Heat Transfer
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Lecture – 06
Resistances in radial systems

So, I would still look at the 1-D system, we will look at 2-D systems later, but let us look at 1-D system in the radial coordinates.

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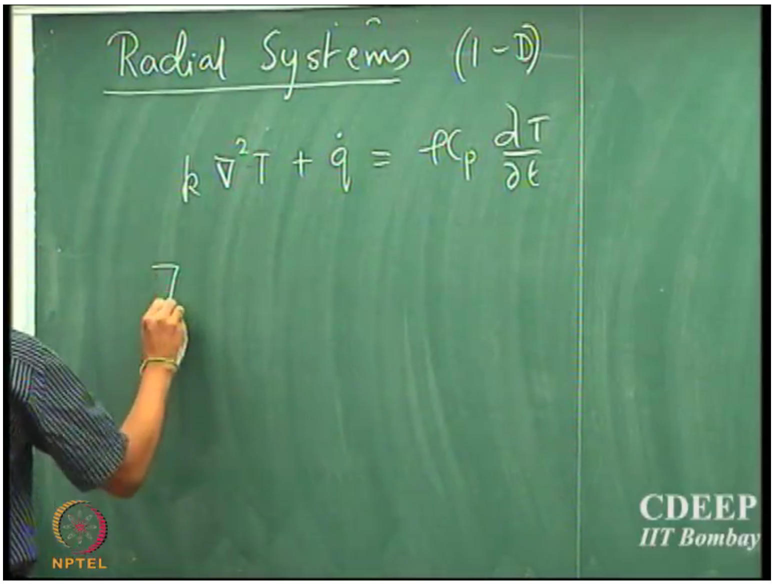

So, the general equation as we derived in the second lecture is that the it is k into del square T plus q dot equal to rho C p into partial derivative doh T by doh t. So, now, we need to write the Laplacian in the radial coordinates and that is not a difficult task to do, people have worked it out.

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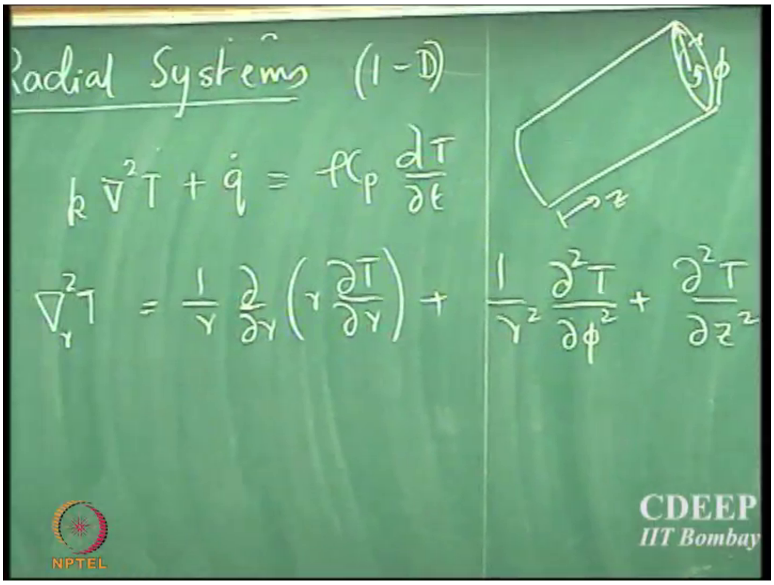

So, del square T in radial coordinates; so, when I use subscript r in Laplacian it means radial coordinate system, so, that will be 1 by r dee by dee r into dee t by dee r plus 1 by r square dee square T by dee phi square plus dee square T by dee z square. Well, in a moment I will draw the coordinate. So, if this is a cylinder, that is the z direction and this is the radial direction and the curve around it, so there is an angle around every radial point and that angle is what is called as phi. So, these are the 3 coordinates of a cylindrical system and this is the general Laplacian in the cylindrical coordinate system.

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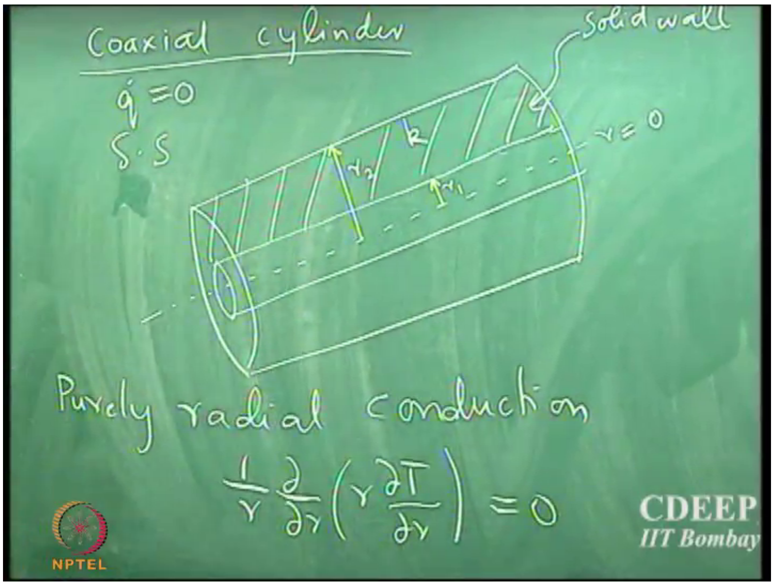

So, now, what we are going to do is, we are going to look at the coaxial cylindrical system. So, the problem we are going to take is coaxial cylinder. So, the centre is here this is r equal to 0 and if the radius of the inner cylinder is r 1 and the radius of the outer cylinder is r 2.

