Heat Transfer
Prof. Ganesh Viswanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture - 08
Heat generation – II Problems; Introduction to extended surfaces

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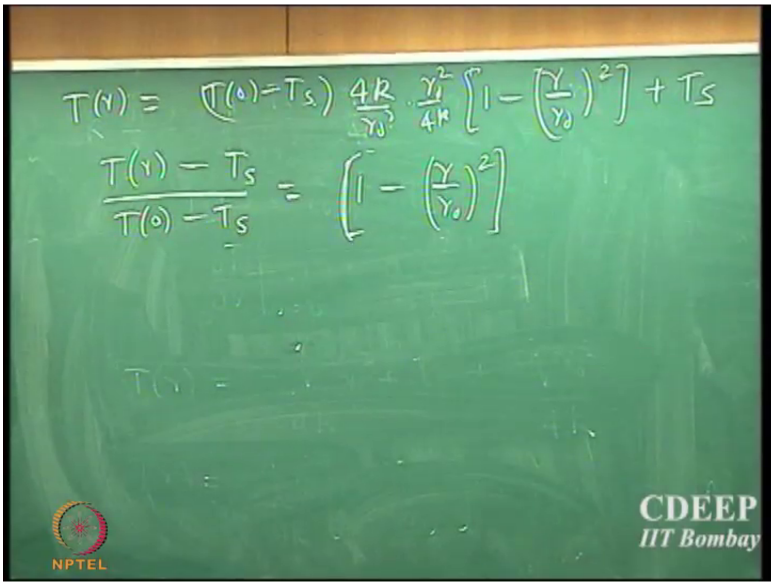

So, let us look at a specific example.

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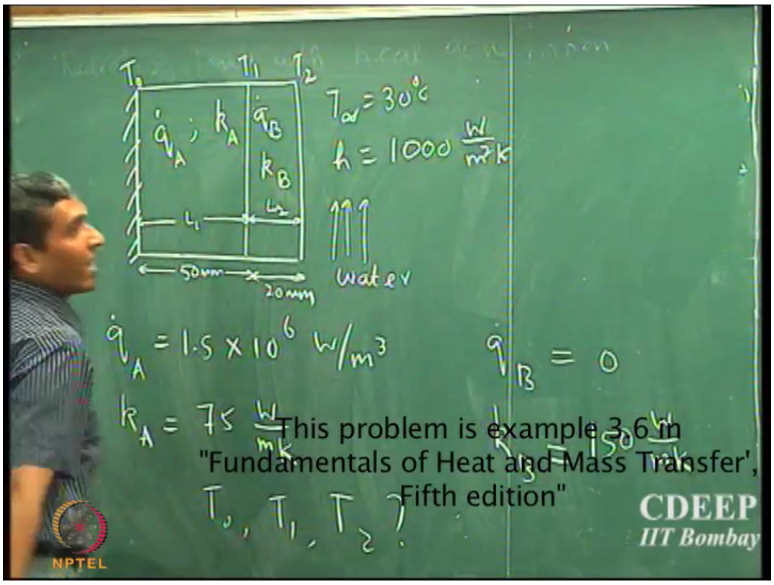

This problem is example 3.6 in "Fundamentals of Heat and Mass Transfer", Fifth edition"

There is a composite slab and one end of this slab is maintained at no flux condition that is adiabatic conditions, and the length is specified as 50 millimeters, and 20 millimeters. So, that is the thickness of the slab. And let us say that the q dot A and q dot B are the volumetric generation of heat in these 2 slabs and if k A and k B are the corresponding conductivities. And if water is flowing outside water is flowing outside with at a temperature of 30 degree c and the heat transport coefficient is 1000 watt per meter squared Kelvin that is the heat transport coefficient and you can throw in some number.

If the temperatures are specified as T naught T 1 and T 2, if q dot a is 1.5 into 10 power 6 watt per meter cube and k A thermal conductivity is given by 75 watt per meter Kelvin and if q dot B is 0 which is which means that there is no heat generation and the second slab and if conductivity is given by 150 watt per meter Kelvin, we need to find the temperatures. So, the question is we need to find the temperatures of these 3 locations T naught is at the location where the adiabatic condition is maintained and T 1 is the interface between the 2 slabs and T 2 is the exterior surface.

