Heat Transfer

Prof. Ganesh Viswanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture - 60
Epsilon – NTU Method

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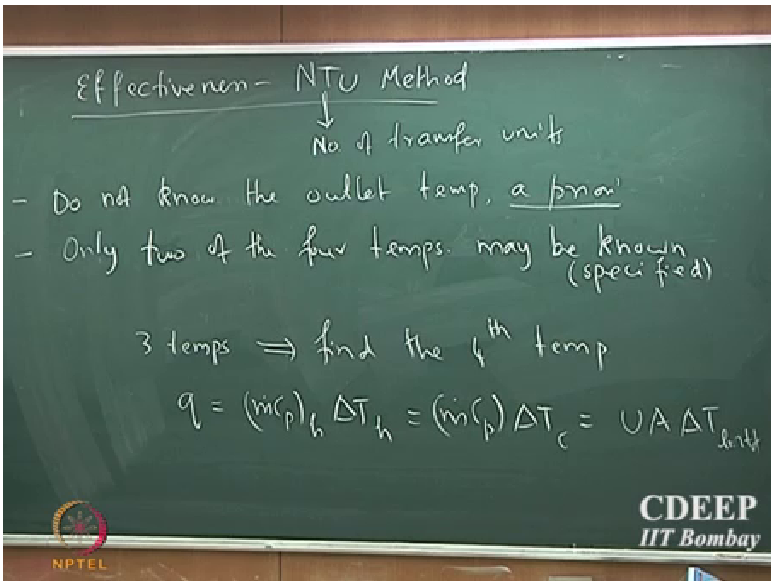

Effectiveness – NTU Method

↓ No of transfer units

- Do not know the outlet temp. a priori
- Only two of the four temps. may be known (specified)

3 temps ⇒ find the 4th temp

$$q = (\dot{m}C_p)_h \Delta T_h = (\dot{m}C_p) \Delta T_c = UA \Delta T_{lmtd}$$

So, it is called the effectiveness NTU method ok. NTU stands for Number of Transfer Units, while we derive this epsilon NTU method you will see the definition of NTU. So, I will not go over it now.

Now, what is important here is that the genesis of this method is based on the fact that in industries often these heat exchangers are connected to a reactor or connected to an outlet stream, corrected to a stiller or something some other process equipment therefore, often we do not know outlet temperature a priori on this is very important we may not know it a priori, ok. So, remember that once you have fabricated a shell and tube heat exchanger there is no point in working out the design after that.

So, you are interested to work out the design before you fabricate the heat exchanger that is one possibility. The second possibility is suppose I want to modify my process remember I told you this battery example where they make gamma m n o 2. So, it is the same plant which manufactures the acidic and the alkaline gamma m n o 2.

So, in order to manufacture different chemicals you do not construct a new plant you always attempt to maximize the equipments and the process that you have built. Therefore, it is often you may know you may realize that you may not know what is the exact outlet temperature or sometimes it could be vice versa your outlet temperatures may have been specified because you may know what should be the initial conditions for thee or the inlet conditions for a reactor, but you may not know what is the inlet conditions for the heat exchanger because it controls some other process. So, it turns out that under many industrial settings only two of the of the 4 temperatures may be known. I may even say that may be specified because remember that you are always interested in designing before fabrication. So, so only two of the 4 temperatures may be specified

Now, if you use delta T lmtd method, remember that when we start use this delta T lmtd example we saw that suppose if we know 3 temperatures we can find the 4th one. So, if we know any 3 you should be able to find the 4th one by simply writing on m dot C p delta T equal to. So, m dot C p delta T for the cold stream. But then if only two temperatures are specified you can still use delta T lmtd it does not mean that you cannot use it the way to do that is.

So, you have q is m dot C p hot fluid delta T hot fluid ok. You actually got 3 equivalences using this relationship. So, you could guess what is one of the temperatures. So, you could guess. So, the first step would be let us say that the T h i, and T c i are given the inlet temperatures of the hot and the cold fluid are given to you, ok.

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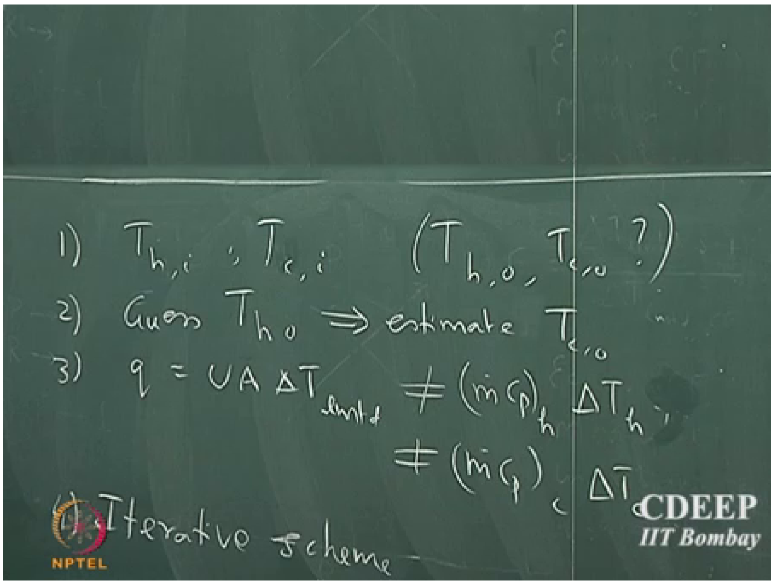