Now, let us assume that it is a solid volume wall here. So, it is a solid wall. So, we are looking at heat transfer between the coaxial cylinders. It could even be filled with a liquid, it could even be filled with other kinds of fluids, but let us consider that it is a solid wall for now and let us assume that the k is the thermal conductivity of the material of the solid wall. And so, what is the model equation?

So, supposing we assume that q dot is 0 and steady state condition what is the model equation. So, it is del square T is 0. So, if we assume that it is 1-D system, that is purely radial conduction then your model is 1 by r dee by dee r equal to 0, that is the model equation and if I specify that the temperature here is T 1 and temperature here is T 2 and I could easily read out the boundary conditions.

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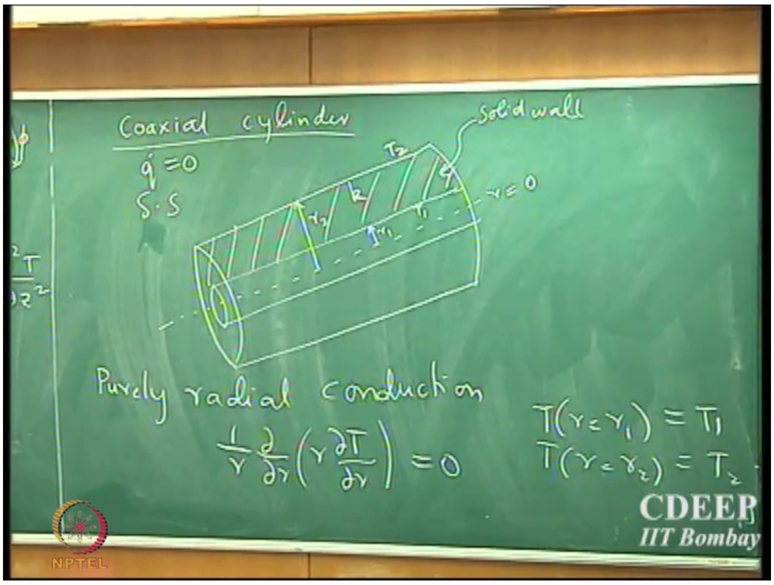

So, the boundary conditions are T at r equal to r 1 is T 1 and T at r equal to r 2 is T 2. So, that is the boundary condition. What is the general solution of this equation, you want to try? Yeah

Student: (Refer Time: 04:50).

Yeah, it is log. What is it? You want to try? C 1 lawn r plus C 2.

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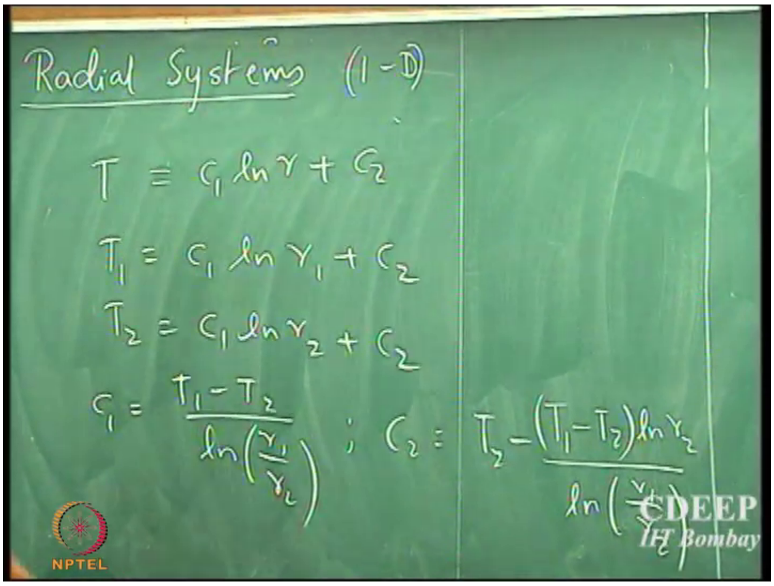

So, the general solution is C 1 lawn r plus C 2, ln stands for the natural logarithm and so, now, we can substitute the boundary conditions. So, t one will be C 1 lawn r 1 plus C 2 and T 2 will be C 1 lawn r 2 plus C 2 and. So, from here we can find out C 1 is T 1 minus T 2 by lawn r 1 by r 2 and C 2 is T 2 minus T 1 minus T 2 lawn r 2 divided by lawn r 1 by r 2. So, it is quite easy, you can almost read out by solving these simultaneous equations, linear equations. And so, when we substitute we can find that the temperature distribution.

