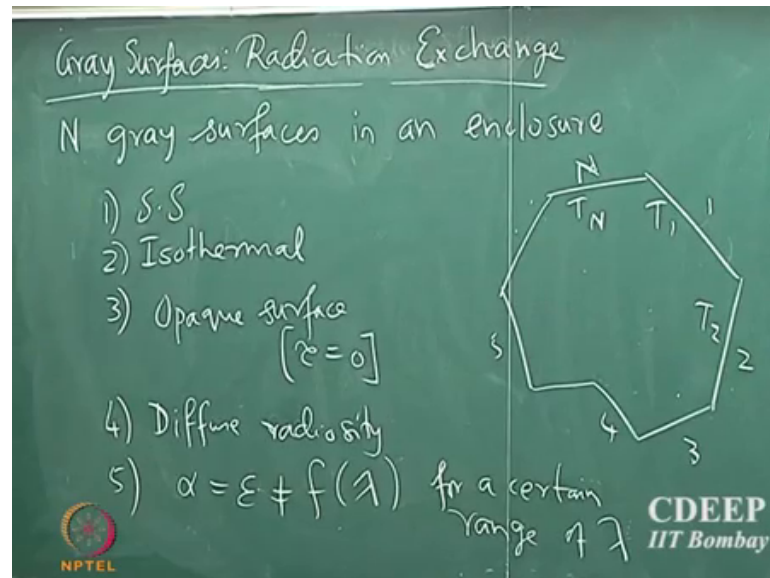


**Heat transfer**  
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**Lecture - 52**  
**Diffuse, gray surfaces in an enclosure**

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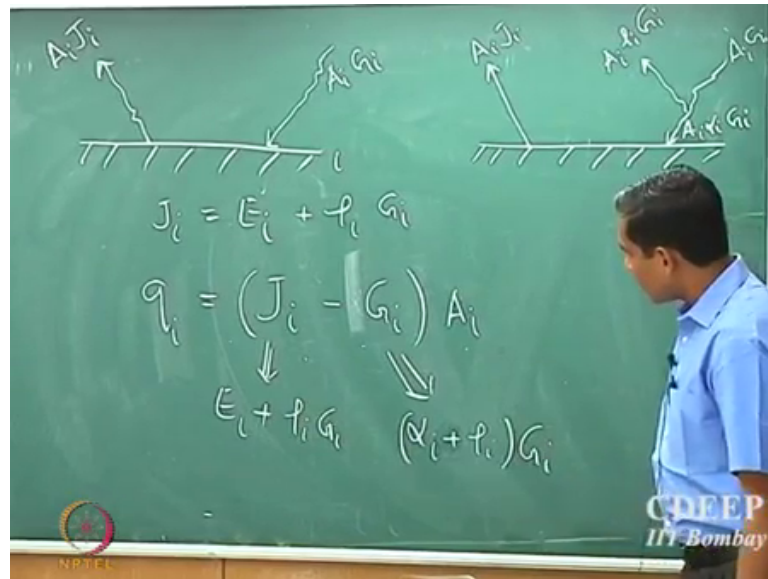


We are going to look at radiation exchange between gray surfaces ok. So, to start with let us say that there are N gray surfaces, in an enclosure ok. If you have N gray surfaces in enclosure; ok N gray surfaces ok.

Now, question is how do we characterize or how do we find the net radiation exchange between one surface and all other surfaces ok. So, that is the question. So, suppose we assume that it is a steady state system ok. Which means that the temperature T 1, T 2 etcetera are maintained constant ok. It is a steady state system and it is isothermal steady state isothermal system and the 3rd one is if we assume that it is a opaque surface. So, which simply means that transmittivity is 0. So, the only mode of splitting of the incident irradiation is by absorption and reflection ok.

And if we assume that it is a, a diffuse radiosity and we said it is gray surface. to which means that alpha and epsilon or not function of lambda for a certain range, for a certain range of lambda alright.

