Heat Transfer
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Lecture - 53
Resistances: Oppenheim Matrix Method

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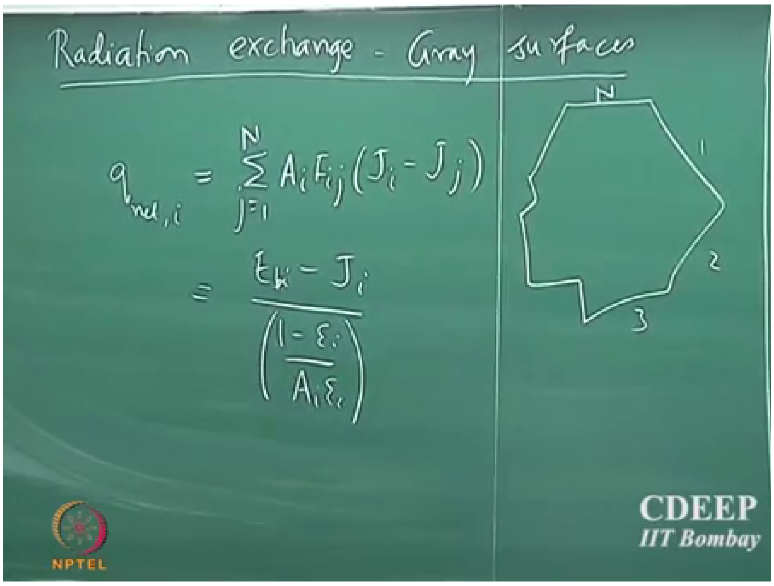

We looked at the energy balance or radiation exchange in gray surfaces. So, we said that the net radiation exchange, if we have an enclosure with N objects with N surfaces and each of them are exchanging radiation with each other. Then we said that the net radiation exchange is given by sum over all Ai F ij, j going from 1 to N and that is also equal to E b i minus J i by 1 minus epsilon i by Ai epsilon ok.

So, we identified that 1 minus epsilon i by Ai epsilon i is the resistance offered by the surface because of its surface properties for radiation exchange from that surface ok. So, now, in a similar way we could also identify resistances for radiation exchange between different surfaces ok. So, note that this is the resistance for radiation from the surface i, and we can identify resistances for radiation exchange between surfaces between surfaces ok.

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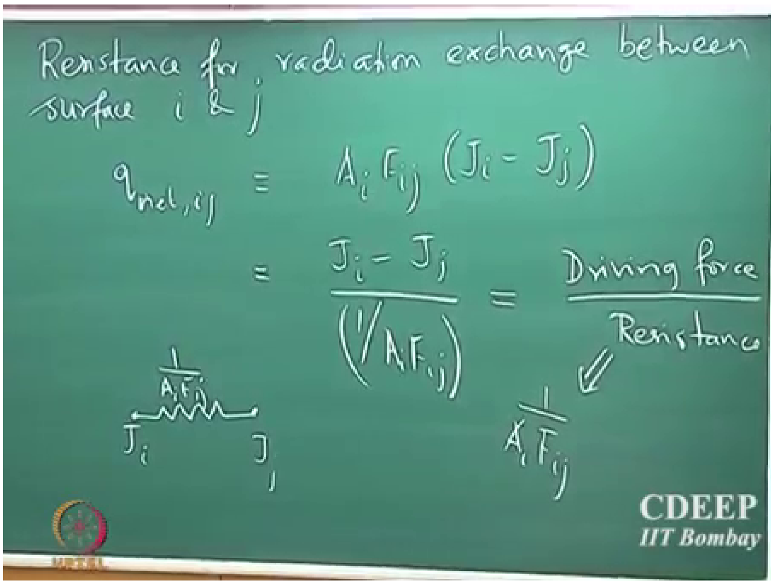

So, radiation exchange between surface i and surface j is given by is Ai Fi j into j i minus j ok. So, now we can rewrite this as Ji divided by 1 by Ai F ij. So, if the driving force for radiation exchange is the difference between the corresponding total emission from each of these surfaces so that is the driving force, driving force divided by the resistance ok.

So, one could identify that the resistance that is offered for radiation exchange between surface i and j, would be 1 by Ai F ij ok. So, this resistance is nothing but 1 by a i. So, note that this resistance is the resistance offered because of orientation this is the geometric resistance that is offered by the system because of the orientation of different surfaces.