1) $T_{h,i}$; $T_{c,i}$ $(T_{h,o}, T_{c,o}\ ?)$

2) Guess $T_{h\,o}$ $\Rightarrow$ estimate $T_{c,o}$

3) $q = UA\,\Delta T_{lmtd} \neq (\dot{m} C_p)_h \Delta T_h$,

$\neq (\dot{m} C_p)_c \Delta T_c$

4) Iterative scheme

Now, we do not know what are the T h o and T c o or not know ok, we do not know these two you only know these two quantities we do not know these two quantities. So, if I use the delta T lmtd method a way to go about it I can guess what should be d T h o I can guess that ok. From this I can estimate T c o I can do that because I can always write up a simple m C p delta T equivalence and I can find out the cold outlet temperature. Then from here I can find out UA delta T lmtd assuming that the area and the transport coefficient is more I can find out delta T lmtd.

Now obviously, this is not going to match with the m dot C p delta T I this is not going to be equal to and that is not and that is and that is not going to be equal to m dot C p cold simply because the initial guessed that you made is not right we know that it is not right. So, guess, so one could actually set up an iterative problem ok. So, one could set up an iterative problem.

How do you set up an iterative scheme? We know what is q based on this and we know what is the UA delta T lmtd. So, we should be able to independently estimate what is T c o based on this gap. So, we could use T c o as an object to temperature and then we can minimize the error that you would get by estimating T c o from these two expressions that is one way there are many ways to do this many ways to post this problem. So, one you need to use an iterative scheme in order to estimate all 4 temperatures. So, that is not a very easy thing to do. So, in fact, in this premise is what the epsilon NTU method was actually discovered where you do not have to follow such cumbersome iterative scheme you can simply use the epsilon NTU method which gives you a very elegant way to design the heat exchanger under this kind of a situation and that is what we are going to see for the rest of the class today.

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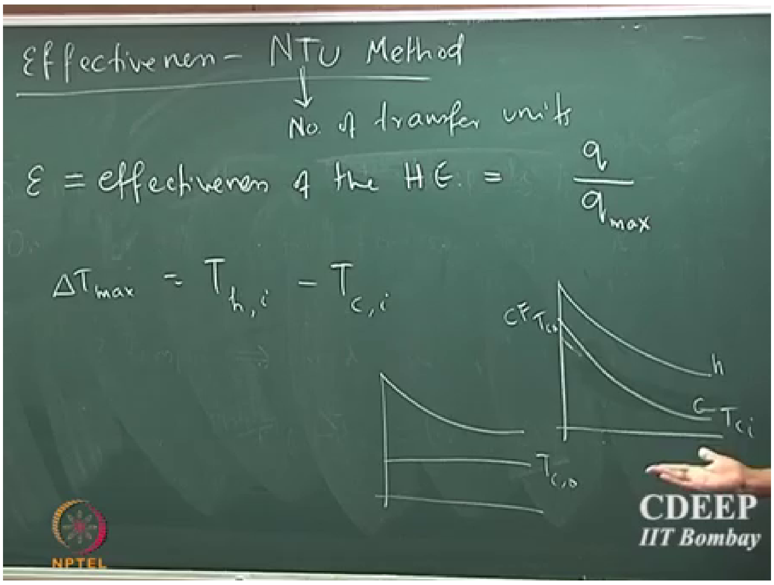

Effectiveness – NTU Method

NTU → No. of transfer units

ε = effectiveness of the HE = $\frac{q}{q_{max}}$

$\Delta T_{max} = T_{h,i} - T_{c,i}$

So, I define epsilon as the efficiency or the effectiveness of the heat exchanger ok. So, epsilon is the effectiveness of the heat exchanger, and that is simply defined as the actual amount of heat that is transporter which is q divided by the maximum possible heat transfer.

So, now, if I know how to estimate q max, I am done I can find out what is the efficiency of heat transfer. How do I find q max? What is the maximum possibility of temperature difference in any heat exchanger, maximum possible.

Student: Delta T (Refer Time: 08:17).

Delta T.

Student: T (Refer Time: 08:20).

Not really we know that. So, for example, if I take a counter flow it is definitely not the case in a parallel flow we know that ok. Suppose I take a counter flow ok. So, this is the temperature profile that you would get it is not parallel let us say this is the temperature profile ok. So, this is the hot stream and this is the cold stream ok, so T c i and T c o, ok.

Now, I can always have a situation the cold the temperature of the cold stream is always going to increase right it is never going to decrease, it is always going to increase because it is always gaining sensibility. Now, what is the maximum possible temperature difference when I maintain the cold stream at a constant temperature? That is the maximum possible temperature difference in a counter flow, where the temperature of the cold fluid does not change how can I achieve this? We looked at some of the special cases I can do the same thing.

Student: (Refer Time: 09:40).

Right. So, if I, yes exactly. So, you can do it either way it does not matter. So, all it simply means that this is the maximum possible temperature difference that you can achieve in the heat exchanger at any location you can look at any location you want and this is the maximum possible this is the theoretical maximum possible temperature difference that you can achieve, ok.