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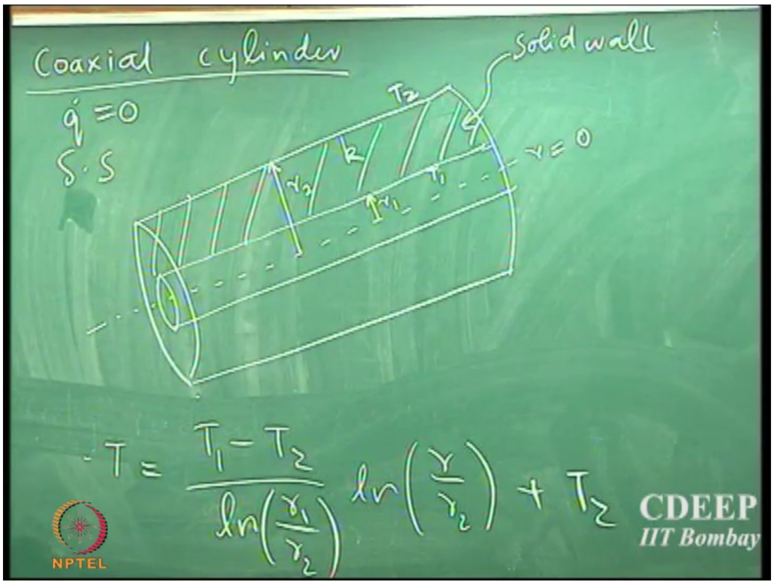

So, the temperature distribution T is given by T 1 minus T 2 by lawn r 1 by r 2 multiplied by lawn r by r 2 plus T 2. So, that is the, all I have done is I have just substituted these constants into the general solution of that equation and this is the temperature profile. So, what should we do next? We know the temperature profile, what is the next step in characterizing? To find the heat transfer rate.

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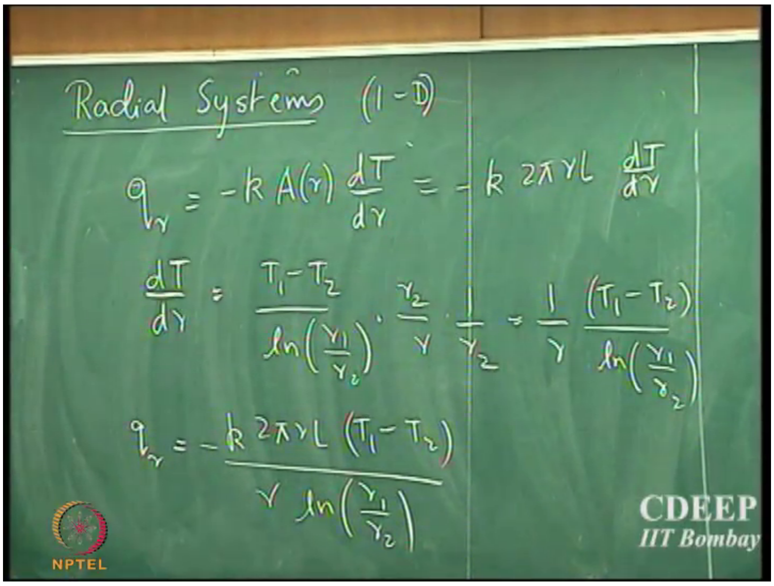

What is q, minus k into A into dT by dr, but the catch here is that area is now a function of the radial position. So, as you go along the radius the cross sectional area of heat transfer is going to be different. So, you have to factor that into the calculation of the heat transfer rate. So, dT by dr. What is A r; minus k 2 pi r L. So, if the length of the cylinder is L, length of the cylinder is L then it is minus k 2 pi r L into dT by dr.

So, dT by dr is T 1 minus T 2 divided by lawn r 1 by r 2 into r 2 by r into 1 by r 2. So, that is the and that is nothing, but 1 by r, T 1 minus T 2 into lawn r 1 by r 2. So, I can substitute the gradient. And, so, that will be q r equal to minus k 2 pi r L into T 1 minus T 2 divided by r into lawn r 1 by r 2.

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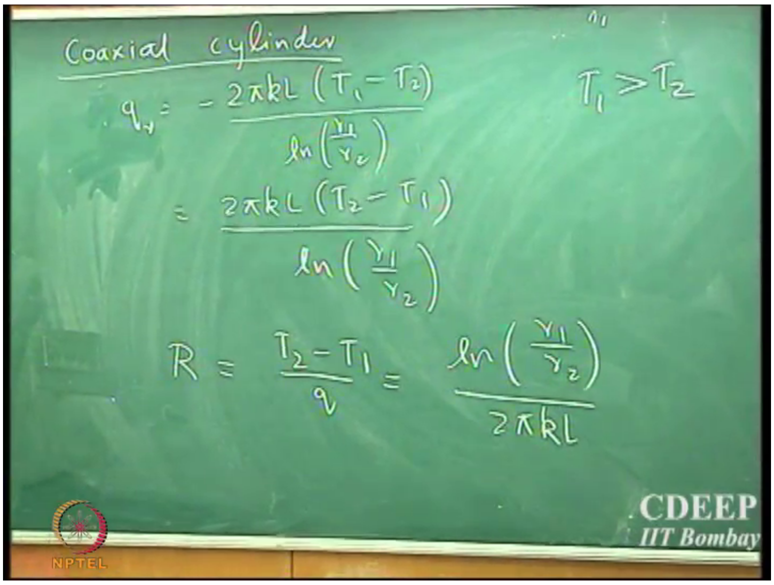

So, that will be q r equal to minus 2 pi k L into T 1 minus T 2 divided by lawn r 1 by r 2. So, why is there is a negative sign here, is the heat transfer rate negative?