So, therefore, one could write a resistance network J 1 by Ai F i j ok. So, that is the resistance which is offered for radiation exchange between surface i and surface j whose total emission are J i and J j ok. So, now based on these 2 resistances, we can now construct the resistance network for the whole system ok.

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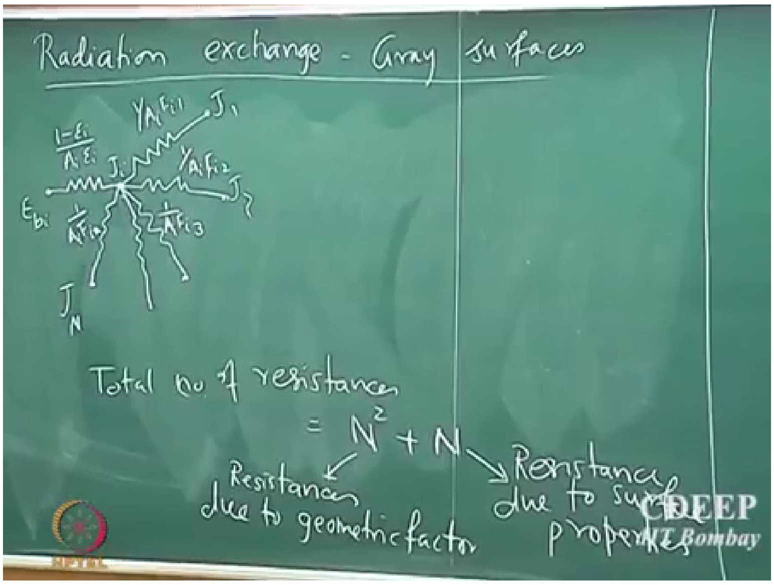

So, supposing E bi is the blackbody radiation for surface i and J i is the corresponding total emission from that surface then the resistance offered is 1 minus epsilon i by Ai epsilon ok. Now this is exchanging radiation with all other surfaces therefore, you have resistance for which are occurring in parallel, because it is simultaneously exchanging radiation with all other surfaces.

Therefore, you have resistances which is appearing parallel, so this is 1 j 1, J 2, spectra up to J N. And these resistances are 1 by Ai F i 1 1 by Ai F i 2 and this will be 1 by Ai F i 3 and this will be 1 by Ai F i N ok. So, that is the resistance network for radiation exchange of surface i with all other surfaces in the enclosure ok.

So, one could similarly write the radiation exchange for all the other surfaces right, one could similarly write radiation exchange network for all the other surfaces going from 1 to N. So, in principle from every node you will have N resistances which are actually coming out of every node.

So, how many resistances are there total number of resistances, for this system total number of resistances what is the total number of the resistances. So, every entity you have N of them so that gives you N square. But you have the resistance that is offered by the properties of the surface right. So, you have 1 resistance for every surface. So, the total number of resistances is equal to N square plus N ok.

So, this N square is the resistance offered because of orientation resistances due to geometric positions or geometric factor. So, these N square resistances are because of the geometric factors and; obviously, they are symmetrical right. And N resistances are because of the resistances due to surface properties; due to surface property what is the resistance offered by the surface due to its surface properties. So, the total number of resistances are N square plus N alright.

So, where do we use all this the question is where do we use all this information. So, there are 2 kinds of questions one who typically ask ok.

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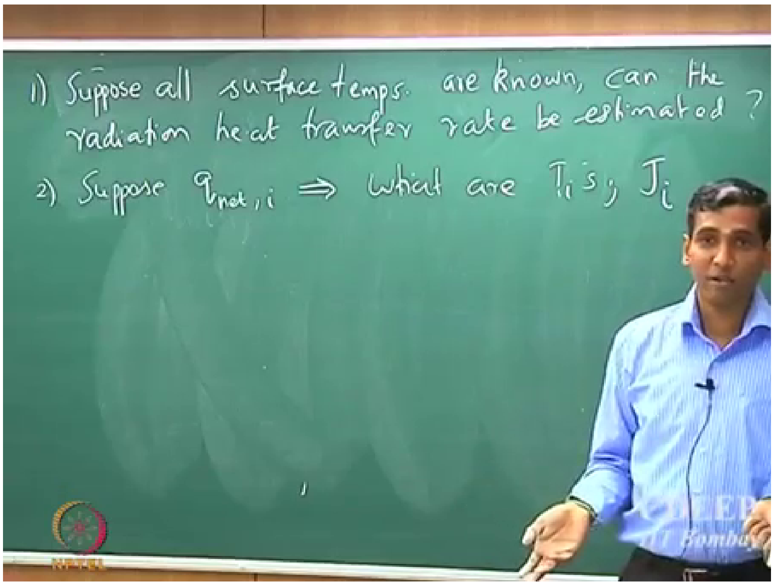