So, now, having said that can we find out what is the. So, if we know the maximum possible temperature difference how do we find the maximum possible heat transport rate. Remember we want q max we do not want delta T max, we want delta T max, but really to define efficiency what we need is q max we need the maximum possible heat transport rate. So, how do we find that?

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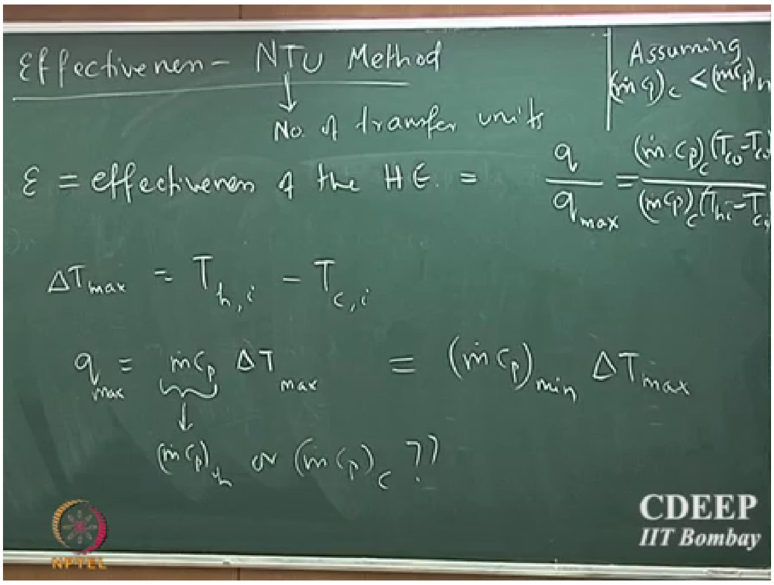

Effectiveness – NTU Method

$\downarrow$

No. of transfer units

Assuming

$(\dot{m} c_p)_c < (\dot{m} c_p)$

$$\varepsilon = \text{effectiveness of the HE} = \frac{q}{q_{max}} = \frac{(\dot{m} \cdot c_p)_c (T_{co} - T}{(\dot{m} c_p)_c (T_{hi} - T}$$

$$\Delta T_{max} = T_{h,i} - T_{c,i}$$

$$q_{max} = \underbrace{\dot{m} c_p}_{\substack{\downarrow \\ (\dot{m} c_p)_h \text{ or } (\dot{m} c_p)_c \,??}} \Delta T_{max} = (\dot{m} c_p)_{min} \Delta T_{max}$$

You remember that q is simply defined as some m dot c p into delta T, ok. So, one could intuitively guess that q max has to be something to do with delta T max, but the question is what m dot C p that you should use. If we have 2 m dot C p here you have m dot C p of hot fluid or m dot C p of cold fluid which one should be used to define the maximum heat transport rate? One that is higher why?.

Student: (Refer Time: 11:08).

No, not really. Suppose I say that the under steady state condition ok. m dot C p of cold fluid T h o, right. So, that is the simple steady state balance.

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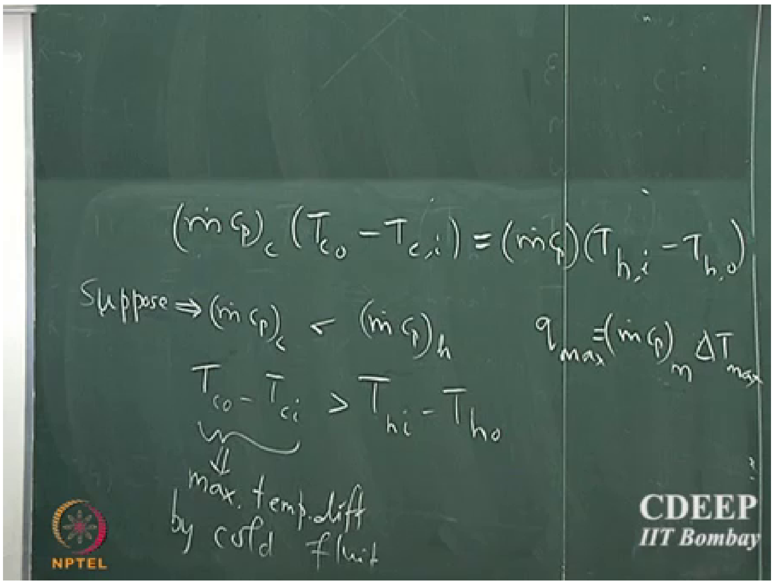

Now, suppose if I say that m dot C p of the cold fluid ok, if it is lesser than m dot C p of the hot fluid, ok. Now, the exercise is to find out theoretically which of these two streams is actually going to be the maximal temperature difference, right. So, really what you need to use to find the maximum heat transport rate is find out theoretically as to which stream is likely to achieve the theoretical temperature difference only that streams m dot C p is what you have to use to find out the maximal heat transport rate ok.