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So, let us zoom in to one surface ok. So, let us say this is surface  $i$  and incident irradiation, the rate at which the irradiation is coming or hitting that surface this  $A_i$  times  $G_i$  ok. So,  $G_i$  is the net incident irradiation and if the net emission that comes from the surfaces is  $A_i$  times  $J_i$ . Where  $J_i$  is the radiosity which includes both emission and reflection. Note that this is a gray surface. So, there will be reflection, there is no reason to assume that reflection is 0 ok.

So, now we could also re sketch the same schematic in a slightly different way. So, supposing if  $A_i G_i$  is the incident irradiation, then there will be some part which is reflected which is given by  $A_i$  into reflectivity which is  $\rho_i$  So,  $\rho_i$  is the reflectivity of that surface multiplied by  $G_i$  right. So remember the definition of reflectivity.

Reflectivity is the amount of radiation which is reflected, divided by the total incident irradiation right. So,  $\rho_i A_i$  into  $G_i$  will tell you what is the irradiation, that is incident is reflected back and then what is absorbed is  $A_i \alpha_i G_i$ . So, that is the fraction of radiation which is absorbed by that surface and this  $J_i$  would essentially be some of the net emission some of the emission from the surface plus the reflection right.

So,  $A_i J_i$  so  $J_i$  will be emission from the surface plus  $\rho_i$  into  $G_i$  ok. So, that is the radiation that is net radiation emitted from that surface ok. So, supposing I want to maintain isothermal conditions ok. So, assumption number 2 is the surfaces are maintained at a certain constant temperature right. So, if I want to maintain at a constant

temperature, then there has to be some amount of heat that is either removed from the surface or supplied to the surface depending upon the radiation exchange ok.

So  $q_i$  which is the net radiation exchange at the surface so that is given by, whatever is emitted net emission minus  $G_i$  into  $A_i$  right. So, this is the net emission  $J_i$  include reflection right. So, keep in mind that  $J_i$  already includes reflection. So, the net radiation exchange is  $J_i$  minus  $G_i$  into  $A_i$  ok. So,  $J_i$  is, so this is emission plus whatever is reflected and  $G_i$  is whatever is  $\alpha_i$  plus  $\rho_i$  into  $G_i$  ok. So, that is already accounted for. So the net radiation exchanges  $J_i$  minus  $G_i$  into the surface area ok.

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Gray Surfaces: Radiation Exchange  $\alpha_i + \rho_i = 1$

$$J_i = E_i + \rho_i G_i = E_i + (1 - \alpha_i) G_i$$

$$\Rightarrow G_i = \frac{J_i - E_i}{1 - \alpha_i}$$

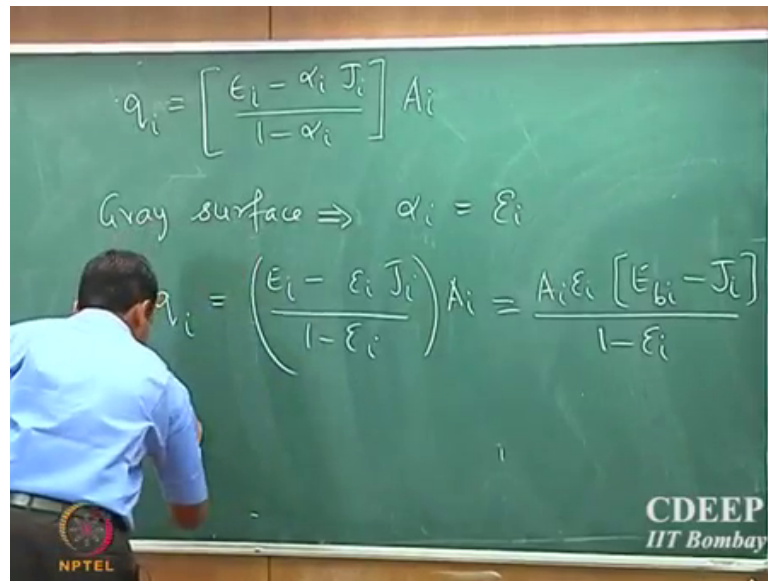
$$q_i = \left[ J_i - \frac{J_i - E_i}{1 - \alpha_i} \right] A_i =$$

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So, now we know that  $J_i$  is  $E_i$  plus  $\rho_i$  into  $G_i$  but we said that the surface is opaque. So,  $\rho_i$  can be expressed in terms of the absorptivity. So we know that  $\alpha_i$  plus  $\rho_i$  equal to 1 alright. So, this will be  $E_i$  plus so that should be  $E_i$  plus  $1 - \alpha_i$  into  $G_i$  ok. So, from here we can find out  $G_i$  equal to  $J_i$  minus  $E_i$  divided by  $1 - \alpha_i$  ok.

