Heat Transfer

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Lecture - 58
LMTD I

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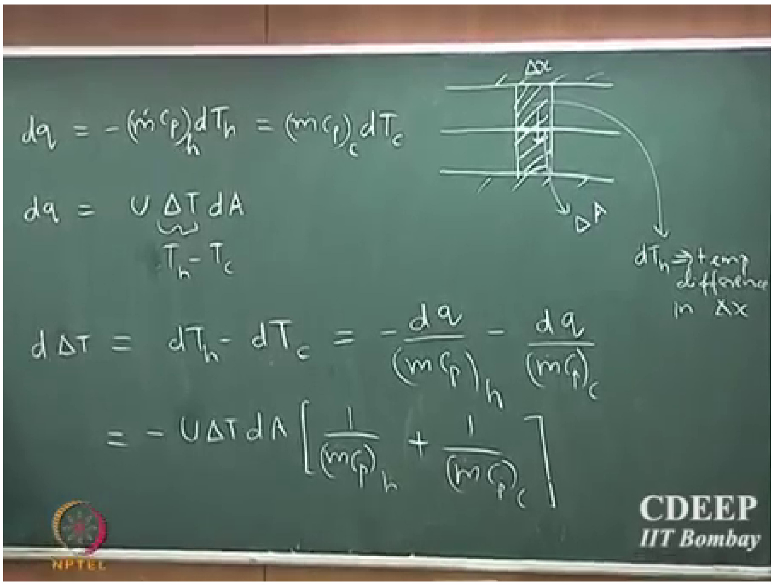

So, we are going to write a simple energy balance. So, here is the parallel flow, I take a small element. So, supposing if it is delta x some whatever small element and let us assume that the area of heat transport in that element ok. So, this area is delta A so, that is the area of heat transport which is available for taking heat from the hot fluid to the cold fluid.

So, now, I can write a so, the d q which is the differential amount of heat that is actually transported from the hot to the cold fluid ok. So, that is given by minus m dot C P into d T h ok. So, d T h is the temperature difference in the hot fluid in this element. So, d T h is the temperature difference in delta x and at steady state because there is no heat that is going out from the top and the bottom right. So, that should be equal to m dot C P multiplied by d T. So, that is the amount of heat that is gained by the cold fluid.

So, now, but we also know that d q is given by U some delta T ok, which is the difference in the temperature between these 2 chambers multiplied by the differential A. So, delta T is nothing, but T h minus T c, it is some representation of T h minus T c we will see that in a short while what it is ok. So, now your d delta T so, the first differential of delta T that is given by d T h minus d T c. So, that is the differential change in the temperature difference between the 2 fluids is given by the difference between the differential temperature differences itself.

So, now, from here we can write that this is minus d q by m dot C P I should I put a subscript h C m dot C P h minus d q by m dot C P cold fluids. So, from here you can write now d q is u into delta T into d A right. So, we can write mu delta T into d A into 1 by m dot C P plus 1 by m dot C P cold. So, that is the differential temperature variation, remember that delta T is the local variation of the temperature between the two fluids local temperature difference. So, that is the differential of that temperature difference ok.

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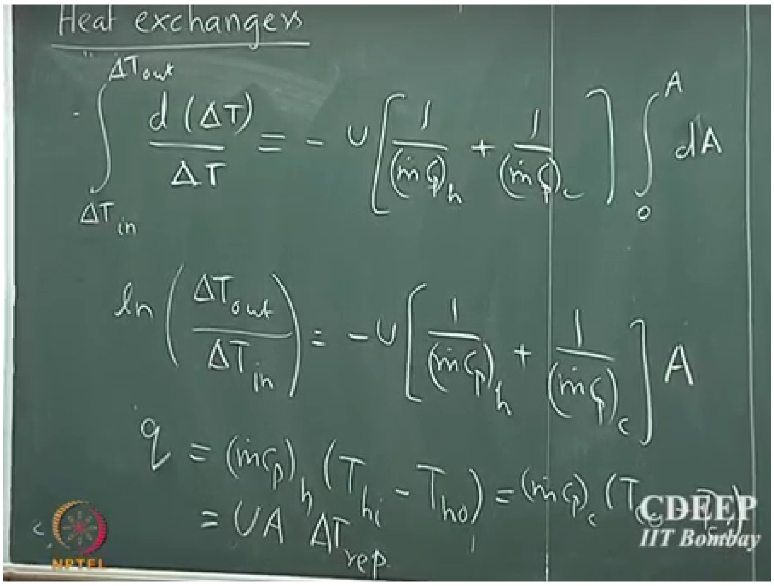

So, now from here it can actually integrate this expression, we can integrate this expression, we will have integral d delta T by delta T that is equal to minus U the properties are constant integral d ok. So, now, we can integrate over the whole heat exchanger. So, this is from the delta T at inlet and this is the delta T at outlet. Now, you can immediately see that we have taught and introducing the measurable quantity, delta T inlet is a measurable quantity, you can measure the temperatures and therefore, you should be able to measure the temperature difference.