So, q max is m dot C p of that stream I call it m let us say delta T max. So, the exercise is to find out which stream is going to theoretically achieve the maximal heat maximal temperature difference. How do we find this? So, if m dot C p of c is less than m dot C p of h ok. Which one will have the maximal temperature difference? It is the is a cold fluid. So, it is very easy to see that. So, T c o minus T c i will always be greater than T h i minus T h o. So, that gives you a very clear way to find out which stream is theoretically possible to have maximal temperature difference. So, if the m dot C p of the cold fluid is less than the m dot C p of the hot fluid then it is the cold fluid which will always achieve the maximal temperature difference, by cold fluid for this example.

If we specify suppose that this is the m dot C p of the cold fluid is lesser than the m dot C p of the hot fluid remember these are the C p is the property of the fluid and m dot is the flow rate So, these are actually control m dot has a controllable parameter because you have a pump which is going to flow the fluid and C p is an intrinsic parameter. So, you should be able to control these which one is maximum which one is minimum, ok. So, the capacity of the fluid multiplied by the mass flow rate whichever is minimum that fluid stream is the one which will likely to have a or maximal temperature difference. So, therefore, you max is essentially defined as m dot C p minimum multiplied by delta T max ok. So, this is an important observation. The maximum possible heat transport rate is essentially the product of the minimum m dot C p between the two multiplied by the maximal temperature difference which is the difference in the inlet temperatures of the two streams ok.

So now, we know how to define q max ok. So, we can simply define efficiency as. So, if I define efficiency in terms of the cold fluid for this example. So, it will be m dot C p of cold fluid multiplied by T c o minus T c i divided by m dot C p of cold fluid T h i minus T c i, ok. So, this is by assuming m dot C p of cold fluid is less than m dot C p of hot all right. So, therefore, suppose if. So, this is a very generic definition. So, he respects you off which fluid is going to have maximal temperature you can always define in terms of the minimum capacity multiplied by the maximal temperature difference that you tell you what will be theoretical maximum possible heat transport rates, ok.

So, therefore, if suppose m dot C p of the hot fluid is less than m dot C p of the cold fluid one can immediately define epsilon asked q by q max m dot C p hot fluid T h i minus T h o divided by m dot C p of hot fluid T h i minus T c i.

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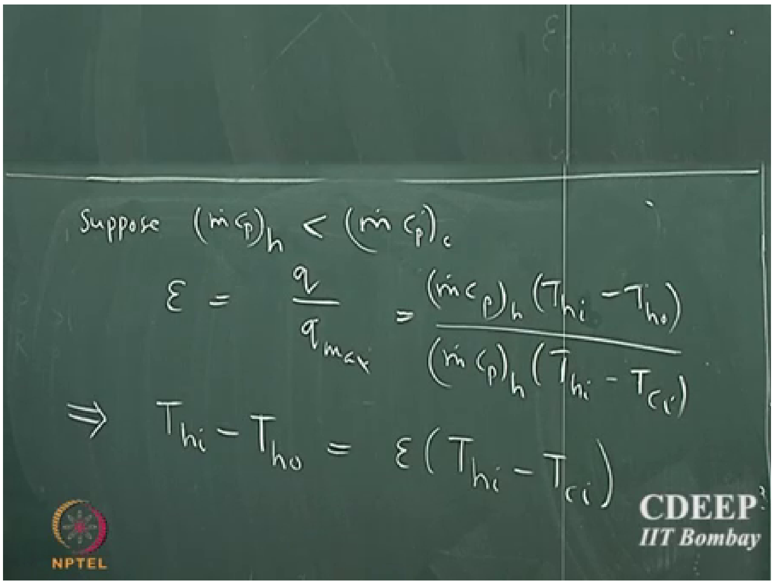

Suppose $(\dot{m} c_p)_h < (\dot{m} c_p)_c$

$$\varepsilon = \frac{q}{q_{max}} = \frac{(\dot{m} c_p)_h (T_{hi} - T_{ho})}{(\dot{m} c_p)_h (T_{hi} - T_{ci})}$$

$$\Rightarrow T_{hi} - T_{ho} = \varepsilon (T_{hi} - T_{ci})$$

So, from here we can see that T h i minus T h o that is equal to epsilon into T h i minus T c i. So, this provides the mechanism to eliminate the one of the temperatures remember that you got in your temperatures in this expression here it provides a mechanism to eliminate the one of the temperatures that is we could eliminate T h o from here we will see how to do that in a short while, ok. So, what it simply means is that we need to know only the efficiency of the heat exchanger. If we know the efficiency of heat exchanger then you are able to independently estimate the out outlet temperature of the hot fluid based on the inlet temperature.

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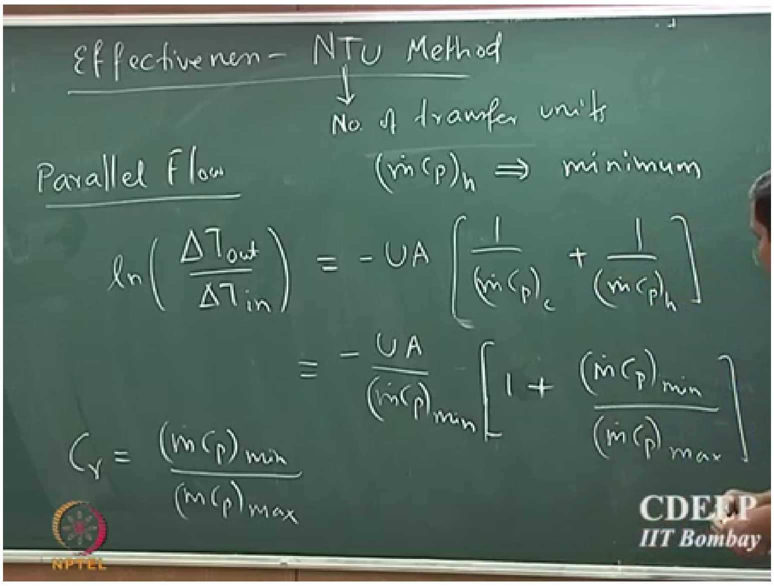