So, now if we substitute that into this expression here, you will find that  $q_i$  equal to  $J_i$  minus  $1 - \alpha_i$  multiplied by  $A_i$  ok. So all I have done is I have now eliminated  $G_i$  from the net radiation exchange ok. So, now I can express this as, so a little bit of algebra.

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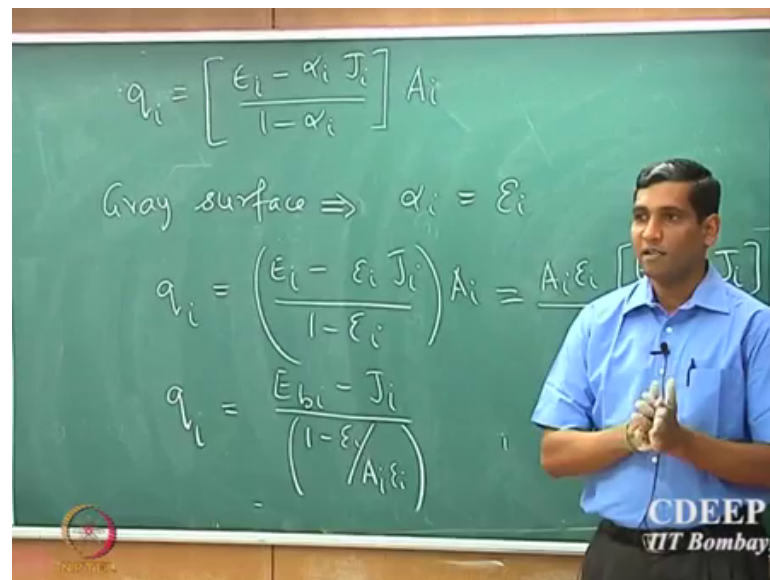


So  $q_i$  will be  $E_i$  minus  $\alpha_i J_i$  divided by  $1 - \alpha_i$  into  $A_i$ . All I have done is have just multiplied  $1 - \alpha_i$  and cancelled the like term and so it is  $E_i$  minus  $\alpha_i J_i$  by  $1 - \alpha_i$ . We also said it is a gray surface.

So, which means that  $\alpha_i$  equal to absorptivity is equal to emissivity of that surface in that particular range of wavelength right. Now if you plug in this we will see that  $q_i$  equal to  $E_i$  minus  $\epsilon_i J_i$   $1 - \epsilon_i$  multiplied by  $A_i$  right. So,  $E_i$  is the net emission this is does not include the reflection. So, it is the net emission from that surface, but that we can express in terms of the properties of the surface right.

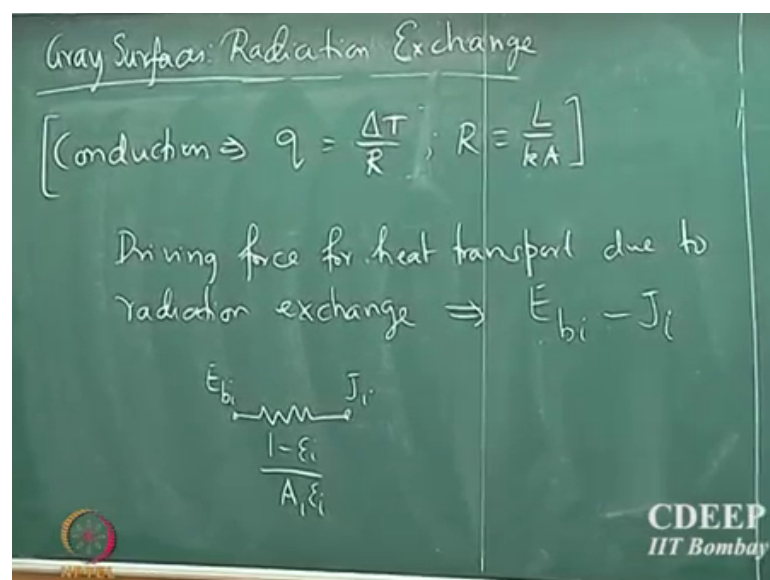
So, we know that emissivity is an intrinsic property of that surface that multiplied by the corresponding blackbody radiation, at that temperature will tell you what is the net emission ok. So, we can write this as  $A_i \epsilon_i$  into  $E_{b i} - J_i$  divided by  $1 - \epsilon_i$  ok.