And similarly delta T out also you should be able to measure. And so, this will be between 0 to the overall area which is available for heat transfer ok. So, that is nothing, but that is ln delta T out divided by delta T in minus U 1 by multiplied by A. So, you can you can always define an overall U average U, why not I can always do that no. So, you should be very careful, when you define your overall U ok.

So, the overall U is a representative heat transport coefficient, for your full heat exchanger, or full surface area which is present. Now, that is multiplied by the fractional surface area it will always give you what is the extent of heat transport and that small section why not? What is wrong in doing that? There is nothing wrong and doing that that is perfectly fine. You are looking at overall heat transport coefficient right, you are looking at average heat transfer coefficient.

So, if you look at the average heat transport coefficient that will be some fraction of the local heat transport coefficient. If I assume that it is a tube, I have not said what the geometry is, but let us say it is a tube ok. Now, if it is fully developed you know that the nusselt number is constant right; so, the heat transport coefficient is constant. So, what do you mean? It is true even for a concentric cylinder, it is true for many different geometries, we did not prove it in the class, but it is true we assuming we have not remember that I have not told you what the geometry is yet. So, we will talk about tubes later the event I have not told you what a geometry is; right now, assume that heat transfer curve in this form it is alright.

So, now so, the overall heat that is lost by the hot fluid ok. So, that is given by m dot C P of the hot fluid multiplied by T h i minus T h o ok. So, that is the overall temperature difference that the sensible heat that is actually lost by the hot fluid and, that should be equal to m dot C P cold fluid into T c o minus d t. So, now, and that is also equal to U into A into some delta T, we do not know what that is some representative delta T. So, I call it Re P we do not know what the delta T is that is what we able to find.

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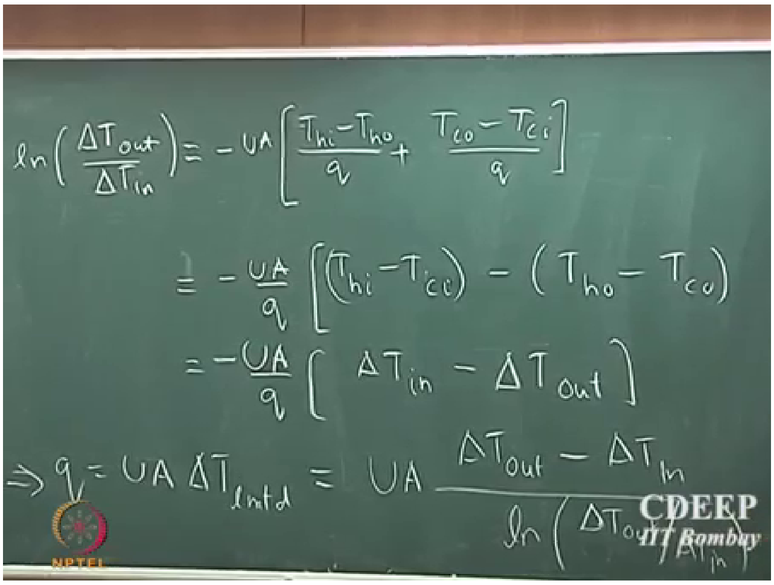

So, learn delta T out by delta T in that should be equal to minus U A into so, I am going to replace m dot C P hot and m dot C P cold. So, that is the T h i minus T h o divided by q plus T c o minus T c i divided by q. So, that will be minus U A by q into so now, I can easily rearrange this. So, this will be T h i; T h i minus T h o over minus 1 T c i minus T h o minus T c u. So, that is nothing, but U A by q into delta T at the inlet with a minus sign, minus delta T at the outlet. So, from here you get that U equal to U into A into delta T lm t d.

Let us define U into A into delta T out divided by lon ratio of the delta T at outlet and delta T at inlet. So, if we know the measurable quantity that is the temperatures of the hot and the cold fluid at the inlet and at the outlet, then we should be able to actually find out what is the overall heat transport rate based on the measurable quantities, if you know what U is. So, U is the universal heat transport coefficient which is based on averages.