Effectiveness – NTU Method

↓ No. of transfer units

Parallel Flow

$(\dot{m}C_p)_h \Rightarrow$ minimum

$$\ln\left(\frac{\Delta T_{out}}{\Delta T_{in}}\right) = -UA\left[\frac{1}{(\dot{m}C_p)_c} + \frac{1}{(\dot{m}C_p)_h}\right]$$

$$= -\frac{UA}{(\dot{m}C_p)_{min}}\left[1 + \frac{(\dot{m}C_p)_{min}}{(\dot{m}C_p)_{max}}\right]$$

$$C_r = \frac{(\dot{m}C_p)_{min}}{(\dot{m}C_p)_{max}}$$

So, let us take the case of a parallel flow ok. So, you may recall from the last lecture. So, delta T that is equal to does anyone remember minus UA into one by m dot C p hold plus one by m dot C p hot ok, because if you modify this what you will get is q equal to UA delta T lmtd what you will get ok.

So, now, suppose I assume that m dot C p of the hot fluid is the minimum ok, assume that that is the that is the smaller than the m dot C p of a cold fluid, ok. So, now, I can rewrite this as minus UA divided by m dot C p min 1 plus ok. So, I have done is I have just pulled out the minimum that is in this case the m dot C p of the hot fluid ok. So, if I define C r as the ratio of the capacities m dot C p min to max, ok. So, I define m dot C p min divided by m dot C p max ok. So, I define that as C r. So, I can plug this in here. So, what I will get is to delta T minus UA divided by T 1 plus C r.

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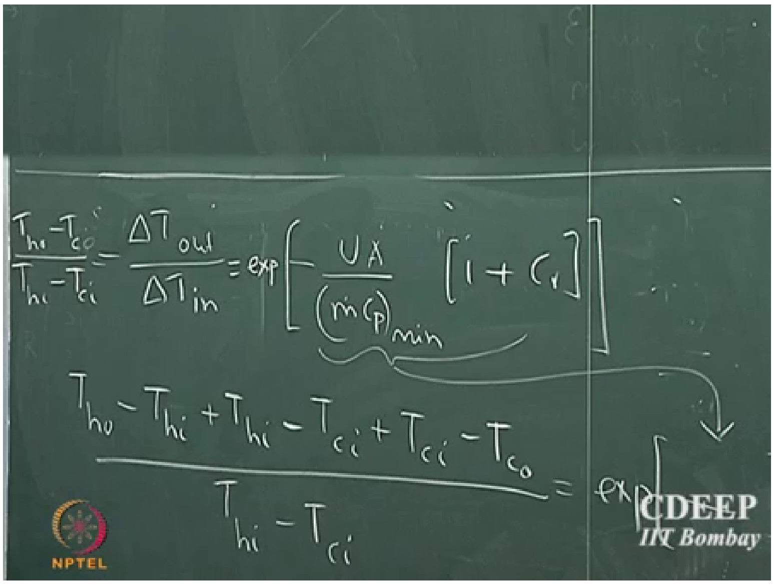

$$\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}} = \frac{\Delta T_{out}}{\Delta T_{in}} = \exp\left[-\underbrace{\frac{UA}{(\dot{m}C_p)_{min}}\left[1+C_r\right]}\right]$$

$$\frac{T_{ho}-T_{hi}+T_{hi}-T_{ci}+T_{ci}-T_{co}}{T_{hi}-T_{ci}} = \exp\left[\right.$$

Now, this is T h i minus T h T c i of exponential please add an exponential ok. So, T out is T h o minus T c o. So, that is why it is a parallel flow we are looking at parallel flow divided by T h i minus T c i that is the inlet stream ok.

So, now, we can rewrite this as T h o minus T h i, all I am doing is I am adding and subtracting the inlet and the outlet stream temperatures plus T h i minus T c i divided by ok, have not done anything spectacular here all I have done is I have just added c i and subtracted c i and that should be equal to the exponential of whatever term that present about that goes. So, we know that T h o T h i minus T h o that is equal to epsilon into T h i minus T c i are simply based on the definition of effectiveness ok.