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So, we can rewrite this as  $q_i$  equal to  $E_{bi}$  minus  $J_i$  divided by  $1$  minus  $\epsilon_i$  ok. What does this form look like? Recall some of the conduction concepts. Net transport, heat transport rate equal to temperature difference divided by resistance. So, it has the same form as what you have seen in resistance concepts in conduction right. So now we can define so again it is a definition.

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So, in conduction, we said that  $q$  is  $\Delta T$  by the resistance offered by that system for conduction right. Now where  $\Delta T$  which is the temperature difference is the driving

force for heat transfer. Similarly for radiation the driving force for supplying heat or removing heat from that object is actually the difference in the blackbody radiation at that temperature minus the net emission.

So, if we define the driving force; for radiation exchange, for heat transport due to radiation exchange. So, if we define the driving force as  $E_{b,i} - J_i$  ok. So, very similar to the way we define driving force in conduction the temperature difference as a driving force in conduction, one could define a driving force for heat transport due to radiation exchange as  $E_{b,i} - J_i$ .

So, once we do that now we can easily represent it in terms of the resistance concept. So, if  $E_{b,i}$  and  $J_i = 1 - \epsilon_i$  by  $A_i \epsilon_i$  ok. What is this resistance signified. So, note that in conduction we said the resistance is the resistance offered by the system for heat transport right due to conduction. What does this offer here with respect to it is capacity the store heat.

So, what does this offer what is this? What resistance does it reflect? Yeah look look at the quantities here maybe I will give you a hint. So, in a 1 d system in conduction resistance is  $l / k A$  right. So, it contains conductivity length scales and conductivity what does it what is this contain?

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Emissivity. So, what does it reflect?

Student: If the resistance (Refer Time: 12:44) everything like (Refer Time: 12:47).

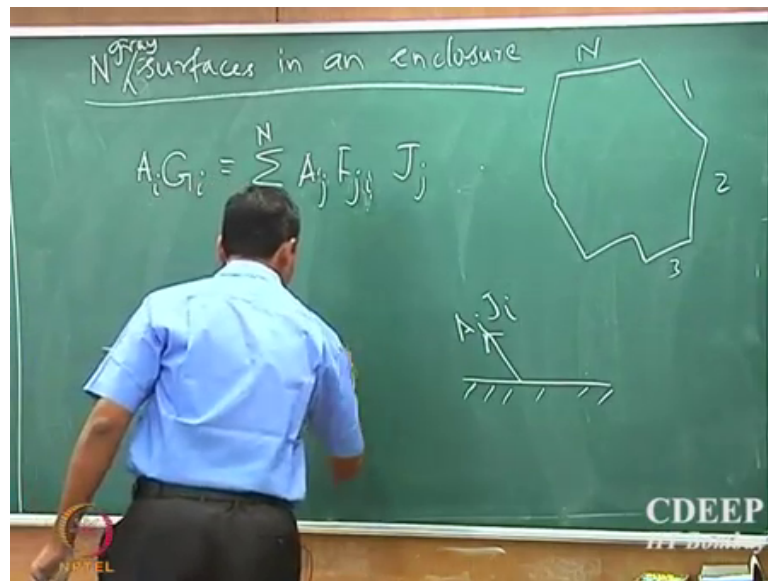
Ok so you have to understand the difference, this is a cross sectional area process right, whereas this is a surface area process. So, this is the resistance that is offered by the properties of the surface. For net radiation that is either absorbed by the surface or emitted by the surface. So, if  $q$  is positive if  $q$  is positive then there is going to be net radiation that is emitted by the surface and if this  $q$  is negative that is going to be net radiation which is absorbed by the surface ok.