Student: (Refer Time: 09:08)

Supposing, if I say T 1 is greater than T 2.

Student: (Refer Time: 09:16)

R 1 is smaller, so, lawn r 1 by r 2 is negative. So, there is now issue with sign here as long as you follow correct convention and you write your model equations properly there will be no sign mistake that you will find in your solution. So, this is very important you must be able to intuit what should be the correct sign of all the terms that you write in your model equations. It is very important to understand this and in fact, I would urge all of you to practice all these model equations as many number of times and convince yourself that this is the correct sign that you have to put at each and every term in the model equation alright.

So, I can simply write it as 2 pi k L into T 2 minus T 1 divided by lawn r 1 by r 2 and once again, I could define the resistances the way I have done before. So, R resistance is given by let us say T 2 minus T 1 by q and so, that will be lawn r 1 by r 2 divided by 2 pi k L. So, that is the resistance between the that is offered by the solid wall for heat conduction in the cylindrical system.

So, note that the form that you get for the resistance is quite different than from what you got in a 1-D system and that is this 2 pi comes because of the change in the cross sectional area and also it is important to note that you got only the length scale which is appearing here, unlike, in the other case where you have A by L, where you have per unit length right. So, here we should note that because of the cross sectional area you have a slightly different form than what you got in a 1-D system. What is the temperature profile? I want to now draw the temperature profile between r 1 and r 2.

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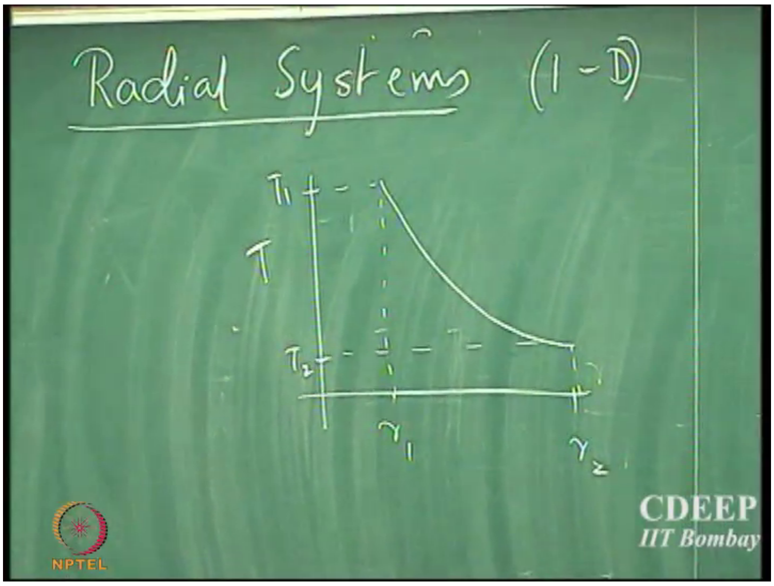

What will be the temperature profile?

Student: (Refer Time: 11:30)

Not negative, but T 2. So, when I say T 1 is greater than T 2. So, I have to define yeah. So, then you have r 2 by r 1. So, depending on how you define, be very careful when you define your quantities, you have to follow the correct sign convention. So, now, what will be the temperature profile, if I say T 1 is greater than T 2? So, it is T 1 and T 2, I said T 1 is greater than T 2, so, what will be the temperature profile. It will be a logarithmically decreasing function there will be a. So, it is quite easy to read it out. So, unlike what you got in Cartesian coordinates where the cross sectional area of heat transport was constant. You had a linear profile here you have a logarithmic profile.

So, remember that you must be able to intuit what the profile is going to be, even before you write the model equations. So, right now we wrote the model equations and we found out what the profile is, but it is also important to do vice versa, otherwise the solution that you get you may not be able to figure out whether the solution you got is right or wrong.

So, it is very important to intuit what is going to be the natural temperature profile that you would get even without writing the model. So, this is very important in any modeling exercise you should be able to intuit what should be the solution or what will be the approximate solution that you would get even before writing the equation solving them. So, this is a very important insight that you should gain while doing various courses in the department alright.