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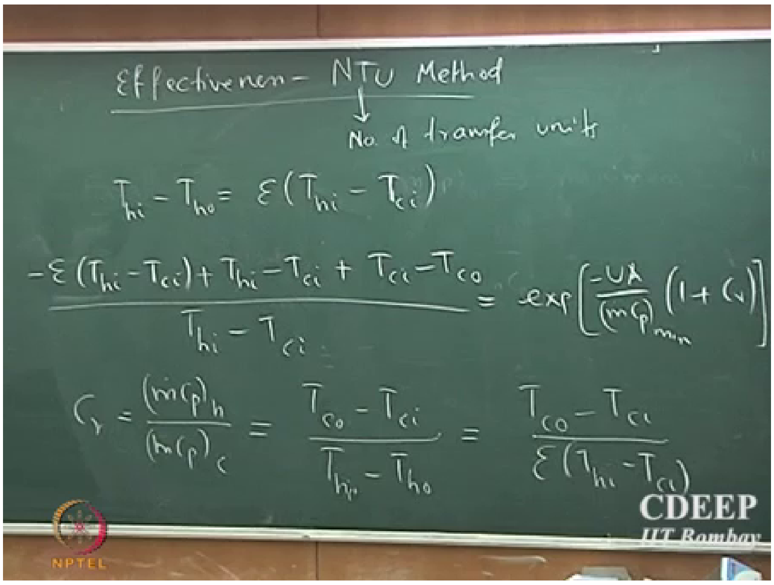

Effectiveness – NTU Method

$\downarrow$ No. of transfer units

$$T_{hi} - T_{ho} = \varepsilon (T_{hi} - T_{ci})$$

$$\frac{-\varepsilon (T_{hi} - T_{ci}) + T_{hi} - T_{ci} + T_{ci} - T_{co}}{T_{hi} - T_{ci}} = \exp\left[\frac{-UA}{(\dot{m}C_p)_{min}} (1 + C_r)\right]$$

$$C_r = \frac{(\dot{m}C_p)_h}{(\dot{m}C_p)_c} = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ho}} = \frac{T_{co} - T_{ci}}{\varepsilon (T_{hi} - T_{ci})}$$

So, from here we can substitute this. So, what you will get is minus epsilon into T h i minus T c i ok, plus T c i minus T c o divided by yeah, ok. So, now so we have eliminated T h o, from this expression we need to eliminate T c o how do we do this how do we eliminate T c o using the heat balance. So, we have m dot C p of hot fluid divided by m dot C p of cold fluid ok, which is basically C r over that that is the definition right that should be equal to T c o minus T c i divided by T h i minus T h o, right.

So, from here can you find out what is T c i minus T c o can we eliminate T c o from here is it possible of course, yes. We know that T h i minus T h o’s epsilon T h i minus T c i that is nothing but T c o minus T c i divided by epsilon into T h i minus T c i, ok. So, we can simply plug this in here. So, that will be minus epsilon T h i minus T c i plus T h i minus T c i minus.

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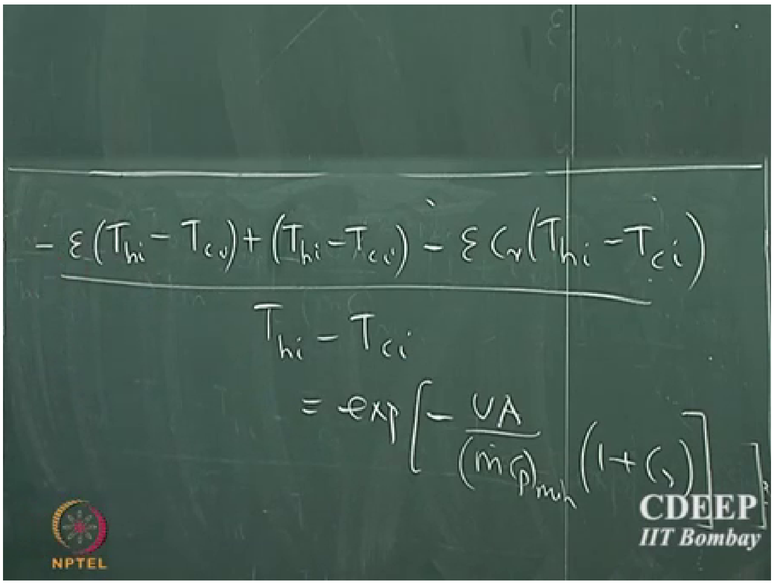

$$\frac{-\varepsilon(T_{hi}-T_{ci})+(T_{hi}-T_{ci})-\varepsilon C_r(T_{hi}-T_{ci})}{T_{hi}-T_{ci}} = \exp\left[-\frac{UA}{(\dot{m}C_p)_{min}}(1+C_r)\right]$$

So, all I have to do is now, to it completely I can eliminate T c i and T c o from here. So, there will be epsilon C r T h i minus T c i that divided by T h i minus T c i that should be equal to exponential minus UA by m dot C p min into 1 plus C r. So, now, I can cancel out T h i, T c i from here and so what I get is will be minus epsilon into 1 plus C r plus 1 that should be equal to exponential of minus UA by m dot C p min into 1 plus C r, ok.