So, this resistance essentially signifies, what is the resistance offered by the surface due to it is surface properties or radiation exchange from that surface. So, you have to

understand the distinction between the resistance concept of resistance in conduction and concept of resistance in radiation ok.

Ok so this is exchange between two objects, now we said there is an enclosure right you have N objects which are actually present in the enclosure. So, how do we represent that in a network concept. We are going to see that next.

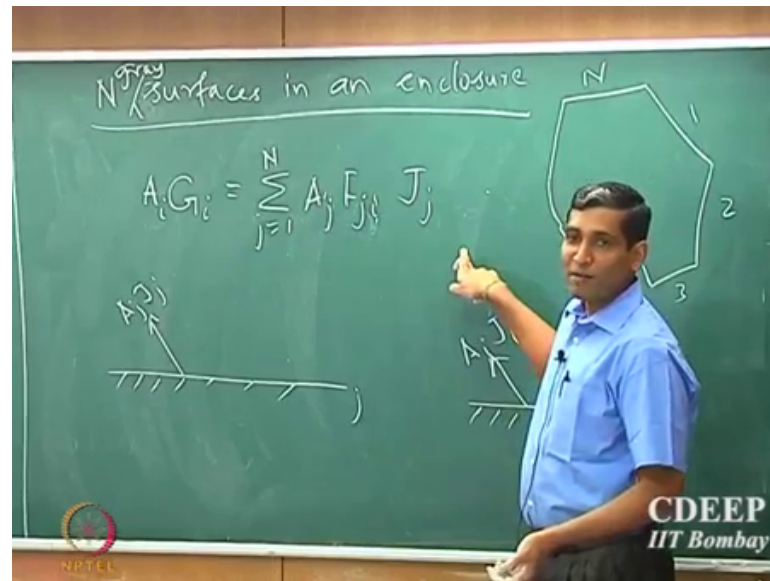
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So, N surfaces, qualified by saying gray surfaces and enclosure ok. So, what is the what is the net irradiation received by a surface? We cannot we cannot quantify the net radiation exchange, from all the N surfaces if we do not know how to find out what is the net incident irradiation. That is actually received by a surface. So, note that this is N surfaces placed in an enclosure ok.

So, we have N surfaces etcetera up to n ok. So, what is the incident radiation irradiation that is received by a surface i ok. So, let us take out 1 surface ok. So, the radiation emitted, the net radiation that is emitted by a surface is given by  $A_i J_i$  right and i goes from 1 to N in a enclosure. So, what is the total irradiation that is received by ith surface? That is right there is the summation that has to come here, because there is some radiation received from 1, sum to sum 2, sum from 3 etcetera there has to be a summation what is that summation. So, that will be summation  $A_j F_{ji}$  into  $J_j$  right.

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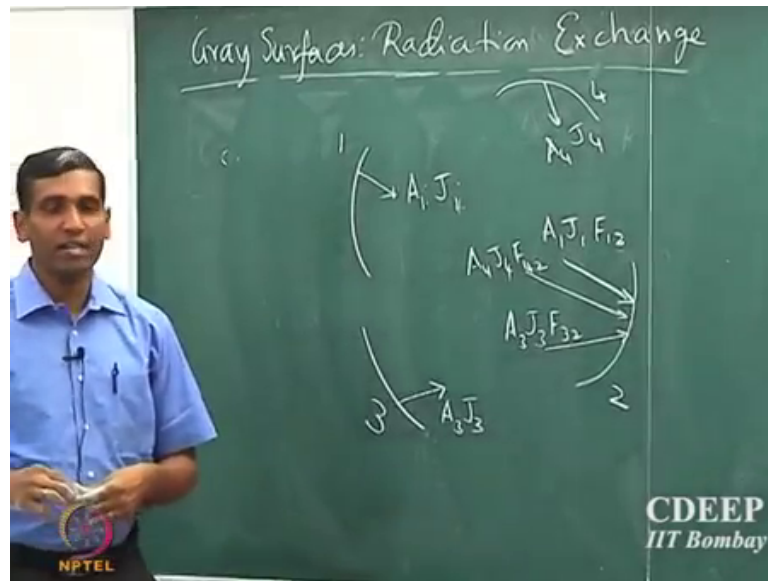
So  $J$  going from one to  $N$  right; so this is the radiation that is emitted by  $j$ th surface and is intercepted by  $i$  summed over all  $J$  right. So, if  $J$  have surface  $J$  So, if this is my surface  $J$  then the radiation that comes out of the phase  $J$  is  $A_j J_j$  and what fraction of that is lead by surface  $i$  that multiplied by the corresponding view factor ok. So like that it is going to receive from all the surfaces which is present in the enclosure.

