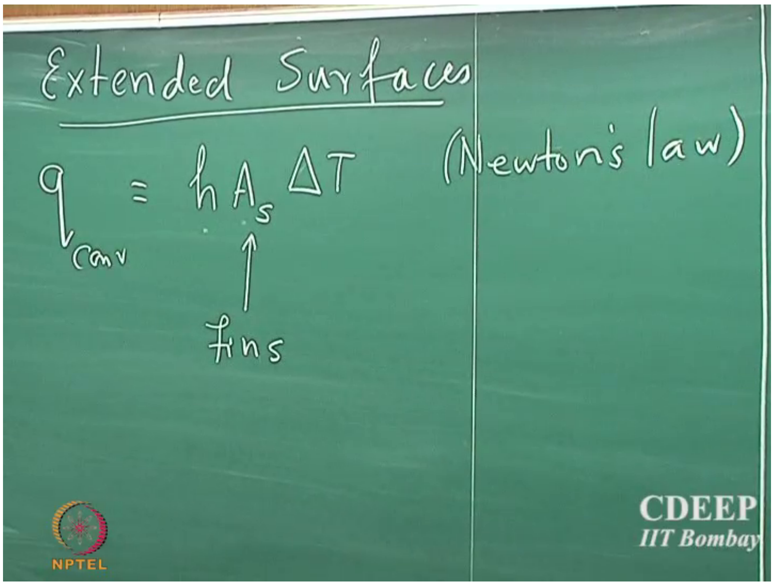

So, we initiated discussion on extended surface in the last lecture. What it really means is that the on account of convective mode of heat transport is essentially given by the product of heat transport coefficient multiplied by the area of heat transport. So, if I put a subscript s, it is basically the surface area of heat transport because it is multi phase system where the heat transport is from the solid to the fluid which is circulating around. So, there has to be it is the surface area is responsible for the heat transport via convection from this fluid around. So, if h is the heat transport coefficient A is the surface area of convective mode of heat transport here and delta T is the corresponding temperature.

So, this is essentially Newton's law of rule, essentially Newton's law of rule is and we said that the purpose of any heat transport equipment or heat transport system is essentially to either increase the heat transfer rate if you want to dissipate the heat or if you do not want to dissipate the heat you want to cut down you want to maintain the to a certain chamber with a certain temperature then you want to cut down the amount of heat that is lost.

So, essentially it boils down the total amount of heat that is transferred from the system to it is surroundings. So, one way to increase as we observed in the last lecture is to increase the surface area of heat transport and that is achieved by a set of systems one class of system called the fins. So, what we are going to see in today's lecture is we are going to look at what are the different ways of fins and how to understand heat transport process in these kinds of surfaces.

So, let us take a little general example so we said that there is a base supposing if there is a base solid.

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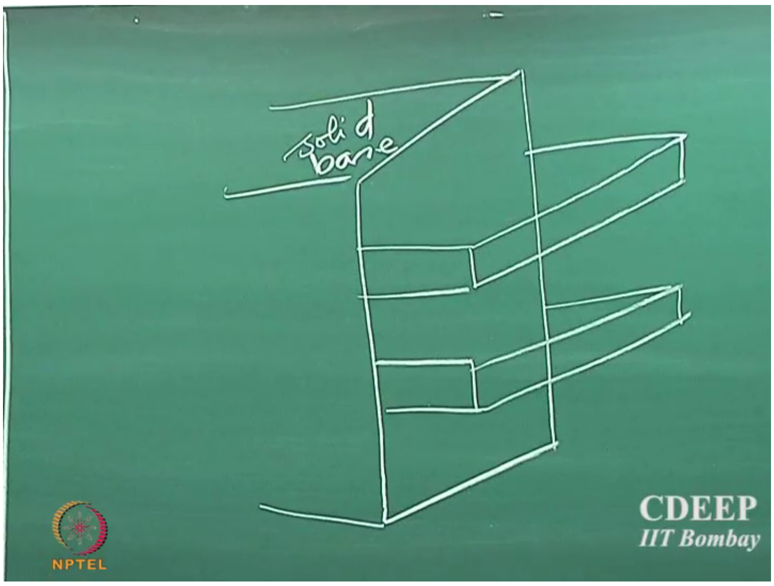

So, note that we purpose is to increase the heat transfer rate into the fluid which is surrounding, one way to do that is to have some sort of a fins which are actually protruding out of these surfaces and it could have many such fins which are protruding out of these base surfaces and there would be heat transfer from the top and the bottom surface of this fin and of course in the sides also. So, let us consider our generalized an arbitrary shape.