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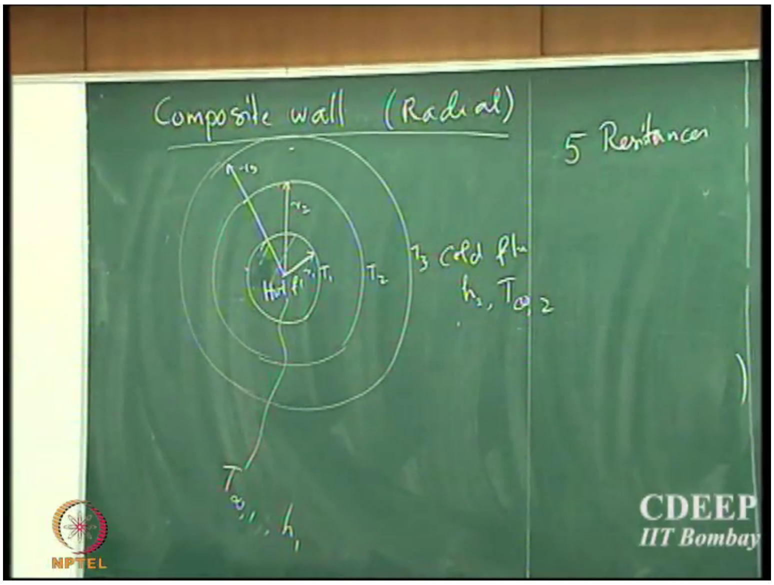

So, let us look at composite wall and radial systems. So, as you would immediately intuit that these are just extensions of what we saw in the last lecture. So, essentially you have many concentric cylinders and what should be the model equation and what should be the resistance network. So, I am going to draw only the cross section here.

So, now if radius is r 1, r 2 and r 3 and let us say we have hot fluid which is flowing here and the cold fluid which is flowing outside and there is let us say the heat transport coefficient is h 2 and T infinity 2 are the temperatures and the temperature here is T infinity 1 and h 1 and if I know the interface temperatures T 1, T 2 and T 3. So, now, I need to draw the resistance network. How many resistances are there? If I assume that the walls are smooth, there is no contact resistance, how many resistances are, they are 5. So, they are totally 5 resistances and what are they? What are the 5 resistances?

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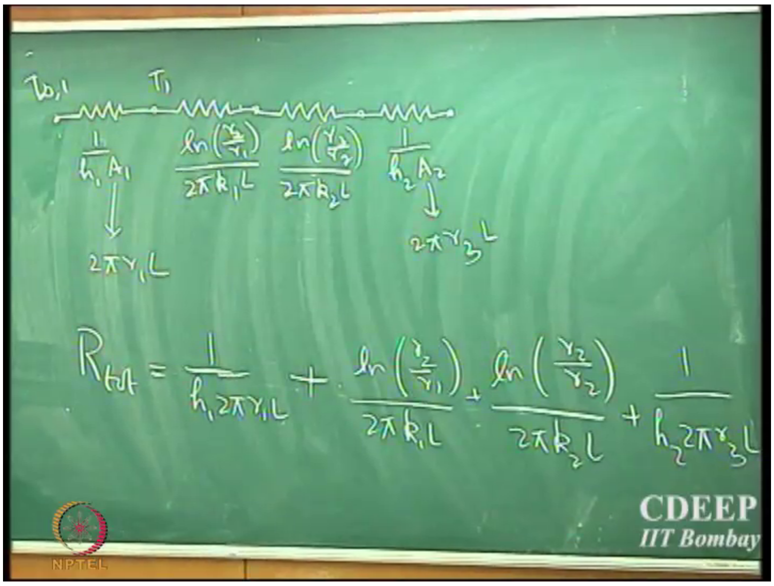

How many are there 4 or 5?

Student: 4.

I am glad you are all awake. It is 4, very good. So, what are the resistances? What is the first one, T infinity 1 and T 1, what is the first resistance?

Student: (Refer Time: 16:03).

1 by h 1 A. Now, what is the area? So, I should say 1 by h 1, A 1. So, note that the area is now different in the inside and the outside 2 pi r 1 L. So, this area is 2 pi r 1 L. So, it is not the cross sectional area, the heat transport is actually occurring along the curved surface.

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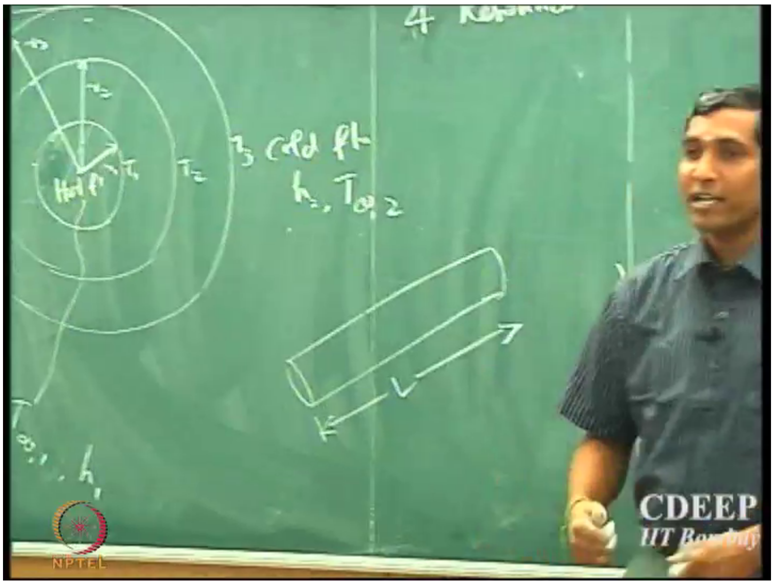