So therefore, the net incident irradiation to  $i$ th surface will be sum over whatever fraction received from each of the surfaces going from one to  $N$  ok. So, the question is how does it include the absorbed part we do not have to include that right whatever is received by let us say surface 3 is essentially what is a fraction of what is actually emitted net emission from surface 1.

We are going to write an energy balance to reflect the amount that is absorbed all we are saying is, what is the net amount of incident irradiation. We have an imposed the condition of isothermal system yet we are going to do that ok. So, we only said that what is the net amount of irradiation that is received by any surface, if I know what is the net emission from a surface that is what is emitted by itself plus whatever is reflected from the incident if I know that, then there has to be some fraction of that which received by surface  $i$  and that summed over all the surfaces is going to tell me what is the total radiation that is actually received by surface  $i$ , that is all  $i$  am saying right.



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So, let me explain again ok. So, supposing you have 2 surface surface 1 and surface 2 ok. So, I call it surface 1 and surface 2 right. So, supposing the net emission that comes out of that surface is  $A_i J_i$  right. So, that is the total amount of radiation that is emitted by this surface correct. Now this includes the emission by itself plus whatever is reflected because of the incident radiation ok.

Now, there has to be some part of it which is received by surface 2 ok. So, what is receive? So, this will be I call it  $A_1 J_1$ . So, that will be  $A_1 J_1$  into  $F_{12}$  ok. The fraction of emission that comes from surface 1 and received by surface 2 is  $A_1 J_1 F_{12}$ . Now let us say I add 1 more surface here, I add 3 ok. Now the emission from here will be  $A_3 J_3$  right.

So, whatever fraction that is received by surface 2, from 3 is  $A_3 J_3$  into  $F_{32}$  right. Now I can add one more let us say 4th one  $A_4 J_4$ . So, there should be  $A_4 J_4$  into  $F_{41}$ ,  $F_{42}$  right. So, sum over all of this is the total incident radiation to surface 2 that is exactly what that summation ok, any other question?

Student: (Refer Time: 19:21)

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Gray Surfaces: Radiation Exchange

$$q_{i,net} = A_i [J_i - G_i]$$
$$= \left[ A_i J_i - \sum_{j=1}^N A_j F_{ji} J_j \right]$$
$$\sum_{j=1}^N F_{ij} = 1$$
$$= \left[ A_i \sum_{j=1}^N F_{ij} J_i - \sum_{j=1}^N A_j F_{ji} J_j \right]$$