Now, what is interesting about this property of heat exchanger where, you can express the measurable quantities, you can use the measurable quantities in the form of delta T lm t d in order to find the total heat transport rate. So, this is a ubiquitous representation. In fact, if you do the same exercise for counter flow, I am not going to derive it again, but if you do the same exercise in fact, I encourage all of you to do that.

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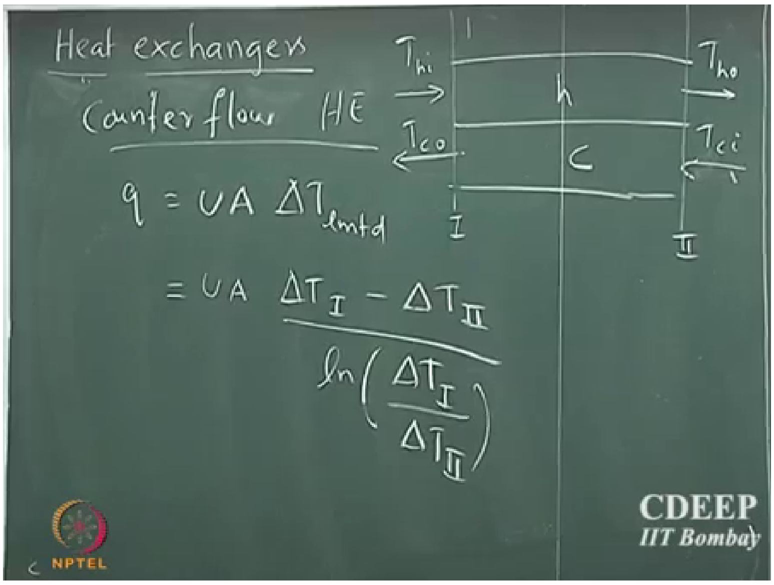

So, you will see that if you do is the same exercise for counter flow, counter flow heat exchanger and I would warn you that be very careful with the sign that you put for heat loss and heat gain, then you have counter flow. So, you should get exactly the same expression delta T lm t d. So, now, the only difference is that supposing if I have counter flow, where the hot fluid is flowing in this direction. And we have cold fluid which is actually flowing the opposite direction. And if T h i is the inlet temperature of hot fluid and T h o is the outlet temperature and similarly T c i and T c o ok.

So, now here delta T lm t d is defined as delta T. So, if I call this location as let us say 1 and this location is 2 because, now you cannot say which is in and which is out ok. So, if I define my inlet based on the hot fluid spring and the location of outlet based on the cold fluid spring. So, this delta T lm t d will simply be delta T at 1, delta T at 2 divided by lon delta T at 1 divided by delta T is 2. I would really encourage all of you to actually derive this and see, that you will it will turn out if you do the derivation carefully.

And correctly you will see that this is exactly the delta T lm t d that you will get, which looks very similar to what you got in parallel flow. So, what really matters is that you need to measure the local time in the hot fluid in the cold fluid temperatures at 1 end of the heat exchanger and a similar temperatures at the other end of the heat exchanger, this is for simple parallel and counter flow and, then you simply define the delta T lm t d.

So, that is why we cracked in all your lab experiments the laminar and turbulent flow experiments the delta T lm t d the expression does not change, whether you had a parallel flow or a counter flow the expression for the lon a delta T lm t d is the same except that you have to replace your temperatures based on, what you define as inlet to the heat exchanger and what you define as outlet of the heat exchanger.

So, that is all you need to know once you define that carefully it does not matter how you write because minus sign will always be absorbed by the numerator and the non numerator it does not matter. So, as long as you define it properly, you should be able to get this change delta T lm t d ok. So, now there is another interesting property, I am not going to derive the algebra. In fact, you should all try this it is it is a little bit complicated algebra and it is doable.

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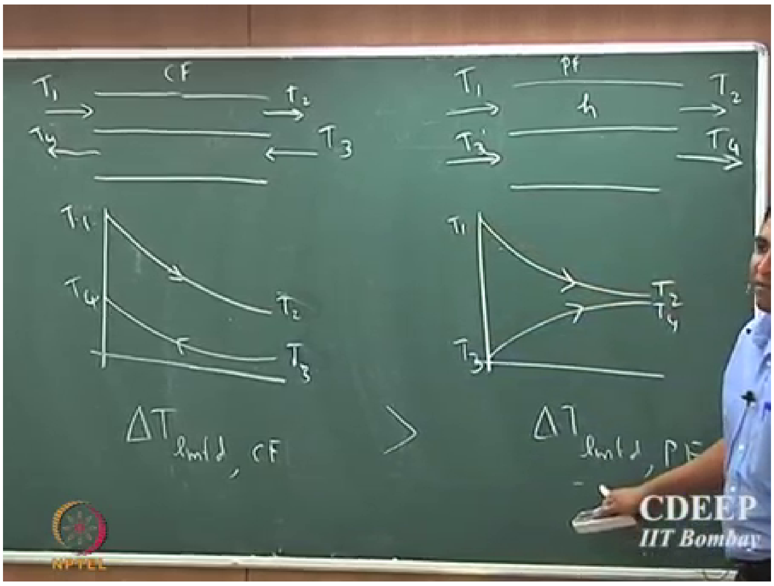