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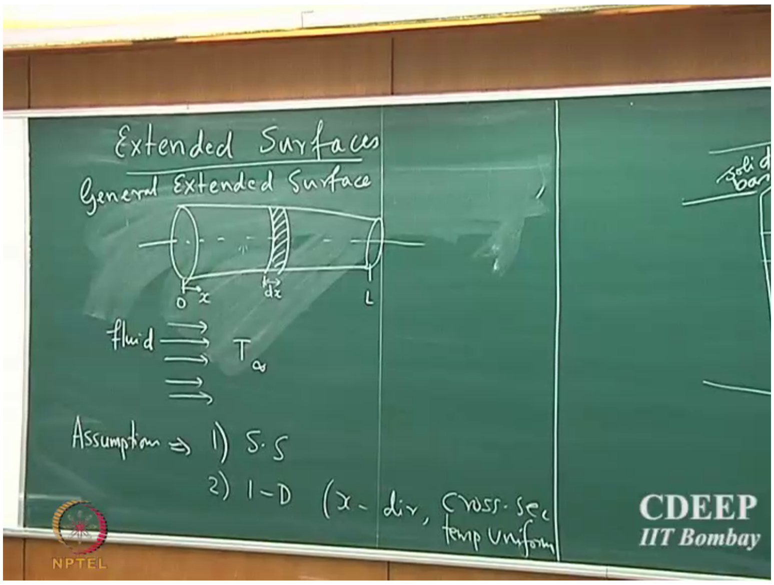

When a general arbitrary shape, let us assume that this is the varying cross sectional system and we are going to take specific examples and specific cases of this generalized a generalized pin to extended surface. So, if we want to quantify heat transport process in such a generalized system we need to write a we need to find the temperature distribution and in order to find the temperature distribution we need to write the energy balance that is what we have been doing.

So, let us say we take a small element, this is d x and let us this b d x direction going to L. And let us assume that there is fluid which is flowing past this object everywhere at temperature T infinity. So, there is some fluid which is flowing and if I assume the steady state process. So, let me may at we make a few assumptions saying that at the steady state.

And let us assume that the heat transfer is primarily one dimensional which means that the heat transfer is only in the x direction and I assume that cross section temperature is uniform. So, I assume that every cross section the temperature is uniform which means that the gradients are 0 in this cross section, but if the aspect ratio that is if the if the radius of the cross section that we are considering if that is substantially small compared to the length of the of the object that we are considering then it is a reasonable assumption to make that the gradients in the cross section is negligible.

If it aspect ratio is significant, if it is close to one certainly it is not correct to assume that the cross sectional temperature is uniform, there will be gradients in the cross section and you will have to consider that and that would re combinational energy balance which we will see a few lectures down the line.

Student: 35 43.

So what is the energy balance, remember the mantra I told you in one of the lectures whatever comes in this element whatever it leaves plus whatever is generated inside the system or whatever is lost from that location should be equal to accumulation.

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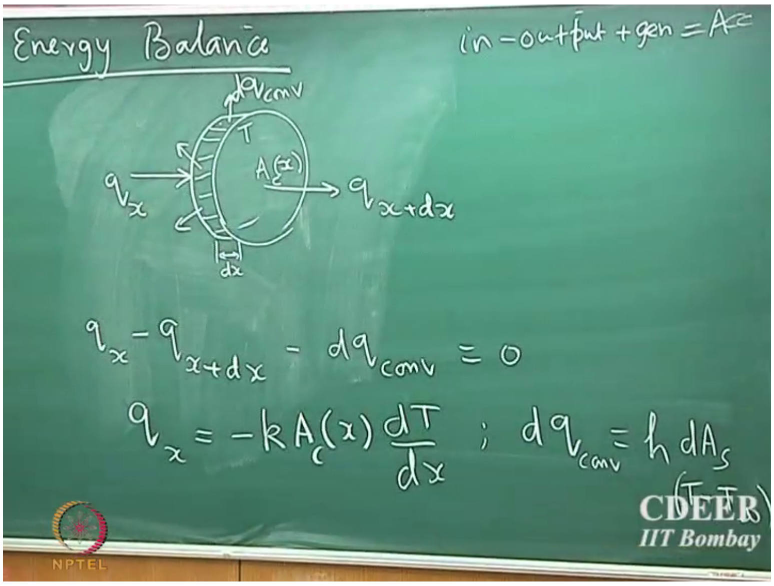