So, how should we proceed?

Student: (Refer Time: 02:35).

Yeah.

Student: (Refer Time: 02:41).

Right, balance equation for both can we do better than that.

Student: (Refer Time: 02:46).

Student: (Refer Time: 02:48).

We know the solution separately of course, we can write the balances and we can find the solutions can we do it better than that. So, look at the system and intuit what is the happening here what is happening here. So, you should not blindly go and write equations all the time you have to see what the system is doing can we do better than that can we do in a simpler fashion can we solve the problem, what is happening at this interface.

Student: (Refer Time: 03:26).

Student: (Refer Time: 03:28).

Heat flux is same because whatever heat that is being transported from slab A is being transported to slab to at that interface what is the flux at that interface. So, note that this is adiabatic, right. There is nothing that is leaving on this end. So, whatever heat that is being generated should actually be the flux at which the heat is being the total amount of heat that is generated because of the heat generation term must be transported only in one direction right. So, that is sort of obvious from the way the problem has been defined, but you must intuit these kinds of things, so the total amount of heat that is generated.

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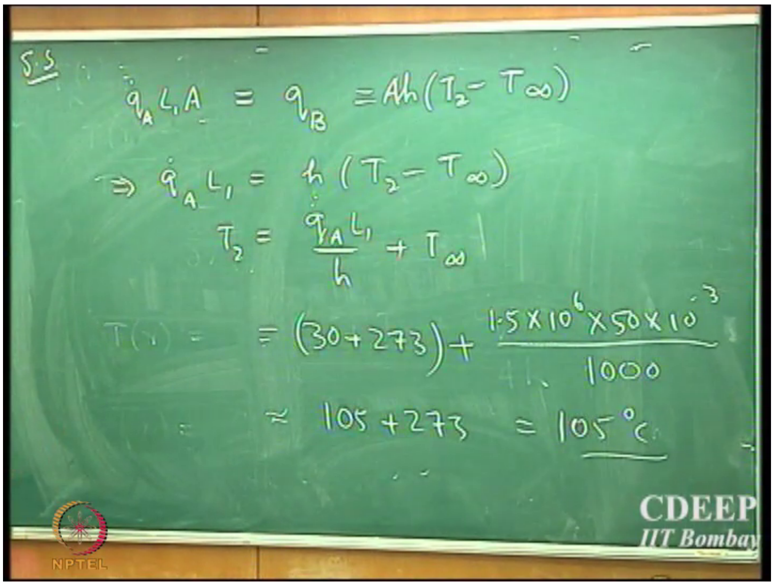

S.S

$$\dot{q}_A L_1 A = q_B = Ah(T_2 - T_\infty)$$

$$\Rightarrow \dot{q}_A L_1 = h(T_2 - T_\infty)$$

$$T_2 = \frac{\dot{q}_A L_1}{h} + T_\infty$$

$$= (30 + 273) + \frac{1.5 \times 10^6 \times 50 \times 10^{-3}}{1000}$$

$$\approx 105 + 273 = \underline{105\ ^\circ C}$$

So, q dot A into the volume of this flat A which is let us say if this is L 1 length L 1 will put the numbers later.

So, this is length L 1 into A, that is the total amount of heat that is generated and if we specify that it is a steady state condition that is what we will be looking at that should be equal to the rate at which heat is being transported in slab B because there is no heat generation there. So, whatever comes in here has to go out if the steady state condition has to be maintained and that should be equal to h into T 2 minus T infinity right very simple. So, because there is constant heat transfer rate in slab B because q B dot is 0 heat transfer rate is constant therefore, whatever heat that is actually flowing at this interface whatever rate at which heat is being flowing from at this interface excuse me should be transported at this interface if you have to maintain steady state conditions.

