Heat Transfer
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Lecture - 07
Heat generation - I
Plane and cylindrical wall

Let us get started.

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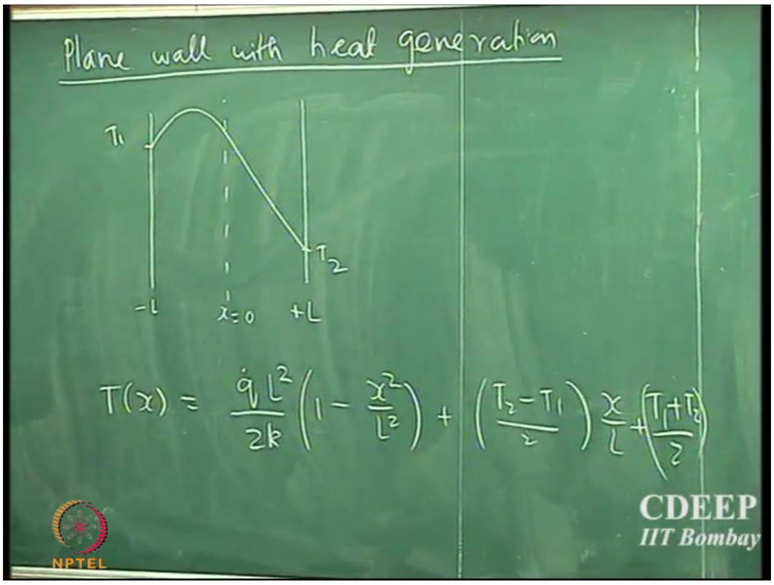

So, we start by making this depiction and we said that the solution of or the temperature distribution is given by q dot L square by 2 k into 1 minus x square by L square plus T 2 minus T 1 divided by 2 into x by L plus T 1 plus T 2 by 2. So, that is the solution what will be the temperature profile.

Student: Parabolic.

Parabolic ok.

Student: (Refer Time: 01:23).

Will it be symmetric, will it be non symmetric, why is it non symmetric because, it will be non symmetric where will be the maxima can you guess.

Student: Closer to T 1.

Closer to T 1 , look I mean without actually going and plotting there are several intuitive things which you will be able to see simply by looking at the system it is very very important to capture these intuitive behaviors of the system. So, we will have a maxima, that is the kind of profile that you would expect for this kind of a solution this is the temperature distribution.

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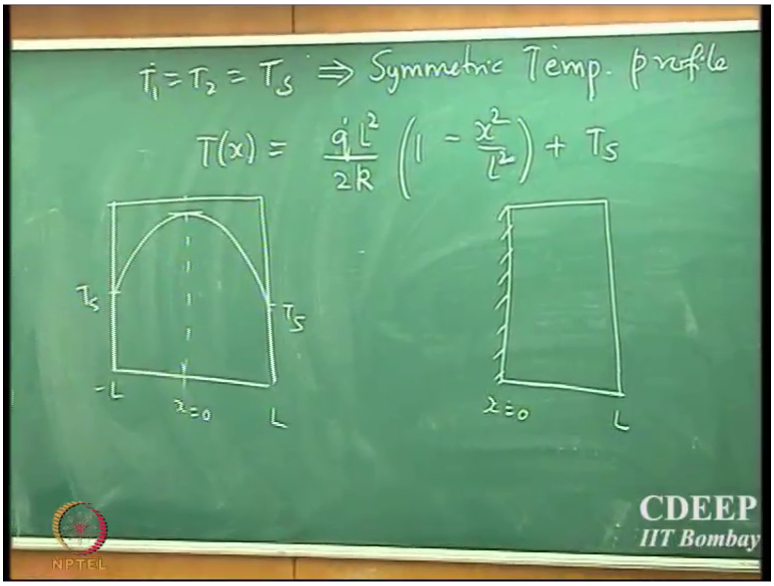

Now, supposing if T 1 is equal to T 2 equal to let us say a surface temperature, then the profile will be symmetric right. So, we should have a symmetric profile we will have a symmetric temperature profile and the solution would be T 2 is equal to T 1. So, the second term will disappear and this will simply be T s. So, the solution will be T x equal to q dot L square by 2 k into 1 minus x square by L square plus T s. So, that is the solution when the temperatures of the 2 surfaces either surfaces are equal to T s.