So, that is the mantra that we wrote, input minus output plus generation equal to accumulation this term could be accounted in the generation by putting appropriate sign. So, I will not write it as a separate term here. So, supposing I draw the cross section, this is dx and A subscript c is the cross sectional area of our conduction and note that the cross sectional area could in principle be a function of the axial position. So, this is in principle A function of x and the convective mode of heat transport is actually occurring from the curved surface area of this object.

So, along the curved surface areas area we see that there is a fluid which is flowing and therefore, there is constant heat exchange from the solid to the fluid along the curved surface area and of course there will be heat transport from the end also.

So, note that the x equal to 0 basically corresponds to the; it is attached to the base it is attached to the base equipment. So, it is not exposed to the fluid and there will be heat exchange from the curves surface and also from the lagging edge of the fin that we are considering. So, the supposing I call d q convection is the differential amount of heat that is exchanged or that that leaves this surface and is sent into the fluid stream. So, that is the amount of rate differential rate at which heat is being exchanged and if the amount of heat that the rate at which heat is transferred to that element input is qx and if the rate at which it leaves this q of x plus d x. So, at steady state accumulation is 0, therefore, the balance is simply q x which is the input minus qx plus dx which is the rate at which heat leaves this element minus the amount of heat that is lost because of exchange of heat from the solid to the fluid which is flowing past it.

So, minus dq convection that is equal to 0 very simple, whatever comes into that element qx minus whatever it leaves that is q of x plus dx minus whatever is lost because of convection from the heat transport from the solid to the fluid which is flowing past that object. So, q from Fourier's law we know that qx is minus k Ac of x into dt by dx. So, note that in principle the cross sectional area could be a function of the position. And so, it is a variable cross sectional area problem and similarly from Newton's law of cooling dq convection is heat transport coefficient into dAs and if the local temperature is T multiplied by T minus T infinity. So, that is the d q convection, this comes from Fourier's law and this comes from Newton's law of cooling.

So, when we substitute all this minus k minus A x into T minus T equal to 0 any questions on this.

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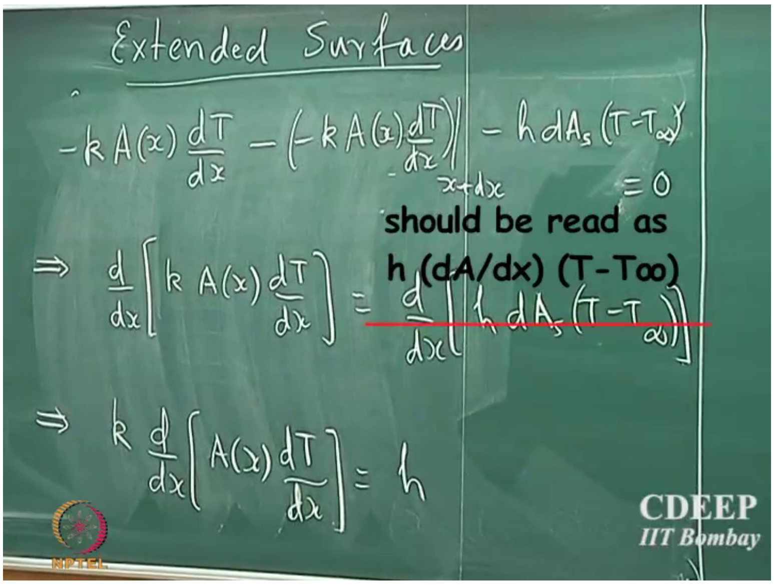

So, far now, I could rewrite this as d by dx k A of x dT by dx equal to d by dx h s into T minus T infinity. So, if I assume that the thermal conductivity and the heat transport coefficient are not function of position then one could pull it out and say k d by dx A of x equal to h is T minus T infinity a function of position in this term here yes or no.