Therefore, q dot into L 1 into A should be equal to h into A right, that is rate we are looking at rate not flux right. So, we know what q dot A is, this means q dot A into L 1 should be equal to h into T 2 minus T infinity. We know q dot A, we know T infinity we know the heat transfer rate. So, we can find out what T 2 is right and that will be q dot A into L 1 by h plus T infinity if I throw in the numbers it will be 30 plus 273 watch the units I am using s I units into 10 power 6 multiplied by 50 into 10 power minus 3 divided by 1000, that is approximately 105 plus 273, that was 105 degrees.

So, that is the temperature T 2 is it clear to everyone how do we find T 1.

Student: (Refer Time: 06:51).

How do we find T 1 we know that the heat transfer rate is constant and we can simply write resistance network right if we know how to write resistance network.

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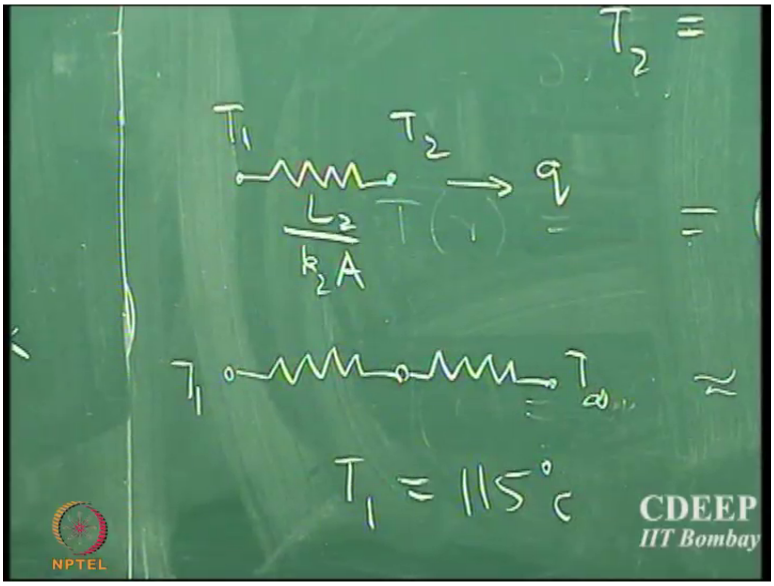

So, the simply based on resistance network T 1 T 2 and that is given by L 2 by k 2 into A right, that is the resistance network and this is q right, can we find T 2 from here.

Student: (Refer Time: 07:42).

No, q B is 0 there is no heat generation in the second slab if there is no heat generation in the seconds slab you would actually write the resistance network right. So, how do we solve this you do not know the area right. So, what do we do we really need the area, how do we solve how do we draw the resistance network correct resistance network to find temperature T 1.

Student: Area.

Area.

Student: (Refer Time: 08:24).

We know q equal to h into A; so there are 2 ways of doing it you know what q is you could use this resistance network or you can draw overall resistance network also T infinity 1 call it T infinity and T 1. So, you can use either of overall networks or you know what q is and you can find out what the temperature T 1 is. So, T 1 will be I will just put the number it is very easy to calculate 115 degree c what about T 0 how do we calculate T 0.

Student: (Refer Time: 09:15).

So, we have to use the solution here we have to write the model if the model is not given to you; obviously, you have to write the model and find the solution because the resistance concept is not valid for that slab. And therefore, we have to write the model equation and find the solution and I am not going to rewrite the equations here.