The question one is suppose I know all the temperatures suppose all surface temperatures are known ok. Can I estimate can the radiation so that is the first class of questions you can answer with some of the things that we have derived so far ok.

The second class of questions you can answer is suppose I know the classes suppose I know q i know the net radiation exchange from every surface. Now that is possible to measure because if you know how much heat you are supplying or removing in order to maintain an isothermal condition and that is what will give you what unit i, then given that what is the temperature.

You can answer different kinds of questions not just that you can also say what are the individual total emission so that and that question also can be answered ok. So, let us look at the first question how to answer the first question with whatever framework that we have developed so far. So, we are going to use what is called the Oppenheim; Oppenheim matrix method to answer question 1 and question 2 ok.

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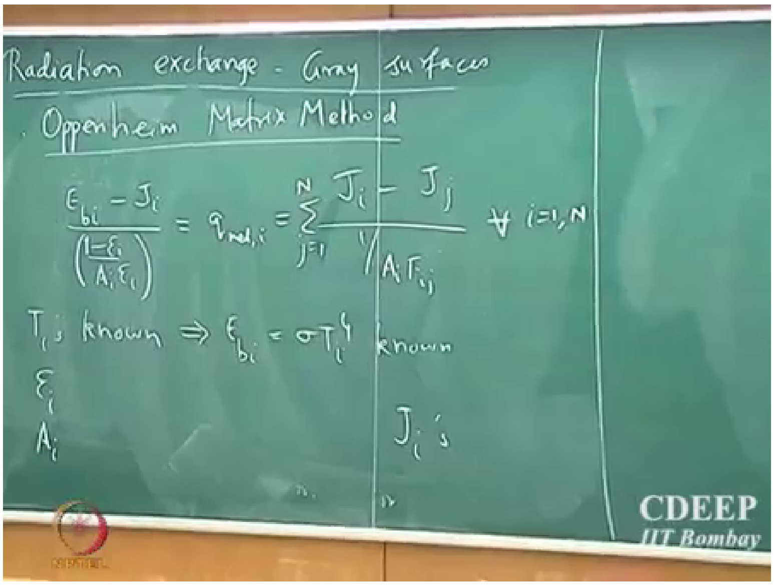

Now, it is nothing, but is transforming all the energy balances that we have written. So, far in a matrix vector form so that it looks elegant to solve the problem and looks easy to solve the problem ok. So, suppose I know all these surface temperatures, I want to find out what is the total emission from every surface right net emission from every surface if I know that I am done I can find out the net heat transfer rate.

So, we have the energy balance so E b i minus J i divided by 1 minus epsilon i a i epsilon i that is equal to q net i equal to J i 1 by summation over j ok. So, this is the emission from the surface with respect to its own properties and this is the radiation exchange between that surface and all the other surfaces in the enclosure ok.

So, supposing I know the temperatures, which quantity in this expression is a known quantity. Suppose I know all the temperatures, suppose I know the temperature of all the surfaces I maintain under isothermal conditions so which quantity is a known quantity here. What are the quantities which are known here? Do I know epsilon i, if I say the these are known then I know epsilon i epsilon i is a known quantity ok. And I can find out what the area of the surface is total area of the surface.

What else do I know? What does do I know? I know the temperatures what else do I know?.

Student: (Refer Time: 12:47).

You know E bi E bi is just the blackbody radiation at that temperature right. So this means E bi equal to sigma T i to the power of 4 is a known quantity right. So, what do we need to estimate we need to find out all the Ji's. We need to find out the total emission from every surface given these quantities is it a solvable problem. How many equations are there? How many?

Student: N.