Student: (Refer Time: 12:23).

Did you watch out, this is the d by dx rate of change of convection convective term, right? So, here what is changing is the area changing with dx or T minus T infinity what should you consider. So, note that this is the differential balance right, you are looking at what is changing in this small element. So, when we say Newton's law of cooling we assume that the cross sectional temperature or the 10 local temperature of that element is constant and therefore, what is changing here with respect to convection from the solid to the fluid is the cross sectional area. Therefore, we can rewrite this as dA s by d x into T minus T infinity.

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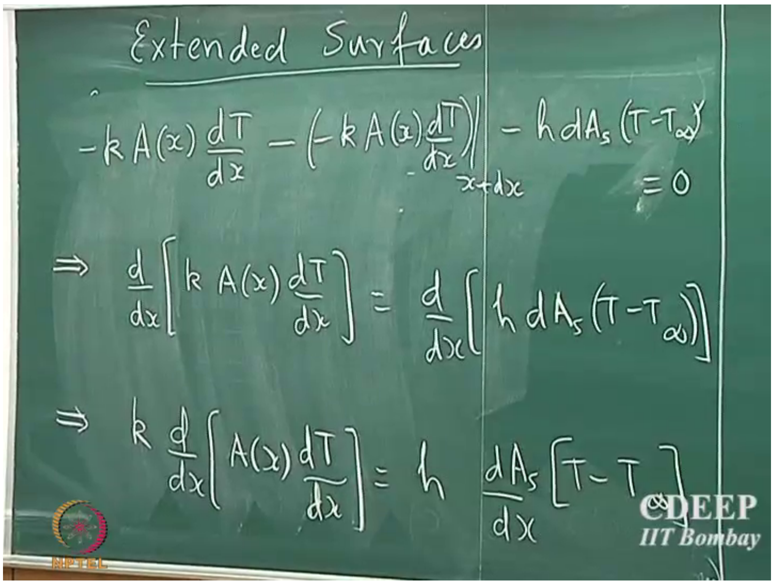

So, keep in mind that what is changing here the, so in this small element whenever you look at convective heat transport from the solid to the fluid along the cross sectional area, in the differential balance. So, note that this is very very important assumption that you make in differential balance, that the local quantity that you are considering in the control volume you assume that the local quantity the element is small enough such that the local quantity remains constant in that element and therefore, the T minus T infinity in this small element must remain constant it is not a function of dx.

Student: (Refer Time: 14:11).

Excuse me.

Student: (Refer Time: 14:13).

That is already a counter here in the gradient, the temperature change that you are seeing here is inside the solid that variation is already accounted with the diffusion term. So, the how what causes the temperature variation inside is the conduction process. So, the variation of temperature inside the solid is already accounted for here, but when you are considering convection what is causing convection the driving force of convection is the local temperature inside the solid and the local temperature in the fluid outside.

Student: (Refer Time: 14:50).

Student: (Refer Time: 14:53) mathematically correct.

They are correct.

Student: Because my input is (Refer Time: 15:01).

That is correct if we use the chain rule, note that look at dq here this is the differential amount of heat that went in this differential amount is because of the differential area and not because of the differential temperature gradient. So, if you use the chain rule the differential temperature gradient is 0, because you assume that the temperature in this small element is same and that is the basic assumption of any differential balance in a control volume you always assume that the temperature is constant. And therefore, the differential conduct amount of heat transfer that occurs because of connections, because of the differential amount of a cross sectional area or surface area which is present.

And therefore, even if we use chain rule you could use even if you use chain rule in this term the differential temperature difference will be 0, because you by nature of the base is the fundamental premise of differential balance is that the local temperature gradient is assumed to be constant. So, if you use that property then even if you use chain rule that temperature gradient will be 0. So, we can simply write this as k into Ac of x I should have put a subscript here.