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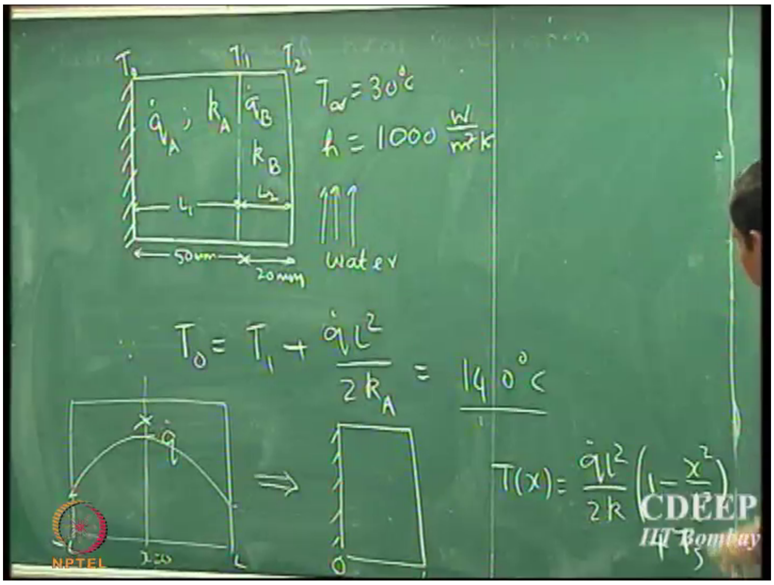

So, we solve the equation just now, you can find out that T naught will be T 1 plus q dot L square by 2 times k is equal to 140 degrees. So, remember that slab A is nothing but if you have if you have a slab which is twice the length of L 1, that is the problem that we just solved right. So, where at the midpoint you have adiabatic condition. So, that is the same problem, if you find what is the.

Student: (Refer Time: 10:22).

Sure, so remember that we solve this problem right at x equal to 0 minus L and plus L and if you have a heat generation we solve this problem. And we found the temperature profile correct and we know what is the temperature distribution what we need to know is what is the temperature at this location right that is the midpoint.

So, remember that we also said solving this problem is equivalent to solving half slab starting from x equal to 0 with the adiabatic condition why is it adiabatic, because if you impose a symmetry condition on this where the temperatures on both sides are same then you have a maxima at the center. So, the profile looks like this, you have a maxima at the center which means that d T by d x is 0 which is exactly the flux condition is exactly the flux boundary condition. So, this problem here is as good as solving a problem which is twice the size, and if you get the temperature profile you can find out what is the temperature at the midpoint and that is what we solved a few moments ago.

So, once you know the temperature this is simply given by this expression here and you will be able to find out. So, remember that the solution is given by I can give you what the solution is.

Student: Sir.

Yes.

Student: (Refer Time: 11:46).

Where?

Student: (Refer Time: 11:50).

Conduction is included in q B.

Student: (Refer Time: 11:54).

This is a balance at the interface the wall that is generated, not that the conduction is happening inside the slab right. So, at the boundary what comes out of the boundary is the total amount of heat that is generated inside the slab of course, it comes through conduction how does heat come here because of conduction inside the slab, but we are looking at what happens with that interface because there is no heat that is leaving on the other side everything has to leave through this side and. So, whatever comes here should be the total amount of heat that is transported the second slab because there is no heat generation in slab B clear.

So, the solution of this equation would simply be T x is q dot L square by 2 k into 1 minus x square by L square plus T s, you put x equal to 0 that is what you would get.

So, T s is the temperature of the 2 surfaces which is same and that is nothing but T 1 and our problem here and, if you put x equal to 0 here what you get is the solution here yes.

Student: (Refer Time: 13:11).

Where?

Student: (Refer Time: 13:13).

You do not need to know you know what q is q is h into A into T 2 minus T infinity so the area will cancel out the area will cancel out when you actually write it is easy. So, we can write, T 1 maybe I will write it here for the benefit of everyone I will write it here. So, what is this?