Now, this is an important observation and in fact, now I will explain why I chose this kind of a coordinate system right. So, this solution is also the solution of the problem where you have a an adiabatic boundary. So, let us say you have a slab where you have an adiabatic boundary at x equal to 0 that is there is no heat loss or there is no heat transfer complete heat transfer at x equal to 0 is 0.

So, remember the second boundary condition we discussed constant flux where here the constant is 0, there is no flux boundary condition. This is also the solution of this problem and the reason why it is this a solution is because if you look at the solution of minus L to plus L because a solution is symmetric it is symmetric at x equal to 0 the flux is the 0 right d T by d x is 0 at that location and therefore, the solution of this problem is also the solution of this problem. So, you got the solution of another problem for free simply by redefining the coordinate system and looking at the problem.

This is an important piece that you have to check, any system that you are looking at you will also have to see how to define your coordinate system. So, that your solution is going to give you much more insight than what you would normally get. So, this is very very important when you are solving engineering problems, how to define a coordinate system correctly, that you would get very good much better insight than what you would normally get otherwise alright. So, can we use resistance concept here can we define resistance for this problem what do you think if the temperatures are not same then there is a overall temperature difference what is the answer yes no why

Student: (Refer Time: 05:17).

That is right, when you have a generation term, this is very important when you have a heat generation term the whole concept of resistance network hinges in the fact that the total amount of heat that is generated inside the system is 0. There is no heat generation or heat loss from the system which means that the net heat transfer rate from the system is constant, when you have heat generation the net heat transfer rate at different locations in the system is not constant. So, this is very important to realize this distinction that you cannot use a resistance network concept whenever there is a heat generation or a heat sink term which is present in the system. So, we cannot use resistance concepts.

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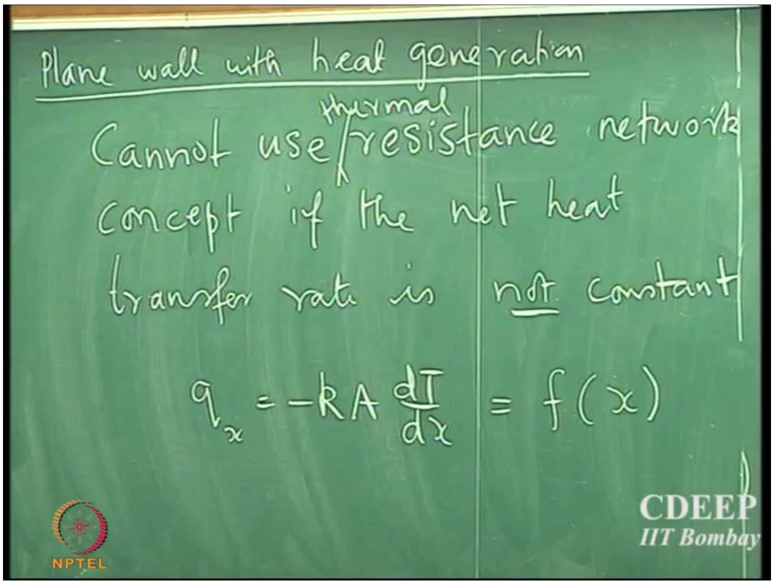

I should qualify by saying thermal resistance network if the net heat transfer rate is not constant and in fact, it is not difficult to realize this the net heat transfer rate depends upon the first derivative of the temperature right. So, q x is minus k A into d T by d x.

So, now that is going to be a function of position as you can clearly see it is a parabolic system and. So, the first derivative is going to be a function of position which means that the q x is now going to be a positionally dependent quantity. So, the heat transfer rate is no more constant at any position inside the slab.