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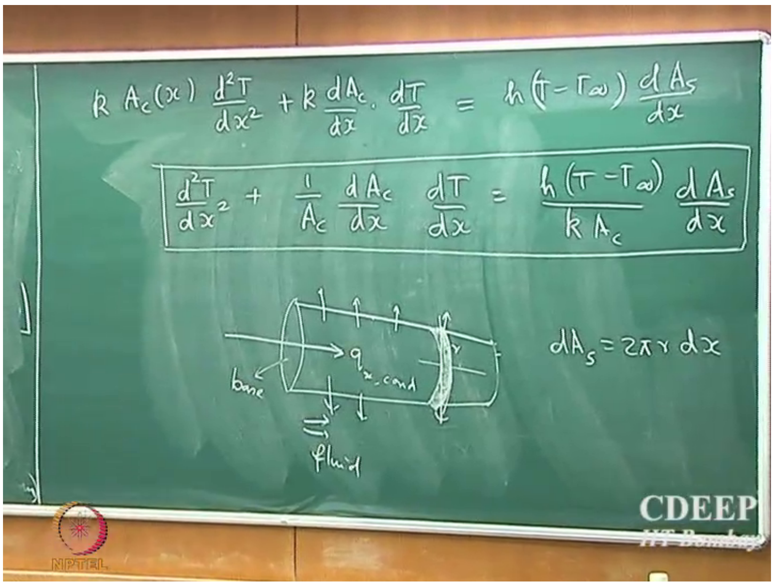

So, Ac of x into d square T by dx square plus k into d Ac by dx; that should be equal to h into T minus T infinity into dA s by d x.

So, note that in the conduction term the gradient in the or the net heat transport rate is now going to be a function of both the temperature gradient and the cross sectional area gradient unlike what you see in convection term because the convection here occurs along the cross sectional area and you assume that the driving force for convection is the local temperature gradient. So, that is the key difference why this kind of why if you use the chain rule the temperature gradient will be 0 any other questions.

This is the general balance, we can simply rewrite this as d square T by dx square plus 1 by Ac into d T by d x that is equal to h into T minus T infinity divided by k into A c by d A s write d x. So, this is the general balance for the arbitrary geometry the ph is considered.

Any questions on this so far, yes.

Student: (Refer Time: 18:21).

Yes, is the curved surface area, curved surface area of the object maybe you came late you did not see the exact picture; yes.

Student: (Refer Time: 18:33).

Let us draw the picture again supposing if this is the arbitrary fin, the conduction occurs in this direction. So, this is the heat transfer rate because of conduction, conduction occurs from this is the base of the object. So, the fin is actually protruding from the base and the heat is now carried from the base to the fin and there is conduction mode of heat transport which is carrying heat across. Now we have fluid which is actually flowing past this object on all directions. So, now, the heat is lost from every location in the curved surface. So, at any location x ax is essentially the supposing if the radius is given by supposing if the radius is given by r. So, at any differential A s any location x the differential area of conductive mode of heat transport it is simply given by 2 pi r time dx if the thickness is d A.

Student: (Refer Time: 19:56).

So, you cannot define because the you need a see for any heat transport locker you need a cross sectional area correct now you need some cross sectional area at which the heat transport is going to occur and at any given location you need a definite thickness in order to define heat transport process. So, as supposing if I look at the, this is the cross sectional area supposing if I know what is the total length that I know what is the total surface area of heat transport, but if I want to know what is the heat transport at a given location where the cross sectional area is changing then you have to define a specific thickness otherwise area is not defined.

If you define a specific thickness then that is the area that you have to consider. So, definitely when you want to define an area of heat transport you need 2 dimensions you cannot define with just one dimension area is not defined. So, you need 2 dimensions in order to define a area, you need to define if it is a varying cross section then you need to have at least a small thickness at which the conductive heat transport is going to occur. Now if it is non-varying cross section that is if the cross section is same then you could actually consider what is the overall area of heat transport is that clear to everyone any.

Student: (Refer Time: 21:19).