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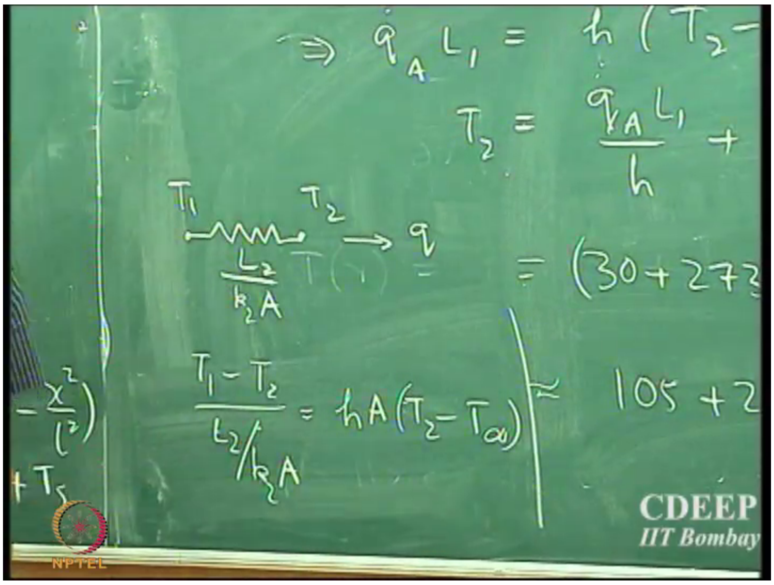

So, this is T 1 minus T 2 divided L 2 by k 2 A h into T 2 minus T infinity right area will cancel out and you know what T 2 is already you know what T infinity. So, you should be able to find T 1 any other questions. So, I mean the take home message of this particular example the reason why I showed this is we should not blindly just go and write equations for a given problem you should first intuit what is happening in the problem can I make some estimate about certain quantities without solving the equations of course, you can solve the equations and verify them, but can I make some entity for example, the intuitive judgment we made about heat transport a second slab such kind of judgments is very very important when you are handling these kinds of engineering problems.

So, you must always pay attention and give utmost importance to your intuition, the next topic that we are going to see is extended surfaces. So, I will go to briefly give introduction to that in the next 5 minutes and then we will go deep into it in the next few lectures.

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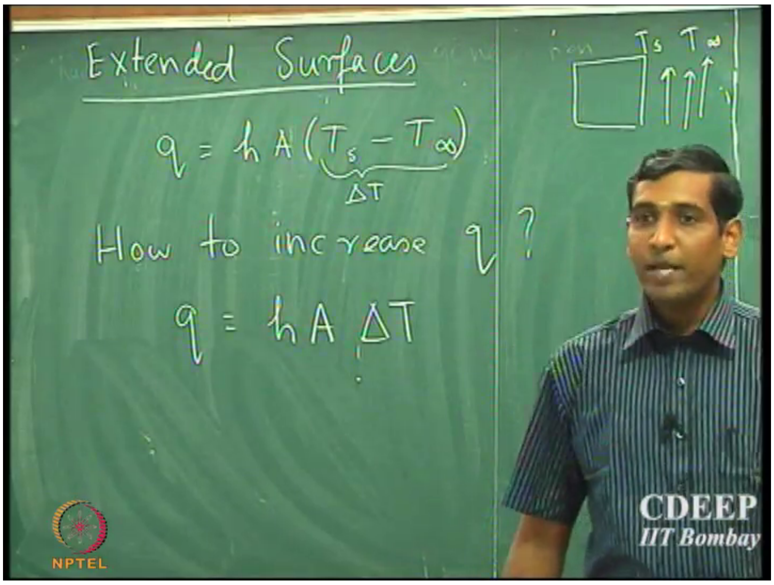

So, far we said by Newton's law of cooling we said that q equal to h A into let us say the surface temperature minus T temperature of the fluid to which the heat is being transported. So, that k let us say it could be a slab like this where the fluid is actually flowing past the slab and this is the surface temperature and this is the temperature of the fluid, now the question comes in most of the engineering problems is how can I increase the total amount of heat transport.

What are the ways by which how to increase it is a very very important question; how to increase q. So, the purpose of heat transport is basically to transport heat from one location to another, the better I can do the better placed I am right. So, if I can design a system which can transport heat better and; obviously, I am much better place. So, if you look at these 3 quantities here, if I call this as delta T some temperature difference. So, I can rewrite it as h into A into delta T. The net heat transfer rate depends upon 3 quantities right one is the heat transport coefficient the other one is the area of heat transport and the third one obviously is the temperature difference. So, these are the 3 quantities that needs to be tweaked. So, if you want to increase heat transfer rate you can increase either of them individually or you can increase multiple of them.