So, this is the length. So, the heat transport is actually occurring along the curved surface of the cylinder and so, the area is 2 pi r 1 into L, what about the resistance here? lawn r 2 by r 1 divided by 2 pi k 1, if I call that k 1 is the thermal conductivity here and k 2 are the thermal conductivity there. So, it is lawn r 3 by r 2 divided by 2 pi k 2 into L and this will be 1 by h 2 into A 2.

So, this is 2 pi r 2 into L, so that is the heat transport area, is that clear to everyone. And so, R total which is the total resistance is given by 1 by h 1 2 pi r 1 L plus lawn r 2 by r 1 divided by 2 pi k 1 L plus lawn r 3 by r 2 divided by 2 pi k 2 L plus 1 by h 2 L r 3 sorry should be r 3 thanks.

Once again, the way we did before we could define an overall heat transport coefficient.

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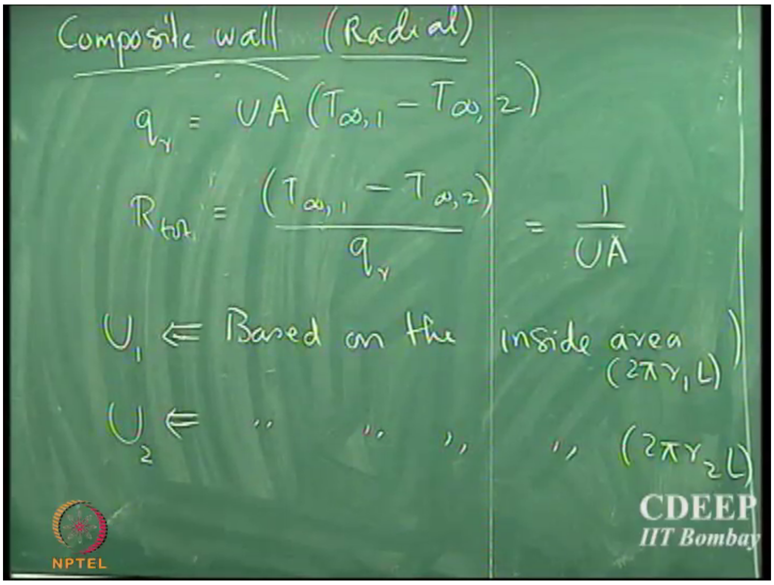

So, q, in the radial direction we can write as U times A into T infinity 1 minus T infinity 2 and. So, 1 by U A, resistance total is given by T infinity comma 1, minus T infinity comma 2 divided by q r and so, that is 1 by U into A.

Student: (Refer Time: 18:54)

So, that is the question. So, here what should be the area that you should choose for defining the overall heat transport coefficient? In a 1-D system in Cartesian coordinates it didn't matter because the area was same. So, here you can now define 2 different heat transport coefficient U 1 and U 2. And, so, this U 1 is based on the inside area that is 2 pi r 1 L and this is based on based on 2 pi r 2 l. So, now, if you have varying cross sectional area system then the overall or the universal heat transport coefficient is defined based on the area of interest.

So, the question comes in which one should I choose? I have 2 options now, which one should I choose. So, it completely depends upon the problem that you are handling and it depends upon what are the quantities that you want to estimate for example, if I want to estimate the temperature of the hot fluid; let us say, I know what the temperature of the hot fluid is, but let us say for some particular problem I want to estimate what should be the temperature of the hot fluid, in order for certain amount of heat to be transferred across a given composite system.

So, in that case I should use the inside area and define find out the overall heat transport coefficient based on the inside area. So, if I want to find a hot fluid properties then I need to use the outside area and find out the overall heat transport coefficient defined based on the outside area, when we.

Student: (Refer Time: 20:53).

Yes, it includes everything. So, we are going to see that in a short while. It includes everything. The only advantage you will have is that it just has some convenience in terms of calculations that is all. It does not matter whichever area you choose, but if you want to have things to be little bit more convenient to do your calculations then you define your heat transport area based on that.