N equations, so you have 1 equation for every value of i right. So, this is so this equation is valid for all i going from 1 to N ok. Is it a linear or a non-linear problem? It is linear; linear in J. So, we should be able to find the solution.

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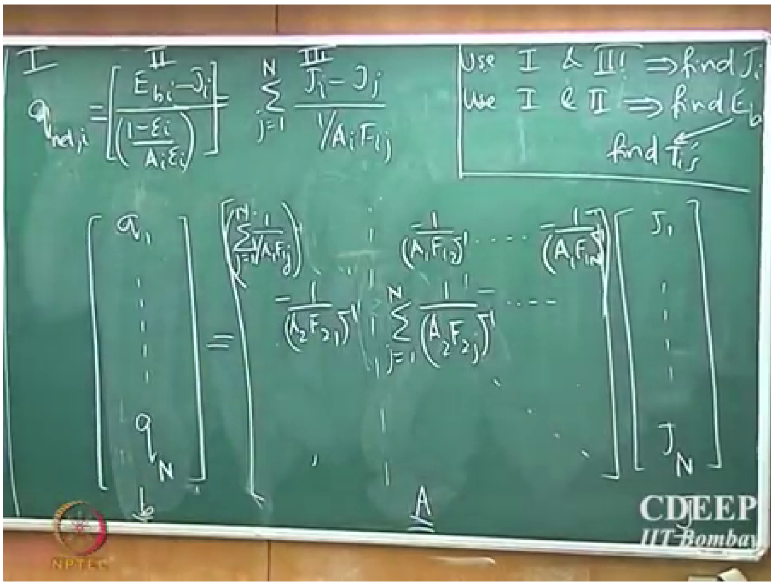

So, the way to represent this is so E bi divided by 1 minus epsilon i by A i epsilon i that is equal to sum j equal to 1 to N, J i divided by 1 by Ai F ij plus j i by 1 minus epsilon i epsilon ok. So, now I can write this in the matrix vector form.

So, note that this is for all i going from 1 to N, now I can write this as a matrix vector problem so that is E b 1 by 1 minus epsilon 1 by A 1 epsilon 1 A N epsilon N and that is equal to we have a matrix multiplied by J 1 all the way up to J N. What is the first element in the matrix that is only for the first term, that is summed over all 1 to N right.

So, similarly you have a summation here j going from 1 to N ok. So, I will open up and write it out write all the terms separately in a minute, but you should understand the structure of this matrix ok. So, let us just quickly so if I call this matrix as A ok, if I call this matrix as A matrix and if I call this a j vector, and if I call this as b vector ok.

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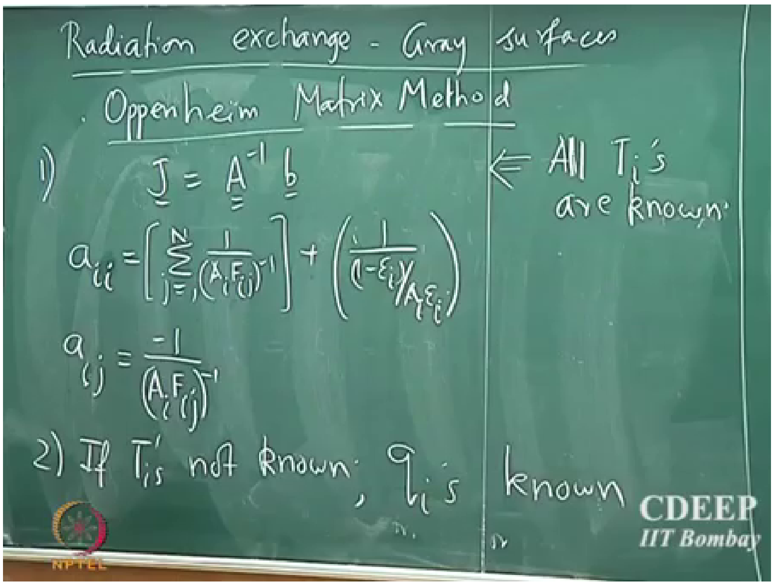

So, the total emission from all the surfaces was just the solution of this matrix problem right. So, J will be A inverse times b ok. So, where if I open up the A matrix a 1 1 the first element in the matrix will be sum over all j 1 by A 1 F 1 j inverse plus 1 by 1 minus epsilon 1 divided by A 1 epsilon 1 so that will be the first term.