So, this is a big difference between what you have with a system where there is no heat generation or heat sink versus the system where there is heat generation or a heat sink this is very important to understand this distinction. So, the resistance concept is not valid for the system where the net heat transfer rate is not constant if it is a function of position you cannot use the resistance network concept alright. So, what do we do, if we cannot use resistance network concept what do we do we have to deal with the original distribution.

So, note that when we use the resistance network concept we did not care about the distribution of the temperature as long as we know the resistances simply based on the resistances we will be able to represent the whole heat transport process in the form of a network and we are able to detect some properties, but if we cannot represent using network then we have to deal with the original equation itself. So, is this is a complete description can we find the temperature profile sure why not if we know q dot we can find it right.

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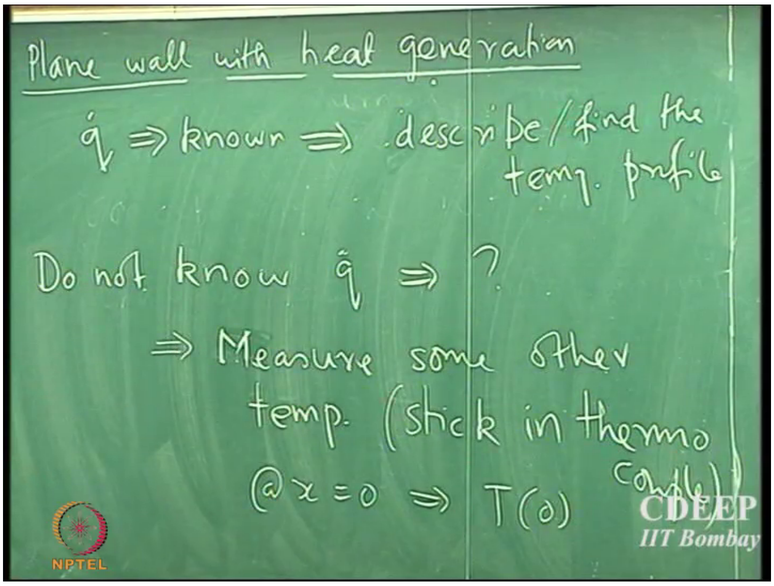

So, if q dot is known then we can describe or find the temperature profile and all the properties in terms of rate etcetera can be simply found by taking the first derivative.

Now, supposing we do not know what q dot is what should we do, supposing we do not know what is the net amount of heat that is generated per unit volume what should we do any suggestions.

Is it still a solvable problem can we still quantify what do you think do we need some additional information. So, the observation is at x equal to 0 when you said x equal to 0 then there is some simplification that occurs on this equation right. So, if q dot is not known then we should be able to measure some other temperature right. So, if q dot is not known then we need to at least know the temperature at some other location.

So, let us say we know how to do that, the rate is done and practice is that. So, there is something called a thermocouple. So, thermocouple is basically a copper wire which has a high conductivity. So, that the amount of heat that is transferred the time it takes for transferring the heat to measure the total amount of heat that is transferred and find the temperature is going to be extremely small. So, it is a fast response system and, you put a thermocouple in some location and then you can measure what is the temperature.

So, you will see lots of thermocouples in the lab course that you will do in this semester most of the experiments will have some measurement of temperature and, you will see lots of thermocouples going in different directions into the thermometer reader.

So, you will see there is a digital thermometer reader and you will see thermocouple going from different locations. So you could use a thermocouple, basically stick in some thermocouple and the appropriate point. So, this is a designed question supposing I have to stick in what is the appropriate point at x equal to 0. So, we can stick in at x equal to 0 and measure the temperature and let us say if that is call T 0.