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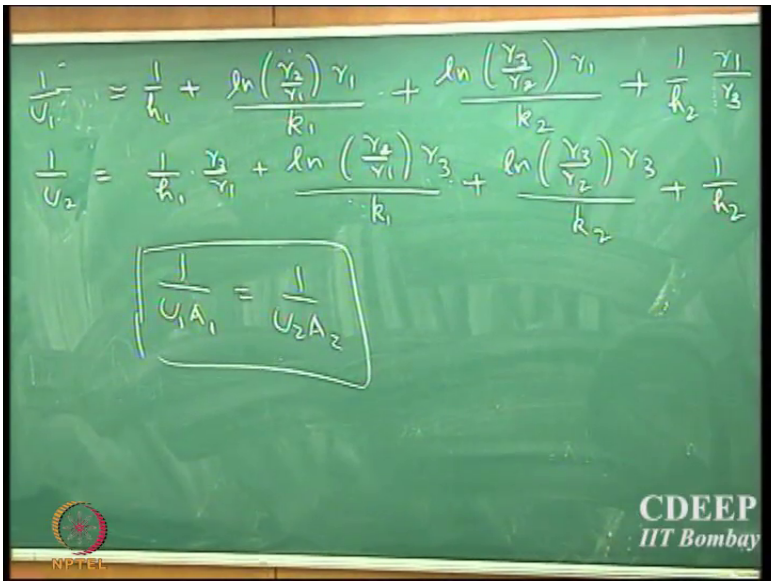

So, supposing if I define, 1 by U A, so, if I define heat transport overall heat transport coefficient based on the inside area and that will simply be 1 by h 1 plus lawn r 2 by r 1 into r 1 by k 1 plus lawn r 3 by r 2 into r 1 by k 2 plus 1 by h 2 into r 1 by r 3. So, that will be the, oops sorry, 1 by U 1 thanks. So, 1 by U 1 all I have done is I have just multiplied the equation by the corresponding area and similarly 1 by U 2 will be 1 by h 1 into r 3 by r 1 plus lawn r 2 by r 1 into r 3 by k 1 plus lawn r 3 by r 2 into r 3 by k 2 plus 1 by h 2. So, which clearly means that, 1 by U 1, A 1 should be equal to 1 by U 2 A 2. So, that sort of, obviously, you can intuit from the way it has been defined.

Any questions on this so far? Yes. So, I am going to quickly summarize in the next couple of minutes what we the first topic that we have finished.

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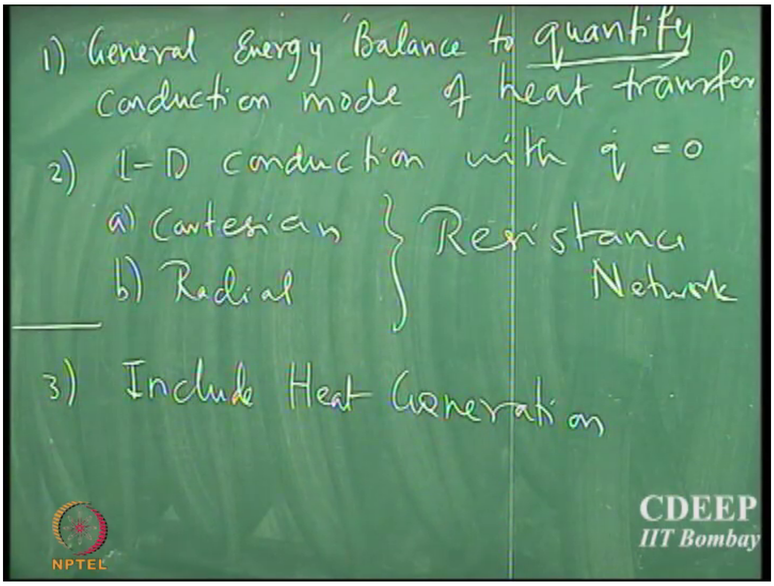

So, we looked at the general energy balance to quantify conduction process. So, its key purpose is to quantify conduction process mode of heat transfer, then we looked at a very simple case of 1-D conduction without heat generation and here we looked at Cartesian coordinate system; we looked at radial coordinates, we looked at radial system. So, these are the things that we have seen. So, this is the sort of the first aspect of conduction process.

So, what we are going to see the next topic is going to be the. So, the third topic in that list which is what we are going to see now is, what happens when we include heat generation and I should also mention that we included resistance concepts along with this, the systems network concepts along with these.

So, what we are going to see now in the next 5 minutes and in the next lecture which starts 10 minutes from now will be, what happens when you include heat generation also? We so far assumed that heat generation is 0, what happens if we include that and how these analyses that we have done so far would look slightly different from what we saw so far.

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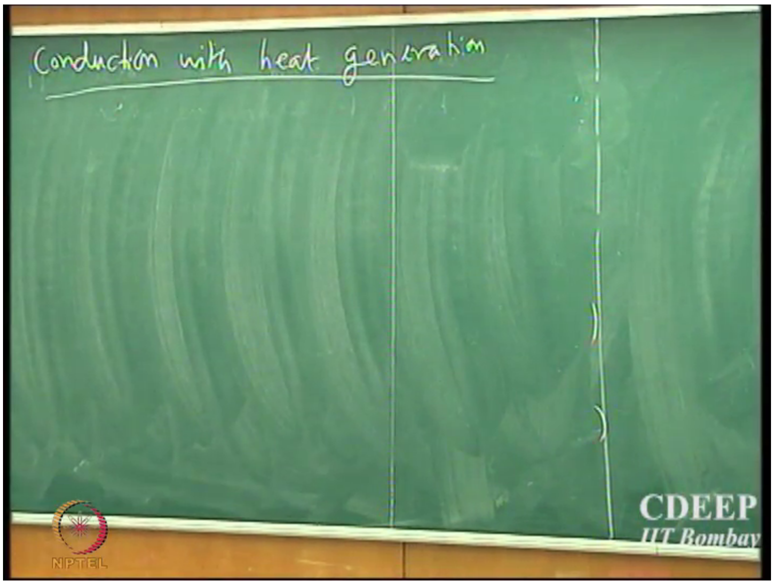

So, supposing we take a slab.