So, suppose I have a heat exchanger and, I have specified the temperatures of the hot and the cold fluid inlet and outlet ok. So, this is for a parallel flow ok. And similarly I have a counter flow and, I define the same inlet and outlet conditions for the hot and the cold fluid ok. So, this is my T 3 and this is my T 4; same fluid, same temperatures question is which 1 is better ok, what is the answer? Counter flow; why is that? It appears very intuitive, but is important to know why ok. So, let me draw the temperature profile ok. So, if this is the hot fluid ok. So, that T h i and T h o and T c i and T c o, or again put T 1, T 2, T 3, and T 4 ok.

So, that is the temperature profile for counter flow and, I can draw a similar temperature profile for parallel flow. So, that is my T 1, T 2, T 3 and T 4, where both are moving parallel. Now, how can you say from this graph that counter flow is always better less to the temperature gradient lessess the heat transfer, but keep in mind that you are also as you go through inside the area of heat transport is also you are having higher area for heat transport right. So, it is possible that there is maximum heat transport here because, the temperature get in this maximum here.

How can you decide which one is better, how can you decide which one is better based on this profile, you really cannot say for sure that counter flow is better. So, the rationale is if you calculate the delta T lm t d of counter flow and, if you calculate the delta T lm t d for a parallel flow ok, it always turns out that for a given condition the delta T lm t d for counter flow is always greater than the delta T lm t d for parallel flow with same condition.

The same inlet temperatures and outlet temperatures of hot and cold fluid, I am not going to show this rigorously, but it is it is definitely possible to show and, it is not very it is not impossible the algebra is a little bit boring, I always want you to derive this and show that the counter flow delta T lm t d is greater than the parallel flow, you take delta T 1 delta T 2 divided by the log of the ratio of both cases and, you fix the temperatures and show that one expression is always going to be greater than the other one ok.

So, it is not a simple exercise there is a little bit tedious algebra and, what I like all of you to first try it alright. So, so that is an important property. So, for a given set of conditions delta T lm t d of counter flow is always greater than that of the parallel flow and, that is the reason why counter flow is preferred.

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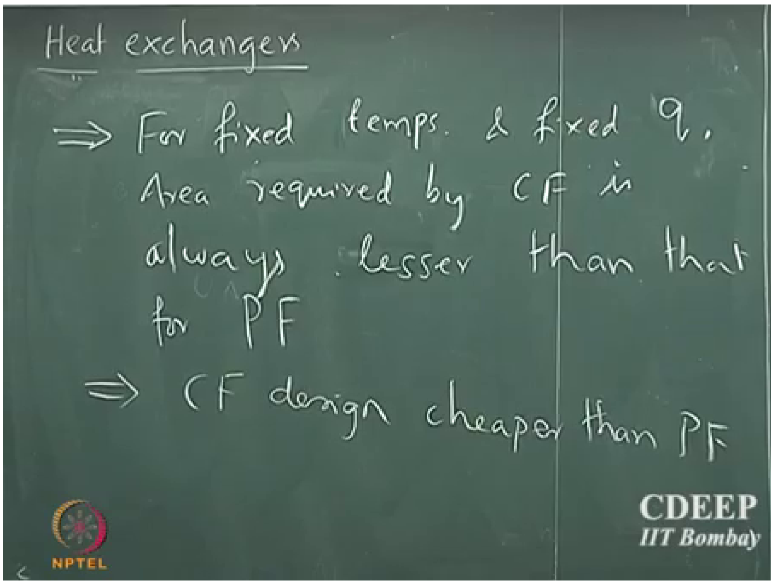

So, which also means implies that for fixed measure of temperature and therefore, fixed heat transport rate ok. The area required for heat transport area required by counter flow is always smaller lesser than that for parallel flow with same temperature and same heat transport rate ok. So, therefore, it also implies that counter flow design is a cheaper design. So, the area of heat transport which is required tells you what is going to be the size of the equipment.

And so, you require a smaller size heat exchanger and therefore, counter flow heat exchanger design is always cheaper, it is cheaper to construct a counter flow heat exchanger than to construct a parallel flow.