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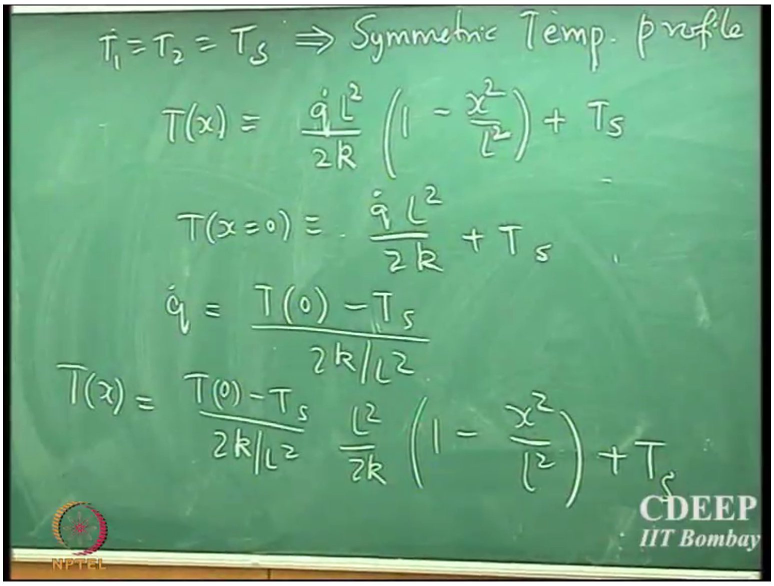

So, T 0 is the temperature at the centre of the system centre of the slab that we are looking at, T at x equal to is nothing, but q dot L square by 2 k plus T s. So, it is; obviously, not a function of the position and, from here we can find out that q dot is T 0 minus T s divided by 2 k by L square. So, that is q dot and, substituting that in the original equation you will find T x that s equal to T 0 minus T s divided by 2 k by L square into L square by 2 k into 1 minus x square by L square plus T s.

So, therefore, if it is unevenly distributed then you will have to know what is the distribution of q dot in the slab and you could consider that in our equation for example, the question is what happens if it is if q dot is unevenly distributed. So, you could simply rewrite your model.

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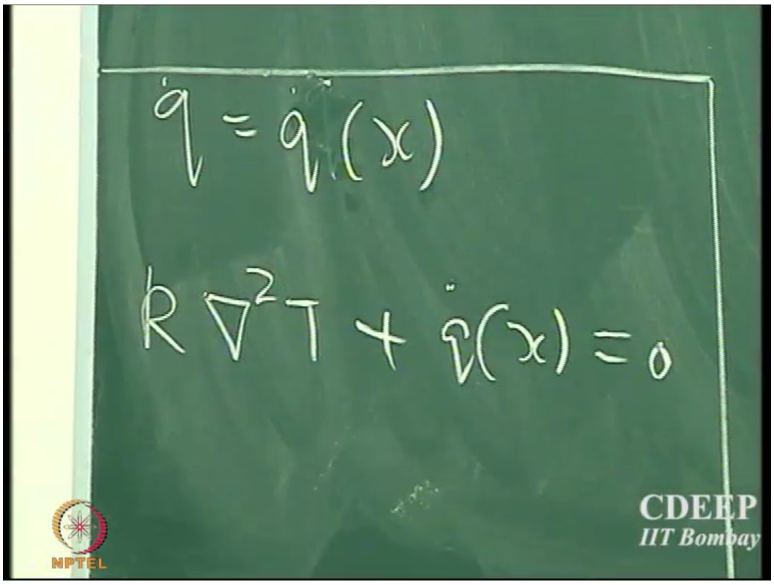

So, if q dot itself is a function of position what if q dot is a function of position. So, you could simply rewrite your model q dot as a function of position and solve. So, it would not matter you can easily take into account. So, when I assume q dot as constant it is completely without loss of generalization. So, if you know what is the function of q dot you could put that and you could solve the equation if it is solvable if it is non-linear then there are some restrictions it is linear you should be able to solve by enlarge most of the linear system.