Boundary condition we want to see that, note that conduction is occurring in the x direction. So, the whatever heat transport that is occurring in this boundary is accounted as the boundary condition that is what we are going to see, it does not matter whether you do integral or the differential balance. In fact, we are going to see in a short while in order to estimate some of the properties there are many different ways to estimate properties whether you use the integral method or whether you use the overall maximal heat transport method you are going to get exactly the same answer. So, the differential and the integral balance can actually be merged together in some sense there are formal ways to do that. So, there is no need to worry about whether if I use integral balance will I get a different answer you will not you get exactly the same answer whether you use an integral or differential balance.

Student: (Refer Time: 22:15).

You want to see that in a short while that is the next case we are going to consider and that is that is rightly pointed out that is the obvious case that you would want to consider right that is the immediate thing you want to check whether if we say cross sectional area is independent of the x direction then what will be the case.

Student: (Refer Time: 22:35).

So, in the original energy balance equations supposing you write multi dimensional energy balance equation. So, let us take that for a moment let us digress for a couple of minutes. So, in principle you could derive the same equation from the generalized energy balance also.

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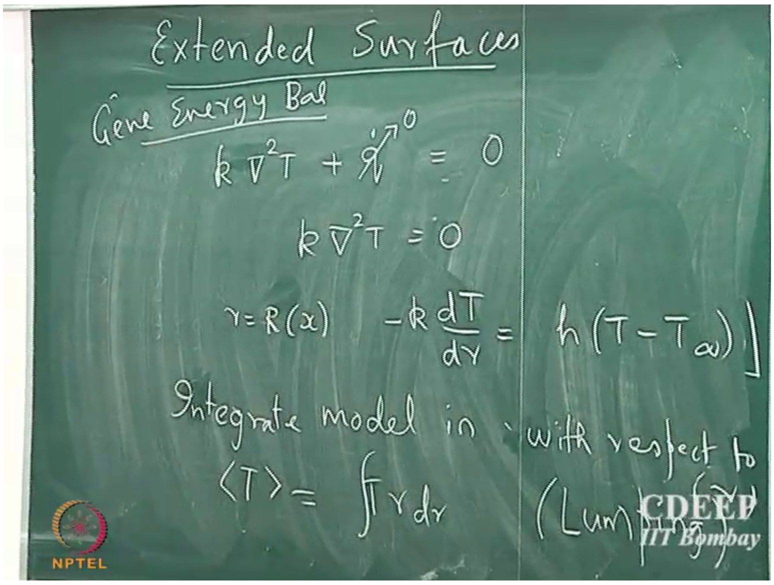

The Generalized energy balance is k del square T, this is the Generalized energy balance is k del square T plus whatever is the amount of heat that is generated inside the solid and that should be equal to the accumulation term. So, if you assume steady state accumulation is 0 and if we assume that there is no heat generation inside that is also 0. So, k del square T equal to 0 is your model, but keep in mind that the heat transport that is occurring from the curved surface is now going to appear as a boundary condition in the radial direction.

So, if I write the boundary condition at r equal to let us say capital R which is a function of position x then I will be minus k d T by d r that is equal to h into T minus T infinity. So, that will appear now as a boundary condition now the way to get this model from here is all you have to do is you say that. So, note that we did not include one of the assumption we said that the cross sectional temperature is uniform. So, how do we incorporate that, you integrate model equation with respect to r if you integrate the schema second if you integrate with respect to r, keep in mind that you have to use the correct weights.

So, note that you are now dealing with these radial coordinates, you have to account for when you integrate the radial coordinates you will have to account for change in the cross sectional area. So, the correct integral is essentially rdr T times rdr, what you will get is a cross sectional averaged temperature. So, this T is essentially we are assuming that the radial distribution is uniform right we said that there is no gradient in the radial direction which is equivalent to saying that it is a cross sectionally average temperature quantity.

Now, if you integrate this equation in the r direction and substitute the corresponding boundary condition you should get that and in fact, it is a good exercise to check. So, account for the varying cross sectional area and you incorporate this integration into your model equation and you will be able to get that. So, this is the way to go from the generalized equation and in fact, we will not discuss this, but this sort of assumption is called lumping or averaging in engineering literature.

So, it is a very very common technique used when you model different engineering systems. So, you can use a lumping method if you know that the aspect ratio of a certain problem is very very small that is I define aspect ratio as the radius divided by the length if that is very very small then you could actually do what is called a lumping assumption and then you get a 1 Dimensional model.