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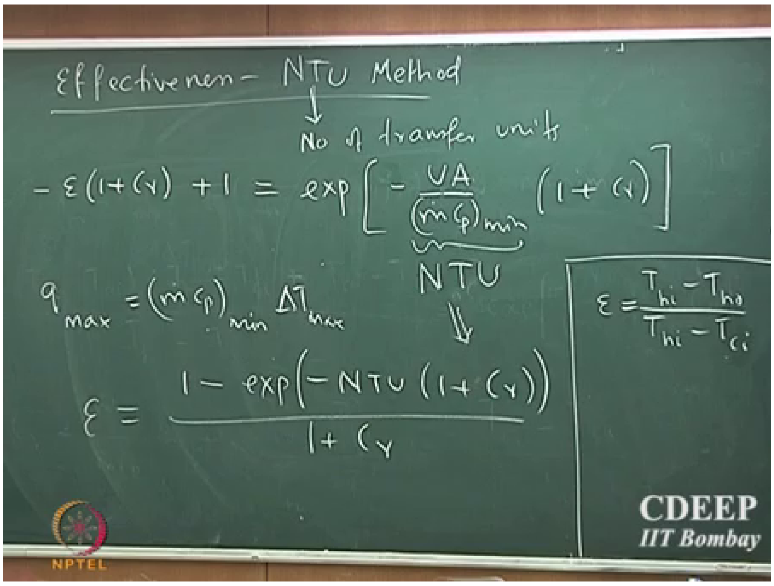

Effectiveness - NTU Method

NTU → No of transfer units

$$-\varepsilon(1+C_r)+1 = \exp\left[-\underbrace{\frac{UA}{(\dot{m}C_p)_{min}}}_{NTU}(1+C_r)\right]$$

$$q_{max} = (\dot{m}C_p)_{min}\,\Delta T_{max}$$

$$\Downarrow$$

$$\varepsilon = \frac{1-\exp(-NTU(1+C_r))}{1+C_r}$$

$$\varepsilon = \frac{T_{hi}-T_{ho}}{T_{hi}-T_{ci}}$$

So, this is UA by m dot C p min is what is called as NTU you called the number of transfer units. So, look at this expression. So, UA what is 1 by UA? What 1 by UA it is the resistance offered for heat transport between the two fluids right. And what is 1 by m dot C p min? It is the resistance offered by the fluid which has a maximum capacity to take heat take sensible heat right maximum. Remember m dot C p min is what is used for defining q max. Remember that q max is defined as m dot C p min multiplied by delta T max. So, this is the resistance that is offered by the fluid which has the theoretical capability to take maximum sensible heat, ok.

So, the number of transfer unit is actually an important design parameter the resistance for heat transport between the two fluid depends upon the area of heat transport. So, the design a meter comes in and NTU and m dot C p min tells you what is the resistance offered by the fluid which is capable of taking maximum possibility, ok.

So, therefore, we can define. So, the final expression would look something like this. So, epsilon equal to one minus exponential of minus NTU into 1 plus C r divided by 1 plus C r, ok. So, I have done is I have just manipulated this algebraic manipulation take this on the left hand side take this on the right hand side and find out what is f c, ok. So, you find no NTU I can find epsilon if I know epsilon I can find in these. So, that is a very powerful method. Remember that note that there is no temperature dependence in this expression at all.

Now, if I know epsilon I can find out what is the one of the temperatures. So, remember that epsilon; epsilon is given by T h i minus T h o divided by T h i minus T c i. So, if I know NTU which is simply dependent on UA and m dot C p min I can find epsilon and if I know epsilon I can find T h o. So, the sequence would be, so the sequence of calculations would be if I first this I find you and second is I find NTU excuse me suppose I know the area of heat transport, already.

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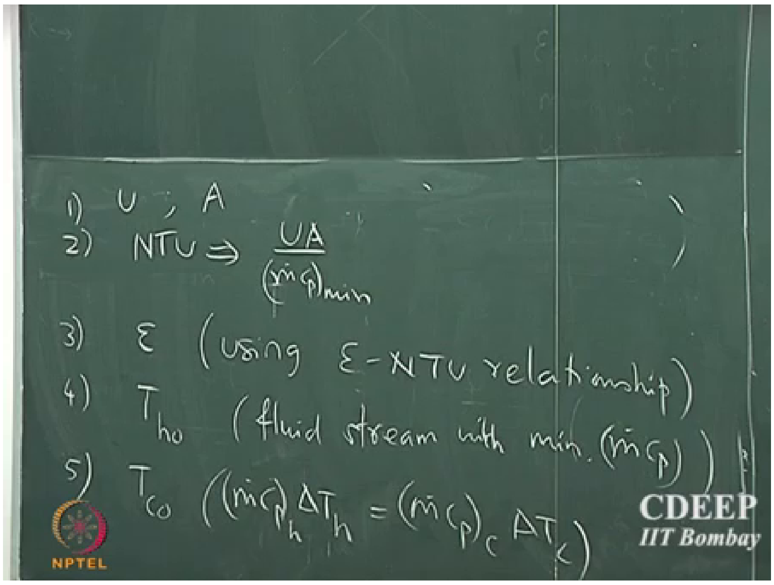

1) U ; A

2) NTU $\Rightarrow \dfrac{UA}{(\dot{m}C_p)_{min}}$

3) ε (using ε-NTU relationship)

4) T_{ho} (fluid stream with min. $(\dot{m}C_p)$)

5) T_{co} ($(\dot{m}C_p)_h \Delta T_h = (\dot{m}C_p)_c \Delta T_c$)

So, I can find UA by m dot C p min then from here I can estimate epsilon. So, using the epsilon NTU relationship epsilon NTU relationship from epsilon I can find out the outlet temperature of fluid stream with minimum m dot C p, ok. I can find out the outer temperature of the fluid stream with minimum m dot C p of and then how do I find the other temperature simply use heat balance, right.