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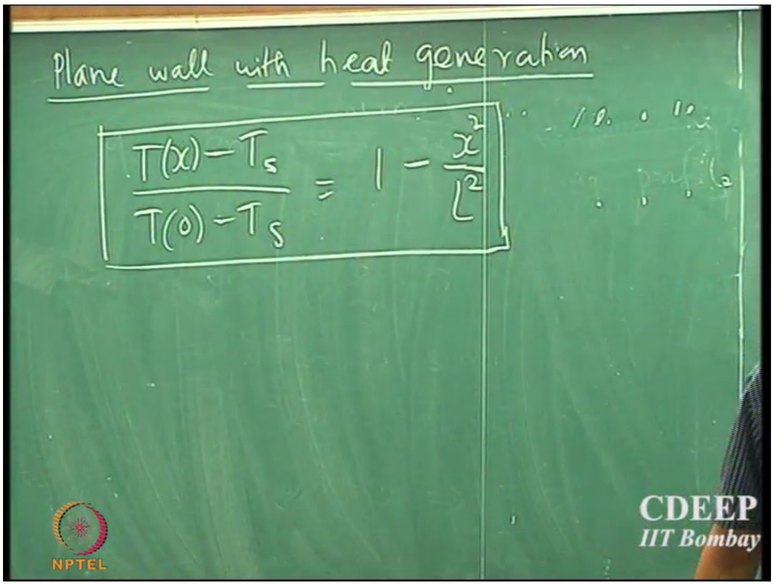

So, T x minus T s, we can rewrite the expression as minus T s that is equal to 1 minus x square by L square.

So, this kind of a reformulation of the solution is sometimes called as similarity solution we will actually see some of the similarity solutions when we talk about convection in a tube laminar flow in a tube, but these kinds of representation by using the maximum value or the symmetric temperature and scaling the solution is sometimes called as the similarity solution.

So, we will look at that sometime, but just for historical purposes I want to mention that this type of representation is called similarity solution. So, this ratio is now going to be a parabolic function and this is a non dimensional temperature. So, note that the quantity on the left hand side is non dimensional and this kind of non dimensionalization is often done in many engineering systems when we will actually use these kinds of non dimensionalization when we actually deal with convection problem any questions so, far.

So, let us move into the next problem, let us look at radial systems or cylinder with heat generation. So, let us represent the cylinder.

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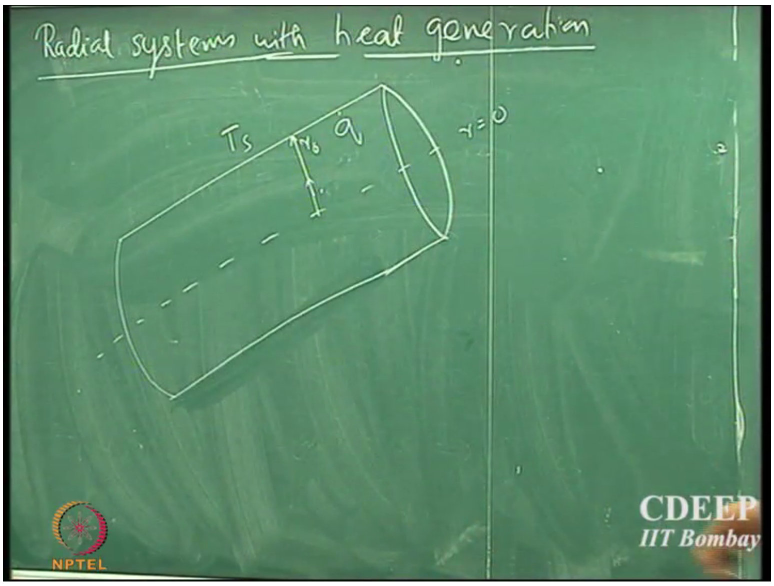

So, let us say that we have a solid cylinder and there is heat that is being generated at a volumetric rate of q dot as T watt per meter cube of heat that is being generated and if r naught is the radius of that cylinder and let us say that T s is the surface temperature. So, one could write a model and it is very simple the way we have done before k and if we assume that it is a 1 d system.