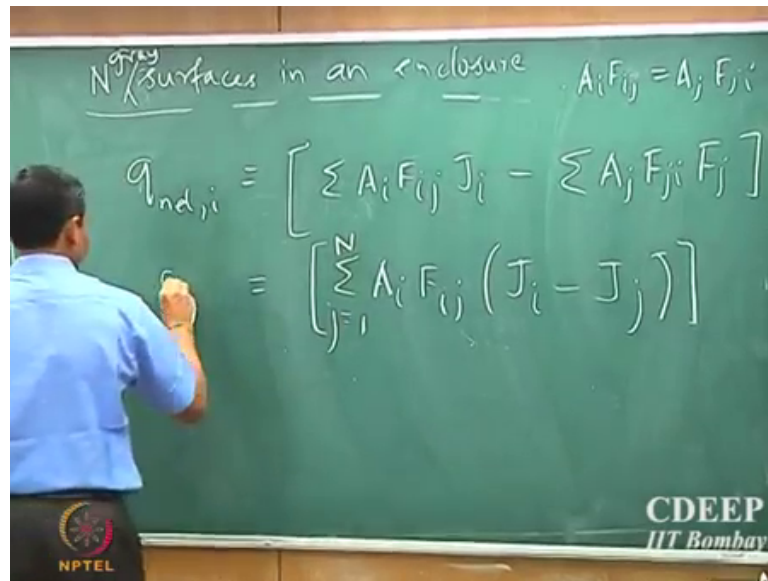
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So, we said that  $q_{i,net}$  is  $A_i$  into  $J_i$  minus  $G_i$  right. So, that is the net radiation exchange between whatever is incident and whatever is emitted by that surface. Now, we know that  $A_i G_i$  is given by the summation that we just derived ok. So, that will be  $A_i$  into  $J_i$  minus  $\sum_{j=1}^N A_j F_{ji} J_j$ . So now, we know that  $\sum_{j=1}^N F_{ij} = 1$  what is this expression, sum of all the view factors is 1 right. That is the conservation property because that is equal to 1.

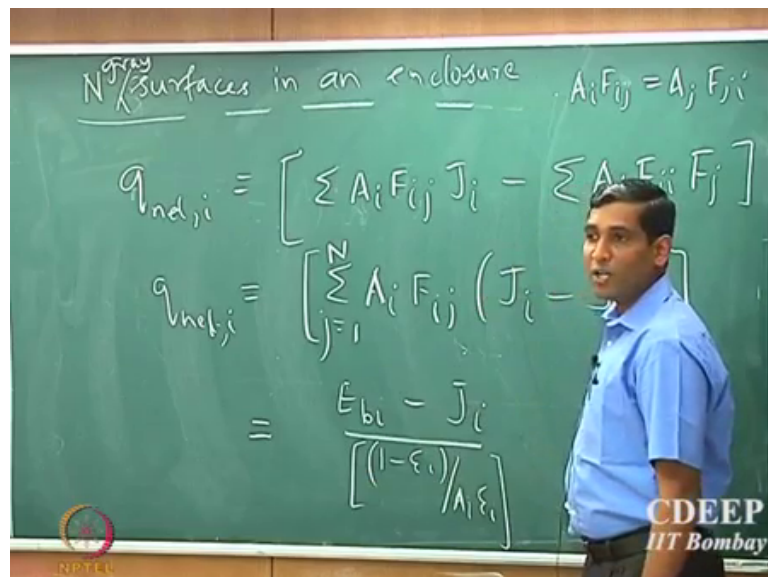
So, now I can throw in that expression here. So, that will be  $A_i \sum_{j=1}^N F_{ij} J_i$  minus this  $A_i$  should be inside minus  $\sum_{j=1}^N A_j F_{ji} J_j$ .

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Therefore,  $q_{net,i}$  will be  $\sum A_i F_{ij} J_i$  minus  $\sum A_j F_{ji} F_j$ , but we know from reciprocity relationship right. We know that  $A_i F_{ij}$  is  $A_j F_{ji}$ . So we plug in the reciprocity relationship, you will see that it will be  $\sum A_i F_{ij} (J_i - J_j)$ . This is sum over all  $J$  ok.

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So, this is the net radiation exchange, with by surface  $i$  with all other surfaces in the enclosure ok. Note that this is also equal to  $E_{bi} - J_i$  divided by  $1 - \epsilon_i$  by  $A_i \epsilon_i$  ok. This is what we derived a short while ago. So, the net radiation

exchange because of radiation exchange between 1 object and all the other objects that balance will give you this expression here and this expression here is where you actually impose the condition that all the surfaces are maintained at an isothermal condition right.

So, when you put in all those expressions you will see that the radiation exchange between individual surfaces in the enclosure can be related to the property surface properties of that surface and that comes from this expression. So, in what we are going to do in the next lecture, we are going to expand the resistance network that we drew for one surface and how to use the resistance concept to find out the radiation exchange with all these surfaces that is present in an enclosure. So, we are going to start with that in the next lecture.