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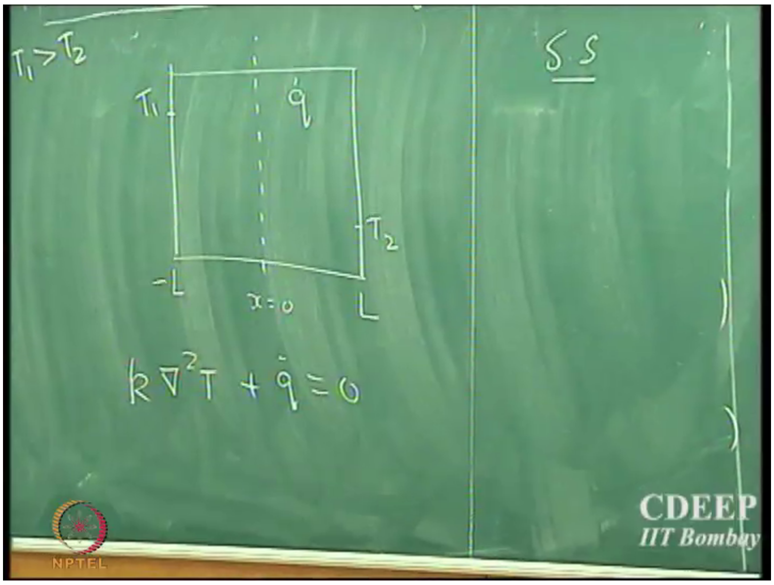

Supposing we take a slab and let us say that q dot is the volumetric heat that is being generated inside the slab. It could be many different ways, for example, it could be like there is an electrical heating system inside and so, that is generating heat inside the slab or it could be some reaction which is inducing heat inside. There are solid reactions. So, it could be that solid reaction is inducing some heat, some exothermic reaction which is inducing some heat inside the slab. So, there could be many different mechanisms by which heat is actually being introduced into a system and so, there is a simultaneous heat generation and heat transport.

So, supposing just for sake of convenience purposes I define midpoint as x equal to 0. In fact, you will see in a short while why I did that and let us say that plus L and minus L are the coordinate systems for this geometry and if I assume that temperature on one end is T 1 and the other end is T 2. So, by representation I have assumed that T 1 is greater than T 2. And, suppose, I proceed, that it is a steady state problem. So, it is a steady state problem, then what is the model equation?

Student: (Refer Time: 27:12).

It is the k del square T, Laplacian plus q dot equal to 0. So, that is the model equation.

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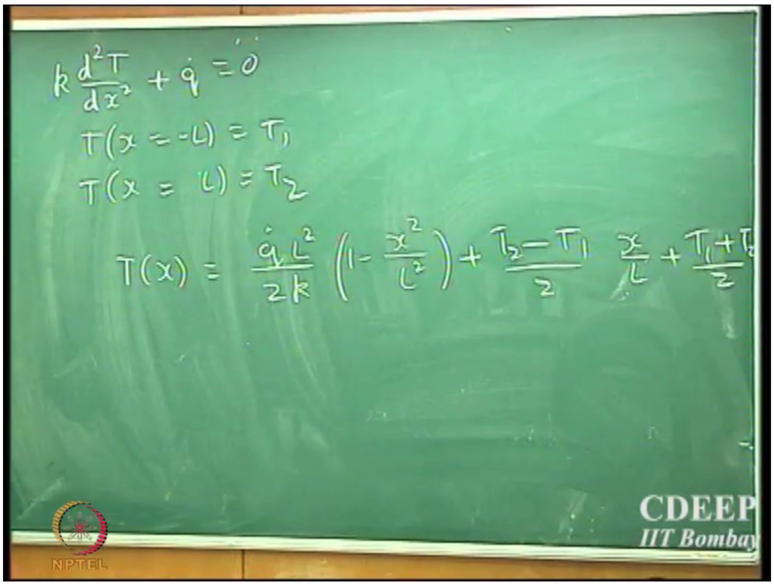

And so, we can write it as k d square T by d x square plus q dot equal to 0 and the boundary conditions are quite trivial minus L is 1 and T at x equal to plus L is T 2. So, I can solve this equation without much effort. So, I am not going to derive the solution here. So, I will give you what the solution is. So, the solution is T of x equal to q dot into L square by 2 k into 1 minus x square by L square plus T 2 minus T 1 by 2 into x by L plus T 1 plus T 2 by 2. So, that is the solution for the model, just a second order differential equation. So, it is not a very difficult exercise to solve them. So, you should actually solve the equation and convince yourself that this is the correct solution.