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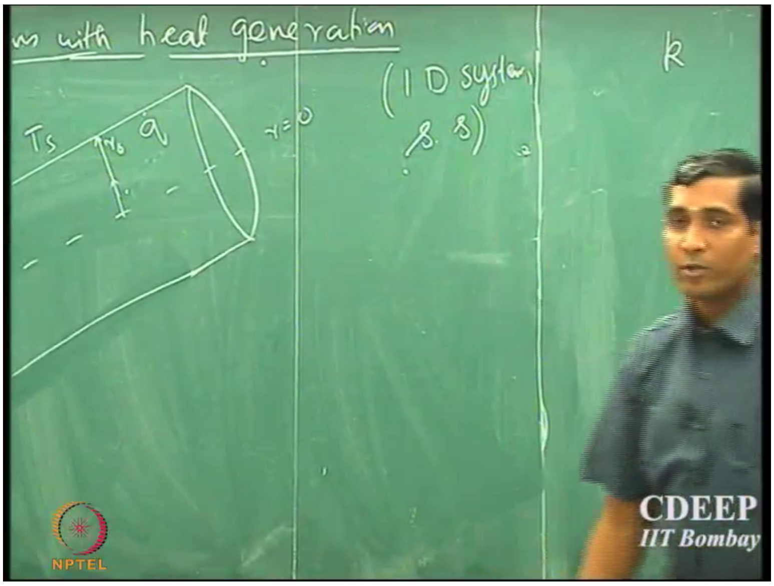

1 D system steady state 1 d steady state system then we can say k into 1 by r d by d r plus q dot equal to 0.

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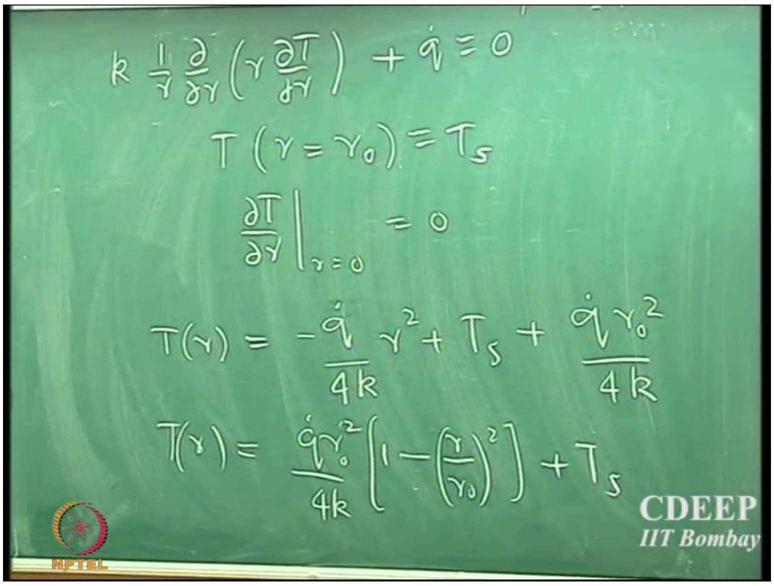

So, that is the model what are the boundary conditions you want to try.

Student: (Refer Time: 16:36).

T at r equal to r equal to r naught that is equal to T s and what about the other boundary condition it is a 2 point boundary value problem.

So d T by d r at r equal to 0 is 0 you can see no flux why it is no flux at r equal to 0. So, it is because there is a symmetry which is present in a natural system, this symmetry imposes the condition that d T by d r equal to 0 at r equal to0. So, again I am not going to solve this problem here solve this equations here.

So, the solution is T r is minus q dot by 4 k into r square plus T s q dot r naught square by 4 k and, that we can simply rewrite as q dot by 4 k into r naught square 1 minus r by r naught square plus T s. So, once again if we introduce a similar scaling as what we did before like looking at the midpoint temperature etcetera. So, supposing we know what is the midpoint temperature?