Many of them simultaneously if you are able to increase then you are going to increase the total amount of heat that is transported. So, when we discuss convection we are going to look at how to play with the heat transport coefficient what are the methods by which heat transport coefficient can be estimated and what are the factors on which the heat transfer coefficient depends upon and how to increase them is what we will see for most of the convection chapter. So, now, what we are going to see is how to increase the area heat transport and thereby increase the net heat transfer rate. So, one way to do that is what is called the extended surfaces or in classical literature it is also called as fins. So, there are several equipments where fins are being placed in order to increase the heat transport.

For example in the radiators in car where you want to dissipate the heat as soon as possible which means you need to have a very high heat transport. So, in order to increase the net heat transport you design a system which has very high surface area in fact, the way it is done is. So, the fins sorts of looks like this, they sort of look like this.

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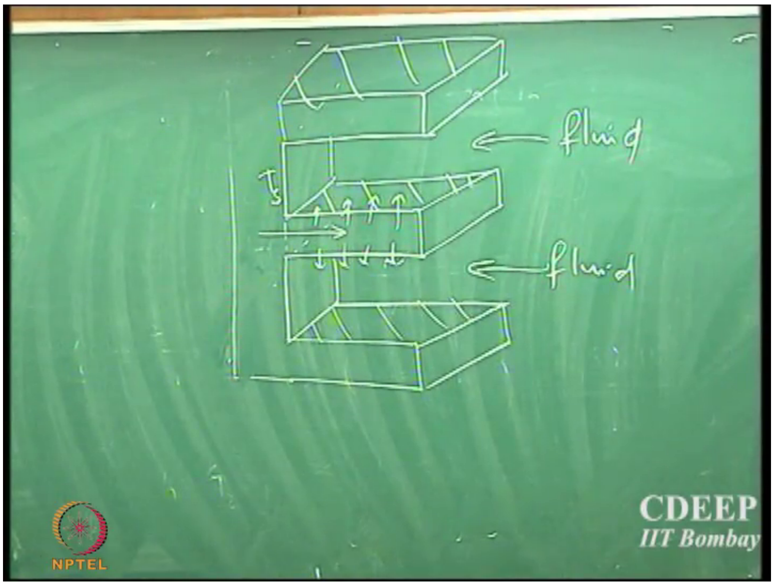

So, you have a base here, there is a base system which is maintained at a certain temperature. So, supposing if the temperature of the base surface is T s. Now you want to increase the heat transport from the base to the fluid which is circulating around. So, there is a fluid which is flowing here it could be any fluid in case of car radiators is basically air which is being circulated. So, you want to maximize the amount of heat that is transported from this surface to the fluid. So, the way to increase the heat transfer area is you create a thin protruding surfaces like this and you have heat transfer occurring from the upper and the lower part of this protrusion that comes out of the surface and you can have many such protrusions and each of these protrusion is what is called as a fin.

So, you have now added some extra area for heat transport and that is how you enhance the net amount of heat that is being transported. Now if you look at it this is a slightly different problem than what we had dealt with so far, compare this kind of a structure with the structure that we had dealt with so far, where the fluid which is being which is flowing is actually at the outer end of the system that you are dealing with and the flow of heat is actually in the direction perpendicular to that of the fluid flow, right.

Unlike here, in this case the conduction is occurring in this direction, while simultaneously there is heat loss from this surface. So, here there is no simultaneous conduction of heat and heat loss from the surface, while in the extended surface you have now introduced the system where there is simultaneous conduction of heat and also loss of heat to the fluid which is surrounding it.

So, in this sense it is a slightly a different problem. And we are going to see how to account for these kinds of systems. In fact, that answers one of the questions we asked a short while ago; what about heat lost from the system. So, here we are going to account for simultaneous loss of heat from the solid which is actually conducting heat in one direction. So, this is what we are going to see for the next couple of lectures, and how to quantify heat transfer process, and how to find out the net amount of heat that is transferred from these kinds of (Refer Time: 21:31).