So, if I now want to find out what is the general diagonal term. So, a i i this will simply be I replace 1 with i so that is the general diagonal term. And then if I look at the off diagonal terms a 1 2 will be 1 by minus 1 by A 1 F 12 inverse right.

So, if I replace if I want to find the general non diagonal term. So, a ij will be A 1 F 1 j ok. And then I replace the first index with I Ai Fi so that is it. So, it is so easy to find out what this matrix is ai i is sum over j equal to 1 to; 1 by Ai Fi i inverse F ij inverse excuse me plus 1 by 1 minus epsilon i divided by Ai epsilon i and the off diagonal terms are Ai j is minus 1 by Ai Fi j inverse.

Student: (Refer Time: 18:03).

Yes it is because of the reciprocity relationship it is symmetric and it has to be symmetric. Because if surface 1 is exchanging with surface 2, surface 2 is in turn exchanging with 1 so it has to be symmetric. So, we can find out the total emission simply by following this matrix problem and so j will be a inverse times b ok. So, this is if all the temperatures are known ok. What if all the fluxes are known or the rate heat transport rate is known. So, this is the first problem.

What about the second one where if temperatures are not known, but we know how to estimate the qi s how do we solve this problem and we still use the same framework.

Student: (Refer Time: 19:20).

No ui minus ji that is right. So, that is the expression.

Student: (Refer Time: 19:31).

We do not know so we want to find out ji you are going to find ji. So, we know q net you are going to find ji now if you know ji you know E bi from this expression. So, there are three terms here there is a first term, there is a second term, and there is a third term right so there are 3 terms here. Now what I can do is I can equate first and the third. So, use the first and third so use 1 and 3 term, 1 and 3 and find j i. Find ji they find all the total emissivity, then use 1 and 2 and find E bi so you can find E bi.

So, once you know E bi from here you can find temperature that is it algorithm is very simple it is still the same framework except that these elements inside the matrix is going to be different. So, that is going to be q 1 and q N and the diagonal elements will not have this summation term this will not have this additional term here. So, that will be so this will be sum j equal to 1 2 N 1 by A 2 F 2 j inverse etcetera and you just complete the matrix ok.

So, this will give you again it is a and this b b vector. So, solving this matrix vector problem will actually give you the total emission from all the surfaces if q net i is known. And then equating 1 and 2 you will be able to find the E bi for every surface and from that you can find out the temperatures ok. So, if we just so this is for problem 2 where all q i's are known.

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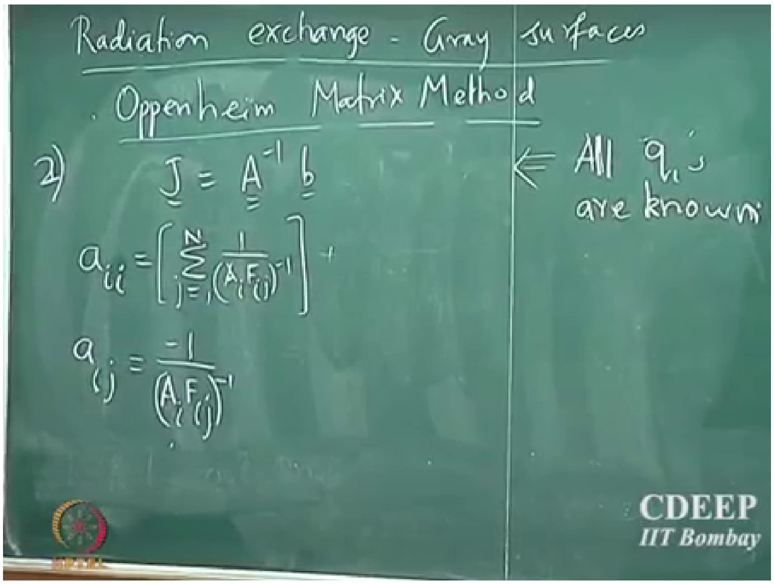

The elements will simply be the diagonal elements will be summation of all the view factors and the off diagonals their terms will remain the same ok. So, this is for the second class of problem where you can still use the same Oppenheim matrix method except that you will have to equate different terms in your energy balance.