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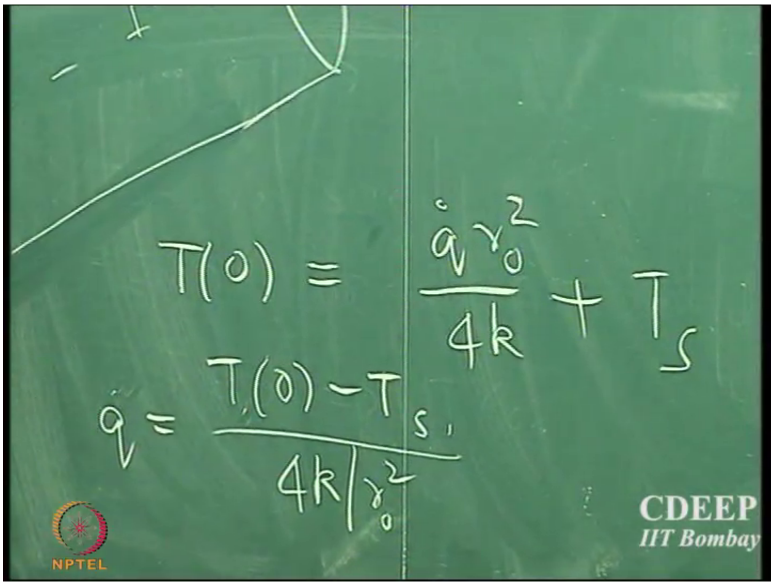

So, T naught is given by q dot r naught square by 4 k plus T s. So, that is the temperature at the midpoint and. So, q dot is given by T naught minus T s into 4 k by r naught square. So, it is exactly what we did before and we can substitute that into the temperature distribution and we will find that.

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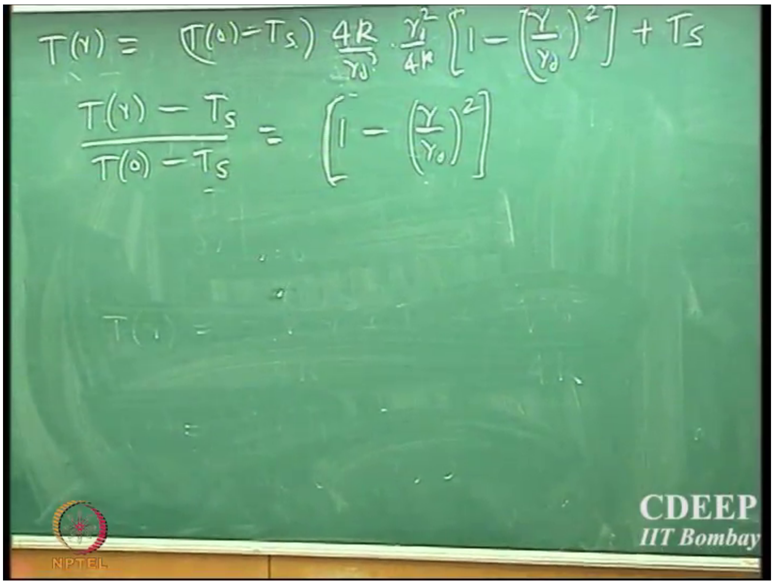

T r equal to T naught minus T s divided by 4 k by r naught square into r naught square by 4 k did I make a mistake

Student: (Refer Time: 19:17).

Yeah it should be the other way into 4 k by r naught square. So, T naught minus T s into 4 k by r naught square multiplied by r naught square by 4 k into 1 minus r by r naught square plus T s. So, we can rewrite this as T r minus T s divided by equal to 1 minus r by r naught the whole square. So, that s the temperature profile and once again we can clearly see that the temperature the heat transfer rate is a function of the radial position.

So, it is not constant anymore unlike what we saw in the radial system with the with no heat generation we saw that the heat transfer rate is constant; however, when there is generation heat transfer rate is not constant. So, whatever you would intuitively guess or whatever you would intuitively identify is something that will fall out from the equation if it does not fall out then what you intuited is most likely wrong or the model equation is wrong either of these two.

So, you have to look at these 2 things whenever you look at any problem both in the problem solving things for this course and for any other engineering system, you have to intuit first and then you write the model equation.