So, we know that we can find T c o by using m dot C p delta T hot fluid equal to m dot C p cold fluid delta T hot, ok. So, what we have suddenly found we have found a powerful method. So, even if you do not know the 2 of the 4 temperatures you do not have to resort to the cumbersome delta T lmtd method you can simply use the epsilon NTU method which provides a nice and simple way to find out the temperatures you can do vice versa.

If you know one of these temperatures you can actually use you can do the reverse if you know one of the temperatures, you can find the other one you can find epsilon you can find it you can find the NTU, and you can find the area. So, you can do the reverse if you want to find the area of heat transport for a given problem you can do the reverse step and you will find the area of heat transport is the area and the parameters of the heat exchanger is given you can go forward and find out all the temperatures.

So, this is very useful particularly if you want to use the same set of equipments or a slightly different process and this is very very common in industry. So, it is never the case that you have individual units for manufacturing individual chemicals they are always shared particularly in Pharma industries and many other places it is all very shared So, if you know that you want to introduce a new process you want to increase manufacture of new chemical you know you cannot dismantle the process. So, with the existing process you should be able to design your parameters such that you can use the existing equivalents in order to achieve the desired output. So, these kinds of reverse calculations are very very useful for that kind of problem.

So, such epsilon NTU has been worked out for all kinds of configurations. In fact, I would recommend that you should derive some of these and check it by yourself I am going to give you these expressions. So, for a counter flow epsilon is equal to one minus potential of minus NTU into 1 minus C r divided by 1 minus C r exponential of minus NTU into 1 minus C r. So, that is for the counter flow ok.

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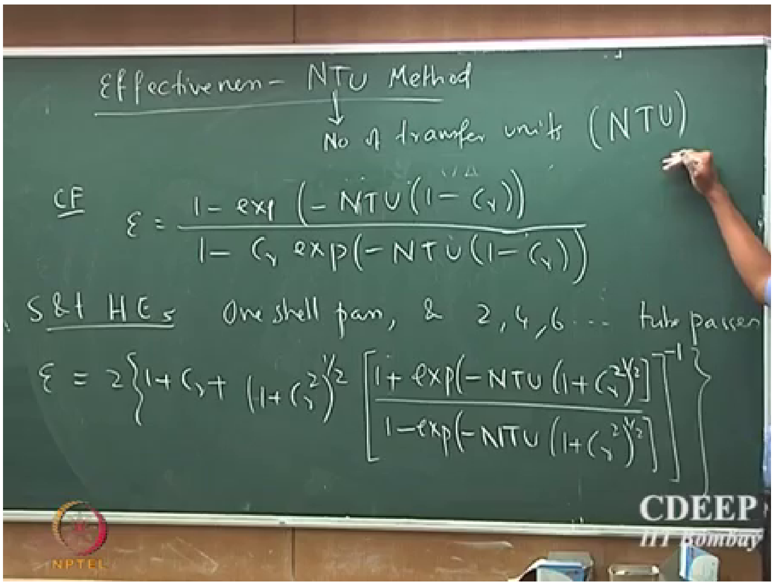

Effectiveness – NTU Method

No of transfer units (NTU)

CF

$$\varepsilon = \frac{1 - \exp\left(-NTU(1 - C_r)\right)}{1 - C_r \exp\left(-NTU(1 - C_r)\right)}$$

S&T HEs One shell pass, & 2, 4, 6 ... tube passes

$$\varepsilon = 2\left\{1 + C_r + (1 + C_r^2)^{1/2}\left[\frac{1 + \exp(-NTU(1 + C_r^2)^{1/2}]}{[1 - \exp(-NTU(1 + C_r^2)^{1/2}]}\right]^{-1}\right\}$$

And for shell and tube heat exchangers for shell and tube heat exchanger, now, this is for one shell pass and 2 4 6 etcetera tube passes; obviously, it has to be even number, ok. So, epsilon will be 2 1 plus C r plus 1 plus C r square to the power of 1 by 2, 2 1 plus exponential of minus NTU divided by 1 minus NTU to the power of minus 1 ok, bracket outside, ok. So, that is the expression for one shell pass and this was been derived for all combinations of heat exchange, ok.

So, remember that, so we sort of finish all the theoretical aspects of heat exchanges. So, what we are going to do next is we are going to look at some example problems and we will march on to the next issue. Now, before we do that I want to emphasize this concept of NTU in the first time you are hearing this course and first time probably you are hearing in a chemical engineering course.

So, this is something that you will see very often in many different calculations. You will see a lot more of the transfer unit concept, it transfer unit concept or mass transport course mass transport operation course and in fact, you will see that when you design a power distillation tower or any column the transfer units is used as a standard units for designing the height designing what should be loading etcetera.

So, NTU this concept is an extremely important concept in chemical engineering and I would strongly urge you to pay special attention to this and try to understand what this means. Remember that it is the ratio of the resistance offered for transport of heat from one fluid together divided by the resistance offered by the fluid which is capable of transporting or which the capable of taking up maximum possible heat transfer, ok. So, that is an important definition and similar definitions you will see under many different contexts, ok.